Università Commerciale
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MSc. Finance/CLEFIN
2018/2019 Edition

Advanced Quantitative Methods for Asset Pricing and Structuring

Project 1 – Structured Products (Prof. Pedio)

This is a <u>sample</u>: the black part is identical for all the groups; the red portion is group specific

A successful completion of this project requires: (a) the delivery (<u>by 10th of May, 2019)</u> of a properly working code (in Matlab, R, VBA, or Python, as you prefer) that performs the tasks described below; (b) the discussion of the results in an individual presentation on the date of the exam (a sample PPT presentation to support such a discussion will be provided in due time). Part (a) can be attended to in a group (max 4 people) and will carry a maximum grade of <u>7 points</u>; part (b) will be individual and will carry a maximum grade of <u>3 points</u>. You can start (if you wish) from my draft code in Matlab, that is made available through the course web site.

Consider an economy where only two assets exist: a risk-free bond and a stock (that is representative of the entire equity market; you can think of it as an ETF on the S&P 500 Index). The model dynamics (slightly simplified with respect to those in Liu and Pan, 2003) under the risk neutral measure are as follows:

$$dS_t = rS_t dt + \sqrt{V_t} S_t \left(\rho dB_t^{(1)} + \sqrt{1-\rho^2} dB_t^{(2)}\right) \tag{1}$$

$$dV_t = k^Q(\bar{v}^Q - V_t)dt + \sigma_V \sqrt{V_t} B_t^{(1)}, \tag{2}$$

where $B_t^{(1)}$ and $B_t^{(2)}$ are independent Brownian motions that represent the volatility and the diffusive shocks, S_t is the stock price at time t, r is the risk free rate, V_t is the variance, $\bar{v}^Q$ is the long run level of the variance, k^Q is the mean-reversion speed, and σ_V is the volatility of volatility; finally, ρ is the instantaneous correlation between the diffusive and the volatility shocks.

Now you would like to consider adding to this economy a Bonus Cap certificate that has the stock as underlying and that has the following characteristics:

Certificate Type	Bonus Cap (cap = bonus)
Issue Price	XX Eur
Barrier Type	American, observed at the end of each day during the life of the product
Barrier Level	$70\% \times S_0$
Strike Level	$100\% \times S_0$

Bonus Level	110 Eur
Maturity	1 year

Use Monte Carlo simulations to compute the fair issue price of this product. In order to approximate the path of the stock price (in 1) use the following Euler discretization scheme (from Broadie and Kaya, 2006):

$$S_t = S_{t-1} + rS_{t-1}\Delta t + \sqrt{V_{t-1}}S_{t-1}\left(\rho\Delta B_t^{(1)} + \sqrt{1-\rho^2}\Delta B_t^{(2)}\right)$$

where $\Delta B_t^{(i)} = B_t^{(i)} - B_{t-1}^{(i)}$ with $i = 1,2$, and these increments are independent of each other and normally distributed with mean zero and variance $\sqrt{t}$. In order to approximate the path of the variance (in (2)), use the following Euler discretization scheme:

$$V_t = V_{t-1} + k^Q(\bar{v}^Q - V_{t-1})\Delta t + \sigma_V\sqrt{V_{t-1}}\Delta B_t^{(1)}$$

where $\Delta B_t^{(1)}$ is defined as above. Similar to Broadie and Kaya (2006) to avoid negative values for variance and stock price, if you encounter negative values during the simulation set these to zero (brute force). Consider the following parameters $S_0 = 100$, $k^Q = 0.5$, $\bar{v}^Q = 0.063$, $V_0 = 0.05$, $\sigma_V = 0.38$, $\rho = -0.321$, and $r = 0.05$. Δt is equal to 1 day. You have to:

- Plot the simulated paths for the stock price and for the variance.
- Print on the screen the no-arbitrage issuance price of the structured product.
- Compute the delta at issuance of this product and display it on the screen.

Now consider the optimization problem of an investor with a time horizon equal to the maturity of the structured product that maximizes a power utility function over terminal wealth,

$$\max_{\omega_{rf}\,\omega_s,\omega_b} \mathrm{E}\left[\frac{W_T^{1-\gamma}}{1-\gamma}\right],$$

where ω_{rf}, ω_s, and ω_b are the weights assigned to the risk free bond, to the stock, and to the structured product, respectively. Using simulations determine the optimal weights. In order to do that, you need to simulate the stock price under the physical probability measure, i.e., use

$$S_t = S_{t-1} + \mu S_{t-1}\Delta t + \sqrt{V_{t-1}}S_{t-1}\left(\rho\Delta B_t^{(1)} + \sqrt{1-\rho^2}\Delta B_t^{(2)}\right)$$

$$V_t = V_{t-1} + k^P(\bar{v}^P - V_{t-1})\Delta t + \sigma_V\sqrt{V_{t-1}}\Delta B_t^{(1)}$$

where $\mu = 0.12$, k^P=1.45, and $\bar{v}^P$= 0.05; all the remaining parameters are set at the same values as before. You need to constraint the weights to sum to one, but also put an upper bound equal to 2 to the value that the weights may take. This upper bound is arbitrary but it avoids incurring into a nasty feature of the power utility function that often prevents convergence.

- Compute (and print on the screen) the optimal weights ω_{rf}, ω_s, and ω_b (i.e., the optimal weights of the risk-free asset, the stock, and the structured product, respectively), for $\gamma = 0.5, 2,$ and 5.
- For the case of γ=2 assess how the weights will change (and plot them) if the risk free rate was 0.01 and $\mu = 0.15$.
- For the case of γ = 0.5, assess how the weights will change if you try different barrier levels for the certificate. In particular, plot the weight assigned to the structured product for values of the barrier equal to 60%, 65%, 70%, 75%, and 80% of the strike. Compare those weights to the optimal weights that the investor will choose in the absence of the structured product.