13. a. If the one-year discount factor is .905, what is the one-year interest rate?

$$DF_1 = \frac{1}{1+r_1} = 0.905 \Rightarrow r_1 = 0.1050 = 10.50\%$$

b. If the two-year interest rate is 10.5 percent, what is the two-year discount factor?

$$DF_2 = \frac{1}{(1+r_2)^2} = \frac{1}{(1.105)^2} = 0.819$$

c. Given these one- and two-year discount factors, calculate the two-year annuity factor.

The one and two-year discount factors of (a) and (b) tell us that the interest rate for both periods = 10,5%. Thus to calculate the two-year annuity factor:

$$Annuity\_factor = \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right] = \left[\frac{1}{0,105} - \frac{1}{0,105(1+0,105)^2}\right] = 1,724$$

An alternative way to calculate the two-year annuity factor is:

$$AF_2 = DF_1 + DF_2 = 0,905+0,819=1,724$$

d. If the PV of \$10 a year for three years is \$24.65, what is the three-year annuity factor?

Using the formula of the Present Value of annuity:

$$PVannuity = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right], where:$$

$$Annuity\_factor = \left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

Hence the three-year annuity factor is:

$$\$24.65 = \$10 \times [AF_3]$$

$$AF_3 = 2.465$$

d. From your answers to (c), and (d), calculate the three-year discount factor.

$$AF_3 = DF_1 + DF_2 + DF_3$$

$$2.465 = 0.905 + 0.819 + DF_3$$

$$DF_3 = 0.741$$

15. A machine costs \$380,000 and is expected to produce the following cash flows:

Year	1	2	3	4	5	6	7	8	9	10
CF(000s)	50	57	75	80	85	92	92	80	68	50

If the cost of capital is 12 percent, what is the machine's NPV?

Period	0	1	2	3	4	5	6	7	8	9	10
Cash Flow	- 380	50	57	75	80	85	92	92	80	68	50
Discount rate	-	12%	12%	12%	12%	12%	12%	12%	12%	12%	12%
Discount factor	-	0,89	0,80	0,71	0,64	0,57	0,51	0,45	0,40	0,36	0,32
DCF	- 380,00	44,64	45,44	53,38	50,84	48,23	46,61	41,62	32,31	24,52	16,10
NPV	23,70										

***Remember not to discount the initial investment (made in period 0 = today).

The above table is equivalent of doing:

$$NPV = -C_o + \sum_{t=0}^{10} \frac{C_t}{(1.12)^t}$$

$$= -\$380,000 + \frac{\$50,000}{1.12} + \frac{\$57,000}{1.12^2} + \frac{\$75,000}{1.12^3} + \frac{\$80,000}{1.12^4} + \frac{\$85,000}{1.12^5}$$

$$+ \frac{\$92,000}{1.12^6} + \frac{\$92,000}{1.12^7} + \frac{\$80,000}{1.12^8} + \frac{\$68,000}{1.12^9} + \frac{\$50,000}{1.12^{10}} = \$23,696.15$$

2

16. Mike Polanski is 30 years of age and his salary next year will be $40,000. Mike forecasts that his salary will increase at a steady rate of 5% per annum until his retirement at age 60.

a. **If the discount rate is 8%, what is the present value of these future salary payments?**

Let S_t = salary in year t

$$PV = \sum_{t=1}^{30} \frac{S_t}{(1.08)^t} = \sum_{t=1}^{30} \frac{40,000\,(1.05)^{t-1}}{(1.08)^t}$$

$$= \sum_{t=1}^{30} \frac{(40,000/1.05)}{(1.08/1.05)^t} = \sum_{t=1}^{30} \frac{38,095.24}{(1.0286)^t}$$

$$= 38,095.24 \times \left[\frac{1}{0.0286} - \frac{1}{0.0286 \times (1.0286)^{30}}\right] = \$760,662.53$$

Alternatively we could have used the growing annuity formula:

$$PV = C\left(\frac{1}{r-g} - \frac{1}{r-g}\left(\frac{(1+g)^T}{(1+r)^T}\right)\right)$$

$$= 40,000\left(\frac{1}{.08-.05} - \frac{1}{.08-.05}\left(\frac{(1+.05)^T}{(1+.08)^T}\right)\right) = 760,662.53$$

b. **If Mike saves 5% of his salary each year and invests these savings at an interest rate of 8%, how much will he have saved by age 60?**

PV(salaries) x 0.05 = $38,033.13

Future value = $38,033.13x $(1.08)^{30}$ = $382,714.3

By saving 5% of his salary each year, by the age of 60 Mark will have $382,714.3.

c. **If Mike plans to spend these savings in even amounts over the subsequent 20 years, how much can he spend each year?**

By using the formula of the Present Value of Annuity: $PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t}\right]$

$$\$382,714.3 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}} \right]$$

$$C = \$382,714.3 \bigg/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}} \right] = \$38,980.30$$

19. (*Tutorial*) As winner of a breakfast cereal competition, you can choose one of the following prizes:

  a) $100,000 now.

PV $= 100,000$

  b) $180,000 at the end of five years.

$$PV = \frac{180,000}{(1.12)^5} = 102,137$$

  c) $11,400 a year forever

PV $= 11,400/0.12 = 95,000$

  d) $19,000 for each of 10 years

$$PV = 19,000 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^{10}} \right] \approx 107,354$$

  e) $6,500 next year and increasing thereafter by 5% a year forever.

PV $= 6,500/(0.12\text{-}0.05) = 92,857.1$

If the interest rate is 12%, which is the most valuable prize?

$19,000 for each of 10 years is the most valuable prize, since it has the highest PV.

22. Kangaroo Autos is offering free credit on a new $10,000 car. You pay $1,000 down and then $300 a month for the next 30 months. Turtle Motors next door does not offer free credit but will give you $1,000 off the list price. If the rate of interest is 10% a year, (about 0.83 a month) which company is offering a better deal?

Kangaroo Autos

Based on the annuity formula: $PV = C \times \left[\dfrac{1}{r} - \dfrac{1}{r \times (1+r)^t} \right]$ we calculate the present value

of the payments to Kangaroo Autos:

$$\$1,000 + \$300 \times \left[\dfrac{1}{0.0083} - \dfrac{1}{0.0083 \times (1.0083)^{30}} \right] = \$8,938$$

Turtle Motors

A car from Turtle Motors costs $9,000 cash ($10,000-$1,000).

Therefore, Kangaroo Autos offers the better deal, i.e., the present value of the payments of the Kangaroo Autos' deal is lower than the one of Turtle motors..

28. What would you prefer?
   a. An investment paying interest of 12% compounded annually.
   b. An investment paying interest of 11.7 percent compounded semiannually.
   c. An investment paying 11.5 percent compounded continuously.

Work out the value of each of these investments after 1, 5 and 20 years.

Hint: For the ease of your calculations use $1,000 as the amount of your investment.

Let A represent the investment at 12 percent, compounded annually.
Let B represent the investment at 11.7 percent, compounded semiannually.
Let C represent the investment at 11.5 percent, compounded continuously.

And remember that an annually compounded rate is computed as: $\left[1 + \left(\dfrac{r}{m} \right) \right]^m - 1$

After one year:

$$FV_A = \$1,000 \times (1 + 0.12)^1 \qquad = \$1,120.00$$

$$FV_B = \$1,000 \times (1 + 0.0585)^2 \qquad = \$1,120.42$$

$$FV_C = \$1,000 \times e^{(0.115 \times 1)} \qquad = \$1,121.87$$

Leaving aside the amount of the investment we can also compare the rates of return as follows:

A: $\left[1+\left(\dfrac{r}{m}\right)\right]^{m}-1=\left[1+\left(\dfrac{0,12}{1}\right)\right]^{1}-1=12,00\%$

B: $\left[1+\left(\dfrac{r}{m}\right)\right]^{m}-1=\left[1+\left(\dfrac{0,117}{2}\right)\right]^{2}-1=12,04\%$

C: $e^{r}-1=e^{0,115}-1=12,18\%$

After five years:

$$FV_A = \$1,000 \times (1 + 0.12)^5 \qquad = \$1,762.34$$
$$FV_B = \$1,000 \times (1 + 0.0585)^{10} \quad = \$1,765.67$$
$$FV_C = \$1,000 \times e^{(0.115 \times 5)} \qquad = \$1,777.13$$

After twenty years:

$$FV_A = \$1,000 \times (1 + 0.12)^{20} \qquad = \$9,646.29$$
$$FV_B = \$1,000 \times (1 + 0.0585)^{40} \quad = \$9,719.29$$
$$FV_C = \$1,000 \times e^{(0.115 \times 20)} \qquad = \$9,974.18$$

Through all the periods the preferred investment is C because it returns a higher future value of the investment.

CHAPTER 3: Challenge questions

38. You own a pipeline which will generate a $2 million cash return over the coming year. The pipeline's operating costs are negligible, and it is expected to last for a very long time. Unfortunately, the volume of oil shipped is declining, and cash flows are expected to decline by 4% per year. The discount rate is 10%.

    a) What is the present value (PV) of the pipeline's cash flows if its cash flows are assumed to last forever?

PV = 2/(0.1-(-0.04)) = 14.29

    b) What is the PV of the cash flows if the pipeline is scrapped after 20 years?

$$PV = 2\times\left[\dfrac{1}{0.14}-\dfrac{1}{0.14}\times\dfrac{(0.96)^{20}}{(1.1)^{20}}\right]\approx 13.35$$