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Capital–skill complementarity and the redistributive effects of Social Security Reform ☆

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Abstract

This paper analyses the general equilibrium implications of reforming pay-as-you-go pension systems in an economy with heterogeneous agents, human capital investment and capital–skill complementarity. It shows that increasing funding, by raising savings, delivers in the long run higher physical and human capital and therefore higher output, but also higher across-group wage and income inequality. It also shows that the general equilibrium effects induced by this reform affect groups' sizes in a way that the higher across-group inequality generated by more funding goes with a larger share of the population against redistribution. © 2007 Elsevier B.V. All rights reserved.

1. Introduction

The discussion over the problems of traditional pay-as-you-go pension systems and on how to change them is by now a long standing one.

A considerable amount of conceptual and empirical work has been directed to identify alternative reform proposals and their impact on different economic variables.[1] Whatever the specific institutional features of these alternative proposals, most of them include some degree of funding. The claimed advantages of introducing or increasing funding with respect to parametric reforms which would maintain the pay-as-you-go nature of traditional social security

[1] See for instance Diamond (1998, 2002) and Sass and Triest (1997).

systems range from higher returns and higher savings to fewer labour market distortions and lower political pressure (see for instance Feldstein, 1998, 2005). Given the general attractiveness of funding, the main concerns stem from transitional, risk and redistributive issues and from the political feasibility of such a change.[2]

Although, according to Gruber and Wise (2002), income redistribution is among the four economic goals which a reform should pursue – the others being to correct the financial imbalance, to increase national saving and to strengthen economic efficiency – the economic literature on pension reform deals only marginally with *intra*generational redistribution. Namely, when considering redistributive issues, it focuses almost exclusively on the *inter*generational redistribution generated by an increase in funding either during the transition period or in the long run.[3] Redistribution within generations is sometimes taken into account by models considering the transition to a fully funded system (see for instance Brunner, 1996 and Feldstein and Liebman, 2002) but it is seldom a long run issue. The absence of an explicit theoretical analysis of the long run intragenerational redistributive implications of introducing more funding[4] is even more critical if one takes into account that, starting from the World Bank (1994) proposal of a three-pillar social security system, the funded component is almost always accompanied by a public, mandatory, pay-as-you-go pillar which should take care of redistributive concerns either via benefit floors, or minimum income guarantees, or flat universal benefits.

This paper tries to fill the gap by analysing the general equilibrium implications of introducing some funding in an economy where there is a pay-as-you-go partially redistributive pension system. It focuses on the intragenerational conflicts that this reform generates both in the short and in the long run. It then studies whether these conflicts can be tackled by changing the degree of intragenerational redistribution performed by the smaller remaining pay-as-you-go pension scheme. The analysis sheds some light on the compatibility between (private) funding and (public) redistribution which is taken for granted by the current policy debate.

We model a two-period OLG closed economy characterised by agents' heterogeneity, human capital investment and capital–skill complementarity. The literature on pension reform commonly assumes that workers are perfect substitutes once productivity differentials are adjusted. Under this assumption, an increase in funding by raising savings and the capital stock, delivers higher real wages for all in the new steady state, leaving relative wages unchanged. The assumption of capital–skill complementarity implies that policy variables affecting physical capital influence across-group wage inequality: namely, changes in the size of the pay-as-you-go system, by modifying capital, also change across-group inequality bringing about new issues in the analysis of pension system reforms. The inclusion of an education decision responds to the need of integrating the analysis of the long run implications of pension reform on physical capital to those on human capital and it offers an endogenous mechanism to offset changes in across-group inequality.

We find that a social security reform based on an increase in funding delivers a higher steady state level of physical and human capital but also a higher across-group wage and income inequality. This is new to the literature on social security reform: with capital–skill complementarity not only pension gaps between the rich and the poor increase but also wage gaps widen, adding to the redistributional problems generated by the switch to funding. When looking at the possibility to compensate the higher income inequality, we find that general equilibrium effects triggered by more funding increase the share of the population against redistribution. This highlights a potential conflict between enhancing redistribution and funding.

The paper is organised as follows: Section 2 provides the basic economic set-up. Section 3 analyses the impact of the social security reform and it discusses the policy implications of our findings. Section 4 concludes.

2. The basic set-up

2.1. Consumers and government

We consider a two-period OLG model of a closed economy populated by a continuum of heterogeneous agents indexed by j. When young, agents consume, save and decide whether or not to invest in human capital: if they do, they

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[2] When considering prefunding of social security, the transition from a pay-as-you-go to a fully funded system is a critical issue and has been the subject of substantial analysis (see for instance Breyer, 1989; Homburg, 1990; Feldstein, 1998). References on risk issues include Diamond and Geanakoplos (2001) and Campbell and Feldstein (2001). On political feasibility see for instance Conesa and Krueger (1999), Leers et al. (2001), Sinn and Uebelmesser (2002).

[3] van Groezen et al. (2002) can be interpreted in this light.

[4] Kotlikoff et al. (2002) simulate the general equilibrium effects of privatising the US Social Security system under agents' heterogeneity in a framework where the key elements of our model are absent.

become skilled workers (type $\mathcal{H}$ agents); if they do not, they remain unskilled (type $\mathcal{L}$ agents). The human capital investment decision depends on the idiosyncratic ability parameter c_j: the latter denotes the time required to become skilled and it is distributed on the interval $[0,1]$ with continuous density function $\varphi(\cdot)$; the more able the agent is, the less time she has to spend investing in human capital, and the lower are her foregone earnings.[5] When old, agents retire and finance their second period consumption out of their savings and pensions.

Formally, agents decide how much to consume and save solving the following maximisation problem:

$$\max U(x_{1t}^j) + \frac{1}{1+\beta} U(x_{2t+1}^j) \tag{1}$$

subject to:

$$x_{1t}^j + \frac{x_{2t+1}^j}{1+r_{t+1}} = y_t^j \tag{2}$$

where U is twice differentiable, concave and increasing in x_{1t}^j and x_{2t+1}^j, with $\lim_{x\to 0} U'(x) = +\infty$. x_{1t}^j and x_{2t+1}^j represent consumption of agent j born at time t respectively when young and old. Time-separability and homotheticity are assumed. β is the rate of time preference; r_{t+1} denotes the interest rate at time $t+1$ and y_t^j represents lifetime income of agent j born at time t which we next specify.

The government operates a balanced pay-as-you-go pension scheme: it collects contributions proportional to wages at a rate τ_t and it pays per capita pensions p_{t+1}^j which are determined according to the following benefit formula:[6]

$$p_{t+1}^j = (1+n)\tau_t w_t^j \alpha_t + \overline{p}_{t+1} \tag{3}$$

where n is the constant rate of population growth, w_t^j is the (gross of payroll tax) wage of agent j at time t, α_t is the contributory share of the scheme applying to generation t (the so-called Bismarckian factor), with $0 \leq \alpha_t \leq 1$, and $\overline{p}_{t+1}$ is the redistributive component of the system paid out at time $t+1$ as a flat universal benefit which is determined according to the social security budget constraint. Namely:

$$\overline{p}_{t+1} = (1+n)[\tau_{t+1}\overline{w}_{t+1} - \alpha_t \tau_t \overline{w}_t] \tag{4}$$

The first term in square brackets represents per capita revenues collected at time $t+1$ with $\overline{w}_{t+1}$ denoting the average wage of the economy at time $t+1$. The second term captures the share of per capita revenues required to finance the contributory pensions. When $\alpha_t = 0$, the pension system is only redistributive; as α_t increases, the contributory share goes up.

By substituting Eq. (4) in Eq. (3), we can write the lifetime income of agent j as follows:

$$y_t^j = w_t^j(1-\tau_t) + \frac{1+n}{1+r_{t+1}}[\tau_t\alpha_t(w_t^j - \overline{w}_t) + \tau_{t+1}\overline{w}_{t+1}] \tag{5}$$

with

$$w_t^j = \begin{cases} w_t^{\mathcal{H}}(1-c_t^j) & \text{if } j \in \mathcal{H} \\ w_t^{\mathcal{L}} & \text{if } j \in \mathcal{L} \end{cases} \tag{6}$$

where $w_t^{\mathcal{H}}$ and $w_t^{\mathcal{L}}$ represent competitive wages of skilled and unskilled labour. w_t^j and therefore y_t^j are linearly decreasing in c_j for skilled agents and independent of it for unskilled agents.

From the solution to problem (1) which is characterised by the first order conditions:

$$\frac{U'(x_{1t}^j)}{U'(x_{2t+1}^j)} = \frac{1+r_{t+1}}{1+\beta} \tag{7}$$

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[5] We do not investigate here the implications of imperfect capital markets on the decision to invest in human capital and on the redistributive effects of a pension reform. This is done in Casarico (1998). Notice however that assuming that education requires the payment of a monetary cost and that capital markets on which agents have to borrow are imperfect would involve further redistributive effects which would add to those generated by our model.

[6] The benefit formula applied here is a common parameterisation of an unfunded pension system which provides some redistribution among retirees. See, for instance, Casamatta et al. (2000).

and by the consumers' budget constraint (2) we can derive the indirect utility functions $V(y_t^j)$ whose maximisation determines the decision to invest in human capital: it is convenient to invest in human capital if $y_t^{\mathcal{H}} \geq y_t^{\mathcal{L}}$. The last agent who finds it profitable to invest is characterised by an education cost c_t^* satisfying the following condition:

$$c_t^* = \frac{w_t^{\mathcal{H}} - w_t^{\mathcal{L}}}{w_t^{\mathcal{H}}} \tag{8}$$

All those whose cost of investing is below c_t^* will become skilled workers while those whose cost is above c_t^* will remain unskilled. It follows that the effective supply of skilled and unskilled labour are:

$$\tilde{H}_t = N_t \int_0^{c_t^*} (1-c)\varphi(c)\mathrm{d}c \tag{9}$$

$$\tilde{L}_t = N_t \int_{c_t^*}^1 \varphi(c)\mathrm{d}c \tag{10}$$

where N_t indicates the size of the population at time t.

Defining the wage-premium z_t as the ratio of skilled to unskilled workers' wages, condition (8) can be rewritten as:

$$c_t^* = 1 - \frac{1}{z_t} \tag{11}$$

In the remaining of the paper we use z_t as a simple measure of across-group wage inequality: it is the ratio between the extreme points of the wage distribution.[7] In order to determine z_t, we introduce production.

2.2. Production

To the best of our knowledge, all the existing literature on social security reform assumes that workers are perfect substitutes, once productivity differentials are adjusted. It follows that a higher (lower) level of capital stock in the economy implies higher (lower) wages for all types of workers, leaving relative wages unchanged. In fact, since the seminal work by Griliches (1969), a large body of empirical studies finds that capital – and technological progress embodied in new investments – better substitutes unskilled labour than skilled labour.[8]

In order to introduce capital–skill complementarity in our model, we assume the following constant return to scale production technology,[9] with δ, b and $\theta \in (0,1)$:

$$F_t = \left[bK_t^\theta + (1-b)L_t^\theta\right]^{\frac{\delta}{\theta}}[H_t]^{1-\delta} \tag{12}$$

where F_t is production, K_t is physical capital, L_t is unskilled labour and H_t is effective skilled labour, all at time t.

In Eq. (12), the Allen–Uzawa partial elasticity of substitution between capital and skilled labour $\sigma_{KH} = 1$, while that between capital and unskilled labour $\sigma_{KL} = \frac{1}{1-\theta}$. Under the condition that θ is strictly greater than zero, $\sigma_{KL} > \sigma_{KH}$ and the production function exhibits capital–skill complementarity.

Dividing by N_t, Eq. (12) can be rewritten in per capita terms:

$$f_t = \left[bk_t^\theta + (1-b)l_t^\theta\right]^{\frac{\delta}{\theta}}[h_t]^{1-\delta} \tag{13}$$

where small letters denote ratios of a given variable with respect to the size of the population N_t.

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[7] Notice that there is no within-group inequality for unskilled agents, as they all have the same wage. Within-group inequality for skilled agents is instead driven by c^j.

[8] For instance, Flug and Hercowitz (2000) use data from a wide range of countries and find evidence that investment in equipment raises the relative demand for skilled labor; similar results are reported by Goldin and Katz (1998), Prasad (1994) and by a number of microeconomic studies, as surveyed in Hamermesh (1993). Krusell et al. (2000) estimate the parameters of a four-factor model using US time-series data and find that the elasticities of substitution between capital equipment and skilled/unskilled labour are consistent with capital–skill complementarity. Here we use the simplifying assumption that there is only one type of physical capital as in Stokey (1996). For additional evidence and references on capital–skill complementarity and capital-embodied skill-biased technological change see, among others, Acemoglu (2000) and Katz and Autor (1999).

[9] See Uzawa (1988), chapter 5 for a detailed discussion.

Profit maximising behaviour of the competitive firms implies that the interest rate is:

$$r_t = \delta b \big[bk_t^\theta + (1-b)l_t^\theta \big]^{\frac{\delta}{\theta}-1} h_t^{1-\delta} k_t^{\theta-1} \tag{14}$$

and that skilled and unskilled wages are:

$$w_t^{\mathcal{H}} = (1-\delta) \big[bk_t^\theta + (1-b)l_t^\theta \big]^{\frac{\delta}{\theta}} h_t^{-\delta} \tag{15}$$

$$w_t^{\mathcal{L}} = \delta(1-b) \big[bk_t^\theta + (1-b)l_t^\theta \big]^{\frac{\delta}{\theta}-1} h_t^{1-\delta} l_t^{\theta-1}. \tag{16}$$

The wage-premium is:

$$z_t = \frac{1-\delta}{\delta(1-b)} \big[bk_t^\theta + (1-b)l_t^\theta \big] l_t^{1-\theta} h_t^{-1}. \tag{17}$$

An easy to verify implication of capital–skill complementarity is that $\frac{\partial z_t}{\partial k_t} > 0$, i.e. the relative productivity of skilled labour is increasing in the amount of capital. In the presence of capital–skill complementarity policy variables affecting the stock of capital do also change across-group wage inequality. As we will see, this is relevant for the analysis of the impact of social security reform.

2.3. Equilibrium

In order to illustrate the working of the model and to set the ground for the analysis of the social security reform, it is convenient to first describe the equilibrium at any period t, taking as given aggregate past savings S_{t-1} and expectations on r_{t+1}, $\overline{w}_{t+1}$ and τ_{t+1}.

Denoting by s_t^j agent j's savings and by I_t aggregate investment, the temporary equilibrium at time t is defined by $\{c_t^*, w_t^{\mathcal{H}}, w_t^{\mathcal{L}}, z_t, r_t, \tau_t, \alpha_t, L_t, H_t, I_t, K_t, F_t, x_{1t}^j, s_t^j, x_{2t}^j\}$ that satisfy for each j the individual maximisation problem on consumption today and tomorrow (1) and (2), the human capital investment decision, the firms' profit maximisation conditions (14), (15) and (16), and the market clearing conditions. Namely, the labour market and the goods market equilibria require respectively:

$$L_t = \tilde{L}_t \text{ and } H_t = \tilde{H}_t \tag{18}$$

$$X_{1t} + X_{2t} + I_t = F_t \tag{19}$$

where

$$X_{1t} = N_t \left[\int_0^{c_t^*} x_{1t}(c; w_t^{\mathcal{H}}, \overline{w}_t, r_{t+1}, \overline{w}_{t+1}, \alpha_t, \tau_t, \tau_{t+1}, n) \varphi(c) \mathrm{d}c \right]$$

$$+ N_t (1 - \Phi(c_t^*)) \cdot x_{1t}(w_t^{\mathcal{L}}, \overline{w}_t, r_{t+1}, \overline{w}_{t+1}, \alpha_t, \tau_t, \tau_{t+1}, n) \tag{20}$$

$$X_{2t} = N_{t-1} \int_0^{c_{t-1}^*} x_{2t}(c; w_{t-1}^{\mathcal{H}}, \overline{w}_{t-1}, r_t, \overline{w}_t, \alpha_{t-1}, \tau_{t-1}, \tau_t, n) \varphi(c) \mathrm{d}c$$

$$+ N_{t-1} (1 - \Phi(c_{t-1}^*)) \cdot x_{2t}(w_{t-1}^{\mathcal{L}}, \overline{w}_{t-1}, r_t, \overline{w}_t, \alpha_{t-1}, \tau_{t-1}, \tau_t, n) \tag{21}$$

denote respectively aggregate consumption of the young and the old at time t and $\Phi(c)$ is the cumulative distribution function and where time $t+1$ variables are in expectation.[10]

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[10] In writing the determinants of the consumption decisions in Eqs. (20) and (21) and, next, of the saving decision in Eq. (22), we take into account that these choices depend on lifetime income and therefore we explicitly introduce the determinants of the latter. In what follows, when it does not create ambiguity, to simplify notation we drop all variables but c.

Notice that, given k_t, the subset {(9), (10), (11), (17), (18)} of the equations defining the equilibrium uniquely determines z_t, c_t^*, L_t and H_t. In turn, conditions (12), (14), (15) and (16) determine F_t, r_t, $w_t^{\mathcal{H}}$, and $w_t^{\mathcal{L}}$. Given that all these variables are a function of k_t only, it follows, in particular, that they do not depend on current and future policy variables.[11] We can also observe that c_t^* and z_t are increasing in k_t,[12] which will be crucial for the analysis of the redistributive implications of social security reform. Turning now to consumption and saving decisions, we notice that, given factor prices, and given the first order conditions (7) plus the individual budget constraint (2), x_{1t}^j and s_t^j are a function of k_t, c^j, r_{t+1}, $\bar{w}_{t+1}$, α_t, τ_t, τ_{t+1} and n, where c^j and n are parameters; α_t, τ_t and τ_{t+1}, are exogenously fixed policy variables; r_{t+1} and $\bar{w}_{t+1}$ are a function of k_{t+1} only, as discussed above with reference to r_t and $\bar{w}_t$. As to x_{2t}^j, it is a function of c^j and n, of past variables k_{t-1}, α_{t-1}, τ_{t-1} and of current variables k_t and τ_t.

Summing up, in the present framework changes to policy variables at time t only affect consumption in the same period and saving of the young,[13] while leaving aggregate factor supplies and their gross rates of return all at time t unaffected. These are indeed, as argued above, determined only by k_t. The impact of policy variables on savings and therefore on capital next period is crucial for the dynamic behaviour of the model.

To study the latter, we observe that the accumulation rule for capital and the assumption of perfect foresight give the link between any two periods, at equilibrium:

$$K_{t+1} = S_t \equiv N_t \int_0^{c_t^*} s_t(c; w_t^{\mathcal{H}}, \bar{w}_t, r_{t+1}, \bar{w}_{t+1}, \alpha_t, \tau_t, \tau_{t+1}, n) \varphi(c) \mathrm{d}c$$

$$+ N_t(1 - \Phi(c_t^*)) \cdot s_t(w_t^{\mathcal{L}}, \bar{w}_t, r_{t+1}, \bar{w}_{t+1}, \alpha_t, \tau_t, \tau_{t+1}, n) \qquad (22)$$

If we denote by $\bar{s}_t$ average per capita savings, we can rewrite the capital market equilibrium condition in per capita terms as:

$$k_{t+1}(1 + n) = \bar{s}_t \qquad (23)$$

To study the effects of a social security reform on the intertemporal equilibrium of the economy and on its steady state[14] it is enough to focus on how policy variables affect savings.

3. Social security reform

Using the model above, we want to study the general equilibrium effects of a reform to the social security system. The policy change we consider is represented by a reduction in the size of the pay-as-you-go pension scheme τ_t. As long as compulsory social security contributions do not exceed individual voluntary savings and savings through a fully funded scheme are perfect substitutes for private voluntary savings, a reduction in τ_t can be used to represent the introduction of some funding in the pension system. We assume that the reduction in τ_t is once and for all, with all subsequent τ_{t+i} remaining unchanged, and that it translates into lower pensions for the old at t.[15] Given that our focus is not on transitional issues, we do not look at alternative ways to finance the switch to more funding and at the different distributions of costs and benefits they may imply.

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[11] This is an implication of the exogeneity of the labour supply and of the fact that the education decision does not depend on the parameters of the pension system. These simplifying assumptions allow us to concentrate on the redistributive effects generated by the distortion on physical capital accumulation, which is the focus of this paper. This distortion is such that –as we will see in Section 3– it takes one period for the general equilibrium effects on factor prices to show up. In this light, the simplified two-period set up wherein education and work take place in the same period of time seems appropriate.

[12] Using Eq. (18), we can substitute Eqs. (9) and (10) in Eq. (17). By Eq. (11) and by applying the implicit function theorem it is easy to show that $\frac{\mathrm{d}z_t}{\mathrm{d}k_t} > 0$ and by Eq. (11) $\frac{\mathrm{d}c_t^*}{\mathrm{d}k_t} > 0$.

[13] In the policy experiment we perform in the paper, changes in τ_t are unanticipated by the old and therefore they translate into changes in second period consumption, leaving past decisions on savings unaffected.

[14] The Appendix discusses the conditions for the existence and uniqueness of the intertemporal equilibrium and of the steady state, and for the stability of the latter.

[15] An alternative way of representing the introduction of some funding is to maintain the total mandatory contribution rate unchanged but to earmark part of it to finance the (new) fully funded portion of the pension system. The mandatory nature of contributions would in this case be unaffected. Under the assumptions specified in the text, it is straightforward to see that the effects shown in the next sections would be unaltered.

3.1. Policy change

3.1.1. Effects at t

A change in τ_t has the following effects: $\frac{\partial z_t}{\partial \tau_t} = 0$; $\frac{\partial r_t}{\partial \tau_t} = 0$; $\frac{\partial c_t^*}{\partial \tau_t} = 0$; $\frac{\partial w_t^j (1-\tau_t)}{\partial \tau_t} < 0$ and $\frac{\partial s_t^j}{\partial \tau_t} < 0$ for each j.

As we illustrated in the previous section, at time t aggregate factor supplies and their gross rates of return only depend on $k_t = \frac{\bar{s}_{t-1}}{1+n}$, which is in turn unaffected by changes in τ_t. It follows that the wage premium is given at t and therefore the decision to invest in education is the same as the one taken at the past level of the contribution rate.

As gross wages at time t are constant, a reduction in τ_t implies higher net wages for all and higher individual savings.[16] Per capita pensions of the old clearly decrease, and so does therefore their second period consumption.

3.1.2. Effects at t + 1

A change in τ_t implies: $\frac{\partial k_{t+1}}{\partial \tau_t} < 0$; $\frac{\partial c_{t+1}^*}{\partial \tau_t} < 0$; $\frac{\partial z_{t+1}}{\partial \tau_t} < 0$; $\frac{\partial f_{t+1}}{\partial \tau_t} < 0$ and $\frac{\partial r_{t+1}}{\partial \tau_t} > 0$.

The higher level of per capita savings associated with a (partially) funded scheme, as seen in the previous section, is such that the amount of per capita physical capital is higher than that observed before the policy change.[17] This in turns translates into higher output and lower interest rates. The presence of capital–skill complementarity and of an education decision adds further implications to the switch to more funding.

First, the higher level of per capita physical capital brings about an increase in the wage premium and therefore it raises across-group wage inequality.[18] This is new to the literature on social security reform which, when allowing for agents' heterogeneity, uniformly assumes perfect substitutability among workers, once adjusted for productivity differentials. The association between more funding and more across-group wage inequality sharpens the redistributional problems associated to this reform. Indeed, the more actuarial the system is, the larger the gap between the pensions received by those at the top and those at the bottom of the wage distribution. With capital–skill complementarity not only pension gaps but also wage gaps widen, reinforcing the increase in across-group income inequality. Utility differentials between the most and the least able unambiguously increase.

Second, the increase in the wage premium caused by the higher level of per capita physical capital induces more people to invest in education, which in turn raises the effective skilled labour of the economy. The endogenous response of the education decision reduces yet not cancels the initial rise in across-group wage inequality. It also unambiguously raises within-group inequality among skilled agents.[19]

3.1.3. Effects in the steady state

A once and for all reduction in the payroll tax rate τ_t from τ to τ', with $\tau' < \tau$, determines a new steady state characterised by: $k_{\tau'}^{SS} > k_\tau^{SS}$; $c_{\tau'}^{*SS} > c_\tau^{*SS}$; $f_{\tau'}^{SS} > f_\tau^{SS}$; $r_{\tau'}^{SS} < r_\tau^{SS}$ and $z_{\tau'}^{SS} > z_\tau^{SS}$.

In the long run, an increase in funding delivers not only higher physical capital but also higher human capital. However, it also raises the wage premium (and pension differentials) generating higher income inequality.

In the next section we study whether the higher across-group inequality can be tackled by changing the degree of intragenerational redistribution performed by the pension scheme. The analysis will throw some light on if and how distributional concerns can be taken care of in the new steady state.

3.2. Varying α

The higher income inequality generated by the increase in funding raises a natural question on the implications of changing the degree of intragenerational redistribution performed by the pension scheme.[20]

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[16] Differentiating individual savings, $s_t^j = w_t^j (1-\tau_t) - x_{1t}^j$, with respect to τ_t, and reminding that in the policy experiment we are considering the change in τ is once and for all, we obtain $\frac{\partial s_t^j}{\partial \tau_t} = -w_t^j \left[1 - \frac{\partial x_{1t}^j}{\partial y_t^j} \right] - \frac{\partial x_{1t}^j}{\partial y_t^j} \cdot \frac{1+n}{1+r_{t+1}} [\alpha_t (w_t^j - \bar{w}_t) + \bar{w}_{t+1}]$. The terms in square brackets are both positive, implying that $\frac{\partial s_t^j}{\partial \tau_t} < 0$ for any j. Recent evidence on Chile seems to confirm that funding increases household saving (Coronado, 2002).

[17] Given that we assume the costs of the policy change to be borne by the old, there is no issue on how the transition affects capital accumulation.

[18] See footnote 12.

[19] Indeed, since H_{t+1} (and c_{t+1}^*) is now higher, by Eq. (11) we can conclude that z_{t+1} has to be higher than the one which would have prevailed in the absence of the reform. As to the rise in within-group inequality, it follows directly from the fact that, for skilled workers, net wages are decreasing in c. See also footnote 7.

[20] Huggett and Ventura (1999) perform steady state comparisons of the intragenerational redistributive effects of introducing a two-tier system for the US economy, maintaining its pay-as-you-go structure.

As clarified above, α_t denotes how large the contributory portion of the pension scheme is. Namely, it determines the fraction of the pension of the old at time $t+1$ which depends on the contributions they have paid at time t and therefore on their past earnings; the remaining part reflects the average contributions in the economy.[21] The higher α_t, the less the pension scheme redistributes resources across heterogeneous agents belonging to the same generation.

First, we show that changing the redistributive portion of the pension scheme, differently from changing the degree of funding, has no implications on aggregate savings and therefore on capital accumulation and prices.[22] Consider the optimal savings of agent j:

$$s_t^j = w_t^j(1 - \tau_t) - x_{1t}^j \tag{24}$$

Differentiating Eq. (24) with respect to α_t and recalling from Section 2.3 that policy changes have no simultaneous effects on factor prices, we find the following expression:

$$\frac{\partial s_t^j}{\partial \alpha_t} = -\frac{\partial x_{1t}^j}{\partial y_t^j} \cdot \frac{\partial y_t^j}{\partial \alpha_t} \tag{25}$$

By the homotheticity of preferences, $\frac{\partial x_{1t}^j}{\partial y_t^j} = \varkappa$ is the same for all agents. It follows that, in aggregate terms:

$$\frac{\partial \overline{s}_t}{\partial \alpha_t} = -\varkappa \int_0^1 \frac{\partial y_t^j}{\partial \alpha_t} \cdot \varphi(c)\mathrm{d}c = 0 \tag{26}$$

i.e., the redistributive parameter has no effect on aggregate savings, irrespective of the distribution of costs.

However, the change in the degree of redistribution does have an impact on lifetime income as captured by:

$$\frac{\partial y_t^j}{\partial \alpha_t} = \frac{1+n}{(1+r_{t+1})}\left\{\tau_t(w_t^j - \overline{w}_t) + \tau_{t+1}\frac{\partial \overline{w}_{t+1}}{\partial \alpha_t} - \frac{1}{1+r_{t+1}}\frac{\partial r_{t+1}}{\partial \alpha_t}\left[\tau_t\alpha_t(w_t^j - \overline{w}_t) + \tau_{t+1}\overline{w}_{t+1}\right]\right\} \tag{27}$$

which, by Eq. (26), reduces to

$$\frac{\partial y_t^j}{\partial \alpha_t} = \frac{1+n}{(1+r_{t+1})}\left\{\tau_t(w_t^j - \overline{w}_t)\right\} \tag{28}$$

Eq. (28) is negative (positive) when $c^j > \overline{c}_t$ ($< \overline{c}_t$), where we denote by $\overline{c}_t$ the cost of investing in education of the agent who is paid the average wage. That is, $\overline{c}_t$ is such that:

$$w_t^{\mathcal{H}}(1 - \overline{c}_t) = \overline{w}_t \tag{29}$$

and the average wage is:

$$\overline{w}_t = w_t^{\mathcal{H}}(\widehat{h}_t - \widetilde{c}_t) + w_t^{\mathcal{L}}(1 - \widehat{h}_t) \tag{30}$$

where $\widehat{h}_t = \int_0^{c_t^*}\varphi(c)\mathrm{d}c$ is the share of skilled individuals in the total population and $\widetilde{c}_t = \int_0^{c_t^*}c_t \cdot \varphi(c)\mathrm{d}c$ denotes the average cost of investing in education for skilled workers.

According to Eq. (28), a higher degree of redistribution, i.e. a decrease in α_t, drives the lifetime income of those whose wage is above the average down. If people form preferences over α_t according to its impact on lifetime income, it is straightforward to conclude from Eq. (28) that the favourite α_t is 1 or 0, depending on whether the individual's wage is above or below the average. A crucial feature of our model is that the size of those who lose on/ are against and those who benefit from/favour a change in α_t is endogenous and, namely, it depends on the degree of funding itself.

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[21] Notice that α_t does not affect the pension of those who are old at time t.
[22] Here the assumption that preferences are homothetic is crucial.

Indeed, denote by $R_t = \int_0^{\bar{c}_t} \varphi(c)\mathrm{d}c$ the share of agents whose wage is above the average. Changes in τ_t, by affecting the wage premium, modify the decision to invest in education and therefore the size of the skilled and unskilled group[23] and the size of the above/below the average wage group of agents. Namely,[24]

$$\frac{\partial \bar{c}_{t+1}}{\partial \tau_t} < 0, \frac{\partial R_{t+1}}{\partial \tau_t} < 0. \tag{31}$$

An increase in funding at time t, given the change in groups' sizes from time $t+1$ onwards, raises the share of those who lose on redistribution in the public pay-as-you-go scheme, while worsening the relative position of those who benefit from it and making therefore redistribution more necessary for them. The *higher* across-group inequality generated by a lower τ goes with a *larger* share of the population against redistribution.

These results may have nontrivial implications for the current debate on pension reform. If one examines the social security reform proposals advanced in the last years, one sees that most of them tend to associate higher funding – possibly via private individual accounts– with smaller public redistributive pay-as-you-go schemes. Although there seems to be a wide consensus in the policy debate on reinforcing the redistributive portion of the smaller pay-as-you-go system, an explicit analysis of the compatibility between enhancing redistribution and funding is still lacking. Our work highlights a link between these two policies which, to our knowledge, has not been addressed before and which may threaten such compatibility.

As a concluding remark, notice that we here focus on a specific way to intragenerationally redistribute income, that is, we use a flat universal pension. We do not allow for means-testing or for any other tax-transfer scheme. Future work should be directed to analyse whether the results reached here hold also in an environment where –for instance– only those who pass a test on means are entitled to receive the state benefit. This would also require to tackle the moral hazard issues both on the saving and on the education decision which means-testing introduces.

4. Conclusions

This paper analyses the general equilibrium effects of increasing funding in an economy with heterogeneous agents, capital–skill complementarity and human capital investment. This reform can generate both inter and intragenerational conflicts. While the theoretical literature and the policy debate have so far mainly focused on the former and on the alternative ways to finance the transition, here we study the intragenerational redistributive flows associated to the reform, assuming that the old generation at the time of the switch bears entirely the costs of financing it.

We show that more funding implies higher physical and human capital but also higher across-group wage and income inequality. This is new to the theoretical literature on social security reform: the next step is to quantify these effects by incorporating capital–skill complementarity and educational decisions in simulation models. We leave this task for future research.

The analysis developed here delivers some policy implications for the current debate on reforming partially redistributive pay-as-you-go systems. Most of the current social security reform proposals involve an increase in funding. Higher funding implies more actuarial equivalence between contributions and benefits and it raises issues on how to take care of distributional concerns in the new reformed system. Although there seems to be an agreement on defending or strengthening the redistributive portion of the smaller remaining pay-as-you-go pillar, the compatibility between (private) funding and (public) redistribution is always taken for granted and never explicitly dealt with. The results of our paper show that changes in groups' size stemming from general equilibrium effects can actually threaten such compatibility as the consensus over redistribution decreases. Public redistribution and private funding may turn out to be at odds.

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[23] This follows from $\frac{\partial c_{t+1}^*}{\partial \tau_t} < 0$ as detailed in Section 3.1.

[24] As to the first inequality, solving Eq. (29) for $\bar{c}_{t+1}$, using Eq. (30) and differentiating with respect to τ_t, we obtain $\frac{\partial \bar{c}_{t+1}}{\partial \tau_t} = -\left[\frac{\partial \hat{h}_{t+1}}{\partial \tau_t}\left(1 - \frac{w_{t+1}^{\mathcal{L}}}{w_{t+1}^{\mathcal{H}}}\right) - \frac{\partial \tilde{c}_{t+1}}{\partial \tau_t} + \frac{\partial \frac{w_{t+1}^{\mathcal{L}}}{w_{t+1}^{\mathcal{H}}}}{\partial \tau_t}(1 - \hat{h}_{t+1})\right]$. Using Eq. (8) and the definition of $\hat{h}_{t+1}$ and $\tilde{c}_{t+1}$, the first two terms cancel out and the above equation can be rewritten as $\frac{\partial \bar{c}_{t+1}}{\partial \tau_t} = -\frac{\partial \frac{w_{t+1}^{\mathcal{L}}}{w_{t+1}^{\mathcal{H}}}}{\partial k_{t+1}}(1 - \hat{h}_{t+1})\frac{\partial k_{t+1}}{\partial \tau_t}$ which is negative by capital-skill complementarity. This is to say that, if τ_t decreases, the agent whose wage coincides with the average is now less able. The second inequality follows from the definition of R_t.

Appendix A

Consider the equilibrium condition (23), where the general expression for average per capita savings $\bar{s}_t$ is:

$$\bar{s}_t = \int_0^{c_t^*(k_t)} s_t(c; w_t^{\mathcal{H}}, \bar{w}_t, r_{t+1}, \bar{w}_{t+1}, \alpha_t, \tau_t, \tau_{t+1}, n)\varphi(c)\mathrm{d}c$$
$$+ [1 - \Phi(c_t^*(k_t))] \cdot s_t(w_t^{\mathcal{L}}, \bar{w}_t, r_{t+1}, \bar{w}_{t+1}, \alpha_t, \tau_t, \tau_{t+1}, n) \tag{32}$$

Recalling that factor prices at any period are a function of the capital stock in the same period only and simplifying notation, we rewrite Eq. (23) as follows:

$$k_{t+1}(1 + n) - \bar{s}_t(k_t, k_{t+1}) = 0 \tag{33}$$

The assumptions on the utility and on the production functions guarantee the existence of the intertemporal equilibrium (see De La Croix and Michel, 2002).

Uniqueness requires $1 + n - \frac{\partial \bar{s}_t}{\partial k_{t+1}} > 0$. In the absence of the pension system, τ_t, τ_{t+1} and α_t in Eq. (32) are 0 and the condition for uniqueness can be written as:

$$1 + n - \frac{\partial r_{t+1}}{\partial k_{t+1}}\left[\int_0^1 \frac{\partial s_t(c)}{\partial r_{t+1}}\varphi(c)\mathrm{d}c\right] > 0. \tag{34}$$

When the pension system is in place, the condition for uniqueness is:

$$1 + n - \left[\int_0^1 \left(\frac{\partial s_t(c)}{\partial r_{t+1}}\frac{\partial r_{t+1}}{\partial k_{t+1}} + \frac{\partial s_t(c)}{\partial \bar{w}_{t+1}}\frac{\partial \bar{w}_{t+1}}{\partial k_{t+1}}\right)\varphi(c)\mathrm{d}c\right] > 0 \tag{35}$$

which is a weaker requirement than the one established in the absence of pensions, given that $\frac{\partial s_t(c)}{\partial \bar{w}_{t+1}}\frac{\partial \bar{w}_{t+1}}{\partial k_{t+1}} < 0$. When Eq. (35) is satisfied, the equilibrium condition (33) implicitly defines k_{t+1} as a function g of k_t.

As to the properties of the $g(\cdot)$ function, we observe that in the absence of the pension system, $g'(k_t) = \frac{\mathrm{d}k_{t+1}}{\mathrm{d}k_t} = \frac{\frac{\partial \bar{s}_t}{\partial k_t}}{1 + n - \frac{\partial \bar{s}_t}{\partial k_{t+1}}} > 0$. To see this, consider that in this case τ_t, τ_{t+1} and α_t in Eq. (32) are 0. Recalling from Eq. (8) that $w_t^{\mathcal{H}}(1 - c_t^*) = w_t^{\mathcal{L}}$, the latter being independent of c, the numerator

$$\frac{\partial \bar{s}_t}{\partial k_t} = \int_0^{c_t^*(k_t)} \left[\frac{\partial s_t(c)}{\partial w_t^{\mathcal{H}}}\frac{\partial w_t^{\mathcal{H}}}{\partial k_t}\right]\varphi(c)\mathrm{d}c + [1 - \Phi(c_t^*(k_t))]\left[\frac{\partial s_t(c)}{\partial w_t^{\mathcal{L}}}\frac{\partial w_t^{\mathcal{L}}}{\partial k_t}\right] \tag{36}$$

is always positive which, together with Eq. (34), guarantees $g'(k_t) > 0$.

When the pension system is in place, the numerator $\frac{\partial \bar{s}_t}{\partial k_t}$ is:

$$\frac{\partial \bar{s}_t}{\partial k_t} = \int_0^{c_t^*(k_t)} \left[\frac{\partial s_t(c)}{\partial w_t^{\mathcal{H}}}\frac{\partial w_t^{\mathcal{H}}}{\partial k_t} + \frac{\partial s_t(c)}{\partial \bar{w}_t}\frac{\partial \bar{w}_t}{\partial k_t}\right]\varphi(c)\mathrm{d}c + [1 - \Phi(c_t^*(k_t))]\left[\frac{\partial s_t(c)}{\partial w_t^{\mathcal{L}}}\frac{\partial w_t^{\mathcal{L}}}{\partial k_t} + \frac{\partial s_t(c)}{\partial \bar{w}_t}\frac{\partial \bar{w}_t}{\partial k_t}\right] \tag{37}$$

where Eq. (37) differs from Eq. (36) for the presence of the derivatives with respect to the average wage. While the second term in Eq. (37), as we will argue next, is always positive, the first term can be either positive or negative. To see these, we focus on agent j. Using Eqs. (5) and (24), $\frac{\partial s_t^j}{\partial k_t}$ can be written as:

$$\frac{\partial s_t^j}{\partial k_t} = \frac{\partial s_t^j}{\partial w_t^j}\frac{\partial w_t^j}{\partial k_t} + \frac{\partial s_t^j}{\partial \bar{w}_t}\frac{\partial \bar{w}_t}{\partial k_t} = (1 - \tau)\frac{\partial w_t^j}{\partial k_t}\left(1 - \frac{\partial x_{1t}^j}{\partial y_t^j}\right) - \frac{\partial x_{1t}^j}{\partial y_t^j}\left[\frac{1 + n}{1 + r_{t+1}}\tau_t\alpha_t\left(\frac{\partial w_t^j}{\partial k_t} - \frac{\partial \bar{w}_t}{\partial k_t}\right)\right] \tag{38}$$

The first term in Eq. (38) is positive for any j. The sign of the second term depends on the sign of $\frac{\partial w_t^j}{\partial k_t} - \frac{\partial \bar{w}_t}{\partial k_t}$. For $j \in \mathcal{L}$, by capital–skill complementarity $\frac{\partial w_t^{\mathcal{L}}}{\partial k_t} - \frac{\partial \bar{w}_t}{\partial k_t} < 0$, and $\frac{\partial s_t^{\mathcal{L}}}{\partial k_t} > 0$. For agents characterised by $\frac{\partial w_t^j}{\partial k_t} - \frac{\partial \bar{w}_t}{\partial k_t} > 0$, the two terms in Eq. (38) are of opposite sign.

To provide an intuition of what is going on, first notice that in the absence of a pension scheme, or with a fully redistributive pension formula, that is $\alpha_t = 0$, the term in square brackets in Eq. (38) would disappear and $\frac{\partial s_t^j}{\partial k_t}$ would be positive for any j. With $\alpha_t > 0$, a higher level of per capita capital has two distinct effects on pensions: it raises the individual wage w_t^j, thus increasing, by Eq. (3), the contributory share of the benefit; it also raises the average wage $\overline{w}_t$, therefore reducing the per capita flat component of the pension as, by Eq. (4), more resources have to be devoted to the contributory share of the scheme. The sum of the two effects determines the sign of the term in square brackets in Eq. (38). For unskilled agents, the reduction in the redistributive share, coupled with an increase in the individual wage which is lower than the one in the average wage, prompts an increase in savings. For the very skilled agents, the increase in wages and in the contributory pension might reduce savings. A sufficient condition for $\frac{\partial s_t}{\partial k_t} > 0$ is that, for agents characterised by $\frac{\partial w_t^j}{\partial k_t} - \frac{\partial \overline{w}_t}{\partial k_t} > 0$:

$$\left| \frac{\partial x_{1t}^j}{\partial y_t^j} \left[\frac{1+n}{1+r_{t+1}} \tau_t \alpha_t \left(\frac{\partial w_t^j}{\partial k_t} - \frac{\partial \overline{w}_t}{\partial k_t} \right) \right] \right| < \left| (1-\tau) \frac{\partial w_t^j}{\partial k_t} \left(1 - \frac{\partial x_{1t}^j}{\partial y_t^j} \right) \right| \tag{39}$$

holds, which, with Eq. (35) guarantees $g'(k_t) > 0$. Eq. (39) imposes that the increase in consumption induced by the rise in pension benefits is not too high. Notice however that what we care about is the behaviour of aggregate savings: for them to react positively to changes in k_t, weaker conditions would suffice.

A steady-state of the economy is a k^{SS} such that $g(k^{SS}) = k^{SS} = \frac{1}{1+n}\overline{s}(k^{SS})$. We first notice that at $k=0$, Eq. (13) reduces to $f = [(1-b)l^\theta]^{\frac{\delta}{\theta}}[h]^{1-\delta} > 0$, which implies that savings are positive and which excludes a corner steady state at 0. We also notice that the properties of the production function guarantee that $\lim_{k\to\infty} \frac{f-rk}{k} = 0$. Given that savings can never exceed labour income – i.e. $\overline{s}(k) \leq f - rk$ –, $\lim_{k\to\infty} \frac{g(k)}{k} = 0$ holds, at the limit $g(k) < k$ and the dynamics is bounded. It follows that there exists at least one stable steady state $k^{SS} > 0$ of the economy. Uniqueness requires $\frac{\overline{s}(k)}{k}$ to be strictly decreasing for any $k > 0$, which is here assumed.

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