Università Commerciale
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20135 Theory of Finance – Part I

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A FEW SAMPLE QUESTIONS, WITH SOLUTIONS – SET 2

WARNING: These are just sample questions. Please do not count or speculate that the actual Part I of your 20135 exam will be identical or closely related to the following questions.

Question 1.

1a. (2 points) Suppose that one of the quant members of your asset allocation team proposes to take asset allocation decisions following the predictive regression for excess stock returns (r_{t+1}):

$$r_{t+1} = \alpha + \beta ep_t + \epsilon_{t+1},$$

where ep_t is the time t earnings-to-price ratio for the stock (index). However, OLS estimation of this model has yielded the following results (standard errors are in parenthesis):

$$\underset{}{r_{t+1}} = \underset{(0.155)}{0.322} + \underset{(0.046)}{0.080}\, ep_t + \underset{(0.196)}{\epsilon_{t+1}} \qquad R^2 = 3.45\%$$

Indicate how the model may be used to predict the equity risk premium. Is this model satisfactory in a statistical dimension?

Answer. The model implies timid and yet possibly useful predictions for the equity risk premium. The predictions are timid because the statistical performance of the model appears to be rather weak, as shown by a t-stat of (0.080/0.046 = 1.739) which falls short (above) a standard p-value of 5%; additionally, also the coefficient of determination of the predictive regression (3.45%) is rather low and illustrates that the majority of the variability of excess stock returns cannot be explained by ep_t. Leaving these features aside, the model yields a simple mechanism to predict the equity risk premium, $E_t[r_{t+1}]$, conditional on the level of the earnings-price ratio at time t:

$$E_t[r_{t+1}] = 0.322 + 0.080 ep_t$$

which means that a higher current ep_t predicts a higher future premium; implicitly, because current prices appear at the denominator of ep_t this means that a strongly bull market in the current period that lowers ep_t (i.e., prices grow faster than earning and the multiple declines) predicts lower expected excess returns in the following periods, which represents a form of revision of returns towards the mean, i.e., recently high returns that have pushed prices high imply that prices ought to grow more slowly in the following periods. However, as noted in the lectures with reference to Giulia Dal Pra's dissertation, it is far from obvious that the statistical performance of such a relationship may represent reliable grounds for asset allocation decisions.

1b. (3 points) Discuss what strategies might be used to assess whether this predictability model may generate any positive economic value to go over and beyond the statistical performance. How would you go about making sure that such economic value tests yield results that are sufficiently robust? Why is such a need for robustness of any relevance in this context?

<u>Answer</u>. One needs to find robust results to avoid that some lucky combination of asset allocation strategies and parametric choices of some convenience (for instance, just mean-variance strategies with a short horizon and omitting transaction costs) may lead to some lucky conclusions concerning the value and usefulness of predictability models. Therefore one would like to use several possible combinations of different asset allocation models/strategies and parametric selections within the former one may deploy to guard against the effects of sheer luck. For instance a few dimensions along which to assess the ability of the model in 1a to generate economic value are:

- The asset allocation model, for instance switching strategies (when one simply invests 100% in stocks when the equity risk premium is predicted to be positive), mean-variance strategies (of which you know everything by now), and (possibly dynamic) power utility based strategies, in which portfolio weights are selected to maximize the expected power utility of future, terminal wealth, when excess stock returns are predictable as in question 1a.
- The investment horizon, at least in the form of investing over a horizon of T periods (in Giulia Dal Pra's dissertation, years) when excess stock returns are predictable in the generalized form $r_{t+T} = \alpha + \beta ep_t + \epsilon_{t+T}$.
- The degree of risk aversion imputed to the benchmark investor considered in your analysis, in the form of selecting some risk aversion parameter κ in the case of mean-variance preferences, or of selection some constant coefficient of relative risk aversion γ in the case of power utility.
- The level of transaction costs imputed to each trade, as trading imposes on investors the need to pay both bid-ask spreads and commission fees.
- The overlapping (when one backtests through the simulation of a sequence of investors whose horizons overlap in time) or non-overlapping (when one simulates the behaviour of one and only investor) nature of the experiments.

1c. (3 points) Consider the following table derived from the recursive implementation ("backtesting") of a switching strategy that invests in the stock when the predicted risk premium is positive ($E_t[r_{t+1}] > 0$) and in cash (1-month T-bills) when the predicted risk premium is non-positive. Every year a different investor, who closes her position at the end of the investment period, selects a different portfolio held for either 1 or 10 years. As you can see three alternative levels of transaction costs have been implemented. Illustrate the effects of the switching strategy that exploits predictability on realized, optimal portfolio variance. Do alternative levels of transaction costs reduce the economic benefits of exploiting the predictability deriving the earnings-price ratio? Does the predictive power of the ratio increase as the horizon grows? Make sure to illustrate your answers with reference to the numbers that appear in the table.

OVERLAPPING STRATEGIES, T=1Y & T=10Y

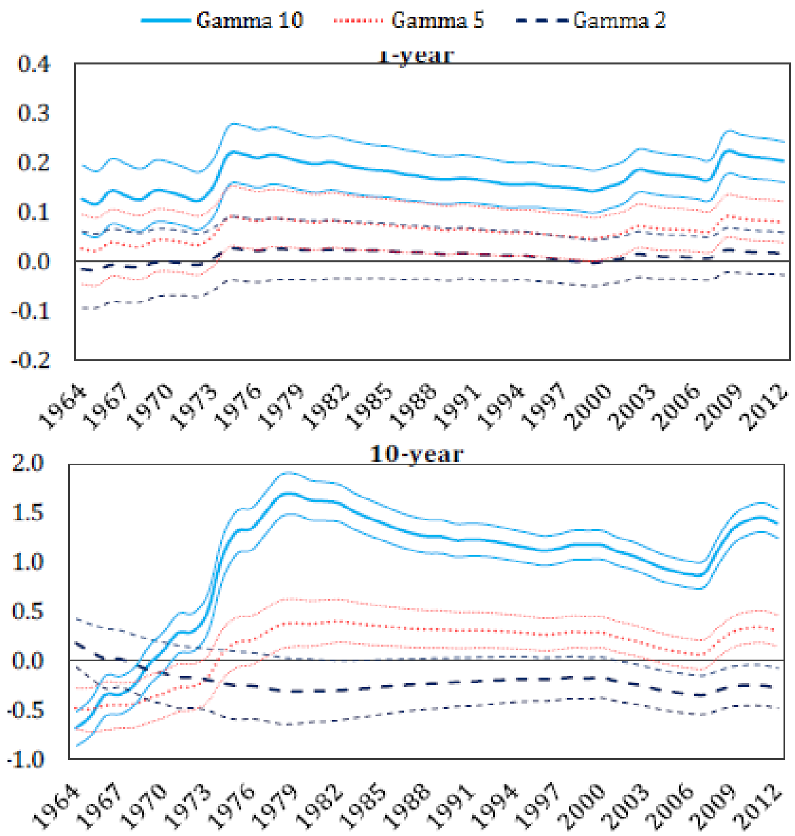
Transaction Costs	Zero		Low		High	
Horizon	1	10	1	10	1	10
Mean	0.091	0.103	0.087	0.101	0.087	0.101
Variance	0.024	0.023	0.023	0.023	0.022	0.023
Sharpe Ratio	0.284	0.370	0.265	0.361	0.274	0.358

BENCHMARK: BUY AND HOLD STRATEGY

Transaction Costs	Zero	Low	High
Mean	0.108	0.108	0.108
Variance	0.029	0.029	0.029
Sharpe Ratio	0.359	0.358	0.358

Answer. Clearly, adopting a switching strategy based on a predictive regression centred on the earnings-price ratio yields realized portfolio variances that are always substantially below those of a benchmark, buy-and-hold strategies in which one simply invests all the time in the S&P 500 U.S. stock index (these numbers obviously come from Giulia Dal Pra's thesis and therefore refer to the dividend yield, but for the sake of argument they did help us here). This occurs independently of the investment horizon and of the level of transaction costs imputed on the strategy implemented by the investor. However, it is also visible that the predictive regression in question 1a does not improve realized mean portfolio returns. Therefore the trade-off occurs between reducing the perceived risk of one's portfolio and bearing a loss in realized mean portfolio returns. Interestingly, predictability does not enable you to make more money: instead you make lower average returns but in a considerably more stable way over time. This is a key aspect often neglected by portfolio managers.
If we take the increase in the Sharpe ratio as a measure of the economic value of the strategy, the yes—exploiting predictability does generate risk-adjusted improvements in performance for long-horizon investors but these decline as the level of transaction costs is increased from zero (when exploiting predictability raises the Sharpe ratio from 0.359 to 0.370) to a high level of transaction costs (in this case exploiting predictability raises the Sharpe ratio from 0.3580 to 0.3582, a really marginal benefit). The intuition is that predictability requires an investor to buy and sell stocks and as such this erodes realized mean returns.
Visibly, the predictive power of ep_t grows the horizon as the results are always better for the T = 10 year horizon investor than for a short term investor.

1d. (2 points) Consider the following two plots derived from the recursive implementation ("backtesting") of a simple mean-variance strategy in which the equity risk premium is derived from the predictive model $r_{t+1} = \alpha + \beta ep_t + \epsilon_{t+1}$. The plots refer to two alternative investment horizons, i.e., every year a different investor, who closes her position at the end of the investment period, selects a different portfolio held for either 1 or 10 years. The plots illustrate the increase in *certainty equivalent return* (CER) that an investor obtains from adopting the strategy that exploits predictability, where a positive ΔCER indicates that adopting the predictability-based strategy generates an increase in risk-adjusted performance. Does the adoption of the earnings-price ratio based model generate positive economic value and under what conditions? Please make sure to closely refer to the plots instead of reporting generic and "over heard" answers.



Answer. The two plots show that the increase in risk-adjusted performance (as measured under a power utility function) is substantial and statistically significant most of the time for a highly risk averse investor with γ = 10. This occurs across both investment horizons and the result is statistically significant starting at least in 1971 for both horizons, as the 90% confidence intervals for both horizons fail to include a zero increase in CER. Especially in the case of a 10-year horizon, the increase in CER is quantitatively large and reaches 150 basis points per year, indicating that a γ = 10 investor ought to be ready to pay an annual fee that may even exceed 1% to access the predictability-based strategy. In the case of investors with intermediate risk aversion of γ = 5 the result is instead less robust for both horizons, and appear to be statistically significant only over the period 1974-2004. In this case, the increase in CER never exceeds 30-40 basis points per year and it is doubtful whether there would exist space for professional asset management to monetize such a difference in CERs. Finally, a moderately risk averse individual with γ = 2 would have little use for the predictability strategy, especially over long horizons. This is perfectly sensible because we have seen that the strategy helps by reducing risk and not by generating trading profits. Therefore investors with γ = 10 will really like it and investors with γ = 2 will find it to be a waste of energies.

Question 2.

2a. (2 points) State the two-fund separation theorem along with the hypotheses needed to obtain such a result. What are the implications of this result for the architecture of the asset/wealth management industry? Suppose that by opening a newspaper you find evidence of the existence of two different global equity mutual funds, that invest in stocks from all over the planet, with different weights and investment strategies. What do you infer from this very fact? Carefully justify your answers. [Note: one-sentence replies without a justification will receive NO partial credit; answers supported by a reasoning that is unrelated to the material covered in the course will receive NO partial credit.]

Answer. The two-fund separation theorem states that for a mean-variance optimizer the optimal structure of her portfolio consists of a linear combination between a fixed/optimal portfolio of risk securities that achieves tangency between the capital market line and the mean-variance efficient frontier and cash, to proxy a riskless asset. The hypotheses that support the separation theorem consists of the typical assumptions backing mean-variance analysis, i.e., mean-variance preferences; increasing absolute risk aversion in the sense that an increasing compensation in terms of expected returns is required to withstand an increasing exposure to risk as measured by the standard deviation of portfolio returns; homogeneous beliefs concerning means, variance, and covariances; a riskless asset at the rate of which it is possible to borrow and invest with limits; no borrowing or short-selling constraints; no taxes or transaction costs, etc.

The implication of the two-fund separation theorem for the architecture of the asset/wealth management industry is that the industry should be extremely concentrated and imply the existence of a single super-mutual fund specialized in investing in all risky assets. There would be no space for competing on performances. In fact all mutual funds—even admitting the existence of more than fund—ought to be achieve the same realized Sharpe ratio equal to the maximal Sharpe ratio typical of the tangency portfolio.

If you were to open the newspaper and find news of the existence of two different global equity mutual funds, that invest in stocks from all over the planet, with different weights and investment strategies, this would indicate that either investors, markets, as well as fund managers are irrational and ignore unexploited risk-adjusted opportunities (in the sense that at least one of the two funds must understand that they are not maximizing their Sharpe ratio) or that these behaviours are rational but the two-fund separation theorem fails. In fact, it is impossible that the two-fund separation theorem applies and two different *risky* portfolio structures appear.

2b. (3 points). Suppose you find two investors who hold different proportions of (the same) risky assets and of cash, considered to be riskless. You verify that they are both risk-averse, with increasing absolute risk aversion, and that they hold homogeneous beliefs concerning means, variances, and covariances of risky asset returns. Is this evidence of any differences across their portfolios sufficient to conclude that the two-fund separation theorem fails to hold? Next you further investigate the portfolios structure for these two investors to discover that not only the overall composition of their portfolios differ, but they also hold quite different *risky* portfolios. Is this evidence of any differences across their risky portfolios sufficient to conclude that the two-fund separation theorem fails to hold? Finally, you manage to determine that while the first investor is long in both cash and the risky assets, the second investor is instead short not only cash (i.e., she is borrowing at the riskless rate to leverage her portfolio) but also a few of the risky assets, say stocks. Does this finding indicate that the second investor is not choosing on the mean-variance efficient frontier? Does this finding indicate that the second investor is not choosing on the capital market line? [Note: one-sentence replies without a justification will receive NO partial credit; answers supported by a reasoning that is unrelated to the material covered in the course will receive NO partial credit.]

Answer. "You verify that they are both risk-averse, with increasing absolute risk aversion, and that they hold homogeneous beliefs concerning means, variances, and covariances of risky asset returns." means that you have checked both investors select in a typical mean-variance set up. Having said this, no the fact that two investor hold different portfolios in terms of the share of *the same* portfolio of risky assets and of cash does not prove that the two-fund separation theorem fails to hold. Indeed we know that this result simply restricts the risky asset portfolio to be the same across investors, not to be held in identical proportions across different investors. However, here the interpretation of the statement by which they "hold different proportions of (the same) risky assets and of cash, considered to be riskless" is key in the sense that the SAME risky assets refers to the fact that both investors do hold the tangency portfolio.
Next, if not only the overall composition of their portfolios differ, but the two investors also hold different *risky* portfolios, then this is evidence that the two-fund separation theorem fails to hold. The reason is obvious from 2a.
However, the finding that the second investor is short not only cash (i.e., she is leveraging her portfolio) but also a few of the risky assets, say stocks, means nothing in itself. Never in the course we said that the tangency portfolio had to be a long-only portfolio. In fact, at this point of the course, the tangency portfolio does NOT yet correspond to the market portfolio and as such it may include short positions. Equivalently, we cannot take this circumstance as an indication that the second investor is inferior to the first investor and not rationally selecting on the mean-variance efficient frontier or even the capital market line (there is no difference, we are really talking about the composition of the tangency portfolio here, which may be any portfolio on the mean-variance frontier and above the GMV, depending on which riskless rate has been fixed in the first instance).

2c. (2.5 points) Consider at this point a generic investor with mean-variance preferences defined over the moment of her portfolio returns, i.e.,

$$V = E_t[\mathrm{R}^{\mathrm{P}}_{\mathrm{t+1}}] - \frac{1}{2}\kappa Var_t[\mathrm{R}^{\mathrm{P}}_{\mathrm{t+1}}].$$

Can you find the expression for the tangency portfolio in this case? Can you verify the two-fund separation theorem on the basis of this formula for the vector of portfolio weights defining the tangency portfolio? Suppose now that the Sharpe ratio on the very first asset doubles. Can you tell whether the weight of the first risky asset will increase, stay constant, or decrease? If you think this is possible, how can it be that a security starts paying out much more than it used to and yet its portfolio weight declines? Carefully justify your answers. [Note: one-sentence replies without a justification will receive NO partial credit; answers supported by a reasoning that is unrelated to the material covered in the course will receive NO partial credit.]

Answer. The expression of the tangency portfolio in this case corresponds to the classical mean-variance formula for the optimal portfolio weights:

$$\widehat{\boldsymbol{w}}_t = \frac{1}{\kappa}\boldsymbol{\Sigma}_t^{-1}E_t\left[\boldsymbol{R}_{t+1} - r_t^f\,\boldsymbol{\iota}_{\mathrm{N}}\right]$$

This has been noted in slide 26, lecture 2.

To verify the two-fund separation theorem on the basis of this formula, note that if you take characterized by different κ, say $\kappa^* = \lambda\kappa$ ($\lambda \neq 1$) then

$$\widehat{\boldsymbol{w}}_t = \frac{1}{\kappa}\boldsymbol{\Sigma}_t^{-1}E_t\left[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}\right]$$

$$\widehat{\boldsymbol{w}}_t^* = \frac{1}{\lambda\kappa}\boldsymbol{\Sigma}_t^{-1}E_t\left[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}\right] = \frac{1}{\lambda}\left(\frac{1}{\kappa}\boldsymbol{\Sigma}_t^{-1}E_t\left[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}\right]\right) = \frac{1}{\lambda}\widehat{\boldsymbol{w}}_t$$

which shows that the second investor adopts the same risky portfolio weights as the first investor, just scaled by some factor $1/\lambda \neq 1$. Yet, one can simply state the second investor simply scales up or down the tangency portfolio defined by the quantity $\boldsymbol{\Sigma}_t^{-1}E_t\left[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}\right]$ which is also the optimal risky portfolio of an investor with $\kappa = 1$. Equivalently, while the first investor invests in cash the percentage

$$1 - \mathbf{1}'\widehat{\boldsymbol{w}}_t = 1 - \frac{1}{\kappa}(\mathbf{1}'\boldsymbol{\Sigma}_t^{-1}E_t[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}]) = 1 - \frac{1}{\kappa}\mathrm{T}$$

the second investor commits:

$$1 - \mathbf{1}'\widehat{\boldsymbol{w}}_t^* = 1 - \frac{1}{\lambda\kappa}(\mathbf{1}'\boldsymbol{\Sigma}_t^{-1}E_t[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}]) = 1 - \frac{1}{\lambda\kappa}\mathrm{T}$$

This shows that only differ in the amount they leave in cash, as claimed.
If the Sharpe ratio on the very first asset doubles this means that

$$\frac{E_t\left[R_{t+1}^1 - r_t^f\right]}{\sigma_t^1}$$

doubles. Unfortunately this ratio involves the first element of the vector $E_t\left[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}\right]$ and one parameter, σ_t^1, that given the general structure of an inverse matrix, may enter in several places of $\boldsymbol{\Sigma}_t^{-1}$ and with ambiguous effects. Therefore, even though the Sharpe ratio of asset 1 increases, the net effect on

$$\widehat{\boldsymbol{w}}_t = \frac{1}{\kappa}\boldsymbol{\Sigma}_t^{-1}E_t\left[\boldsymbol{R}_{t+1} - r_t^f \boldsymbol{\iota}_{\mathrm{N}}\right]$$

will be very difficult to assess and specific calculations will have to be carried out to provide some definitive answer. So the answer is "it depends". The intuition for the indeterminacy is that when the Sharpe ratio changes we have no information on what happens to correlations. Moreover, a security gives a contribution to portfolio risk that depends also on its correlation with other risky assets and such it will not always be trivial to assess whether securities with the highest Sharpe ratios are also the ones for which portfolio weights are the highest.

2d. (2.5 points) Discuss the following statement: "the only way to obtain the mean-variance formula for portfolio weights shown in the reply to question 2c is by assuming either a quadratic utility function of terminal wealth or by directly specifying preferences as being of a mean-variance type." Can you provide examples of mean-variance *type* weights deriving from assumptions different from the ones listed above? If so, carefully the differences between the weights derived under the alternative framework and the classical mean-variance formula in question 2c. Can you detect any implications of these two different paths to mean-variance analysis for the asset/wealth management industry? Carefully justify your answers. [Note: one-sentence replies without a justification will receive NO partial credit; answers supported by a reasoning that is unrelated to the material covered in the course will receive NO partial credit.]

Answer. The statement "the only way to obtain the mean-variance formula for portfolio weights shown in the reply to question 2c is by assuming either a quadratic utility function of terminal wealth or by directly specifying preferences as being of a mean-variance type." is incorrect. We have seen that a mean-variance *type* formula may be obtained when preferences take the form of a power utility function, the investor is an expected utility maximize, and the benchmark portfolio follows a lognormal distribution. In this case, however, the formula for the mean-variance weight is not

$$\widehat{w}_t^{MV} = \frac{E_t\left[R_{t+1}^P - r_t^f\right]}{\kappa\sigma_t^2}$$

where R_{t+1}^P is the discretely compounded portfolio return and κ trades-off mean and variance in the function

$$V = E_t[\mathrm{R}_{t+1}^{\mathrm{P}}] - \frac{1}{2}\kappa Var_t[\mathrm{R}_{t+1}^{\mathrm{P}}].$$

(the case of quadratic utility is even more complex and boring), but instead

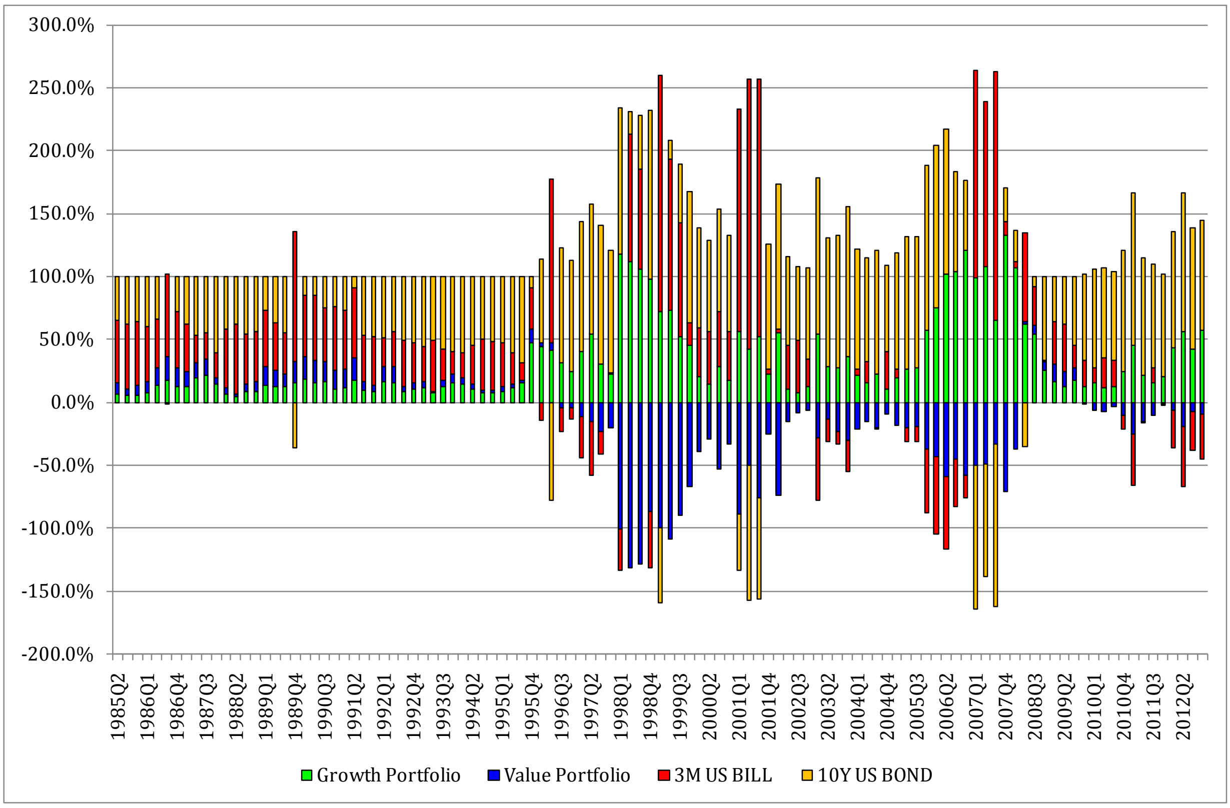
$$\widehat{w}_t^{logN} = \frac{lnE_t[\ (1 + R_{t+1}^P)]}{\gamma\sigma_t^2}$$

where γ is the constant coefficient of relative risk aversion. Of course the differences lie in the portions $lnE_t[\ (1 + R_{t+1}^P)] \neq E_t\left[R_{t+1}^P - r_t^f\right]$ although the two quantities may get very close as the time frequency increases and the absolute scale of realized returns goes to zero and for a small value of the riskfree rate, and in the fact that the parameters κ and γ have rather different interpretations.

The implications for the asset/wealth management industry are positive on net, in the following sense. While economists have long criticized mean-variance approaches because these would be based on either a proper, but non-monotonic utility function of terminal wealth (the quadratic one) or on some functional that cannot really be capturing preferences over monetary payoffs (the standard MV criterion $E_t[\mathrm{R}_{t+1}^{\mathrm{P}}] - \frac{1}{2}\kappa Var_t[\mathrm{R}_{t+1}^{\mathrm{P}}]$), the log-normal case reveals that a mean-variance result obtains also under a proper, expected power utility objective. However, as discussed in the lectures, the downside is clear: while the simple return on a portfolio is a linear combination of the simple returns on the risky and riskless assets, the log portfolio return is not the same as a linear combination of logs.

Question 3.

3a. (4 points) Consider the following plot depicting the effects of predictability in (expected) risky asset returns on optimal mean-variance weights obtained with reference to a quarterly US sample spanning the period 1985-2012. Carefully explain what causes the variation of optimal weights over time making sure to write down the type of model you have estimated in Excel. Is the variation sensible in the light of your knowledge of boom/bust cycles in the U.S. market (bear periods have characterized 1998-2001 and then again 2007-2008). Carefully justify your answers. [Note: one-sentence replies without a justification will receive NO partial credit; answers supported by a reasoning that is unrelated to the material covered in the course will receive NO partial credit.]



Answer. The variation in optimal weights over time is caused by time-varying predictions of the risk prima on the four risky assets (here also 3-month T-bills are taken as risky) coming from regressions models of the type:

$$r_{t+1}^i = \alpha^i + \beta^i x_t + \epsilon_{t+1}^i,$$

where r_{t+1}^i measures the excess return on asset i = 1, 2, ..., N and x_t is generic predictor, for instance the current dividend yield.

The variation in recorded weights sensibly tracks the sequence of boom/bust cycles in the U.S. financial markets: during bear periods (such as 1998-2001 and 2007-2008), a mean-variance investor tends to hold almost zero net investment positions in stocks, in the form of portfolios that typically go long in growth stocks (which during a bear period is sensible, one bets on future opportunities) and short in value stocks, with lots of assets already in place. Considerable amounts are invested in short-term 3-month T-bills, although some positive net holding of 10-year Treasury Notes are also held. During bull periods, an investor tends to go long in all the assets with some prevalence of growth stocks and 10-year Treasury Notes, probably betting on future rate reductions during bear periods characterized by central bank interventions.

3b. (2 points) Can you notice any special structure in the composition and size of the equity component of the optimal portfolio in correspondence to bear market states?

Answer. During bear periods (such as 1998-2001 and 2007-2008), a mean-variance investor tends to hold almost zero net investment positions in stocks, in the form of portfolios that typically go long in growth stocks (which during a bear period is sensible, one bets on future opportunities) and short in value stocks, with lots of assets already in place. This is the origin of a famous long-short portfolio that you have encountered in the second part of the course, the HML portfolio, long in value stocks and short in growth stocks. Clearly, our finding is that during recessions, a mean-variance investor would go short in HML which is equivalent to short value stocks to buy growth stocks.

3c. (2 points) Explain and provide intuition for the static vs. the dynamic effects of predictability on optimal portfolios. Make sure to relate your answer to the notion of hedging demands. Do the results shown in question 3a reflect any hedging demands? Carefully justify your answer in the light of the work you have performed in homework 2.

Answer. As we have seen in lecture 3, predictability has two effects on portfolio choice. In a static dimension, predictability makes some or all the inputs of any asset allocation decision tool a function of time and imply that optimal weights change recursively as new information becomes available. In practice this means that in the standard mean-variance formula, the conditional risk premia and possibly also conditional portfolio variance do become a function of time. In a dynamic dimension, predictability creates interesting and often non-negligible hedging demands, even though one needs a horizon of at least two periods to see hedging demands in action. Hedging what? Future changes in the nature of investment opportunities that derive from the fact that not only asset returns are risky, but also their statistical features characterizing risk, become time-varying. When the long-term weights are twisted in directions to protect us against future adverse movements in investment opportunities, this represents a source of a hedging demand. See slides 5 and 6 of lecture 3 for one example that hopefully reveals the intuition for what hedging demands are, even though here just defining them would have been sufficient.
No, the results shown in the graph in 3a do not reflect any hedging demands because these had been obtained in the homework by simply predicting future risk premia and plugging them inside the standard mean-variance formula to obtain portfolio weights. Much more complex calculations along the lines of (but not identical to) those performed in slides 5 and 6 of lecture 3 would be needed to derive hedging demands.

3d. (2 points) You know that the results presented in the plot of question 3a are based on the statistical outputs copied below. Comment on the *statistical* strength/accuracy of the predictability patterns that appear in the data. Would a decision to pursue an asset allocation system based on the predictive regressions reported below be supported by the statistical results you have obtained? Equivalently, what is the expected relationship between the strength of the statistical results and the potential economic value that may be extracted from such an asset allocation system? Carefully explain your answers in the light of our lectures [Note: one-sentence replies without a justification will receive NO partial credit; answers supported by a reasoning that is unrelated to the material covered in the course will receive NO partial credit.]

Predictive Regresssion Results of Growth Portfolio Returns

$$r_{t+1}^{GP} = \alpha_1 + \beta_1 pd_t + \varepsilon_{1,t}$$

Input Y (dependent) variable: Growth Portfolio returns, from 1985Q2 to 2012Q4.
Input X (dependent variable): log Price-Dividend, from 1985Q1 to 2012Q3.

Regression Statistics	
Multiple R	0.1578672
R Square	0.024922
Adjusted R Square	0.0159764
Standard Error	0.0954205
Observations	111

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.0253661	0.0253661	2.7859334	0.0979651
Residual	109	0.9924532	0.0091051		
Total	110	1.0178194			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.1991498	0.0997357	1.9967751	0.0483421	0.0014768	0.39682281	0.00147685	0.396822814
log Price-Dividend	-0.043423	0.0260155	-1.6691116	0.0979651	-0.0949846	0.0081391	-0.09498464	0.008139103

Predictive Regresssion Results of Value Portfolio Returns

$$r_{t+1}^{VP} = \alpha_2 + \beta_2 pd_t + \varepsilon_{2,t}$$

Input Y (dependent) variable: Value Portfolio returns, from 1985Q2 to 2012Q4.
Input X (dependent variable): log Price-Dividend, from 1985Q1 to 2012Q3.

Regression Statistics	
Multiple R	0.2096543
R Square	0.0439549
Adjusted R Square	0.0351839
Standard Error	0.0921234
Observations	111

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.0425301	0.0425301	5.0113633	0.0272131
Residual	109	0.9250528	0.0084867		
Total	110	0.9675828			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.2426053	0.0962895	2.5195398	0.0132008	0.0517626	0.43344797	0.051762577	0.433447971
log Price-Dividend	-0.056226	0.0251166	-2.2386074	0.0272131	-0.1060064	-0.00644592	-0.10600638	-0.00644592

Predictive Regresssion Results of US 3M BILL Returns

$$r_{t+1}^{3Mbill} = \alpha_3 + \beta_3 spr_t + \varepsilon_{3,t}$$

Input Y (dependent) variable: US 3M BILL returns, from 1985Q2 to 2012Q4.
Input X (dependent variable): Temr Spread, from 1985Q1 to 2012Q3.

Regression Statistics	
Multiple R	0.0785664
R Square	0.0061727
Adjusted R Square	-0.002945
Standard Error	0.0064699
Observations	111

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.834E-05	2.834E-05	0.6770008	0.4124178
Residual	109	0.0045627	4.186E-05		
Total	110	0.0045911			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95,0%*	*Upper 95,0%*
Intercept	0.0111798	0.0010189	10.972419	2.551E-19	0.0091604	0.01319921	0.009160358	0.013199206
Term Spread	-0.562127	0.6831877	-0.8228006	0.4124178	-1.9161831	0.79192856	-1.91618307	0.791928556

Answer. The statistical strength/accuracy of the predictability patterns that appear in the data according to the regression models presented below is modest. The R-square ranges between 0.6 and 4.4% at best. Of the three regressions, only one implies a beta coefficient that is statistically significant with p-value below 0.05 (one p-value falls between 0.05 and 0.10, though). Many indicators are on the whole rather weak. However, a decision to NOT pursue—given the modest evidence of a strong predictive association between predictors and subsequent excess asset returns—an asset allocation system based on the predictive regressions reported below could be tragically incorrect. As we have seen in lecture 3 with reference to the case of Giulia Dal Pra's thesis there is no compelling link between the strength of the statistical results and the potential economic value that may be extracted from such an asset allocation system. For instance, despite the beta coefficients tend to be small and imprecisely estimated, the time series of portfolio weights in question 3a shows that these tend to change considerably in response to the moves of the predictors that were selected in your homework.