Investing in Bargaining Power

Sarah Auster Nenad Kos Salvatore Piccolo *

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Abstract

We study a model of surplus division with costly investments in a bilateral trade environment. Investments determine the probability with which agents get to make an offer and thereby the surplus they are able to secure, however, it may also signal their private information, something that the rival can use to his advantage. We characterize the equilibria of the game and show that the seller's payoff can be non-monotonic in the share of high value buyers. The seller faced with an entry decision might, therefore, find it propitious to enter a market mostly populated by low value customers. *JEL Code*: C72, D82, D83

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*Auster: Bocconi University, Department of Decision Sciences and IGIER. E-mail: sarah.auster@unibocconi.it. Kos: Bocconi University, Department of Economics, CEPR and IGIER. E-mail: nenad.kos@unibocconi.it, Piccolo: University of Bergamo, E-mail: salvapiccolo@gmail.com.
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1 Introduction

Situations in which agents exert effort or make investments to improve their bargaining position are ubiquitous: potential buyers search the internet for information on how to bargain with car salesmen, companies and individuals hire lawyers or other experts when selling or buying valuable assets, parties in a troubled marriage find divorce lawyers to negotiate on their behalf, just to name a few.

The amount of resources people invest in improving their bargaining position depends not only on their individual characteristics, such as their innate ability to negotiate and the value attached to the deal they bargain over, but also on the rival's expected strength, and the possible information that these actions may signal. Highlighting the forces that shape these investments and the underlying trade-offs is an important step towards a better understanding of how people bargain and why they choose to do so in certain environments (markets).

We address these issues by studying a model in which parties can exert costly effort in order to improve their bargaining position—e.g., the amount of resources spent to find and hire the right lawyer (or other representatives) negotiating on their behalf, the amount of attention and time they devote to specialized courses on 'the art of negotiation' (see Cialdini and Garde (1987), Fisher et al. (2011)), etc. We consider a stylized bilateral trading problem between a seller (she) and a buyer (he) who is privately informed about his valuation. The seller's value is commonly known and normalised to zero, the buyer's value is either low or high, but always above the seller's. The agents choose between exerting or not exerting effort. Effort comes at a cost and determines the probability with which either party gets to make an offer through a success function. Naturally, exerting effort increases the agent's chance of proposing a price. The opponent accepts or rejects the proposal and the game ends.

We do not claim that this is precisely how bargaining unfolds. Rather, our environment should be interpreted as a reduced form model capturing two salient features. First, if agents invest effort, they are able to obtain a higher share of surplus. This is captured through an increase in the probability that they win the opportunity to make an offer. Second, agents learn about their opponents during the bargaining process. Specifically, if a certain type of buyer exerts less effort compared to others, then the seller is more likely to secure the chance to make an offer against that type. The revelation that the seller is the one who gets to make an offer can, therefore, be a signal of the buyer's willingness to pay. Our objective is to analyze how these two elements interplay, and what novel insights their interaction delivers.

We first consider a simplified version of the model in which the seller's effort is fixed; the seller's only choice is which price to offer. Whenever the buyer gets to make an offer, he optimally offers price zero. On the other hand, if the seller is to make an offer, she chooses between a low price that both types of the buyer accept and a high price that is accepted only by the high value buyer. Whether the low or the high price is optimal depends on the seller's posterior belief—the probability that she attaches to the buyer being the high type—, which is determined by the equilibrium effort decision of the buyer. The analysis proceeds by observing that the high valuation buyer always has at least as high an incentive to exert effort as the low valuation buyer—a monotonicity results. The difference in incentives is strict only if the buyer expects the seller to offer the high price with strictly positive probability. When the high type exerts effort more often than the low type, the seller is more likely to secure the opportunity to make an offer against the low type, thus, the right to make an offer depresses the seller's belief. As a consequence, conditionally on making an offer, the seller's posterior is never above her prior.[1]

Of interest is the domain where the buyer's cost of effort is not too extreme. The low type buyer optimally refrains from effort, while the high type's effort choice depends on the seller's behavior. The analysis is split according to the seller's prior. When the seller's prior is low he offers the low price, neither type exerts effort, and consequently, the seller learns nothing. The most interesting case arises for intermediate values of the prior. In this region, if the seller were to naively follow the prior she would offer the high price. This would incentivize the high type buyer to exert effort and the seller's posterior would fall. Subsequently the seller would prefer to offer the low price. The seller offering the low price, on the other hand, would make the high type buyer best respond by refraining from effort. The seller's posterior then would coincide with the prior, and the seller would in fact best respond with the high price. To escape such cycling of beliefs, the two players need to randomize: the high type buyer randomizes over the effort choices while the seller randomizes over the two prices. Finally, for high priors, the seller offers the high price and the high type buyer exerts effort. The seller learns and revises her belief downward, but the prior is so high that even after the belief revision the high price is preferable.

We then explore how the seller's payoff varies with the proportion of the high types. Our main finding shows that the seller's payoff is non-monotonic: it is constant in the fraction of high types for low priors, decreasing for intermediate priors and increasing for high priors. The striking feature that a higher proportion of high value buyers can be detrimental to the seller's payoff arises as a consequence of the strategic interaction. In the intermediate region of priors, where buyer and seller use a mixed strategy, the

[1] On the other hand, when the buyer gets to make an offer, the seller's posterior is weakly greater than her prior. Due to the private value assumption, however, the seller's posterior belief in this case is irrelevant.

probability with which the high type buyer exerts effort increases in the seller's prior: as the prior increases, the high type buyer has to separate himself further from the low type in order to keep the seller's posterior constant. The seller, therefore, gets to make an offer less often. At the same time, the seller is indifferent between the two prices, so her payoff conditionally on making an offer does not change with the prior. As a result, her total payoff is declining. The result has interesting consequences for entry games. If the seller could choose to enter one of two markets, she might indeed enter a market with a larger proportion of the low value buyers. Although the buyers with low valuations offer less surplus to be shared, they also put up less of a fight when splitting the said surplus.

In the richer version of the model the seller can also choose to exert effort. The seller's effort has two effects. First, the effort influences the probability that the seller gets to make an offer. Second, it affects the seller's learning. We show that the seller learns less when she exerts effort, or with other words, when exerting effort her posterior declines to a smaller extent. Several results are qualitatively similar to the environment where the seller does not exert effort, however a novel behavior arises in a region of parameters: the seller randomizes over pairs of efforts and prices—no effort coupled with the low price and effort coupled with the high price. Such mixing is a consequence of the fact that the seller's posterior is higher when she exerts effort than when she does not. Intuitively, when the seller does not exert effort, having the right to make an offer suggests to a higher degree that the buyer has not exerted effort himself and is therefore more likely to have a low valuation.

We also explore the welfare effects of bargaining and, in particular, discuss the role of the buyer's and seller's effort costs. In equilibrium each party's probability of exerting effort decreases in their own effort cost, as one might expect. Effort cost then affects welfare through three channels. First, it directly affects the size of the loss incurred in equilibrium through the cost of effort. Second, it changes the relative probability of the informed and uninformed party making an offer. Whenever the buyer makes an offer, trade happens with probability one, whereas if the seller makes an offer, trade may fail. Third, effort has an effect on how much the seller learns in equilibrium. After exerting effort the seller is more likely to be inclined to offer the high price which can result in trade breaking down.

Our results have bearing on the so called probabilistic bargaining models. In such models the buyer and the seller have exogenously fixed probabilities of making an offer. Moreover, these probabilities tend to be interpreted as bargaining powers; see for example Merlo and Wilson (1995), Okada (1996), Zingales (1995), Inderst (2001), Krasteva and Yildirim (2012) and Münster and Reisinger (2015). Our findings imply that in some environments it would be more realistic to assume that bargaining power is increasing in the buyer's valuation.[2] We show that taking the probability of making an offer exogenously increasing in the buyer's type provides the researcher with a simpler model than ours, yet captures some of its arresting features.

Last but not least, we show that non-monotonicity of the seller's payoff does not depend on the discreetness of the buyer's effort choice. We explore the model where the buyer can choose any non-negative effort and show that the main results extend to this environment.

Related Literature: The foundation for models of non-cooperative surplus division under complete information was set by Stahl (1972) and Rubinstein (1982). Stahl (1972) proposed a finite time model in which two agents alternate in proposing a division of a commonly known surplus until an offer is accepted, Rubinstein (1982) solves for the infinite horizon version of the game. Our model is closer in spirit to incomplete information bargaining models as in Fudenberg and Tirole (1983) and Gul, Sonnenschein, and Wilson (1986). It shares with the dynamic models the feature that the seller's belief declines after some information is revealed, however, the structure of the equilibria and the shape of equilibrium payoffs—the non-monotonicity of the seller's payoff—are specific to our work.[3]

Most related to our paper are models by Yildirim (2007), Yildirim (2010), Board and Zwiebel (2012) and Ali (2015). Yildirim (2007) and Yildirim (2010) study a sequential bargaining model in which agents exert efforts to be the proposer of a split of a pie of a fixed and commonly known size. After the proposer is chosen and the proposal is made, the agents vote on whether to accept it. The focus is on how different voting rules affect the bargaining outcome. Board and Zwiebel (2012) study a similar model with two agents who repeatedly compete to make an offer to each other for a fixed pie until one of the offers is accepted. The competition is in the form of a first price auction, and the agents are endowed with a limited bidding capital. They characterize how the bargaining outcomes depend on the size and distribution of the bidding capital. Ali (2015) studies a multilaterial bargaining model in which the right to make an offer is sold to the highest bidder. In his model the first proposer captures the entire surplus, unless some player has a veto power. Unlike our model, these papers analyse environments where the surplus is of commonly known size.[4]

[2]Grennan (2014) provides evidence that heterogeneity of bargaining power across agents plays an important role in the market for medical devices.

[3]In the two period model of Fudenberg and Tirole (1983) the seller makes all the offers. The seller never randomizes on equilibrium path, while the high type buyer randomizes over his decisions only in the highest region of priors.

[4]A somewhat different, but related, literature–see Crawford (1979), Evans (1997) and Pérez-Castrillo

Lauermann and Wolinsky (2017) study a first price auction in which the seller can solicit buyers. While the model and the focus of that paper are quite different from ours, costly solicitation bears resemblance to our effort. Heinsalu (2017) studies a market with adverse selection, where a seller, privately informed about the quality of his good, invests into access to a market. The key differences to our framework are the assumptions that values are interdependent and that the uninformed party has no bargaining power.

2 The Baseline Model

A seller and a buyer want to trade an object. The seller's valuation is commonly known and normalised to zero, while the buyer's valuation v is either high or low, denoted by v_H and v_L respectively. We assume $v_H > v_L > 0$ and denote the probability that the buyer's valuation is v_H by μ. After learning his type, the buyer decides whether to exert effort; his effort choice is denoted by $e_b \in \{0, 1\}$. Effort affects the buyer's probability of making an offer to the seller. Let $k_b e_b$ denote the cost of effort and let ρ_1 (resp. ρ_0) denote the buyer's probability of making an offer to the seller when he does (resp. does not) exert effort. We assume $0 < \rho_0 < \rho_1 < 1$. The seller gets to make an offer with the remaining probability. After the offer is accepted/rejected the game ends.

In the baseline model we keep the seller's effort fixed. The seller potentially learns about the buyer's type through the possibility of making an offer and moreover the extent to which the seller learns may depend on her choice of effort. Shutting down the seller's effort enables us to distill the direct effect of learning on the equilibrium outcome. Later we relax this assumption in order to see how the effects of the seller's effort interplay with learning.

A pure strategy for the buyer is a tuple (e_b, p_b) for each type, where $p_b \in \mathbb{R}$ denotes the price the buyer proposes when called upon. The seller only chooses a price offer $p_s \in \mathbb{R}$.

Remark 1. An example to keep in mind is that of a Tullock contest in which agents' chances to make an offer are commensurate to their effort investments; Tullock (1980). More precisely, suppose that the buyer can choose between two actions $\{e^L, e^H\}$, where $e^H > e^L > 0$, and the seller's action is fixed to e^s. The probability that agent i gets to make an offer is $\frac{e_i}{e_i+e_j}$, where e_j is the opponents effort. The buyer incurs cost $\tilde{k}_b(e_b - e^L)$ from exerting effort e_b, while the seller incurs no cost. Our model corresponds to setting $\rho_1 \triangleq \frac{e^H}{e^H+e^s}$ and $\rho_0 \triangleq \frac{e^L}{e^L+e^s}$.

We call a profile of strategies and a belief function an equilibrium if the strategies are

and Wettstein (2001)–studies the effects of competition on coalitional bargaining games.

sequentially rational given the beliefs and the beliefs are updated using Bayes rule whenever possible. In addition, we assume that each agent after their own deviation presumes that the other agent has followed his equilibrium strategy. Two types of off-equilibrium paths are possible. First, an agent could be the one making an offer after an own deviation in effort. In such a case, he computes the posterior using his deviation strategy and equilibrium prescribed strategies for the opponent. Second, an out of equilibrium price could be announced. The beliefs at such a node, however, are irrelevant due to the private values structure of the model.

We maintain two interpretations for the effort decision in this environment. First, agents often exert effort during or prior to the bargaining. They read a book that (presumably) increases their bargaining skills, e.g. Cialdini and Garde (1987), Trump and Schwartz (2009), attend a course on bargaining, or search the internet for advice.[5] Second, agents find specialists to negotiate on their behalf, especially when an asset has significant value. These specialists can take the form of lawyers and/or negotiation consultants. Marital dissolution, for example, is a multibillion dollar business.[6]

The outlined model, as always, is a simplified representation of reality. We do not claim that bargaining proceeds precisely as specified above. However, the model does capture two important features that we would like to explore: 1.) A larger investment in bargaining enables the agent to secure a higher share of the surplus. This is captured by the probability with which an agent gets to make an offer. In fact, the existing literature commonly uses the probability that an agent makes a take-it-or-leave-it offer as a proxy for bargaining power (see for example Zingales (1995) and Inderst (2001)). 2.) By observing the opponent's bargaining stance, an agent may learn something about the opponent's type. The seller faced with an opportunity to make an offer can infer something about the buyer's valuation due to different types of buyer making (possibly) different effort decisions.

2.1 Optimal Bargaining Effort

It is useful to start with some preliminary observations. If the buyer gets to make an offer, he optimally offers price $p_b = 0$. The buyer's strategy can therefore be reduced to the probability with which he exerts effort. Let this probability be denoted by β_i when the buyer's type is $i = L, H$. On the other hand, if the seller gets to make an offer she either offers the pooling price v_L or the separating price v_H. The seller's strategies can

[5]Data shows that people spend on average more than eight hours researching how to bargain before they buy a car in the US. See http://agameautotrader.com/agame/pdf/2016-car-buyer-journey.pdf

[6] See http://www.huffingtonpost.com/susan-pease-gadoua/divorce-is-big-business-a_b_792271.html

thus be reduced to the probability with which she offers price v_H; denoted by $\sigma \in [0, 1]$.

The buyer type v_i's expected payoff from exerting effort $e_b \in \{0, 1\}$ when the seller uses the strategy σ is

$$\rho_{e_b} v_i + (1 - \rho_{e_b}) \mathrm{E}_\sigma[1_{[p_s \leq v_i]}(v_i - p_s)] - k_b e_b,$$

where E_σ is the expectation operator with respect to the seller's strategy. After making an effort choice e_b the buyer gets to make an offer with probability ρ_{e_b} and obtains a payoff equal to v_i. With the complementary probability the seller makes an offer, which the buyer accepts as long as his valuation is not below the price.

Of particular interest will be the buyer's (expected) benefit from exerting effort (rather than not exerting it), which can be written as

$$\Delta u_b(v_i) = \underbrace{\Delta\rho}_{\triangleq \rho_1 - \rho_0} \big(v_i - \mathrm{E}_\sigma[1_{[p_s \leq v_i]}(v_i - p)]\big) - k_b. \tag{1}$$

The term $\Delta\rho$ measures the increase in the probability of the buyer making an offer due to exerting effort, while the second factor is the increase in the buyer's payoff when he makes the offer instead of the seller. If the buyer proposes the price, his payoff is v_i, whereas if the seller makes an offer, the buyer's payoff is $v_i - p$ if he accepts and 0 otherwise.

An important consequence of the above derived benefits is that the high type buyer's incentives to exert effort are at least as high as those of the low type.

Lemma 1. *For any strategy σ, we have $\Delta u_b(v_H) \geq \Delta u_b(v_L)$, where the inequality holds with equality if and only if $\mathrm{E}_\sigma[p_s] = v_L$.*

The above result follows after a closer inspection of the buyer's benefits from exerting effort. For the low type buyer expression (1) simplifies to

$$\Delta u_b(v_L) = \Delta\rho\, v_L - k_b. \tag{2}$$

The increase in the buyer's payoff when making an offer is v_L; if the seller gets to make an offer, the low type buyer's payoff is zero as he either rejects the seller's offer or pays a price equal to his valuation. The low type buyer, therefore, benefits from expending effort if and only if $k_b \leq \Delta\rho\, v_L$. As a result, the low type buyer's optimal strategy is independent of the price the seller charges. If his cost k_b is sufficiently small he exerts effort, otherwise he does not.

The high type buyer's benefits from switching to exerting effort is

$$\Delta u_b(v_H) = \Delta\rho\, \mathrm{E}_\sigma[p_s] - k_b. \tag{3}$$

The high type always accepts the seller's offer, so his gain from making the offer himself is the price he does not have to pay. Given that the expected price the seller charges, $\mathrm{E}_\sigma[p_s]$, is weakly greater than the valuation of the low type buyer, Lemma 1 follows. Moreover, when $\mathrm{E}_\sigma[p_s]$ is equal to v_L, that is, when the seller offers the low price with certainty, the incentives to exert effort are the same for both types of buyer.

As to the high type, since $\mathrm{E}_\sigma[p_s]$ belongs to the interval $[v_L, v_H]$, we have:

$$\Delta\rho\, v_L - k_b \leq \Delta u_b(v_H) \leq \Delta\rho\, v_H - k_b.$$

The high type buyer thus has two dominance regions: he always exerts effort when the cost of effort is sufficiently small; in particular $k_b < \Delta\rho\, v_L$.[7] In that case, the buyer finds it worth to expend effort even if he expects the seller to charge price v_L. On the other hand, when the cost of effort very high, $\Delta\rho v_H < k_b$, the dominant action for the high type buyer is not to exert effort. In the intermediate range of costs, the high type's behavior depends on the seller's expected price; see Figure 1. The following lemma summarizes the above analysis.

Lemma 2. *Exerting effort is conditionally dominant for both types of buyer if $k_b < \Delta\rho\, v_L$, while not exerting effort is conditionally dominant for the low type buyer if $k_b > \Delta\rho\, v_L$ and for the high type buyer if $k_b > \Delta\rho\, v_H$.*

The fact that the low type never exerts effort with a higher probability than the high type has consequences for the seller's behavior. If the seller believes that type v_i exerts effort with probability β_i ($i = L, H$) and the seller gets to make the offer herself, her posterior belief that the buyer is of high type is

$$\hat{\mu} \triangleq \frac{\mu\,[\beta_H(1-\rho_1) + (1-\beta_H)(1-\rho_0)]}{\mu\,[\beta_H(1-\rho_1) + (1-\beta_H)(1-\rho_0)] + (1-\mu)\,[\beta_L(1-\rho_1) + (1-\beta_L)(1-\rho_0)]},$$

where $\beta_H(1-\rho_1) + (1-\beta_H)(1-\rho_0)$ is the probability that the seller wins against the high type, and $\beta_L(1-\rho_1) + (1-\beta_L)(1-\rho_0)$ is the probability that she wins against the low type. Since the high type buyer is more likely to exert effort, that is $\beta_H \geq \beta_L$, he is also the one who is more likely to make an offer. In other words, the seller has a higher chance to make an offer when she is facing the low type. Her posterior $\hat{\mu}$ is, therefore, at most as high as her prior.

[7]More precisely, the buyer's strategies are conditionally dominated after one removes strategies that are dominated for the seller.

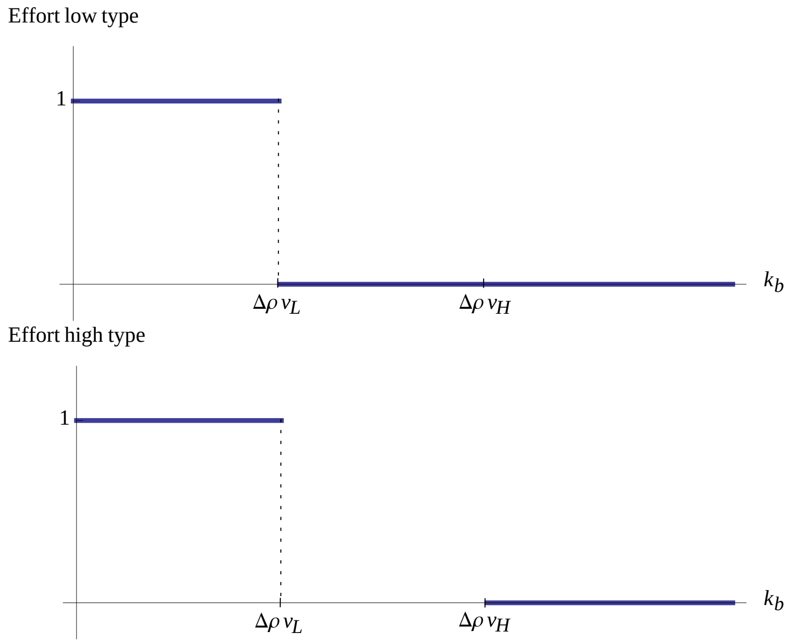

Figure 1: Conditional dominance regions for low and high type buyer

Lemma 3. *In equilibrium $\hat{\mu} \leq \mu$.*

While being able to make an offer is good news for the seller, she understands that she is more likely to win against the low type buyer. Winning, therefore, depresses her belief.[8]

The above result echoes Coasian dynamics where the prices fall over time – a result closely related to the fact that the seller's belief is declining; see Fudenberg and Tirole (1991). The reason for the declining beliefs, of course, is different. In the Coasian dynamics high types are more eager to accept a price, therefore the likelihood of encountering the high type in the market is falling over time. In our model, the seller is less likely to be able to make an offer against the high type.

Equilibria. In what follows we characterize equilibria. We divide the analysis into cases on the basis of the prior. Before proceeding we should point out that the threshold v_L/v_H plays a prominent role in the analysis. The seller optimally offers the low price if her posterior is low enough, $\hat{\mu} \leq \frac{v_L}{v_H}$, and the high price otherwise.

We start with the case where the seller's prior is low: $\mu \leq v_L/v_H$.

[8] The reader should note that the above lemma does not contradict the fact that beliefs are a martingale. Indeed, when the buyer is making an offer, the seller revises her beliefs upwards. The seller's beliefs when the buyer is making an offer are, however, of no interest to us due to the private-value nature of our environment.

Proposition 1. *If $\mu \leq \frac{v_L}{v_H}$, there is a generically unique equilibrium with the property that both types of buyer undertake the same effort choice and the seller offers price v_L.*

The seller's decision to offer price v_L is a consequence of Lemma 3: if her prior is below the threshold v_L/v_H, her posterior will be below the threshold too. Moreover, once the seller is charging the low price, Lemma 1 implies that both types of buyer have the same incentive to exert effort and, therefore, make the same effort choice. The seller, in turn, learns nothing from gaining the opportunity to make an offer. Her posterior thus coincides with her prior.[9]

Turning to high values of the seller's prior, $\mu > v_L/v_H$, Lemma 2 established that both types of buyer make the same effort choice when the cost of effort is extreme—either very high or very low. In those cases, the two types have the same dominant strategy and the seller cannot infer anything from the bargaining process. She, therefore, offers the high price.

Proposition 2. *Let $k_b \leq \Delta\rho\, v_L$ or $k_b \geq \Delta\rho\, v_H$ and $\mu > v_L/v_H$. In the (generically) unique equilibrium both types of the buyer make the same effort choice and the seller offers price v_H.*

The seller learns. Hereafter we focus on the environment in which the high type buyer's optimal effort choice depends on the seller's behavior, that is, when the buyer's cost is intermediate:

$$\Delta\rho\, v_L < k_b < \Delta\rho\, v_H.$$

As a reminder, in this region the low type buyer prefers to exert low effort, while the high type buyer's optimal effort decision depends on the price he expects the seller to charge. The potential difference in the strategies of the two types of buyer gives rise to a possibility for the seller to learn about her competitor. To this end, we will maintain the assumption $\mu > \frac{v_L}{v_H}$, so that if the seller were to learn nothing, she would optimally propose the separating price v_H.

Before stating the result, it is useful to define additional notation. Let

$$m \triangleq \frac{(1-\rho_0)\frac{v_L}{v_H}}{(1-\rho_0)\frac{v_L}{v_H} + (1-\rho_1)(1-\frac{v_L}{v_H})}, \tag{4}$$

be the prior belief such that if the high type buyer exerts effort and the low type does not, the seller's posterior is precisely v_L/v_H. As a consequence, for all priors above m, the

[9] Genericity in the above proposition refers to the case where both types of buyer are indifferent between exerting effort and not exerting effort when the seller is expected to offer the low price.

seller's posterior will be above the threshold v_L/v_H, regardless of the buyer's behavior, and the seller optimally offers price v_H.

Proposition 3. *Let $k_b \in (\Delta\rho\, v_L, \Delta\rho\, v_H)$ and $\mu > \frac{v_L}{v_H}$. There is a unique equilibrium:*

- *when $\mu < m$, the high type buyer exerts effort with probability $\beta_H = \frac{(1-\rho_0)(\mu v_H - v_L)}{\Delta\rho\,\mu(v_H - v_L)}$ and the low type exerts no effort; the seller offers price v_H with probability $\sigma = \frac{k_b - \Delta\rho\, v_L}{\Delta\rho(v_H - v_L)}$;*
- *when $\mu \geq m$, the high type buyer exerts effort and the low type does not; the seller offers price v_H.*

The equilibrium exhibits the most interesting behavior when the prior is only slightly above the threshold $\frac{v_L}{v_H}$; more precisely, when the prior is between $\frac{v_L}{v_H}$ and m. If the seller was to learn nothing during bargaining, she would offer the high price v_H. In anticipation of the seller's behavior, the high type buyer would find it optimal to exert effort, while the low type buyer would not. The seller would then revise her beliefs downward and optimally offer the low price v_L. Foreseeing the seller's intention to offer the low price neither type of the buyer would have an incentive to exert effort. But then the seller would not learn and be tempted to offer the high price.

As a result, no pure strategy equilibrium exists. Instead, the high type buyer randomizes between exerting and not exerting effort in such a way that the seller is indifferent between the two prices. The seller, on the other hand, offers the high price with the probability that makes the high type buyer indifferent between the two efforts.

2.2 Comparative Statics

Of particular interest is how the seller's payoff varies with her prior μ.

Corollary 1. *The seller's equilibrium expected payoff is*

$$u_s = \begin{cases} (1-\rho_0)v_L & \text{if } \mu < \frac{v_L}{v_H} \\ (1-\mu)(1-\rho_0)\frac{v_L v_H}{v_H - v_L} & \text{if } \frac{v_L}{v_H} \leq \mu \leq m \\ \mu(1-\rho_1)v_H & \text{if } m < \mu. \end{cases}$$

The striking property of the seller's payoff is that it is non-monotone in the probability of the high type, in particular, that it is decreasing on the interval $[v_L/v_H, m]$. A closer inspection of the structure of the equilibrium reveals what lies behind the peculiar behavior of the seller's payoff. For low priors, $\mu < v_L/v_H$, the seller invariably offers the pooling price v_L and neither type of buyer exerts effort. Her payoff in this region of priors is, therefore, independent of the prior.

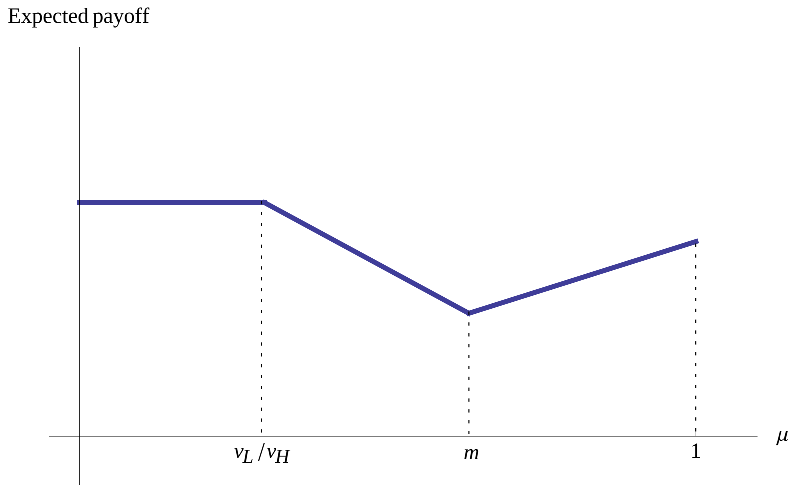

Figure 2: Seller's equilibrium expected payoff

To see why the seller's payoff is decreasing when her prior is in the intermediate region, recall that in the relevant region of priors the seller is indifferent between the two price offers. Her payoff from offering the low price is

$$u_s = [\mu (\beta_H(1-\rho_1) + (1-\beta_H)(1-\rho_0)) + (1-\mu)(1-\rho_0)]\, v_L.$$

The term in the brackets is the probability that the seller gets to make an offer, while v_L is the seller's payoff conditionally on making an offer. Notably, the seller's payoff conditionally on making the offer is independent of her posterior or the prior. Since she is randomizing over the two prices and the price v_L is accepted with certainty, her conditional payoff is v_L. As to the probability of the seller making an offer, when the prior rises, the high type buyer must exert effort with higher probability to distinguish himself from the low type, and thereby bring the seller's posterior down to the threshold belief. With other words, β_H is increasing in μ. The probability that the seller gets to make an offer is, therefore, falling for two reasons: 1) as μ increases, the high type is exerting effort with higher probability, and 2) the seller faces the high type more often. The seller's probability of making an offer, consequently, decreases in μ, while her payoff conditionally on making the offer is constant. Together this implies that an increase in the seller's prior leads to a decrease in her expected payoff.

In the third region, where $\mu > m$, the seller optimally offers the separating price v_H, which is accepted only by the high type buyer. Here the high type buyer exerts effort, implying that the seller's probability of making an offer is $1-\rho_1$. In contrast

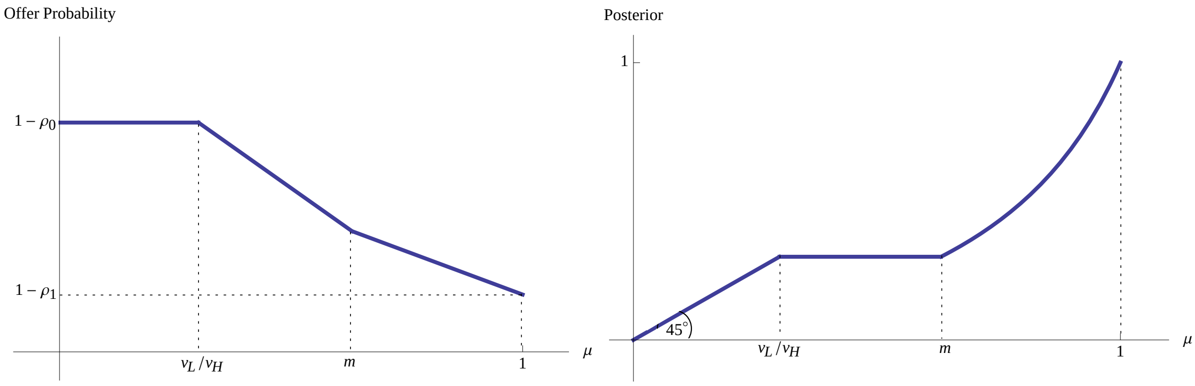

Figure 3: Seller's probability of making an offer and posterior belief in equilibrium

to the first two parameter regions, the seller's payoff in the third parameter region is strictly increasing in the fraction of high type buyers. This follows from the fact that the probability of the seller making an offer against the high type is constant in μ, while her chances of having price v_H accepted are strictly increasing.

The seller's expected payoff is therefore non-monotonic in the prior: it is constant for the low priors, decreasing for the intermediate priors and finally increasing for the high priors.[10] Whether the seller's expected payoff increases above the value it attains for low priors depends on the parameters of the problem. The seller is better off facing the low type buyer with certainty ($\mu = 0$) than the high type buyer with certainty ($\mu = 1$) if $(1 - \rho_0)v_L \geq (1 - \rho_1)v_H$. A high type buyer promises higher gains from trade but the probability that the seller can appropriate these gains are smaller than the respective probability when facing a low type buyer. A seller therefore obtains a higher expected payoff when faced with a low type buyer if the difference in the buyer's valuation is small relative to the difference in probabilities with which the seller gets to make the offer.

Finally, notice that the case for the seller's payoff being decreasing could have been made even in the environment with perfect information. This is the case when the seller's payoff at $\mu = 1$ is smaller than at $\mu = 0$. However, our result is stronger than that: even when the seller's payoff at $\mu = 1$ is larger than at $\mu = 0$, there is always an intermediate region of priors under which the seller is strictly worse off than when she faces the low value buyer with certainty.

The non-monotonicity of the seller's expected payoff stands in stark contrast to models where the probability of each agent making an offer is exogenously determined and fixed over types; more about this in Section 4.2. The fact that the seller's payoff can be decreasing in the probability of the high type has potentially interesting implications for applied work. Suppose that the seller has to decide on entering one of two markets:

[10]The non-monotonicity implies the seller would benefit if she could commit ex ante to a price.

the first populated almost exclusively by low value buyers, the second consisting of a fair share of high value customers. The above non-monotonicity result shows that the seller might indeed prefer to enter the market with low value customers and go for the proverbial low hanging fruit. The higher proportion of the high value buyers offers higher surplus to be split between the two parties. However, the high value buyers fight harder for the said surplus, which can result in a worse outcome for the seller.

3 Two-Sided Effort

We now augment the analysis with the possibility of the seller exerting effort $e_s \in \{0, 1\}$ at a cost $k_s e_s$. The seller's effort, in addition to affecting the probability with which the seller gets to make an offer, plays a prominent role in the seller's learning about the buyer's valuation. We show that this interaction gives rise to novel and interesting equilibrium behavior.

We assume that the probability of making an offer as a function of the buyer's and seller's effort choice takes the following simple form:

$$\begin{cases} \rho & \text{if } e_i > e_{-i}, \\ \frac{1}{2} & \text{if } e_i = e_{-i}, \\ 1-\rho & \text{if } e_i < e_{-i}, \end{cases}$$

where $\rho \in (\frac{1}{2}, 1)$. The effect of effort on the probability of making an offer is symmetric for the buyer and the seller, while differences in bargaining skills are captured by the relative costs of effort. Similarly as in the previous section, a canonical example of our environment is a Tullock contest, though here both agents face effort choices.

3.1 Equilibrium Analysis

The seller optimally chooses a price p_s from the set $\{v_L, v_H\}$. Her strategy can, therefore, be described by a probability distribution σ over pairs (e_s, p_s) with $e_s \in \{0, 1\}$ and $p_s \in \{v_L, v_H\}$. The buyer optimally offers a price equal to zero, thus his decision problem boils down to the choice of effort.

We start again by considering the buyer's benefit from exerting effort. Notice that the increase in the probability with which the buyer gets to make an offer when switching to exerting effort does not depend on whether or not the seller exerts effort: $\Delta\rho =$

$\frac{1}{2} - (1-\rho) = \rho - \frac{1}{2}$. The high type buyer's benefit from exerting effort can therefore be written as

$$\begin{aligned}\Delta u_b(v_H) &\triangleq \Pr_\sigma[e_s = 1]\mathrm{E}_\sigma\left[\Delta\rho\, p_s - k_b | e_s = 1\right] + \Pr_\sigma[e_s = 0]\mathrm{E}_\sigma\left[\Delta\rho\, p_s - k_b \mid e_s = 0\right] \\ &= \Delta\rho\, \mathrm{E}_\sigma[p_s] - k_b,\end{aligned}$$

exactly as in Section 2. Similarly, for the low type buyer:

$$\Delta u_b(v_L) \triangleq \Delta\rho\, v_L - k_b.$$

Due to the fact that the buyer's benefit of exerting effort does not depend on the seller's effort choice, some of the lemmata that we developed in Section 2 carry over to this environment. In particular, the high type buyer has at least as high an incentive to exert effort as the low type. This implies that (generically) the high type buyer exerts effort with at least as high probability as the low type. Another implication, corresponding to Lemma 3, is that the seller's posterior is never above her prior.

The seller's benefit of exerting effort when planning to offer a price p_s is

$$\Delta u_s(p_s) = \Delta\rho\left[\mu p_s + (1-\mu)1_{[p=v_L]}p_s\right] - k_s. \tag{5}$$

The seller's optimal effort choice is determined by the price she intends to propose. At the same time, it does not directly depend on the buyer's behavior. The seller's optimal pricing strategy, and through that her effort, however, do depend on the buyer's conduct. Vice versa, the seller's choice of effort affects how much she learns about the buyer and, as a result, her optimal pricing strategy.

Lemma 4. *When the high type buyer exerts effort with a higher probability than the low type, the seller's posterior $\hat{\mu}$ is smaller when $e_s = 0$ than when $e_s = 1$.*

Exerting effort diminishes the extent to which the seller updates her belief, i.e., learns. All else equal, exerting effort favors the separating price v_H.

There are several regions of parameters to be considered. Here we focus on the one that demonstrates the effect of the seller's learning through her own effort. The rest of the analysis is outlined in Appendix B.

We consider the case where neither the buyer nor the seller have a dominant effort choice, that is $k_b, k_s \in (\Delta\rho v_L, \Delta\rho v_H)$. In addition, define m_0 as the analogue of m of the previous section when the seller exerts no effort: m_0 is the prior such that if the high type buyer was to exert effort while the low type and the seller would not, the seller's posterior would be precisely v_L/v_H:

$$m_0 = \frac{\frac{1}{2}\frac{v_L}{v_H}}{\frac{1}{2}\frac{v_L}{v_H} + (1-\rho)(1-\frac{v_L}{v_H})}. \tag{6}$$

Finally, let $m^H(k_s)$ be the prior at which the seller is indifferent between exerting effort and not conditionally on charging the high price,

$$m^H(k_s) = \frac{k_s}{\Delta\rho v_H}, \tag{7}$$

and let $\tilde{m}(k_s)$ be the prior at which the seller is indifferent between no effort followed by the low price and effort followed by the high price when the high type buyer is exerting effort and the low type is not.

Proposition 4. *Assume $k_b, k_s \in (\Delta\rho\, v_L, \Delta\rho\, v_H)$ as well as $m^H(k_s) < m_0$. There is a generically unique equilibrium with the following properties:*

- *if $\mu \leq \frac{v_L}{v_H}$, nobody exerts effort and the seller offers the low price;*
- *if $\frac{v_L}{v_H} < \mu \leq m^H(k_s)$, the high type buyer randomizes over the two effort choices, the seller does not exert effort and randomizes over the two prices;*
- *if $m^H(k_s) < \mu < \tilde{m}(k_s)$, the high type buyer randomizes over the two effort choices, while the seller randomizes over the two pairs $(0, v_L)$ and $(1, v_H)$;*
- *for $\tilde{m}(k_s) \leq \mu$, the high type buyer and the seller exert effort and the seller offers the high price;*

where $\tilde{m}(k_s) = \frac{k_s + \frac{1}{2}v_L}{\frac{1}{2}v_H + \Delta\rho\, v_L}$. In all cases the low type buyer does not exert effort.

At the priors below $m^H(k_s)$ the equilibrium behavior mimics the behavior from the model where the seller did not have an effort choice. The seller refrains from exerting effort and offers the low price if $\mu < v_L/v_H$. She randomizes over the prices, instead, if the prior is between v_L/v_H and $m^H(k_s)$. At the prior $m^H(k_s)$, however, the seller starts to randomize across the effort-price pairs $(0, v_L)$ and $(1, v_H)$. Such randomization can be sustained due to Lemma 4: when the seller exerts effort, her posterior is higher than when she does not. Indeed, in the relevant region of priors refraining from effort is optimally followed by the low price, while exerting effort is optimally followed by the high price. The seller's randomization preserves the expected price, leaving the high type buyer indifferent between the two effort choices. The high type buyer, in response, adjusts his randomization to make the seller indifferent. In contrast to the case when μ is just below $m^H(k_s)$, the probability with which the buyer exerts effort does not make the seller indifferent between prices for a given effort but instead between the two strategies $(0, v_L)$ and $(1, v_H)$. The threshold $\tilde{m}(k_s)$ is the highest prior at which the seller can indeed be made indifferent between the two effort-price pairs, thereafter she exerts effort and offers the high price.

3.2 Comparative Statics

We explore how the seller's payoff varies as a function of her prior μ for the case described in Proposition 4.

Corollary 2. *Assume $k_b, k_s \in (\Delta\rho\, v_L, \Delta\rho\, v_H)$ and $m^H(k_s) < m_0$. The seller's equilibrium expected payoff is*

$$u_s = \begin{cases} \frac{1}{2} v_L & \text{if} \quad \mu \leq \frac{v_L}{v_H} \\ \frac{1}{2}(1-\mu)\frac{v_H v_L}{v_H - v_L} & \text{if} \quad \frac{v_L}{v_H} \leq \mu \leq m^H(k_s) \\ \frac{(\frac{1}{2} - \mu\rho)v_H + k_s}{v_H - v_L} v_L & \text{if} \quad m^H(k_s) \leq \mu \leq \tilde{m}(k_s) \\ \frac{1}{2}\mu v_H - k_s & \text{if} \quad \tilde{m}(k_s) \leq \mu. \end{cases}$$

For priors below $m^H(k_s)$ the seller exerts no effort, so the comparative statics are the same as in the case where only the buyer faced the effort choice. At the threshold $m^H(k_s)$, the seller's benefit of exerting effort when offering the price v_H becomes positive. In order to keep her indifferent between this option and exerting no effort followed by v_L, the high type buyer's probability of exerting effort must increases in μ faster than when the seller's effort is fixed. This exhibits downward pressure on the seller's posterior, thereby reducing the value of the separating price v_H. Notably, the seller's posterior after exerting no effort now decreases in her prior: the high type buyer's probability of exerting effort increases sufficiently fast so that the learning effect overpowers the direct effect of the increase in μ, resulting in a lower posterior; the seller's posterior is depicted in Figure 4.

In the region of priors $(m^H(k_s), \tilde{m}(k_s))$ the seller's expected payoff is also decreasing in her prior. Namely, she is indifferent between not exerting effort followed by the low price and exerting effort followed by the high price. Evaluating her payoff at the first tuple confirms the result, just as in the region where she randomizes only over the prices. Moreover, since her probability of making an offer now decreases in μ even faster, so does her payoff (see Figure 5). Finally, when μ reaches the threshold $\tilde{m}(k_s)$, the seller's prior is sufficiently high so that she and the high type buyer exert effort with probability one. From here on the seller's payoff is again increasing.

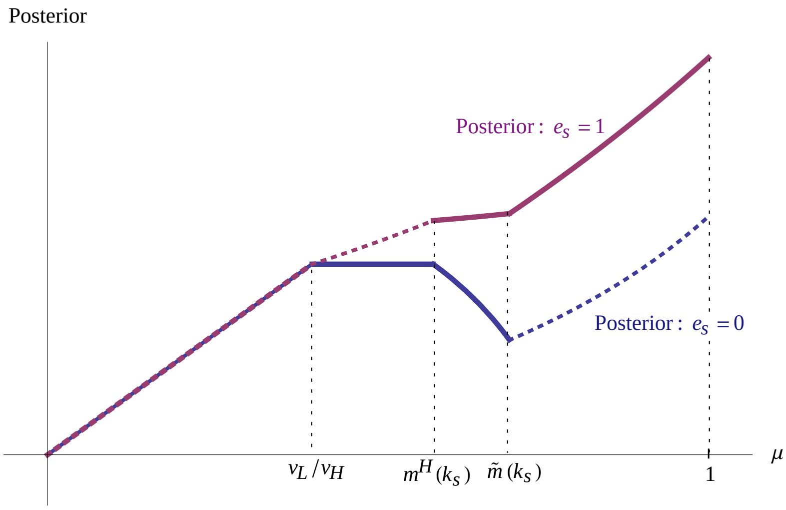

Figure 4: Seller's equilibrium posterior after exerting and not exerting effort

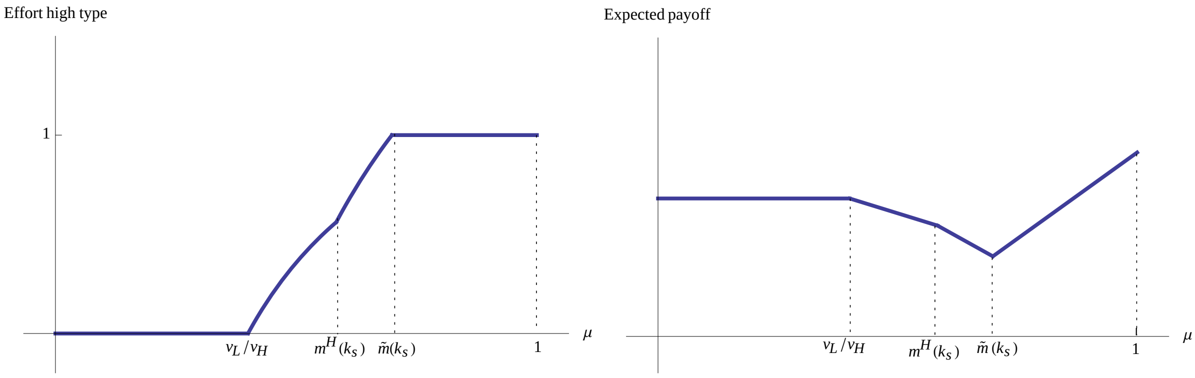

Figure 5: High type buyer's effort and seller's expected payoff in equilibrium

4 Discussion

4.1 Effort Costs and Welfare

In what follows we will discuss how total surplus depends on the effort cost parameters k_b and k_s. The two parameters affect the probability with which either party exerts effort in equilibrium and through that welfare. As can be verified from Propositions 4, 7, and 8, the seller's and buyer's equilibrium efforts are non-increasing in their own costs, k_s and k_b. However, since the probability with which agents exert effort in equilibrium varies with the cost parameters in a non-continuous way, effects on welfare are rather erratic. Instead of giving a full account, we focus on three important channels through which effort costs influence welfare.

The first channel is the direct effect of an increase in k_s or k_b on the total cost of effort that is incurred in equilibrium. As long as an increase in the cost of effort does not lead to a change of the equilibrium effort choice of the respective agent, such increase clearly lowers total surplus. A higher effort cost might, however, deter an agent from exerting effort, thus lowering the incurred cost of effort and potentially increasing welfare. The realized equilibrium effort costs for agent $i = b, s$ are therefore lowest either when k_i vanishes or when k_i is sufficiently large.

The second channel comes from the one-sidedness of the private information in our model. If the buyer was always the one to make an offer, trade would take place with probability one and the gains from trade would be maximized. In contrast, when the seller makes an offer, she sometimes finds it optimal to offer the separating price v_H, thereby forgoing the possibility of trade with the low type buyer. By implication, the realized gains from trade cannot decrease when the probability that the buyer makes an offer increases; all else equal. This channel therefore suggests that the realized gains from trade are greater when the buyer's cost of effort is small relative to the seller's.

Lastly, effort costs have an indirect effect on how much the seller learns in equilibrium. Lemma 4 shows that the seller's posterior is lower when she exerts no effort. This makes her more likely to charge the pooling price v_L, which, holding the buyer's effort fixed, increases trading surplus. Given that the seller's equilibrium effort decreases in her cost, this implies that a higher value of k_s not only makes it more likely for the buyer to make an offer but also less likely that trade is forgone in case the seller makes an offer. The three channels imply that surplus is maximal when k_b is small and k_s is large.

Finally, we want to point out that an increase in k_s can lead to an increase in total surplus also when the seller's equilibrium effort choice does not change. Even more perplexing, not only surplus but also the seller's payoff can increase with k_s. This effect arises in the parameter region where the seller randomizes across the pairs of strategies $(0, v_L)$ and $(1, v_H)$; see Proposition 4. In this region the buyer's payoff does not depend on the seller's cost parameter since the seller's randomization between $(0, v_L)$ and $(1, v_H)$ is independent of k_s.[11] On the other hand, the seller's expected payoff, given by

$$u_s = \frac{(\frac{1}{2} - \mu\rho)v_H + k_s}{v_H - v_L} v_L,$$

is increasing in k_s; see Corollary 2. To gain some intuition for why this is the case, notice that, as k_s increases, exerting effort and offering the high price becomes less attractive for the seller. As a consequence, the high type buyer can exert effort with a smaller probability, without violating the seller's indifference condition. This implies that the payoff associated to exerting no effort and offering the low price must increase: the seller

[11] The seller randomizes to keep the buyer indifferent, who clearly does not care about k_s.

incurs no cost but gets to make an offer with a strictly higher probability. In equilibrium the payoff from choosing $(0, v_L)$ is equal to the one from choosing $(1, v_H)$, which means that also the latter increases. The positive effect on the probability of making an offer therefore outweighs the higher effort cost the seller incurs. It follows that the seller's expected payoff (and in consequence total surplus) strictly increases in k_s.

4.2 Probabilistic Bargaining Model

The probabilistic bargaining model is a model in which the buyer and the seller get to make an offer with a fixed probability; the probability reflecting the buyer's bargaining power. Unlike in the above proposed model, the probability of making an offer is exogenously given and independent of the agents' valuations. The elegance of probabilistic bargaining models makes them a popular tool in economics and finance; for examples see Inderst (2001) and Zingales (1995), though probabilistic bargaining goes back at least to Rubinstein and Wolinsky (1985). One might wonder whether a generalization of the probabilistic bargaining model could capture some of the interesting feature of our model without having to deal with the added dimension of efforts.

In the probabilistic model of bargaining curtailed to our environment the buyer gets to make an offer with probability ρ while the seller makes an offer with the remaining probability. If the offer is rejected the game ends. Securing the right to make an offer conveys no information to the seller; she makes an offer on the basis of her prior distribution. This model corresponds to the one presented in the previous sections only in the case where the buyer has a dominant strategy.[12] The interesting properties—learning and non-monotonicity of the seller's payoff—, therefore, cannot be replicated with a simple probabilistic bargaining model.

Our model with efforts showed that high value buyers have a higher propensity to exert effort and through that higher bargaining power. To capture this feature we propose a richer version of the probabilistic bargaining model in which the high type buyer gets to make an offer with probability ρ_H and the low type with probability ρ_L, where $\rho_H > \rho_L$. The following proposition characterizes the equilibrium.

Proposition 5. *The seller optimally offers the high price if*

$$\mu \geq \underbrace{\frac{(1-\rho_L)\frac{v_L}{v_H}}{(1-\rho_H)(1-\frac{v_L}{v_H})+(1-\rho_L)\frac{v_L}{v_H}}}_{\triangleq \mu^*} \tag{8}$$

[12] With a slight difference that in our model some surplus can be burnt due to the cost of effort.

and the low price otherwise.

It is easy to see that the seller's threshold μ^* (the right hand side of the above inequality) is larger than v_L/v_H. Being given the opportunity to make an offer is a negative signal for the seller; she has an easier time making an offer against the low type. Winning the chance to make an offer, therefore, lowers her belief. Consequently, she is willing to offer the high price only when her prior sufficiently favors the high type.

To highlight the comparison between the probabilistic bargaining model with varying bargaining power and our model with efforts, we examine how the seller's payoff changes with the prior.

Corollary 3. *The seller's equilibrium expected payoff is*

$$u_s = \begin{cases} [\mu(1-\rho_H) + (1-\mu)(1-\rho_L)]v_L & \text{if} \quad \mu \leq \mu^* \\ \mu(1-\rho_H)v_H & \text{if} \quad \mu > \mu^*. \end{cases}$$

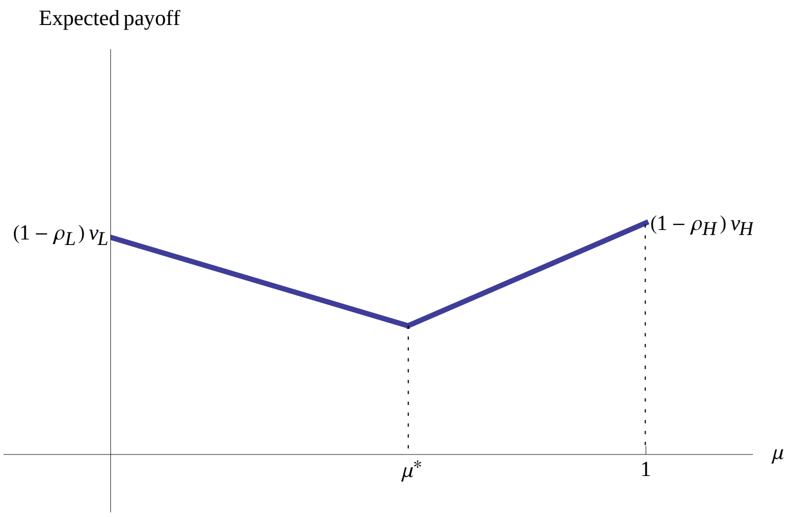

Figure 6: Seller's equilibrium payoff: random proposal model

The assumption $\rho_H > \rho_L$ implies that the seller's payoff is decreasing as long as $\mu \leq \mu^*$ and increasing for $\mu > \mu^*$. In the region of low priors the seller offers price v_L and both types accept it. The seller, however, suffers from the incidence of high types, as this decreases her probability of making an offer. This property is different from our model where the probability of making an offer is endogenously determined. In the latter model, when expecting the low price, both types of buyer make the same effort choice, and therefore, win with the same probability against the seller. In the region of high priors where the seller charges the high price, the occurrence of high types also decreases her chances of making an offer. The effect is, however, overpowered by the fact that the seller's price offer is accepted more often. The sum of the two effects is an unequivocal benefit of facing more high types for the seller.

The version of the probabilistic bargaining model with different probabilities for different types approximates the model with efforts relatively well—it can capture the non-monotonicity of the seller's payoff—, but does not match it exactly. The main distinction is that in the model with effort the difference between the probabilities of the high and the low type making an offer increases in the prior. This accounts, among other things, for the difference between the two models in the seller's payoff below the threshold v_L/v_H. We conclude that despite of some of its shortcomings, a researcher who wishes to use a more manageable tool will be well suited with the probabilistic model of bargaining proposed here.

4.3 Continuous Effort

In this section we show that our comparative statics result concerning the seller's expected payoff does not rely on the buyer's randomization across efforts. To this end we modify the baseline model of Section 2 by allowing the buyer to choose an effort e_b from the set $[0, +\infty)$. We let $\rho(e_b)$ denote the probability that the buyer gets to make the offer when exerting effort e_b and assume that ρ is differentiable, strictly increasing and strictly concave. Type v_i's expected payoffs as a function of e_b is then given by

$$\rho(e_b)v_i + (1 - \rho(e))\mathrm{E}_\sigma[1_{[p_s \leq v_i]}(v_i - p_s)] - k_b e_b, \quad i = L, H.$$

The following result holds.

Proposition 6. *Assume $k_b < \rho'(0)v_H$. In the unique equilibrium of the game, there exists a threshold $\hat{m} > v_L/v_H$ such that:*

- *the seller offers the price v_L if $\mu \leq v_L/v_H$;*
- *the seller randomizes over the prices v_L and v_H if $\mu \in (v_L/v_H, \hat{m})$;*
- *the seller offers price v_H if $\mu > \hat{m}$.*

Moreover, the high type buyer exerts at least as high effort as the low type buyer.

The assumption $k < \rho'(0)v_H$ assures that exerting zero effort is not the dominant effort choice for both types of the buyer. Given the assumption, there exists a range of intermediate values of μ where the seller's expected payoff is decreasing in the probability that the buyer's value of the object is high. In this parameter region the high type buyer exerts strictly more effort than the low type buyer and, conditional on having the opportunity to make an offer, the seller is indifferent between the low and the high price. The latter implies that the seller's payoff conditional on making the offer is equal to v_L and therefore does not depend on the seller's prior belief. His probability of making an offer, on the other hand, strictly decreases in μ.

5 Concluding Remarks

We propose a model that endogenizes bargaining power in a bilateral trade environment through costly effort. We show that higher types endogenously acquire higher bargaining power. Interestingly, this can lead to the seller's payoff being non-monotonic in the proportion of high value buyers. Therefore, if the seller were to choose between the entry into two markets, one with a low proportion of high value customers, the other with a higher, she might choose the one with the lower incidence of high value customers. While the high value buyers offer a higher potential surplus, they also more actively negotiate the terms of trade. The two countervailing incentives can drive the seller to pick the market that is predominantly populated by low value customers—the seller focuses on the low hanging fruit.

Flexibility of our model offers several avenues for future research. Worthy of scrutiny might be the environment where efforts are observable, and thus serve as a signalling device. One might also wonder how the seller's private information would influence bargaining. We plan to explore these models in future work.

6 Appendix A

Proof of Lemma 3. The seller's posterior can be written as

$$\begin{aligned}
\hat{\mu} &= \frac{\mu\left[\beta_H(1-\rho_1)+(1-\beta_H)(1-\rho_0)\right]}{\mu\left[\beta_H(1-\rho_1)+(1-\beta_H)(1-\rho_0)\right]+(1-\mu)\left[\beta_L(1-\rho_1)+(1-\beta_L)(1-\rho_0)\right]} \\
&\leq \frac{\mu\left[\beta_H(1-\rho_1)+(1-\beta_H)(1-\rho_0)\right]}{\mu\left[\beta_H(1-\rho_1)+(1-\beta_H)(1-\rho_0)\right]+(1-\mu)\left[\beta_H(1-\rho_1)+(1-\beta_H)(1-\rho_0)\right]} \\
&= \mu,
\end{aligned}$$

where the inequality follows from $\beta_H \geq \beta_L$ and $\rho_1 > \rho_0$. □

Proof of Proposition 1. Due to Lemma 3, we know that $\mu < v_L/v_H$ implies $\hat{\mu} < v_L/v_H$. Conditional on making an offer, the seller thus strictly prefers the pooling price v_L. Given this, we have $\Delta u_b(v_L) = \Delta u_b(v_H)$, so that the low type buyer optimally exerts effort if and only if the high type optimally exerts effort. □

Proof of Proposition 2. Under the stated conditions, both types of buyer undertake the same effort choice. This implies that when the seller gets to make the offer her posterior is equal to her prior μ. If μ is smaller (greater) than v_L/v_H, the payoff associated to the pooling price, v_L, is greater (smaller) than the payoff associated to the separating price, μv_H. □

Proof of Proposition 3. Consider first the case $\mu \geq m$. Recollect that m is the prior at which the seller's posterior is v_L/v_H when the high type buyer exerts effort and the low type does not. When starting from a prior $\mu > m$, the seller's posterior is, therefore, above v_L/v_H irrespective of the buyer's strategy. Consequently, the seller offers the high price. On the other hand, given $k_b \in (\Delta\rho\, v_L, \Delta\rho\, v_H)$, for $E_\sigma[p_s] = v_H$: $\Delta u_b(v_L) < 0$ and $\Delta u_b(v_H) > 0$. The low type buyer optimally exerts no effort, while the high type buyer does.

Consider next the case $\mu \in (v_L/v_H, m)$. By the argument outlined after the statement of the theorem, there is no pure strategy equilibrium. The seller must therefore randomize between offering the pooling and the separating price. This requires that her posterior when she gets to make the offer is v_L/v_H. Setting $\beta_L = 0$, the condition $\hat{\mu} = v_L/v_H$ becomes

$$\frac{\mu[\beta_H(1-\rho_1) + (1-\beta_H)(1-\rho_0)]}{\mu[\beta_H(1-\rho_1) + (1-\beta_H)(1-\rho_0)] + (1-\mu)(1-\rho_0)} = \frac{v_L}{v_H}$$

Solving this equality for β_H yields $\beta_H = \frac{(1-\rho_0)(\mu v_H - v_L)}{\Delta\rho\, \mu(v_H - v_L)}$. The value of β_H lies in the interval $(0, 1)$ if and only if $\mu \in (v_L/v_H, m)$. Finally, in equilibrium it must be optimal for the high type buyer to randomize over exerting and not exerting effort. This requires $\Delta u_b(v_H) = 0$, or equivalently

$$\Delta\rho\, [\sigma v_H + (1-\sigma)v_L] = k_b$$

Solving the above equality for σ delivers expression (10). The value of σ as in (10) belongs to the interval $(0, 1)$ if and only if $k_b \in (\Delta\rho\, v_l, \Delta\rho\, v_H)$. Under the stated conditions, the equilibrium as characterized in Proposition 3 thus exists and is (generically) unique. □

Proof of Lemma 4. Setting $\beta_L \leq \beta_H$, the difference in the seller's posterior between $e_s = 1$ and $e_s = 0$ is given by

$$\begin{aligned}
& \frac{\mu\left[\beta_H \frac{1}{2} + (1-\beta_H)\rho\right]}{\mu\left[\beta_H \frac{1}{2} + (1-\beta_H)\rho\right] + (1-\mu)\left[\beta_L \frac{1}{2} + (1-\beta_L)\rho\right]} \\
& - \frac{\mu\left[\beta_H(1-\rho) + (1-\beta_H)\frac{1}{2}\right]}{\mu\left[\beta_H(1-\rho) + (1-\beta_H)\frac{1}{2}\right] + (1-\mu)\left[\beta_L(1-\rho) + (1-\beta_L)\frac{1}{2}\right]} \\
= \; & \frac{\mu(1-\mu)\Delta\rho^2(\beta_H - \beta_L)}{\left(\mu\left[\beta_H \frac{1}{2} + (1-\beta_H)\rho\right] + (1-\mu)\rho\right)\left(\mu\left[\beta_H(1-\rho) + (1-\beta_H)\frac{1}{2}\right] + (1-\mu)\frac{1}{2}\right)} \\
\geq \; & 0
\end{aligned}$$

The inequality is strict when $\beta_L < \beta_H$. $\square$

Proof of Proposition 8. Recall that $m^H(k_s) = \frac{k_s}{\Delta\rho\, v_H}$ is the prior at which the seller is indifferent between the two effort choices when planning to offer price v_H. Since we assume that $k_s > \Delta\rho\, v_L$, we have $m^H(k_s) > \frac{v_L}{v_H}$. In turn

$$\Delta\rho\, v_L - k_s \leq \Delta\rho\, m^H(k_s) v_H - k_s = 0.$$

In words, the seller's benefit from exerting effort when planning to offer the low price, v_L, is smaller than the same benefit at the prior $m^H(k_s)$ when planning to offer the high price, v_H. The latter is equal to 0 by definition of $m^H(k_s)$. At priors below $m^H(k_s)$ the seller thus optimally refrains from exerting effort irrespective of the price she is planning to charge or the buyer's behavior. We can thus apply the analysis from Proposition 3 after setting $\rho_1 = \rho$ and $\rho_0 = \frac{1}{2}$. Notice also that m_0 corresponds to m in the before-mentioned proposition. This takes care of the first three bullet points.

The case that remains to be considered is $\mu > m^H(k_s)$. By definition of m_0 and Lemma (4), the seller optimally charges the high price for the priors above m_0, regardless of her effort choice. The definition of $m^H(k_s)$ and the assumption $m^H(k_s) > m_0$ then imply that for priors above $m^H(k_s)$ the seller prefers to exert effort, followed by the high price. Given the assumption $k_b < \Delta\rho\, v_H$, this incentivizes the high type buyer to exert effort too. $\square$

Proof of Proposition 4. As shown in the proof of Proposition 8, for priors below the threshold $m^H(k_s)$, the seller optimally refrains from exerting effort, so we can apply the analysis from Proposition 3 after setting $\rho_1 = \rho$ and $\rho_0 = \frac{1}{2}$. This covers the first two bullet points.

Assume next $\mu > m^H(k_s)$, or equivalently, $k_s < \Delta\rho\, \mu v_H$. The assumption $k_b > \Delta\rho\, v_L$ implies $\Delta u_b(v_L) < 0$. The low type buyer, therefore, optimally refrains from exerting effort regardless of the seller's behavior. Consider then the seller's optimal effort choice. Since $k_s \in (\Delta\rho\, v_L, \Delta\rho\,\mu v_H)$, we have

$$\begin{aligned}\Delta u_s(v_L) &= \Delta\rho\, v_L - k_s < 0,\\ \Delta u_s(v_H) &= \Delta\rho\,\mu v_H - k_s > 0.\end{aligned}$$

The above inequalities imply that $(0, v_L)$ dominates $(1, v_L)$ and $(0, v_H)$ is dominated by $(1, v_H)$. The seller, thus, chooses between $(0, v_L)$ and $(1, v_H)$. Her benefit when switching from $(0, v_L)$ to $(1, v_H)$ is given by

$$\tilde{\Delta} u_s = \mu\left[\beta_H \frac{1}{2} + (1-\beta_H)\rho\right] v_H - k_s - \left[\mu\beta_H(1-\rho) + \mu(1-\beta_H)\frac{1}{2} + (1-\mu)\frac{1}{2}\right] v_L.$$

This term is decreasing in β_H and equal to zero when β_H takes the value

$$\beta_H = \frac{\frac{1}{2}(v_H - v_L) + \Delta\rho\,\mu v_H - k_s}{\Delta\rho\,\mu(v_H - v_L)}. \tag{9}$$

We next argue that in equilibrium the seller cannot choose $(0, v_L)$ with probability one. Recalling the assumption $k_b > \Delta\rho\, v_L$, if the seller would choose $(0, v_L)$ with probability one, the high type buyer would prefer not to exert effort:

$$\Delta u_b(v_H) = \Delta\rho\, v_L - k_b < 0.$$

We would thus have $\beta_H = 0$ and hence

$$\begin{aligned}\tilde{\Delta} u_s &= \mu\rho v_H - k_s - 1/2 v_L,\\ &> \mu\rho v_H - k_s - 1/2\mu v_H,\\ &= \Delta\rho\,\mu v_H - k_s,\\ &> 0,\end{aligned}$$

where the first inequality follows from the assumption $\mu v_H > v_L$ and the second from $k_s < \Delta\rho\,\mu v_H$. By implication, the seller would find it optimal to deviate to $(1, v_H)$.

Consider next the possibility of an equilibrium where the seller chooses $(1, v_H)$ with probability one. This makes it optimal for the high type buyer to exert effort:

$$\Delta u_b(v_H) = \Delta\rho\, v_H - k_b > 0,$$

where the inequality follows from the assumption $k_b < \Delta\rho\, v_H$. Given $\beta_H = 1$, we then have

$$\tilde{\Delta} u_s = \frac{1}{2}(\mu v_H - v_L) + \Delta\rho\,\mu v_L - k_s.$$

For $(1, v_H)$ to be optimal, the above term needs to be non-negative. This is the case if

$$\mu \geq \underbrace{\frac{k_s + \frac{1}{2}v_L}{\frac{1}{2}v_H + \Delta\rho\, v_L}}_{=\tilde{m}(k_s)}.$$

Hence, when $\mu \geq \tilde{m}(k_s)$, there exists a pure strategy equilibrium where the seller and the high type buyer exert effort, while the low type does not, and the seller offers price v_H.

When the above inequality is not satisfied, the seller must randomize between $(0, v_L)$ and $(1, v_H)$ in equilibrium. Indifference between the two tuples requires $\tilde{\Delta} u_s = 0$. As we showed above, this condition is satisfied if β_H takes the value in (9). Given the assumption $\mu > m^H(k_s)$, this value of β_H lies in $(0, 1)$ if and only if $\mu < \tilde{m}(k_s)$. Finally, randomizing is optimal for the high type buyer if

$$\Delta u_b(v_H) = \Delta\rho\, [\sigma v_H + (1 - \sigma)v_L] - k_b = 0.$$

Solving the equality for σ yields

$$\sigma = \frac{k_b - \Delta\rho\, v_L}{\Delta\rho\, (v_H - v_L)}. \tag{10}$$

It can be verified that under the imposed parameter restrictions, the above term lies in $(0, 1)$.

Taken together, this shows that the equilibrium, as described in Proposition 4, exists and that it is unique.

□

Proof of Proposition 5. If the seller gets to make an offer, her posterior belief about the buyer's type is

$$\hat{\mu} = \frac{\mu(1 - \rho_H)}{\mu(1 - \rho_H) + (1 - \mu)(1 - \rho_L)}.$$

She optimally offers price v_H if her posterior belief exceeds v_L/v_H, i.e. if

$$\frac{\mu(1 - \rho_H)}{\mu(1 - \rho_H) + (1 - \mu)(1 - \rho_L)} \geq v_L/v_H.$$

Solving this inequality for μ, yields expression (8). □

Proof of Proposition 6. As before let σ denote the probability with which the seller offers price v_H. The low type buyer solves the problem

$$\max_{e_b} \quad \rho(e_b)v_L - k_b e_b.$$

Let e_L^* denote the solution of this problem. If $\rho'(0)v_L \leq k_b$, we have $e_L^* = 0$, otherwise the solution is uniquely defined by the first-order condition $\rho'(e_L^*)v_L = k_b$. The high type buyer solves the problem

$$\max_{e_b} \quad v_H - (1-\rho(e_b))[\sigma v_H + (1-\sigma)v_L] - k_b e_b.$$

The solution of this problem depends on the seller's strategy σ and will be denoted by $e_H^*(\sigma)$. If $\rho'(0)[\sigma v_H + (1-\sigma)v_L] \leq k_b$, we have $e_H^*(\sigma) = 0$, otherwise $e_H^*(\sigma)$ is implicitly defined by the first-order condition $\rho'(e_H^*)[\sigma v_H + (1-\sigma)v_L] = k_b$. Let $\hat{\sigma}$ denote the smallest value of σ such that the solution of the high type buyer's problem is interior for all $\sigma > \hat{\sigma}$. Notice that $e_H^*(\sigma)$ is strictly increasing on $[\hat{\sigma}, 1]$ with $e_H^*(\hat{\sigma}) = e_L^*$.

We will characterise the equilibrium by distinguishing three cases:

- $\sigma = 0$: suppose in equilibrium the seller offers price v_L with probability one. In this case the effort choice of both buyers is the same, so the seller does not learn in equilibrium. Price v_L is then optimal if and only if $\mu \leq v_L/v_H$. Hence, if and only if $\mu \leq v_L/v_H$, there exists a pure strategy equilibrium where the seller offer the low price v_L.

- $\sigma = 1$: suppose in equilibrium the seller offers price v_H with probability one. The assumption $k < \rho'(0)v_H$ implies that $e_H^*(1) > 0$. Furthermore, we have $e_H^*(1) > e_L^*$, hence the high type buyer exerts strictly more effort than the low type buyer. The seller's posterior when he makes the offer is

$$\hat{\mu} = \frac{\mu(1-\rho(e_H^*(1)))}{\mu(1-\rho(e_H^*(1))) + (1-\mu)(1-\rho(e_L^*))}$$

  Let $\hat{m}$ denote the value of μ at which the term on the right-hand side equals v_L/v_H. By $\rho(e_L^*) < \rho(e_H^*(1))$, we have $\hat{m} > v_L/v_H$. Since the posterior is increasing in μ, it is greater than v_L/v_H if and only if $\mu \geq \hat{m}$. Hence, there exists an equilibrium where the seller offers the high price v_H with probability one if and only if $\mu \geq \hat{m}$.

- $\sigma \in (0,1)$: suppose now the seller randomizes across prices. Ignoring the degenerate case $\mu = v_L/v_H$, this requires $\mu > v_L/v_H$ and σ such that the high type buyer's effort, $e_H^*(\sigma)$, is such that after updating the seller is indeed indifferent between both prices. That is:

$$\frac{\mu(1-\rho(e_H^*(\sigma)))}{\mu(1-\rho(e_H^*(\sigma))) + (1-\mu)(1-\rho(e_L^*))} = v_L/v_H \tag{11}$$

  Since $e_H^*(\sigma)$ is continuous and strictly increasing on $[\hat{\sigma}, 1]$, the term on the left-hand side of (11) is continuous and strictly decreasing in σ. Given $e^*_H(\hat{\sigma}) = e^*_L$ and $\mu > v_L/v_H$, at $\sigma = \hat{\sigma}$ it is strictly greater than v_L/v_H. Taken together, this implies that (11) can be solved by some $\sigma \in (0, 1)$ if and only if at $\sigma = 1$, the left-hand side is strictly smaller than the right hand side. By definition of $\hat{m}$, this is the case if and only if $\mu < \hat{m}$. Hence, there exists an equilibrium where the seller randomizes over both prices if $\mu \in (v_L/v_H, \hat{m})$.

To argue formally that for $\mu \in (v_L/v_H, \hat{m})$ the seller's payoff is decreasing notice that in the mixed strategy equilibrium the seller's payoff conditional on making an offer is constant in μ and given by v_L. His probability of making an offer in equilibrium is $\mu(1-\rho(e^*_H(\sigma)))+(1-\mu)(1-e^*_L)$, where σ solves (11) and therefore depends on μ. Notice that the left-hand side of (11) is increasing in μ and decreasing in σ. This implies that, as μ increases, the value of σ solving (11) increases. Taking the first derivative of the seller's offer probability with respect to μ, we obtain

$$\frac{\mathrm{d}(\mu(1-\rho(e^*_H(\sigma))) + (1-\mu)(1-e^*_L))}{\mathrm{d}\mu} = -(\rho(e^*_H(\sigma)) - \rho(e^*_L)) - \mu\rho'(e^*_H(\sigma)){e^*_H}'(\sigma)\frac{\mathrm{d}\sigma}{\mathrm{d}\mu}.$$

The first term is strictly negative. The fact that ρ and e^*_H are increasing functions, together with the property that the seller's mixing probability σ increases in μ, implies that also the second term is negative. Hence, in the parameter region $\mu \in (v_L/v_H, \hat{m})$ the seller's expected payoff is strictly decreasing in μ. □

7 Appendix B

Here we provide a comprehensive analysis of the case where both the buyer as well as the seller can exert effort. When the seller's cost of effort is sufficiently extreme, she has a dominant effort choice. If $k_s > \Delta\rho v_H$, the benefit from exerting effort is negative regardless of the seller's prior and her choice of prices. The equilibrium analysis of Section 2 applies after setting $\rho_1 = \rho$ and $\rho_0 = \frac{1}{2}$. Similarly, if $k_s < \Delta\rho\, v_L$, the seller optimally exerts effort for all μ. The equilibrium analysis of Section 2 can be applied with the parameters $\rho_1 = \frac{1}{2}$ and $\rho_0 = 1 - \rho$. These cases are summarised in the following proposition.

Proposition 7. *The equilibrium is generically unique and satisfies the following properties:*

- *If $k_s \leq \Delta\rho\, v_L$, the seller exerts effort and the seller's pricing strategy as well as the buyer's effort strategy are described by Propositions 1-3 with $\rho_1 = 1/2$ and $\rho_0 = 1 - \rho$.*

- *If $k_s \geq \Delta\rho\, v_H$, the seller exerts no effort and the seller's pricing strategy as well as the buyer's effort strategy are described by Propositions 1-3 with $\rho_1 = \rho$ and $\rho_0 = 1/2$.*

More interesting is the analysis of the case where the seller does have an incentive to change her effort choice with the prior. In addition, we restrict attention to the case where the two types of buyer do not have a dominant effort choice, so that learning is indeed possible. All together, $k_b, k_s \in (\Delta\rho\, v_L, \Delta\rho\, v_H)$.

We show that there is a threshold prior such that below this prior the seller refrains from effort regardless of the buyer's behavior, then we determine the behavior above the threshold. The relevant threshold, $m^H(k_s)$, is the prior at which the seller is indifferent between exerting effort and not when she charges the high price; equation (7).

Recall that, in the environment without the seller's effort choice, the seller's pricing decision was to offer the low price for low priors, randomize over the two prices at intermediate priors and offer the high price for high priors. Given $\rho_1 = \rho$ and $\rho_0 = \frac{1}{2}$, the threshold betweenthe last two regions, as determined in (4), is given by (6).

Given that $m^H(k_s)$ is the threshold where the seller wants to switch to exerting effort when planning to offer the high price, we break the analysis into the cases when it falls in the region below m_0, where the seller randomizes over prices, or above m_0, where the seller offers the high price (when not exerting effort).[13]

Starting with the case $m^H(k_s) > m_0$, just below $m^H(k_s)$ the high type buyer exerts effort and the seller offers the high price after not exerting effort. At the prior $m^H(k_s)$ the seller switches to expending effort. Her posterior jumps up (see Lemma 3), implying that the seller indeed prefers to offer the high price. Since the buyer's effort choice does not depend on the seller's, the buyer's behavior remains optimal. To sum up, for $\mu \geq m^H(k_s)$ the seller exerts effort and offers price v_H, while for the remaining values of μ the equilibrium is as characterized in Proposition 3 when the seller does not exert effort. The following proposition summarizes this analysis.

Proposition 8. *Let $k_b, k_s \in (\Delta\rho\, v_L, \Delta\rho\, v_H)$ and $m^H(k_s) > m_0$. There is a generically unique equilibrium with the following properties:*

- *if $\mu \leq \frac{v_L}{v_H}$, nobody exerts effort and the seller offers the low price;*
- *if $\frac{v_L}{v_H} < \mu \leq m_0$, the high type buyer randomizes over the two effort choices, the seller does not exert effort and randomizes over the two prices;*

[13] Given the assumptions on k_s and k_b, $m^H(k_s) > v_L/v_H$.

- *if $m_0 < \mu < m^H(k_s)$, the high type buyer exerts effort, while the seller does not; the seller offers price v_H;*
- *it $m^H(k_s) \leq \mu$, the seller and the high type exert effort, and moreover, the seller offers price v_H.*

The low type buyer never exerts effort.

It remains to investigate the case where $m^H(k_s)$ falls into region where the seller would randomize over prices after not exerting effort, $m^H(k_s) \in (v_L/v_H, m_0)$. For priors just below $m^H(k_s)$ the seller not exerting effort and randomizing over prices remains the only equilibrium. At the threshold $m^H(k_s)$ the seller would, however, prefer to exert effort if she intended to offer the high price but not if she wanted to offer the low price. Moreover, since just below $m^H(k_s)$ the seller is randomizing over the prices, she must be indifferent between the tuples $(0, v_L)$ and $(0, v_H)$. By definition of $m^H(k_s)$, at the threshold the tuple $(1, v_H)$ becomes preferable to $(0, v_H)$, presenting a profitable deviation from the equilibrium with randomization over prices. Thus, the strategy profile where the seller exerts effort and charges the high price becomes the natural candidate for equilibrium above $m^H(k_s)$. In that case, the high type buyer would optimally exert effort as well. This is an equilibrium only if the seller has no incentives to deviate to not exerting effort and offering the low price. That is, if

$$\mu\frac{1}{2}v_H - k_s \geq \left(\mu(1-\rho) + (1-\mu)\frac{1}{2}\right) v_L.$$

Letting $\tilde{m}(k_s)$ denote the prior at which the above condition is satisfied with equality, the described equilibrium exists for all $\mu \geq \tilde{m}(k_s)$. Under the assumed parameter conditions, the threshold $\tilde{m}(k_s)$ is strictly larger than $m^H(k_s)$. This implies that there is an additional parameter region, between $m^H(k_s)$ and $\tilde{m}(k_s)$, where there is no equilibrium in which the seller has a fixed effort. This brings about a rather interesting new type of behavior as demonstrated in Proposition 4.

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