A simple derivation of the Capital Asset Pricing Model from the Capital Market Line

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ABSTRACT

This paper demonstrates a simple way of deriving both the Capital Asset Pricing Model (CAPM) and a capital asset's beta value from the Capital Market Line (CML). The CML model is extended to include a series of *isocorrelation* curves along which the returns of any portfolio can be plotted according to its total risk and the degree to which its return correlates to that of the market. This approach is simpler than methods currently available in the relevant literature and may be useful for teaching purposes.

Keywords: *Capital Market Line, CML, Capital Asset Pricing Model, CAPM, Security Market Line, SML, isocorrelation curves, isobeta curves.*

JEL classification: *G11*.

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Introduction

The Capital Market Line (CML) and Capital Asset Pricing Model (CAPM) are standard features of finance and investments textbooks. Brailsford and Faff (1993, p. 54) claim that the link between the two is not "intuitively obvious" and that "students often fail to grasp this concept". They provide what they claim is a "simple derivation of the CAPM". This paper aims to provide a simpler derivation, which may be useful for teaching purposes.

The paper commences with a simple description of the CML, noting that it applies only to efficient portfolios; i.e. portfolios with returns that are perfectly correlated to that of the market and therefore plot along the CML according to their level of total risk. It demonstrates both a simple algebraic and schematic derivation of the CAPM from the CML. The latter approach involves extending the CML to apply to all portfolios through the inclusion of *isocorrelation* curves. The paper describes the nature of such curves and explains how their upper and lower bounds can be located within the CML framework.

The paper also shows how *isobeta* curves can be included in the CML. The beta value of a given portfolio can then be visually identified with reference to its *isocorrelation* curve and total risk. This approach provides an innovative schematic link between the CML and CAPM models hitherto unavailable in the literature. The paper concludes with a standard caveat about models that may lack decision usefulness in spite of their internal algebraic logic and mathematical precision.

The Capital Market Line

The CML describes a model of portfolio construction, as depicted in Figure 1. It assumes that there is a single market portfolio *M* and that investors are able to lend and borrow at a risk-free rate and therefore able to construct efficient portfolios at any point along the CML.[1] Risk-averse investors can reduce risk and return by splitting their investment between the market portfolio and the risk-free asset. Those with higher risk tolerance can increase risk and return by using funds borrowed at the risk-free rate to leverage their investment in the market portfolio. Whichever strategy is adopted, reward per unit of risk is both constant and the best available.

[1] In this paper "efficient portfolio" refers to a portfolio lying on the CML, as opposed to one lying on a Markowitz efficient frontier.

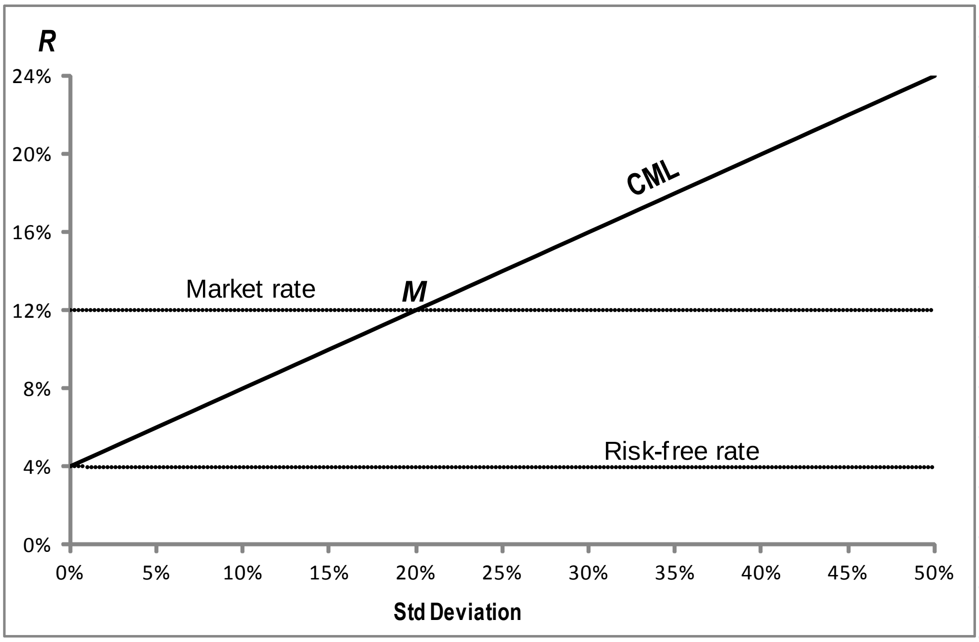

Figure 1. The Capital Market Line

Equation (1) defines the return on an efficient portfolio as a function of its total risk.

$$R_p = R_f + \frac{R_m - R_f}{\sigma_m} \sigma_p \qquad (1)$$

where: R_p is the return on an efficient portfolio

R_f is the risk-free rate

R_m is the return on the market portfolio

σ_m is the standard deviation of returns on the market portfolio

σ_p is the standard deviation of returns on efficient portfolio p.

In the current context Equation (1) is assumed to describe a closed system, within which the value of a single dependent variable R_p is precisely defined by values attributed to four independent variables. The model derives its validity from algebraic logic, as opposed to the verifiability or plausibility of attributed values, whether realised or expected. It explains the assumed risk-return characteristics of efficient portfolios while providing a platform from which to derive the CAPM.

Extending the CML to derive the CAPM

An algebraic derivation

Efficient portfolios plot along the CML and are perfectly correlated with the market portfolio.[2] Equation (2) extends Equation (1) to define the return on any portfolio as a function of its total risk.

$$R_p = R_f + \frac{(R_m - R_f)\rho_{pm}}{\sigma_m}\sigma_p \tag{2}$$

where: ρ_{pm} is the correlation coefficient between the return of portfolio p and that of the market.

We can now transform Equation (2) into the CAPM, as follows:

$$R_p = R_f + \left(\frac{\rho_{pm}\sigma_p}{\sigma_m}\right)(R_m - R_f) \tag{3}$$

$$= R_f + \left(\frac{\rho_{pm}\sigma_p\sigma_m}{\sigma_m^2}\right)(R_m - R_f) \tag{4}$$

$$= R_f + \left[\frac{Cov(R_p, R_m)}{\sigma_m^2}\right](R_m - R_f) \tag{5}$$

$$= R_f + \beta_p(R_m - R_f) \tag{6}$$

The transformation is completed by substituting capital asset j for portfolio p, so that that CML can now be restated as the Security Market Line (SML), as illustrated in Figure 2.

A schematic derivation

This demonstrated link between the CML and CAPM can be made more attractive for educational purposes by adding *isocorrelation* and *isobeta* curves to the CML model, as shown in Figure 3. Both sets of curves are straight lines; the former radiating out from the CML's point of origin, the latter running parallel to the horizontal axis. Positive *isocorrelation* curves are bounded by the CML (ρ_{pm} = 1) and the risk-free rate (ρ_{pm} = 0), with intermediate values intercepting any perpendicular between those limits at

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[2] Throughout this paper, correlation refers to the degree of correlation between the return of an asset or portfolio and that of the market.

proportional intervals. Negative *isocorrelation* curves plot below the risk-free rate and mirror those above.

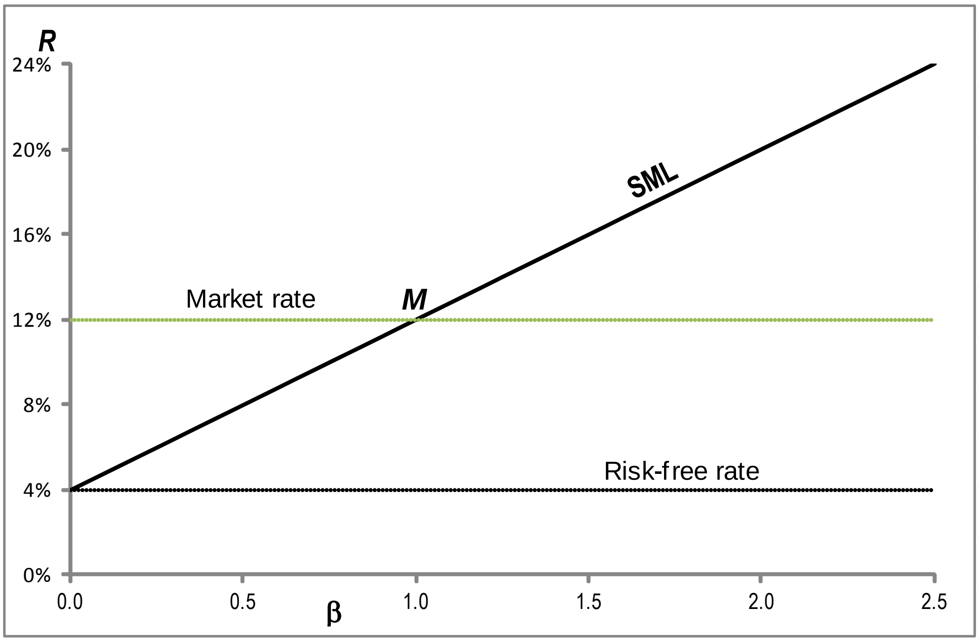

Figure 2. The Security Market Line

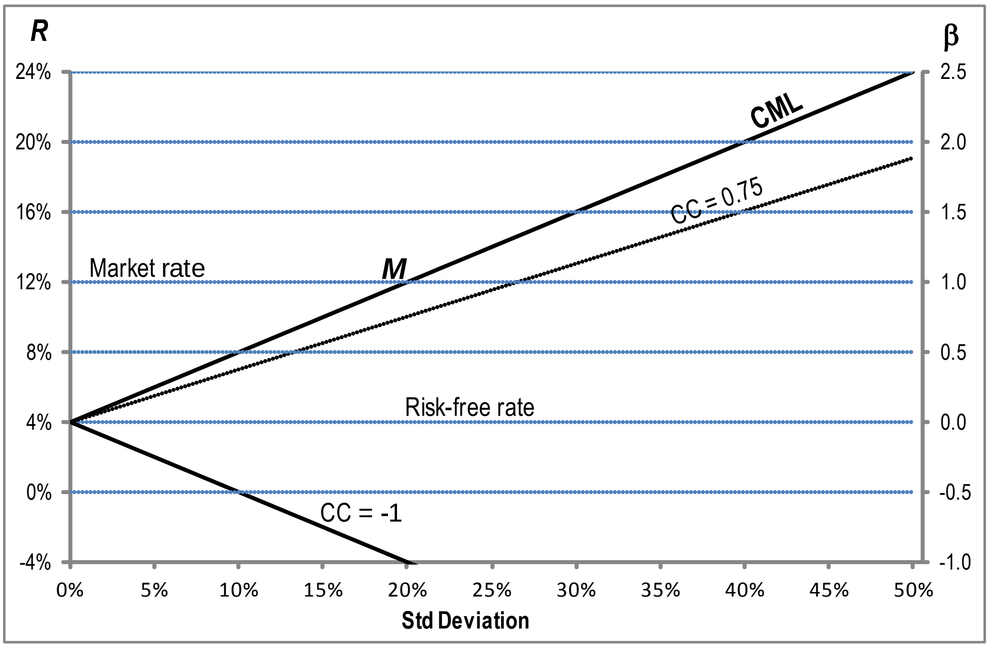

Figure 3. The Capital Market Line - showing *isobeta* and *isocorrelation* curves

Figure 3 shows the upper and lower bounds of *isocorrelation* curves, together with an exemplar curve with a correlation coefficient of 0.75.

Isobeta curves are unbounded and scaled with reference to the market risk premium: zero-beta and unity-beta curves coinciding respectively with the risk-free and market rates. All *isobeta* curves intercept the vertical axis at proportional intervals, as illustrated in Figure 3, which shows *isobeta* curves at intervals of 0.5 on the secondary vertical axis. Figure 3 also shows how *isocorrelation* and *isobeta* curves intercept at R_p to define a portfolio's beta value. For example, the R_p of portfolio p with $\rho_{pm} = 0.75$ will plot on the illustrated *isocorrelation* curve according to its level of total risk σ_p. A standard deviation of 40% defines a return of 16% and a coinciding beta value of 1.5. The SML in Figure 2 identifies the same return for a capital asset with a beta value of 1.5, thereby confirming the link between the two models. Similar equalities can be obtained by inserting the required *isocorrelation* curve into the CML model.

Conclusion

This paper demonstrates a simple way of deriving both the CAPM and a capital asset's beta value from the CML. The CML model is extended to include a series of *isocorrelation* curves along which the returns of any portfolio can be plotted according to its total risk and the degree to which its return correlates to that of the market. This approach is simpler than methods encountered in the relevant literature[3] and may therefore have instructional merit.

A caveat remains regarding the realism and consequent usefulness of the CML and CAPM. Both models fall within the second stage of Markowitz's portfolio selection paradigm (Markowitz, 1952, p. 77); i.e. they incorporate sets of given values or "relevant beliefs", without addressing their decision usefulness.

References

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[3] See, for example, Brailsford and Faff, 1993; Copeland and Weston, 1992, pp. 195-198; Yip, 2005, pp. 108-110.