Monetary Policy, Bond Risk Premia, and the Economy

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Abstract

This paper develops an affine model of the term structure of interest rates in which bond yields are driven by observable and unobservable macroeconomic factors. It imposes restrictions to identify the effects of monetary policy and other structural disturbances on output, inflation, and interest rates and to decompose movements in long-term rates into terms attributable to changing expected future short rates versus risk premia. The estimated model highlights a broad range of channels through which monetary policy affects risk premia and the economy, risk premia affect monetary policy and the economy, and the economy affects monetary policy and risk premia.

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1 Introduction

With their traditional instrument of monetary policy, the short-term federal funds rate, locked up against its zero lower bound since 2008, Federal Reserve officials have resorted to other means for influencing long-term interest rates in order to provide further stimulus to a struggling US economy. Some of these non-traditional policy measures, such as the provision of "forward guidance," aim to lower long-term interest rates by shaping expectations about the future path of short-term rates, in particular, by creating expectations that the federal funds rate will remain at or near zero even as the economy continues to recover. Other new programs, including multiple rounds of "large-scale asset purchases," known more popularly as "quantitative easing," attempt to lower long-term interest rates more directly by reducing the term, or risk, premia that ordinarily cause long-term rates to exceed the average expected value of the short-term policy rate and thereby generate a yield curve with its most typical, upward slope. As former Federal Reserve Chair Ben Bernanke (2013, p.7) explains: "To the extent that Treasury securities and agency-guaranteed securities are not perfect substitutes for other assets, Federal Reserve purchases of these assets should lower their term premiums, putting downward pressure on longer-term interest rates and easing financial conditions more broadly."

In addition to the assumption, stated clearly by the Chair, that Federal Reserve bond purchases work to lower long-term rates by reducing the size of term or risk premia, a second assumption, equally important but left implicit, that provides the rationale for those policy actions is that reductions in risk premia are effective at stimulating the private demand for goods and services and thereby work to increase aggregate output and inflation in much the same way that more traditional monetary policy actions do. Yet, as Rudebusch, Sack, and Swanson (2007) astutely note, although this "practitioner view" that smaller long-term bond risk premia stimulate economic activity is quite widely held, surprisingly little support for the view can be found in existing theoretical or empirical work. In textbook New Keynesian models such as Woodford (2003) and Galí's (2008), for instance, the effects of monetary policy actions on aggregate output arise only to the extent that they have implications for current and future values of the short-term interest rate. Thus, as Eggertsson and Woodford (2003) show, these models offer a rationale for the provision of forward guidance but not for large-scale asset purchases. Andrés, López-Salido, and Nelson (2004) elaborate on the New Keynesian framework, introducing features that imply the imperfect substitutability referred to in Chair Bernanke's comment from above, to demonstrate how downward movements in long-term yields can stimulate aggregate demand even holding the path of short rates fixed. More recently, however, Chen, Cúrdia, and Ferrero (2012) have estimated this model with US data from 1987 through 2009 and concluded that the extra effects running through this additional channel are of limited practical importance. In a similar exercise, Kiley (2014) finds somewhat stronger effects of changes in risk premia on aggregate demand, but mainly when the long-term interest rates used in the estimation are those on corporate bonds instead of Treasury securities.

In the meantime, using a variety of empirical approaches, Ang, Piazzesi, and Wei (2006) and Dewachter, Iania, and Lyrio (2014) find that changes in bond risk premia do not help forecast future output, while Hamilton and Kim (2002), Favero, Kaminska, and Söderström (2005), and Wright (2006) obtain estimates associating larger bond risk premia with *faster* future output growth, exactly the opposite of what the practitioner view asserts. Jardet, Monfort, and Pegoraro (2013), by contrast, detect evidence of the expected, inverse relation between risk premia and future output, but estimate the effect to be short-lived, reversing itself after less than one year. Rudebusch, Sack, and Swanson (2007) also find some evidence of an inverse relation between term premia and future output, although, as they point out, this result appears quite sensitive to both the specification of the forecasting equation and the choice of sample period used to estimate the model. Finally, Bekaert, Hoerova, and Lo Duca (2013) find stronger links between monetary policy actions, financial market measures of risk, and economic activity that are consistent with the practitioner view, but derive their risk measures from the stock-option-based VIX instead of from risk premia embedded into the prices of the government bonds that the Federal Reserve has been purchasing.

Motivated by the weak and often conflicting results reported in previous studies, this paper develops and estimates a model designed specifically to explore the interplay between monetary policy, bond risk premia, and the economy. Rather than imposing a strong set of theoretical assumptions about how these channels of transmission arise, as, for example, Andrés, López-Salido, and Nelson (2004) do in their extension of the tightly-parameterized New Keynesian model, the approach taken here uses a more flexible, multivariate time series model to assess the extent to which, operating through a wider range of mechanisms, changes in monetary policy affect bond risk premia and the economy and changes in bond risk premia influence aggregate output and inflation and lead the Federal Reserve, in turn, to adjust its monetary policy stance relative to what purely macroeconomic conditions would otherwise dictate. The paper's goal, therefore, is to add to the existing empirical literature, cited above, in hopes of highlighting more clearly the regularities in the data that future theoretical work, perhaps along the same lines as Andrés, López-Salido, and Nelson (2004), might try to explain more fully.

Of course, even with a more flexible empirical specification, *some* assumptions must be drawn from theory in order to identify the effects that different fundamental shocks have on endogenous variables. Here, those assumptions are borrowed from three sources. First, following Ang and Piazzesi (2003), cross-equation restrictions implied by no-arbitrage in an affine model of the term structure of interest rates are used to identify the unobserved risk premia built into observable bond yields. But while Ang and Piazzesi's (2003) original model allows macroeconomic variables to affect the behavior of the yield curve, by design it omits channels through which changes in the yield curve can feed back on and affect their macroeconomic drivers. Here, as in Ang, Piazzesi, and Wei (2006), Diebold, Rudebusch, and Aruoba (2006), and Pericoli and Taboga (2008), the model allows for such feedback effects. Going further than those previous studies, however, the model developed here draws, second, on identifying assumptions like those used in more conventional vector autoregressions for macroeconomic variables alone to isolate the effects of monetary policy shocks on bond risk premia and the effects of shocks to bond risk premia on output and inflation. Similar assumptions are also employed by Bekaert, Hoerova, and Lo Duca (2013) but, as noted above, using observed movements in the equity options-based VIX measure of stock market volatility rather than movements in bond risk premia implied by no-arbitrage. Third, as in the New Keynesian models outlined by Woodford (2003) and Galí (2008), Federal Reserve policy is described here by a monetary policy rule like that proposed by Taylor (1993), according to which the short-term interest rate adjusts in response to movements in output and inflation. Once again going beyond previous work, however, the analysis here adds a bond risk premium term, identified with the help of the affine term structure model, to the short list of variables to which the policy rate potentially responds. Estimates of the model's key parameters provide evidence of a rich set of multi-directional channels linking monetary policy, bond risk premia, and the economy, while impulse responses and forecast error variance decompositions highlight the quantitative importance of these various channels.

In addition to its three core macroeconomic variables – the short-term nominal interest rate, the output gap, and inflation – and five longer-term bond yields, the model developed here also includes two unobserved state variables. Inspired by Cochrane and Piazzesi (2008), time-variation in bond risk premia within the affine pricing framework is driven by a single factor. Rather than measuring this factor using the observable combination of forward rates isolated by Cochrane and Piazzesi (2005) in their earlier work, however, the specification here follows Dewachter and Iania (2011), Dewachter, Iania, and Lyrio (2014), and Cieslak and Povala (2015) by treating this "risk" variable as unobservable, identified through the comparison of long-term rates and the expected path of future short-term rates implied by the affine model's cross-equation restrictions. This more flexible approach leaves the model free to focus on the possible linkages between monetary policy, bond risk premia, and the economy, while still imposing enough structure to avoid the overparameterization that, as Bauer (2015) explains, often blurs the view of bond risk premia provided by less highly-constrained term structure models.

The model features, in addition, an unobservable long-run trend component of inflation, interpreted as a time-varying target around which the Federal Reserve has used its interest rate policy to stabilize actual inflation. A fluctuating, but unobserved, inflation target of this kind is introduced into the New Keynesian macroeconomic model by Ireland (2007) and into models that include both macroeconomic and term structure variables by Kozicki and Tinsley (2001*a*, 2001*b*), Dewachter and Lyrio (2006), Hördahl, Tristani, and Vestin (2006), Spencer (2008), Doh (2012), Hördahl and Tristani (2012), and Rudebusch and Swanson (2012). Implied time paths for these unobservable risk premium and inflation target variables, generated using the same Kalman filtering and smoothing algorithm used to estimate model's parameters via maximum likelihood, provide additional insights into the broader effects of monetary policy and other shocks to the US economy. They are examined and discussed below, together with the model's implications for the interplay between monetary policy, bond risk premia, aggregate output, and inflation.

2 Model

Bond yields in this affine pricing model get driven by five state variables: two unobservable and three observable. The first unobservable, denoted by v_t, is a "risk" variable, so called because, as explained below, it governs all variation in bond risk premia. The second unobservable is the central bank's inflation target τ_t, which follows the autoregressive process

$$\tau_t = (1 - \rho_\tau)\tau + \rho_\tau \tau_{t-1} + \sigma_\tau \varepsilon_{\tau t}, \tag{1}$$

where τ measures the average, or steady-state, value of the target, the persistence and volatility parameters satisfy $0 \leq \rho_\tau < 1$ and $\sigma_\tau > 0$, and the serially uncorrelated innovation $\varepsilon_{\tau t}$ has the standard normal distribution. The observable state variables are the short-term (one-period) nominal interest rate r_t, the inflation rate π_t, and the output gap g_t^y.

Although the equations of the model could be specified directly in terms of r_t and π_t, it is more convenient to define the interest rate and inflation gap variables as

$$g_t^r = r_t - \tau_t$$

and

$$g_t^\pi = \pi_t - \tau_t.$$

In Ireland's (2007) extension of the New Keynesian macroeconomic model, a random walk specification for the inflation target generates nonstationary behavior in nominal interest rates and inflation, so that the transformations introduced in these definitions of the interest rate and inflation gaps are needed to obtain an empirical model cast in terms of stationary variables. Here, by contrast, the stationary autoregression (1) for the inflation target implies that interest rates and inflation remain stationary as well. This change in specification works to sidestep the technical problem, noted by Campbell, Lo, and MacKinlay (1997, p.433) and discussed further by Spencer (2008), that asymptotically long-term bond yields become undefined in models, like this one, with homoskedastic shocks when the short-term interest rate follows a process containing a unit root. Of course, settings for the parameter ρ_τ very close to one can – and will – allow the model to explain much of the persistence in nominal variables seen in US data. However, the model also allows for serially correlated movements in inflation π_t away from the central bank's target, implying that the one-step-ahead expectation of inflation, $E_t\pi_{t+1}$, will not generally coincide with τ_t and, by extension, the nominal interest rate gap g_t^r will not generally equal the one-period real interest rate. Instead, the definition $g_t^r = r_t - \tau_t$ of the interest gap reflects the idea that when the central bank raises its inflation target τ_t, it should eventually increase the short-term nominal rate r_t by an equal amount so as to leave the interest rate gap unchanged, but when the central bank wishes to stabilize actual inflation π_t around a given target τ_t, it should raise or lower the nominal rate r_t or, equivalently, increase or decrease the interest rate gap itself.

More specifically, the central bank manages the interest rate gap according to the policy rule

$$g_t^r - g^r = \rho_r(g_{t-1}^r - g^r) + (1-\rho_r)[\rho_\pi g_t^\pi + \rho_y(g_t^y - g^y) + \rho_v v_t] + \sigma_r \varepsilon_{rt}. \tag{2}$$

In (2), ρ_r, satisfying $0 \leq \rho_r < 1$, governs the degree of interest rate smoothing and $\rho_\pi \geq 0$ and $\rho_y \geq 0$ measure the strength of the central bank's policy response when inflation deviates from target or an output gap opens up. The volatility parameter satisfies $\sigma_r > 0$, and the serially uncorrelated monetary policy shock ε_{rt} has the standard normal distribution. Different from those in previous studies, the rule in (2) also allows for a systematic response of monetary policy to changes in the risk variable v_t. While, in the estimation procedure described below, the parameters ρ_π and ρ_y are constrained to be nonnegative, as they are in more conventional Taylor (1993) rule specifications, the response coefficient ρ_v attached to the risk variable is left unconstrained in sign. Thus, the estimate of ρ_v – positive, zero, or negative – will summarize both whether and how the Federal Reserve has reacted to changes in bond risk premia by adjusting its short-term policy rate. Finally, in (2), g^r and g^y denote the steady-state values of the interest rate and output gaps. The inflation gap is assumed to have zero mean, so that actual inflation π_t equals the central bank's target on average, and the risk variable v_t is normalized to have zero mean as well. Thus, the policy rule implies that when inflation equals the central bank's target and the output gap and risk variable equal their own steady-state values, the interest rate gap will gradually converge to its steady-state value, with the speed of convergence determined by the smoothing parameter ρ_r.

Given (1) and (2), describing the conduct of monetary policy, the inflation and output gaps are allowed to depend on their own lagged values and lagged values of the model's other variables, as they would in a more conventional macroeconomic vector autoregression, with

$$g_t^\pi = \rho_{\pi r}(g_{t-1}^r - g^r) + \rho_{\pi\pi} g_{t-1}^\pi + \rho_{\pi y}(g_{t-1}^y - g^y) + \rho_{\pi v} v_{t-1} + \sigma_{\pi\tau}\sigma_\tau \varepsilon_{\tau t} + \sigma_\pi \varepsilon_{\pi t}, \tag{3}$$

and

$$g_t^y - g^y = \rho_{yr}(g_{t-1}^r - g^r) + \rho_{y\pi} g_{t-1}^\pi + \rho_{yy}(g_{t-1}^y - g^y) + \rho_{yv} v_{t-1} + \sigma_{y\pi}\sigma_\pi \varepsilon_{\pi t} + \sigma_{y\tau}\sigma_\tau \varepsilon_{\tau t} + \sigma_y \varepsilon_{yt}, \quad (4)$$

where the volatility parameters satisfy $\sigma_\pi > 0$ and $\sigma_y > 0$ and the serially and mutually uncorrelated innovations $\varepsilon_{\pi t}$ and ε_{yt} both have standard normal distributions. Although (3) and (4) allow for considerable flexibility in the behavior of the macroeconomic state variables, they do, nevertheless, impose some restrictions and identifying assumptions. In particular, (3) and (4) permit innovations in the inflation target τ_t to impact immediately on the inflation and output gaps, but allow for further effects of changes in the inflation target only to the extent that they are not met by proportional changes in the nominal interest rate and inflation rate and therefore affect the interest rate and inflation gaps; these restrictions are meant to impose a form of long-run monetary neutrality that limits the extent to which changes in the inflation target influence the other variables. Equations (3) and (4) also impose the timing restrictions typically incorporated into the specification of more conventional macroeconomic vector autoregressions: they assume, in particular, that shocks to monetary policy and bond risk premia have no contemporaneous effects on the inflation and output gaps and that the innovation ε_{yt} to the output gap has no contemporaneous effect on the inflation gap. These assumptions, similar to those invoked by Bekaert, Hoerova, and Lo Duca (2013), for example, help disentangle the effects of changes in monetary policy and bond risk premia on inflation and output from the effects of changes in inflation and output on monetary policy and bond risk premia. Importantly, however, (3) and (4) allow movements in the risk variable v_t to affect inflation and output with a lag; the signs and magnitudes of the key parameters $\rho_{\pi v}$ and ρ_{yv} from these equations will measure the direction and strength of the macroeconomic effects of shifts in bond risk premia.

Finally, the risk variable v_t's own dynamics are described by

$$v_t = \rho_{vv} v_{t-1} + \sigma_{vr}\sigma_r \varepsilon_{rt} + \sigma_{v\pi}\sigma_\pi \varepsilon_{\pi t} + \sigma_{vy}\sigma_y \varepsilon_{yt} + \sigma_{v\tau}\sigma_\tau \varepsilon_{\tau t} + \sigma_v \varepsilon_{vt}, \quad (5)$$

where the persistence and volatility parameters satisfy $0 \leq \rho_{vv} < 1$ and $\sigma_v > 0$ and the serially uncorrelated innovation ε_{vt} has the standard normal distribution. Though inspired by Cochrane and Piazzesi (2005), Dewacher and Iania (2011), and Dewachter, Iania, and Lyrio's (2014) success in attributing movements in bond risk premia to a single variable, the specific form of (5) resembles most closely Cieslak and Povala's (2015) purely autoregressive specification for this term-structure factor. Equation (5) adds flexibility to Cieslak and Povala's (2015) specification, however, by allowing all of the model's other shocks – to monetary policy, inflation, output, and the inflation target – to have immediate effects on bond risk premia, as they should if asset prices react quickly to all developments in the economy. But (5) merely permits, and does not require, movements in risk premia to have policy or macroeconomic origins, since variations in v_t may also be triggered by the exogenous shock ε_{vt}. Thus, estimates of the correlation and volatility parameters σ_{vr}, $\sigma_{v\pi}$, σ_{vy}, $\sigma_{v\tau}$, and σ_v, together with an analysis of the impulse responses and forecast error variance decompositions implied by those estimates, will be used below to assess the extent to which movements in bond risk premia are driven by monetary policy and macroeconomic shocks or whether they reflect, instead, disturbances that appear purely financial in origin.

Part one of the appendix shows that (1)-(5) can be written more compactly as

$$X_t = \mu + PX_{t-1} + \Sigma\varepsilon_t, \tag{6}$$

by collecting the five state variables into the vector

$$X_t = \begin{bmatrix} g_t^r & g_t^\pi & g_t^y & \tau_t & v_t \end{bmatrix}'$$

and the five innovations into the vector

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{rt} & \varepsilon_{\pi t} & \varepsilon_{yt} & \varepsilon_{\tau t} & \varepsilon_{vt} \end{bmatrix}'.$$

The short-term nominal interest rate r_t anchoring the yield curve can be expressed as a linear function of the state vector by inverting the transformation defining the interest rate gap:

$$r_t = \delta' X_t, \tag{7}$$

where

$$\delta = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix}'.$$

Prices of risk are assigned to each of the state variables, but are allowed to vary over time only in response to movements in the single unobserved factor v_t. Inspired by the work of Cochrane and Piazzesi (2005, 2008), which attributes the bulk of all movements in long-term bond risk premia to variation in a single combination of forward rates, this assumption implies that all variation in risk premia implied by this model will, likewise, be driven by changes in v_t. Unlike the return forecasting factor that Cochrane and Piazzesi (2008) incorporate into their affine term structure model, but similar to the ones used by Dewachter and Iania (2011), Dewachter, Iania, and Lyrio (2014), and Cieslak and Povala (2015) in theirs, the risk-driving variable v_t is treated here as being unobservable in the data. This specification, therefore, is designed to reflect the observation, made implicitly by Cochrane and Piazzesi (2005, 2008) and more explicitly by Bauer (2015), that the large number of parameters included in less highly constrained affine term structure models more frequently lead to overfitting that blurs, rather than sharpens, their interpretation of movements in bond risk premia. At the same time, however, treating the single risk factor v_t as unobservable permits it to move in line with Cochrane and Piazzesi's observable combination of forward rates, but also leaves the estimation procedure free to account for the links, if any, not only between this risk variable and long-term interest rates, but also between bond risk premia, monetary policy, and the behavior of output and inflation.

Thus, in this specification, as in other members of Duffee's (2002) essentially affine class of dynamic term structure models, the log nominal asset pricing kernel takes the form

$$m_{t+1} = -r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}, \tag{8}$$

where the time-varying prices of risk

$$\lambda_t = \begin{bmatrix} \lambda_t^r & \lambda_t^\pi & \lambda_t^y & \lambda_t^\tau & \lambda_t^v \end{bmatrix}'$$

satisfy

$$\lambda_t = \lambda + \Lambda X_t. \tag{9}$$

But while the vector of constant terms in (9),

$$\lambda = \begin{bmatrix} \lambda^r & \lambda^\pi & \lambda^y & \lambda^\tau & \lambda^v \end{bmatrix}', \tag{10}$$

is left unconstrained, the assumption that the unobserved variable v_t is the exclusive source of time-variation in risk premia requires that all but the final column of the matrix

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 & 0 & \Lambda^r \\ 0 & 0 & 0 & 0 & \Lambda^\pi \\ 0 & 0 & 0 & 0 & \Lambda^y \\ 0 & 0 & 0 & 0 & \Lambda^\tau \\ 0 & 0 & 0 & 0 & \Lambda^v \end{bmatrix} \tag{11}$$

consist entirely of zeros.

Equations (6)-(11) imply that the log price p_t^n of an n-period discount bond at time t is determined as an affine function

$$p_t^n = \bar{A}_n + \bar{B}_n' X_t \tag{12}$$

of the state vector by the no-arbitrage condition

$$\exp(p_t^{n+1}) = E_t[\exp(m_{t+1})\exp(p_{t+1}^n)], \tag{13}$$

where the scalars $\bar{A}_n$ and 5×1 vectors $\bar{B}_n$ for $n = 1, 2, 3, \ldots$ can be generated recursively, starting from the initial conditions $\bar{A}_1 = 0$ and $\bar{B}_1' = -\delta'$ required to make (12) for $n = 1$ consistent with (7) for $r_t = -p_t^1$, using the difference equations

$$\bar{A}_{n+1} = \bar{A}_n + \bar{B}_n'(\mu - \Sigma\lambda) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n \tag{14}$$

and

$$\bar{B}_{n+1}' = \bar{B}_n'(P - \Sigma\Lambda) - \delta' \tag{15}$$

obtained, as shown in part two of the appendix, by substituting (7), (8), and (12) into the right-hand side of (13), taking expectations, and matching coefficients after substituting (12) into the left-hand side of the same expression. Once bond prices are found using (14)-(15), the yield y_t^n on an n-period discount bond at time t is easily computed as

$$y_t^n = -\frac{p_t^n}{n} = A_n + B_n' X_t, \tag{16}$$

where $A_n = -\bar{A}_n/n$ and $B_n = -\bar{B}_n/n$ for all values of $n = 1, 2, 3, \ldots$.

Cochrane and Piazzesi (2008) define and discuss various measures of the risk premia incorporated into long-term interest rates. The most familiar, and the one preferred by Rudebusch, Sack, and Swanson (2007) as well, is given by the yield on a long-term bond, minus the average of the short-term rates expected to prevail over the lifetime of that long-term bond:

$$q_t^n = y_t^n - \frac{1}{n}E_t(r_t + r_{t+1} + \ldots + r_{t+n-1}). \tag{17}$$

The n-period bond yield implied by the model used here has already been found using (16).

To compute the expected future short-term rates, use (6) and (7) to obtain

$$E_t r_{t+j} = \delta' E_t X_{t+j} = \delta' \bar{\mu} + \delta' P^j (X_t - \bar{\mu}). \tag{18}$$

where $\bar{\mu} = (I - P)^{-1}\mu$. Combining (16)-(18) yields

$$q_t^n = A_n - \delta' \left(I - \frac{1}{n} \sum_{j=0}^{n-1} P^j \right) \bar{\mu} + \left(B_n' - \delta' \frac{1}{n} \sum_{j=0}^{n-1} P^j \right) X_t. \tag{19}$$

When even the last column of (11) consists of zeros, so that $\Lambda = 0$, (15) implies that the term multiplying X_t on the right-hand side of (19) vanishes and the bond risk premium is constant. Similarly, without variation in the risk variable v_t, the restricted form of Λ in (11) will imply that bond risk premia are constant. Thus, to the extent that evidence of time-variation in bond risk premia does appear in the data, this variation will be attributed by the estimated model to variation in the otherwise unobservable variable v_t.

3 Estimation

Interpreting each of the model's periods as a quarter year in real time, its parameters can be estimated with US data on the short-term nominal interest rate r_t, the inflation rate π_t, the output gap g_t^y, and yields y_t^4, y_t^8, y_t^{12}, y_t^{16}, and y_t^{20} on discount bonds with one through five years to maturity. Figures for inflation and the output gap are drawn from the Federal Reserve Bank of St. Louis' FRED database, with inflation measured by quarter-to-quarter changes in the GDP deflator as reported by the US Department of Commerce and the output gap as the percentage (logarithmic) deviation of the Commerce Department's index of real GDP from the Congressional Budget Office's estimate of potential GDP. The interest rate data are those most commonly used in empirical studies of the term structure. The short-term interest rate is the three-month rate from the Center for Research on Security Prices' Monthly Treasury/Fama Risk Free Rate Files and the long-term discount bond rates are from the CRSP Monthly Treasury/Fama-Bliss Discount Bond Yield Files. To match the quarterly frequency of the inflation and output gap series, quarterly averages of the monthly interest rate observations from the CRSP files are taken.

The dataset begins in 1959:1. Since the model does not impose the zero lower bound on short-term nominal interest rates that has constrained the Federal Reserve since 2008, most of the results are obtained with data running through 2007:4. Thus, the estimation exercise sheds light mainly on the interlinkages between monetary policy, bond risk premia, and the economy as they have appeared during more normal periods of expansion and recession. Nevertheless, some of the model's implications when estimated with data continuing through 2014:4 are discussed below, and a full set of results obtained from data spanning 1959 through 2014 are provided at the end of the appendix.

With eight variables treated as observable and only five fundamental disturbances, at least three of the observables must be interpreted as being measured with error in order to avoid the problem of stochastic singularity discussed by Ireland (2004) for macroeconomic models and Piazzesi (2010, pp.726-727) for affine models of the term structure. Thus, the analysis here follows the general approach first used by Chen and Scott (1993), treating exactly three of the longer-term interest rates as being subject to measurement error, so as to obtain a variant of the model with the same number of observables as shocks. The choice of exactly which rates to view as error-ridden instead of perfectly observed is, admittedly, somewhat arbitrary, but attaching measurement errors to the one, two, and four-year rates forces the estimation procedure to track the three and five-year rates without error; since the short-term interest rate is also taken as perfectly measured, the model's fundamental shocks must then account for most broad movements along the yield curve.

The model can be made to match the average values of the macroeconomic variables together with the average slope of the yield curve, and the estimation exercise can thereby be simplified by using de-meaned data and dropping the constant terms that appear in (6) and (9). To accomplish this, the steady-state value of τ is set equal to the mean inflation rate

π over the sample period, reflecting the assumption made previously that actual inflation equals the central bank's target on average. The steady-state value of the interest rate gap g^r is pinned down by subtracting $\tau = \pi$ from the average value of the short-term nominal interest rate, and the steady-state value g^y is set equal to the average value of the output gap in the data. Part three of the appendix shows that, likewise, steady-state values for the five long-term bond yields can be pinned down through appropriate choices of the five elements of the vector λ that appears in (9) and (10), so as to match the average yields in the data.

Thus, the empirical model consists of (6) with μ set to zero for the state and

$$d_t = UX_t + V\eta_t, \tag{20}$$

for the observables, where

$$d_t = \begin{bmatrix} r_t & \pi_t & g_t^y & y_t^4 & y_t^8 & y_t^{12} & y_t^{16} & y_t^{20} \end{bmatrix}'$$

keeps track of the now de-meaned data and

$$\eta_t = \begin{bmatrix} \eta_t^4 & \eta_t^8 & \eta_t^{16} \end{bmatrix}'$$

is the vector of measurement errors in the one, two, and four-year rates, assumed to be mutually and serially uncorrelated with standard normal distributions. In (20), the matrix U links the observables in d_t to the state vector X_t and imposes the cross-equation restrictions implied by the bond-pricing recursion (15), and the matrix V picks out the three yields that are subject to measurement error and contains the parameters $\sigma_4 > 0$, $\sigma_8 > 0$, and $\sigma_{16} > 0$ measuring the volatility of those errors. Part four of the appendix describes the construction of U and V in more detail.

Equations (6) and (20) are in state-space form, allowing maximum likelihood estimates of model's parameters to be obtained using the Kalman filtering methods outlined by Hamilton

(1994, Ch.13). Two sets of parameter constraints are imposed during estimation. First, for the unobserved variable v_t that, as explained above, is responsible in the model for driving all fluctuations in bond risk premia, if the value of σ_v in its law of motion (5) is scaled up or down by multiplying by some number $\alpha > 0$, then multiplying the parameters σ_{vr}, $\sigma_{v\pi}$, σ_{vy}, and $\sigma_{v\tau}$ in (5) by α and dividing the parameters ρ_v, $\rho_{\pi v}$, ρ_{yv}, Λ^r, Λ^π, Λ^y, Λ^τ, and Λ^v in (2)-(4), (9), and (11) by α leaves the model's implications for the dynamic behavior of all observable variables unchanged. Hence, the constraint $\sigma_v = 0.01$ is imposed as a normalization, to pin down the scale of movements in v_t. Likewise, the sign restriction $\Lambda^\pi < 0$ is imposed during the estimation since no other feature of the model works to determine the direction, positive or negative, in which an increase in v_t changes bond risk premia and all other variables. And while this additional restriction is not needed for normalization, imposing the constraint $\Lambda^v = 0$ implies that the variable v_t works solely, as in Cochrane and Piazzesi (2008) and Cieslak and Povala (2015), to move prices of risk associated with the model's remaining four factors and is not itself a source of time-varying priced risk.

Second, for the inflation target, when the persistence parameter ρ_τ in (1) is left unconstrained, the estimation procedure pushes the value of this parameter very close to its upper bound of one, leading to convergence problems when numerically maximizing the likelihood function. While this result is suggestive of possible specification error, the random walk formulation for the inflation target in Ireland's (2007) New Keynesian model would, as noted above, result in undefined asymptotically long-term bond yields in the affine term structure model used here. In practice, imposing the restriction $\rho_\tau = 0.999$ avoids these problems while remaining consistent with the observation that data strongly prefer an extremely high degree of persistence in the inflation target. Related, but more generally, the estimation procedure also constrains the eigenvalues of the matrix P in (6), governing the "physical" persistence of the state variables, and $P - \Sigma\Lambda$ in (15), governing the "risk neutral" dynamics and hence the pricing of long-term bonds, to be less than one in absolute value, so that the entire system of macroeconomic and bond-pricing equations remains dynamically stable.

Thus, with these normalizations made and restrictions imposed, estimates are obtained for the model's remaining 31 parameters: the coefficients ρ_r, ρ_π, ρ_y, and ρ_v from the monetary policy rule (2), the coefficients $\rho_{\pi r}$, $\rho_{\pi\pi}$, $\rho_{\pi y}$, $\rho_{\pi v}$, ρ_{yr}, $\rho_{y\pi}$, ρ_{yy}, ρ_{yv}, ρ_{vv}, σ_τ, σ_r, σ_π, σ_y, $\sigma_{\pi\tau}$, $\sigma_{y\pi}$, $\sigma_{y\tau}$, σ_{vr}, $\sigma_{v\pi}$, σ_{vy}, and $\sigma_{v\tau}$ governing the persistence, volatility, and comovement between the inflation gap, output gap, and risk variable v_t in (3)-(5), the coefficients Λ^r, Λ^π, Λ^y, Λ^τ describing time-variation in the prices of risk in (9) and (11), and the coefficients σ_4, σ_8, and σ_{16} measuring the volatility of the measurement errors in (20).

4 Results

Table 1 displays the maximum likelihood estimates of the parameters just listed, together with their standard errors, computed using a boostrapping method outlined by Efron and Tibshirani (1993, Ch.6), according to which the model, with its parameters fixed at their estimated values, is used to generate 1,000 samples of artificial data on the same eight variables found in the actual US data. These artificial series then get used to re-estimate the 31 parameters 1,000 times; the standard errors reported in table 1 correspond to the standard deviations of the parameter estimates taken over the 1,000 replications. This bootstrapping procedure thereby accounts for the finite-sample properties of the maximum likelihood estimates as well as all constraints that are imposed during estimation.

Most notable in the table are the estimated parameters from the interest rate rule (2) for monetary policy. The estimate of $\rho_r = 0.62$ implies a considerable amount of interest rate smoothing, a finding that is consistent with many other studies that estimate Taylor (1993) rules in various ways. The point estimates of $\rho_\pi = 0.19$ and $\rho_y = 0.16$ measure monetary policy responses to changes in inflation and the output gap that are roughly balanced, though slightly stronger for prices than output. Both of these policy response coefficients are considerably smaller than estimates reported in studies that use macroeconomic data alone. Ang, Dong, and Piazzesi (2007), on the other hand, estimate values for Taylor rule coefficients in an affine term structure model that are more similar to those found here.

In New Keynesian models, the forward-looking "IS curve" is a log-linearized Euler equation implied by the assumption that consumers have additively-time separable utility functions of the constant relative risk aversion form. Here, the no-arbitrage condition (13), with the more flexible specification for the nominal asset pricing kernel given by (8)-(11), takes the place of the New Keynesian IS curve and the parameters of the modified Taylor rule (2) are identified, in part, by the timing assumptions, reflected in (3) and (4), that monetary policy shocks affect the output gap and inflation with a one-quarter lag. Thus, the comparison between the estimated coefficients of the Taylor rule obtained here and those reported in previous studies speaks directly to the practical importance of issues examined from a variety of different angles by Sims and Zha (2006), Ang, Dong, and Piazzesi (2007), Atkeson and Kehoe (2008), Cochrane (2011), Joslin, Le, and Singleton (2013), and Backus, Chernov, and Zin (2015), each of which finds that the identification of the parameters of interest rate rules for monetary policy is complicated by the similarities between the Taylor (1993) rule, which links the nominal interest rate to output and inflation, and the Euler equation, which in models without investment does much the same thing. Changes in the specification of one of these equations, therefore, can easily change the estimated values of coefficients in the other, implying vastly different behavior on the part of consumers and the central bank.

Of course, (2) differs from the standard Taylor (1993) rule by including the risk variable v_t among those to which the Federal Reserve can respond by adjusting the short-term nominal interest rate. In fact, the positive and statistically significant estimate of $\rho_v = 0.09$ reveals that the Fed has consistently tightened monetary policy in response to shocks that increase bond risk premia. McCallum (2005) embeds a monetary policy rule that moves short-term rates higher after a positive shock to bond risk premia into a model designed to account for the pattern of regression coefficients that Campbell and Shiller (1991), among many others, have obtained when testing the expectations hypothesis of the term structure, by assuming that the Fed responds more directly to the slope of the yield curve when adjusting its policy rate. The rationale for this policy response remains hazy – McCallum speculates that it could arise if policymakers view a steepening yield curve as an indicator that inflation and output growth are due to accelerate and tighten policy as a result – but the positive estimate of ρ_v obtained here provides evidence that the Fed has operated in this way.

Other noteworthy estimates from table 1 are those of $\rho_{\pi v}$ and ρ_{yv} from (3) and (4), measuring the effects of changes in bond risk premia on inflation and the output gap, and σ_{vr}, $\sigma_{v\pi}$, σ_{vy}, and $\sigma_{v\tau}$ from (5), capturing the effects of macroeconomic disturbances on bond risk premia. The former appear small, both in absolute terms and relative to their standard errors, but the latter are more sizable, pointing to statistically significant interactions, in particular, between monetary policy shocks and shocks to output on bond risk premia. The implied relationships, however, can be seen more clearly by plotting impulse response functions and tabulating forecast error variance decompositions than by trying to interpret each coefficient individually. Hence, figures 1 through 5 plot impulse responses to each of the model's five shocks, and tables 2 and 3 report on the variance decompositions. In the graphs, the output gap is shown as a percentage deviation from its steady state, while the inflation and interest rates are all expressed in annualized, percentage-point terms.

The left-hand column of figure 1 shows how a one-standard deviation monetary policy shock ε_r raises the short-term nominal interest rate by slightly less than 60 basis points on impact; the short rate then converges back to its initial value over the following six quarters. The output gap falls and, after a brief and very small increase that resembles the "price puzzle" that frequently appears in more conventional vector autoregressive models of monetary policy shocks and their effects, inflation declines persistently. The risk variable v_t rises in response to the monetary policy shock, so that the long-term interest rates shown in the figure's middle column rise by more than the average of expected future short rates. The right-hand column of the figure confirms, therefore, that the rise in v_t is mirrored by a rise in risk premia built into all five of the longer-term bond rates. Thus, monetary policy shifts the yield curve by affecting risk premia as well as the expected path of short rates.

Figure 2 displays impulse responses to a one-standard deviation shock to v_t, which as shown in the right-hand column, gives rise to increases in all bond risk premia. The output gap and inflation both fall quite persistently in response to this shock, providing evidence consistent with the "practitioner view" described by Rudebusch, Sack, and Swanson (2007) that higher long-term interest rates, reflecting larger bond risk premia, work to slow aggregate economic activity in the same way that more traditional aggregate demand shocks do. As noted above, the positive estimate of ρ_v in the policy rule (2) causes monetary policy to tighten when bond risk premia rise.

Figure 3 plots impulse responses to shocks to the inflation target τ_t. With the persistence parameter ρ_τ in (1) fixed at 0.999, this is the model's most persistent shock, and the simultaneous and roughly equal upward movements in interest rates on bonds of all maturities shown in the figure's middle column indicate that this shock plays the role of the "level factor" that appears in more traditional, affine models of the term structure without macroeconomic variables. The figure's left-hand column shows how actual inflation rises gradually to meet the new, higher target that results from this shock, while the output gap increases, reflecting the implied monetary expansion. The risk variable v_t falls, but only by a small amount, so that changes in the inflation target affect long-term rates mainly by revising the expected future path of short rates; bond risk premia remain nearly unchanged.

In figure 4, the shock ε_π to inflation has small effects on the model's other variables: its effect are mainly on inflation itself although, consistent with the interpretation of this as a "cost-push" shock, the disturbance works as well to decrease the output gap. In figure 5, meanwhile, the shock ε_y to output has effects that might be expected from a non-monetary shock to aggregate demand: it increases both the output gap and inflation and causes interest rates to rise. The risk variable v_t declines following this shock, however, so that bond risk premia fall. Taken together, all these impulse responses are indicative of important multi-directional effects running between monetary policy, bond risk premia, output, and inflation.

Table 2 decomposes the k-quarter-ahead forecast error variance in the output gap, inflation, the short-term interest rate, and bond risk premia into components attributable to each of the model's five fundamental shocks. Since (1) makes the inflation target evolve as an exogenous process, unrelated to any of the model's other shocks or variables, all of its forecast error variance is by assumption allocated to the shock ε_τ; hence, it is excluded from the table. In addition, the law of motion (5) for the risk variable v_t, coupled with the restrictions imposed on the matrix Λ in (11), imply that the forecast error variance for bond risk premia is invariant both to the specific maturity of the bond and the forecast horizon.

The various panels of table 2 show that the monetary policy shock ε_r accounts for sizable components of the variation in the output gap, the short-term interest rate, and bond risk premia. According to the estimated model, in fact, nearly one fifth of all historical movements in bond risk premia are related to monetary policy shocks. Meanwhile, the "practitioner view" referred to by Rudebusch, Sack, and Swanson (2007) is still reflected, but less strongly so, in the variance decompositions: exogenous shocks to bond risk premia account for between 4.9 and 7.7 percent of the variance in the output gap and between 3.7 and 5.1 percent of the variance in inflation at forecast horizons between 3 and 5 years. On the other hand, stronger effects run from the shock ε_y, which, as noted above, acts in the model like a non-monetary aggregate demand disturbance, to bond risk premia: accounting for one quarter of their variance, this shock is even more important than monetary policy in driving movements in risk premia. In total, about 46 percent of all variation in bond risk premia are attributed by the estimates to macroeconomic disturbances, with the remaining 54 percent allocated to purely financial factors, modeled here as exogenous shocks to the risk variable v_t.

Table 3 breaks down, in a similar manner, the forecast error variance in bond yields into components attributable to the five fundamental shocks and, in the cases of the one, two, and four-year bonds, to the measurement errors added to the empirical model to facilitate maximum likelihood estimation. Reassuringly, those tables reveal that measurement errors are quite small, soaking up 4 percent of the one-quarter-ahead variance in the one-year rate, slightly more than 2 percent of the one-quarter-ahead variance in the two-year rate, and only 1 percent of the one-quarter-ahead variance in the four-year rate. Consistent with the association, made through the impulse response analysis, of the model's inflation target with the level factor in more traditional affine models, shocks to the inflation target are shown in table 3 to account for the largest movements in interest rates up and down the yield curve. The monetary policy shock also plays an important role in affecting bond rates, particularly at shorter horizons and for the bonds with shorter terms to maturity. The shock ε_v to bond risk premia, meanwhile, also appears as a key factor in driving sizable movements, especially in the two through four year bond rates, over horizons extending out one to two years.

Returning to table 1, it is also of interest to make note of the estimated parameters from the matrix Λ in (11), governing how movements in the variable v_t translate into changes in the prices of risk attached to the model's fundamental shocks. While Cochrane and Piazzesi (2008) find that the single, observable factor that they associate with time-variation in bond risk premia works to change the pricing of their model's level factor – which, as already noted, seems to resemble most closely the inflation target in the model used here – table 1 shows that the estimate of Λ_τ is small and statistically insignificant. Instead, time variation appears most important in the prices of risk attached to the monetary policy shock ε_r and the inflation and output shocks ε_π and ε_y. Again, the impulse response analysis makes both of these shocks look like traditional, monetary, cost-push, and non-monetary aggregate demand disturbances. These results join with others from above, therefore, to suggest that macroeconomic shocks feed through financial markets and the economy as a whole through multiple channels, most of which are simply not present in existing theoretical models.

Figure 6 provides another view of the model's implications, by plotting estimates of the inflation target τ_t and the five-year risk premium q_t^{20}, obtained using the Kalman smoothing algorithm that is also described by Hamilton (1994, Ch.13). After remaining stable at an annualized rate of about one percent through the mid-1960s, the inflation target rises to a peak of 10 percent in 1981. Comparing the top and bottom panels of the left-hand column shows how the inflation target remains elevated through the end of 1984, even as actual inflation declines. Hence, the model attributes the persistence of high bond yields into the early to mid-1980s in large part to continued high expected inflation during that period, indicative of credibility problems associated with the Federal Reserve's fight against inflation. The inflation target begins its long-run trend downward in 1985 and stabilizes back at a rate of one percent by the end of the sample.

The two panels on the right-hand side of figure 6, meanwhile, exhibit evidence of shifting cyclical patterns in bond risk premia, with the estimated risk premium in the five-year bond rate appearing as highly countercyclical (correlation -0.86 with the output gap) from 1959 through 1989, approximately acyclical (correlation -0.14) from 1990 through 1999, and procyclical (correlation 0.40) from 2000 through 2007. The model can account for these shifting correlations since, as shown in figures 1-5, different shocks give rise to different patterns of comovement between the output gap and bond risk premia, with monetary policy shocks, shocks to the risk variable v_t itself, and shocks to output pushing these variables in opposite directions and shocks to inflation moving them in the same direction.

Campbell, Sunderam, and Viceira (2013) focus on similarly shifting patterns of nominal and real correlations evident in data on nominal and real bond yields and stock returns over the same time periods, suggesting that the preponderance of supply-side shocks hitting the economy during the 1970s and 1980s may explain the positive comovement between bond and stock returns during those decades and the prevalence of demand-side shocks may explain the negative comovement across bond and stock returns in more recent years. Compared to Campbell, Sunderman, and Viceira's, the empirical analysis here excludes data on stock prices and inflation-indexed bond yields but includes data on output itself; moreover, the analysis here uses restrictions on the empirical model to identify shocks with specific, structural interpretations. It is of interest to note, therefore, that the results here seem to point to aggregate demand shocks as drivers of countercyclical bond risk premia both during the inflationary period of the 1960s and 1970s and the disinflationary episode of the 1980s and to shocks to inflation itself and therefore to aggregate supply as a source of procyclical bond risk premia since 2000. Clearly, more detailed structural modeling, both theoretical and empirical, is needed to better understand and reconcile these findings.

Finally, figure 7 repeats the analysis from figure 6 after the model is re-estimated with data running all the way through 2014:4. These additional results need to be interpreted with caution: since the model does not account for the zero lower bound on the short-term nominal interest rate, forecasts of future short rates implied by (18) may differ from expectations of future short rates held by financial market participants over the period since 2008 when this constraint has been a binding one for the Fed. The figure's top two panels reveal, however, that the model attributes the very low long-term bond yields observed over the last eight years of the sample period not to unusually low risk premia but instead to further reductions in the inflation target, which is estimated to be negative from the end of 2011 through the middle of 2013 and close to zero thereafter. Again, more detailed modeling, to account for both the effects of the zero lower bound on expectations of future short rates and the effects of the Federal Reserve's large-scale asset purchase programs on bond risk premia, seems needed to interpret these movements more fully.

5 Conclusion

The Federal Reserve's recent policies of large scale asset purchases, more popularly known as "quantitative easing," rely on the widely-held view that monetary policy actions can influence the risk premia built into long-term bond rates and that changes in bond risk premia can then have impacts, working through aggregate demand channels, on output and inflation as well. As Rudebusch, Sack, and Swanson (2007) explain, however, surprisingly little evidence has been compiled to support this "practitioner view," even in data from more normal times. Using an affine model of the term structure with observable and unobservable macroeconomic factors, the empirical analysis here looks for – and finds – such evidence. Monetary policy shocks, identified using restrictions borrowed from the literature that works with more conventional, macroeconomic vector autoregressions but imposed here, instead, on the driving processes for the macroeconomic state variables in a term structure model, do appear to influence bond risk premia, with monetary policy tightenings working to increase those premia and, consistent with the goals of quantitative easing, monetary policy easings working to decrease them. In addition, purely exogenous shocks to bond risk premia, identified by restricting the determinants of those risk premia in a manner that is inspired by the work of Cochrane and Piazzesi (2006, 2008), Dewachter and Iania (2011), Dewachter, Iania, and Lyrio (2014), and Cieslak and Povala (2015), do appear to work like aggregate demand disturbances, with higher risk premia associated with slower output growth and inflation and, again consistent with the intended workings of quantitative easing, lower risk premia associated with faster output growth and inflation.

The estimated model, however, also allows for and provides evidence of other channels through which monetary policy, bond risk premia, and the macroeconomy interact. The extended version of the Taylor (1993) rule, for example, that is included in the estimated model indicates that, historically, the Federal Reserve has moved to raise the short-term interest rate, not only in response to shocks that increase output and inflation, but also when bond risk premia rise, in a manner that is consistent with McCallum's (2005) earlier analysis. In addition, different structural disturbances identified by the model move output, inflation, and bond risk premia in a variety of directions, helping to account for the shifting correlations between these variables seen in the data.

Thus, monetary policy affects bond risk premia and the economy; bond risk premia affect monetary policy and the economy, and the economy affects monetary policy and bond risk premia. Standard, textbook New Keynesian models like Woodford (2003) and Galí's (2008) do not even begin to consider the channels through which all of these connections are made; and even the most ambitious extensions thus far, such as Andrés, López-Salido, and Nelson's (2004), account only for a small subset. Much more research along these lines is needed, to fully understand how the workings of monetary policy and financial markets have and will continue to interact to shape the performance of the American economy.

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Table 1. Maximum Likelihood Estimates and Standard Errors

Parameter	ML Estimate	Std Error
ρ_r	0.6211	0.0433
ρ_π	0.1922	0.0894
ρ_y	0.1563	0.0301
ρ_v	0.0874	0.0161
$\rho_{\pi r}$	0.1325	0.1301
$\rho_{\pi\pi}$	0.8537	0.0490
$\rho_{\pi y}$	−0.0010	0.0221
$\rho_{\pi v}$	−0.0213	0.0165
ρ_{yr}	−0.9646	0.4151
$\rho_{y\pi}$	0.0459	0.1690
ρ_{yy}	1.0128	0.0722
ρ_{yv}	0.0250	0.0492
ρ_{vv}	0.9001	0.0341
σ_τ	0.0012	0.0001
σ_r	0.0013	0.0001
σ_π	0.0026	0.0001
σ_y	0.0077	0.0004
$\sigma_{\pi\tau}$	−0.4437	0.1868
$\sigma_{y\pi}$	−0.2267	0.2206
$\sigma_{y\tau}$	0.9585	0.5964
σ_{vr}	4.6064	1.7366
$\sigma_{v\pi}$	−0.9421	0.7623
σ_{vy}	−0.8876	0.2814
$\sigma_{v\tau}$	−0.0945	1.4534
Λ_r	−7.2018	3.6793
Λ_π	−16.7834	8.6905
Λ_y	−26.3763	10.3782
Λ_τ	−0.0103	3.1824
σ_4	0.0004	0.0000
σ_8	0.0002	0.0000
σ_{16}	0.0001	0.0000

Note: The table reports the maximum likelihood estimate and bootstrapped standard error of each parameter listed.

Table 2. Forecast Error Variance Decompositions

Output Gap					
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	0.0	0.0	2.0	0.6	97.4
4	5.8	0.2	2.0	0.7	91.3
12	16.6	4.9	1.7	1.6	75.2
20	18.1	7.7	1.6	1.9	70.7
∞	18.1	8.1	1.6	1.9	70.2
Inflation					
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	0.0	0.0	6.0	94.0	0.0
4	0.1	1.0	10.7	86.7	1.5
12	0.7	3.7	23.8	61.6	10.2
20	2.1	5.1	33.9	46.0	12.8
∞	0.3	0.6	93.7	4.1	1.3
Short-Term Interest Rate					
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	57.2	3.0	38.2	0.1	1.4
4	33.0	7.9	52.0	0.3	6.8
12	15.1	5.2	68.2	0.2	11.3
20	11.1	3.8	75.9	0.2	8.9
∞	0.6	0.2	98.6	0.0	0.5
Bond Risk Premia					
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	18.1	53.7	0.0	3.1	25.1
4	18.1	53.7	0.0	3.1	25.1
12	18.1	53.7	0.0	3.1	25.1
20	18.1	53.7	0.0	3.1	25.1
∞	18.1	53.7	0.0	3.1	25.1

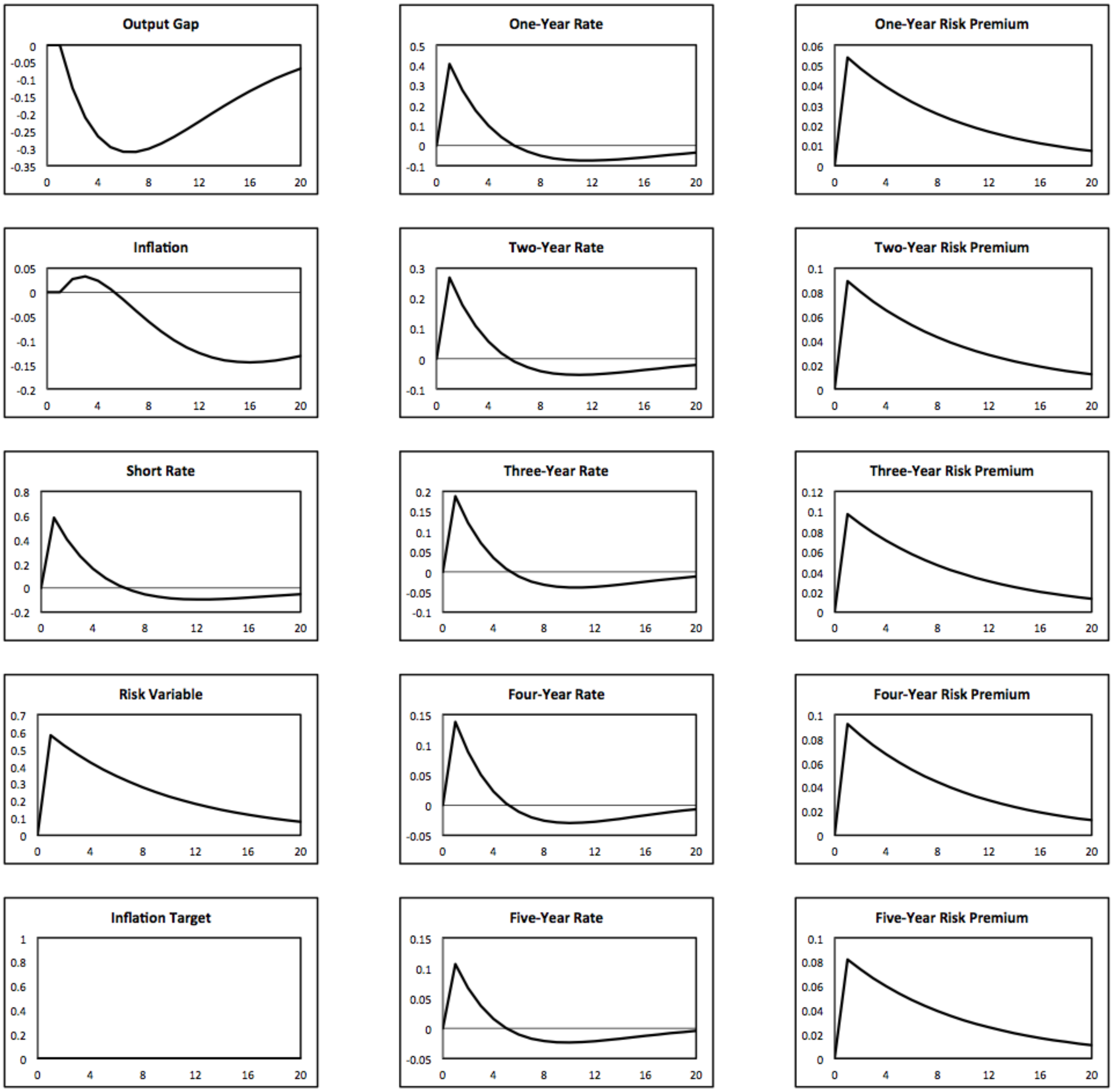
Note: Each row decomposes the forecast error variance in the indicated variable at the indicated horizon into percentages attributable to each of the model's five shocks.

Table 3. Forecast Error Variance Decompositions

One-Year Bond Rate						
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	32.5	15.3	46.0	0.0	2.1	4.0
4	17.2	17.0	58.6	0.0	5.9	1.2
12	7.9	9.4	74.1	0.1	7.9	0.5
20	5.8	6.5	81.2	0.2	5.9	0.4
∞	0.3	0.3	99.0	0.0	0.3	0.0
Two-Year Bond Rate						
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	16.9	22.9	55.8	0.0	2.1	2.2
4	8.3	20.1	66.7	0.1	4.2	0.6
12	3.8	10.3	80.8	0.3	4.5	0.3
20	2.7	6.9	86.7	0.3	3.2	0.2
∞	0.1	0.3	99.4	0.0	0.2	0.0
Three-Year Bond Rate						
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	9.9	21.8	66.3	0.2	1.8	0.0
4	4.5	17.3	75.0	0.3	2.9	0.0
12	2.0	8.4	86.5	0.4	2.7	0.0
20	1.4	5.5	90.9	0.4	1.8	0.0
∞	0.1	0.3	99.6	0.0	0.1	0.0
Four-Year Bond Rate						
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	6.0	17.1	74.1	0.4	1.4	1.0
4	2.6	13.0	81.7	0.4	2.0	0.3
12	1.1	6.1	90.6	0.5	1.6	0.1
20	0.8	3.9	93.8	0.4	1.1	0.1
∞	0.0	0.2	99.7	0.0	0.0	0.0
Five-Year Bond Rate						
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	4.0	12.8	81.6	0.5	1.1	0.0
4	1.6	9.4	87.0	0.5	1.5	0.0
12	0.7	4.2	93.5	0.5	1.1	0.0
20	0.5	2.7	95.8	0.4	0.7	0.0
∞	0.0	0.1	99.8	0.0	0.0	0.0

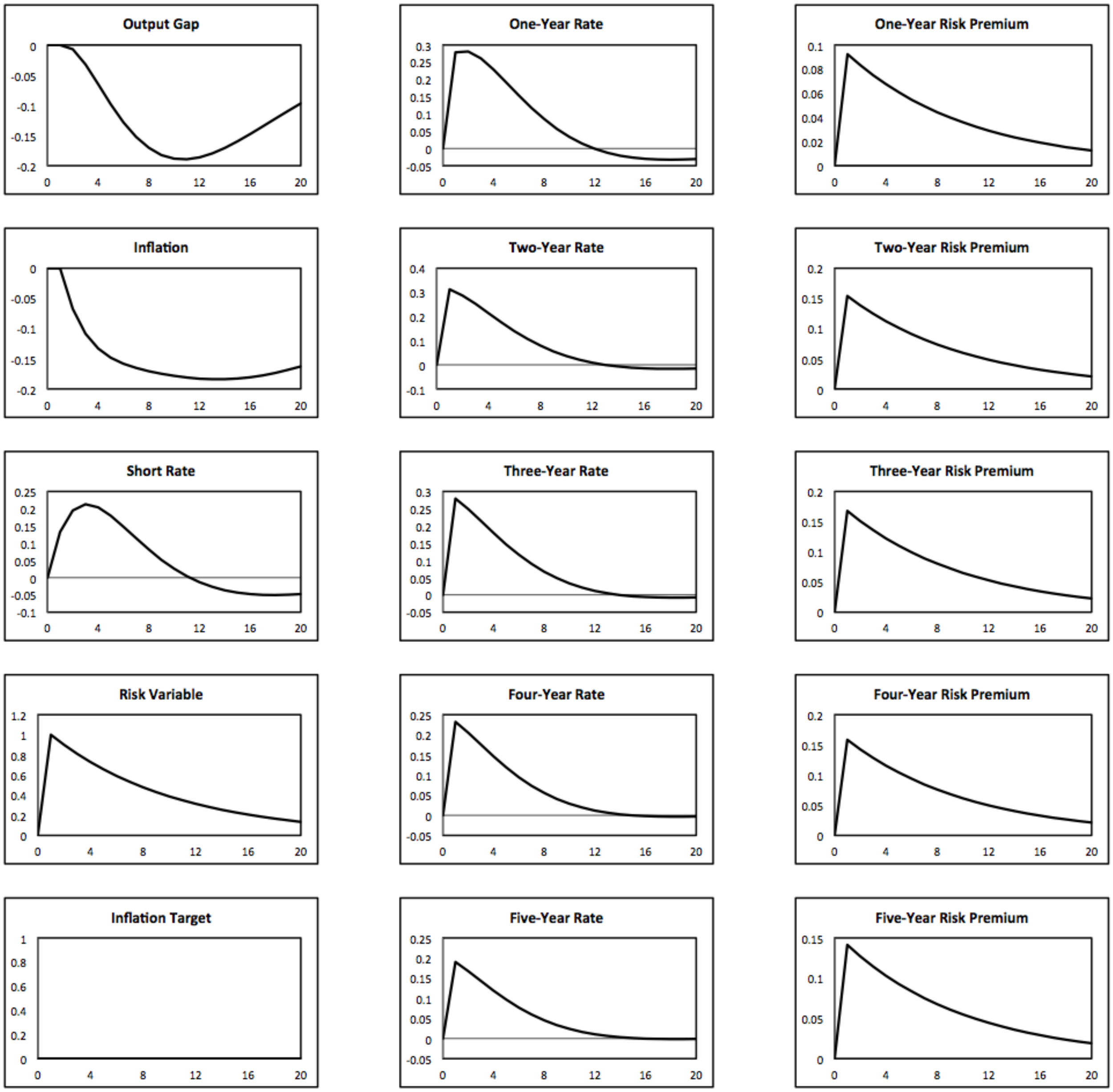
Note: Each row decomposes the forecast error variance in the indicated bond yield at the indicated horizon into percentages attributable to each of the model's shocks and measurement error.

Figure 1. Impulse Responses to a Monetary Policy Shock



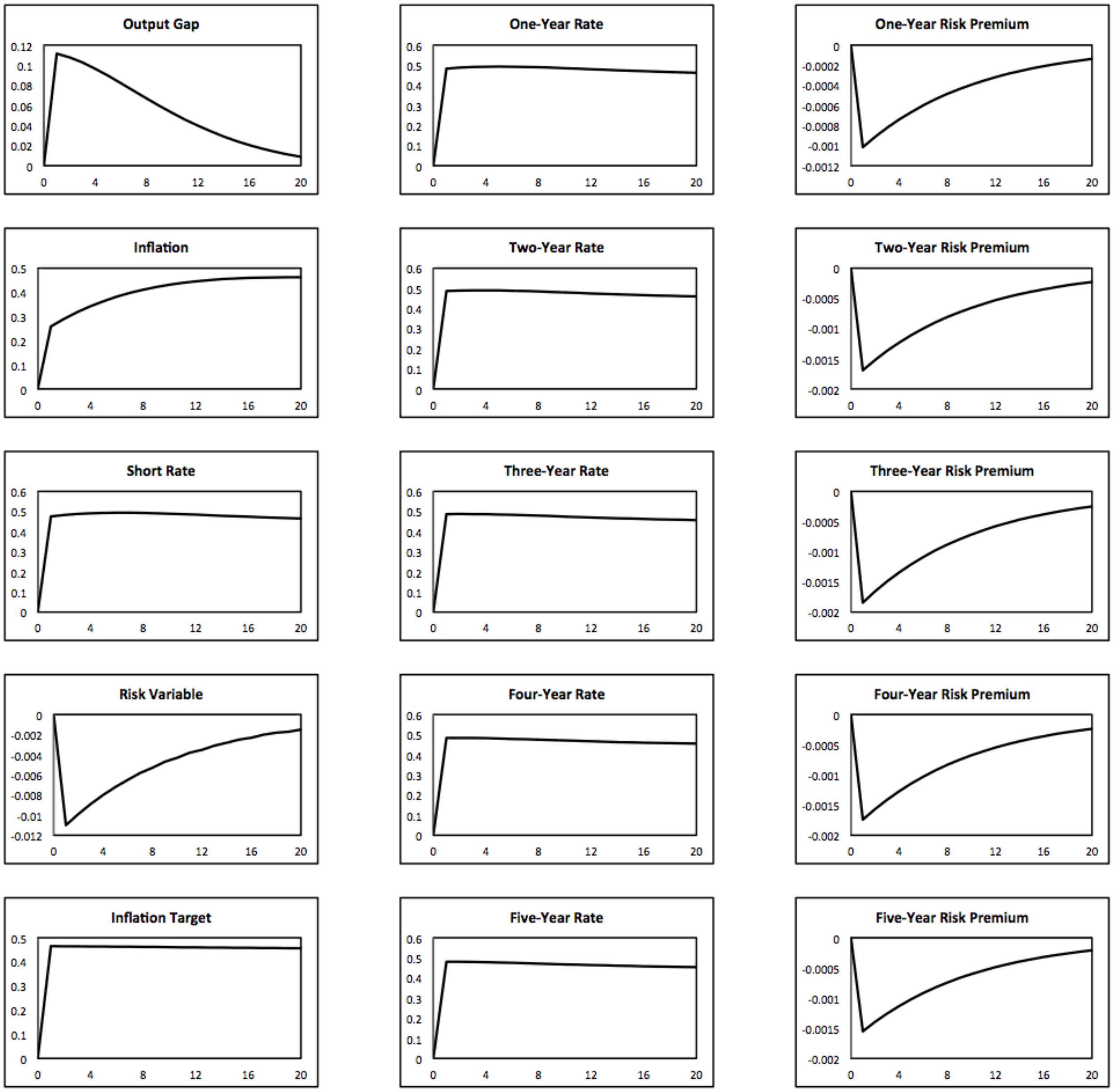
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation monetary policy shock ε_r. The inflation and interest rates are in annualized terms.

Figure 2. Impulse Responses to a Risk Premium Shock



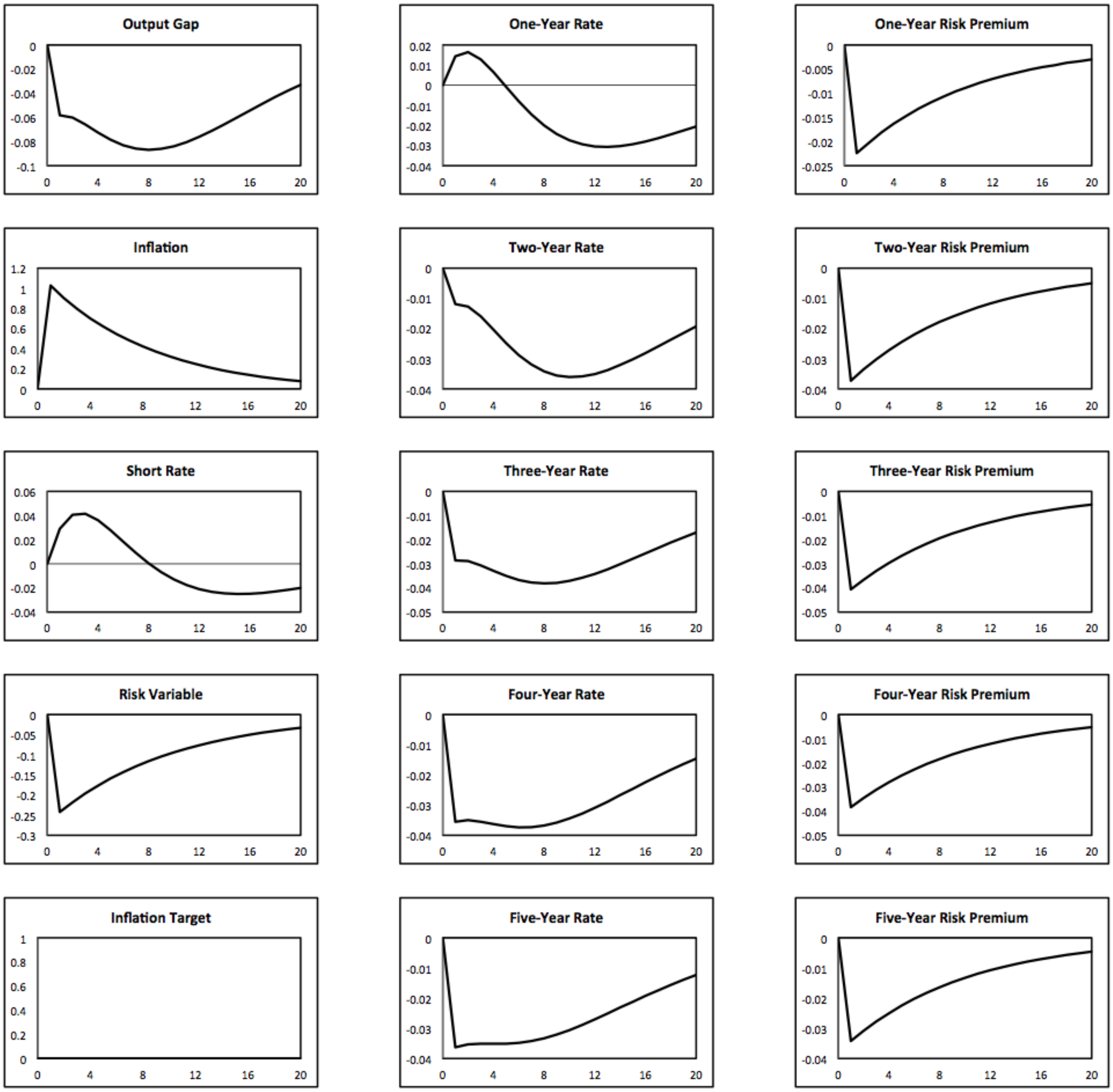
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation risk premium shock ε_v. The inflation and interest rates are in annualized terms.

Figure 3. Impulse Responses to an Inflation Target Shock



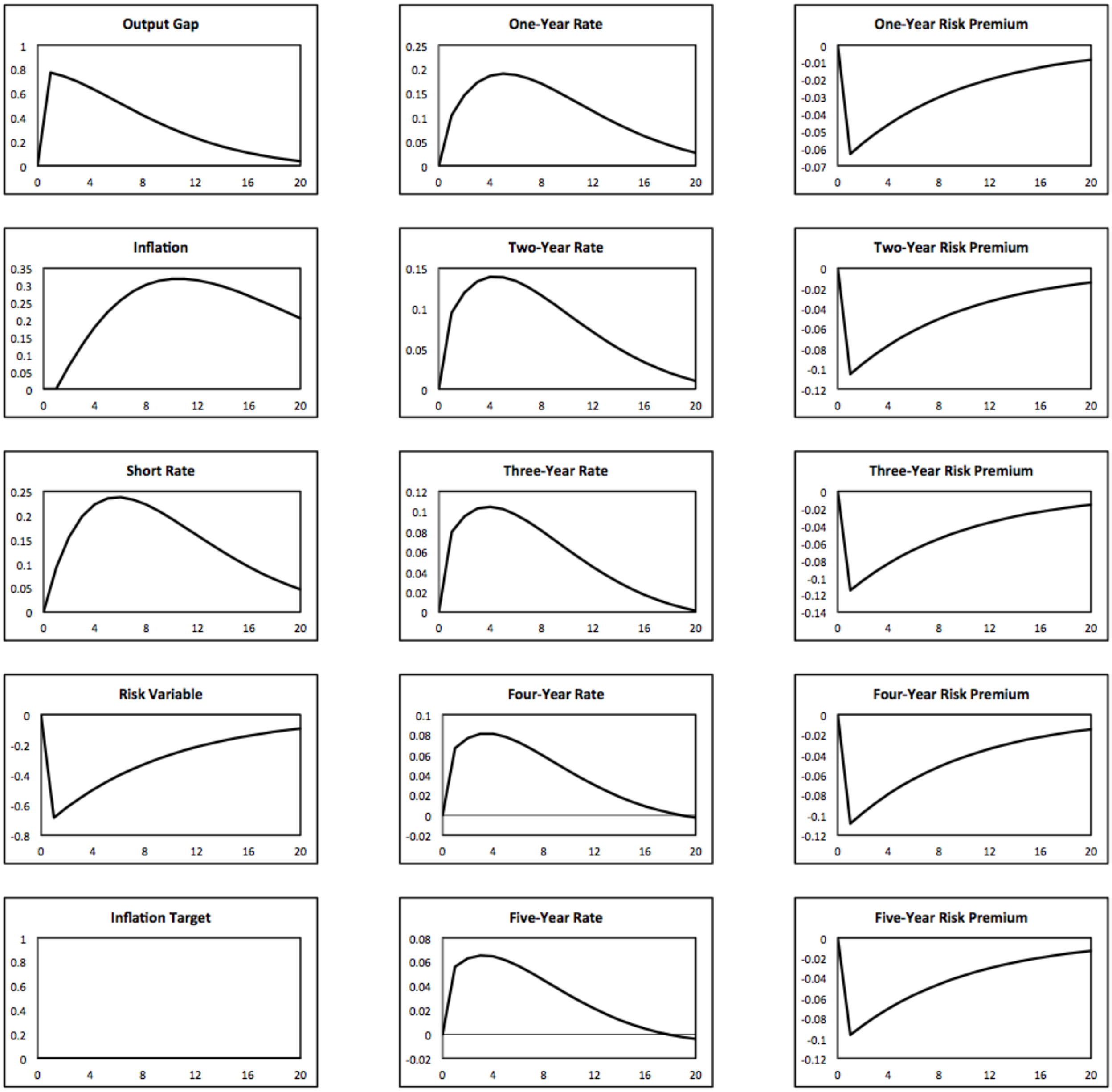
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation inflation target shock ε_{τ}. The inflation and interest rates are in annualized terms.

Figure 4. Impulse Responses to an Inflation Shock



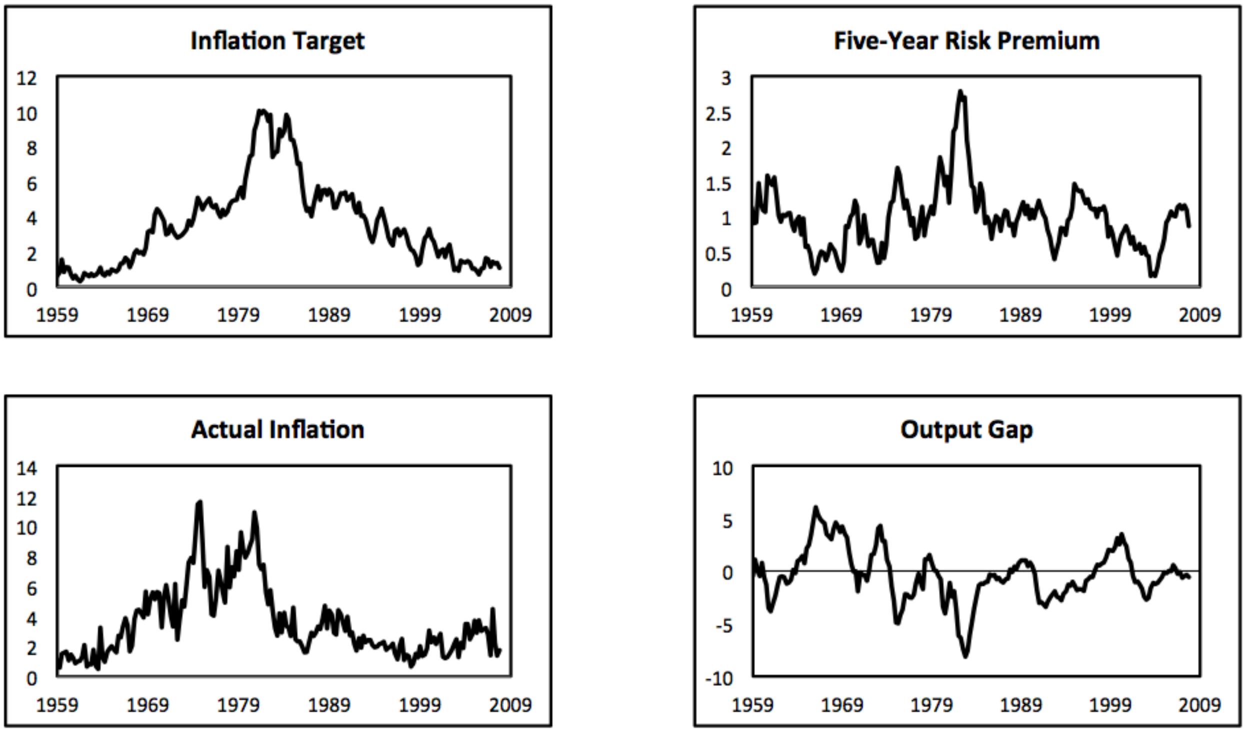
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation inflation shock ε_π. The inflation and interest rates are in annualized terms.

Figure 5. Impulse Responses to an Output Shock



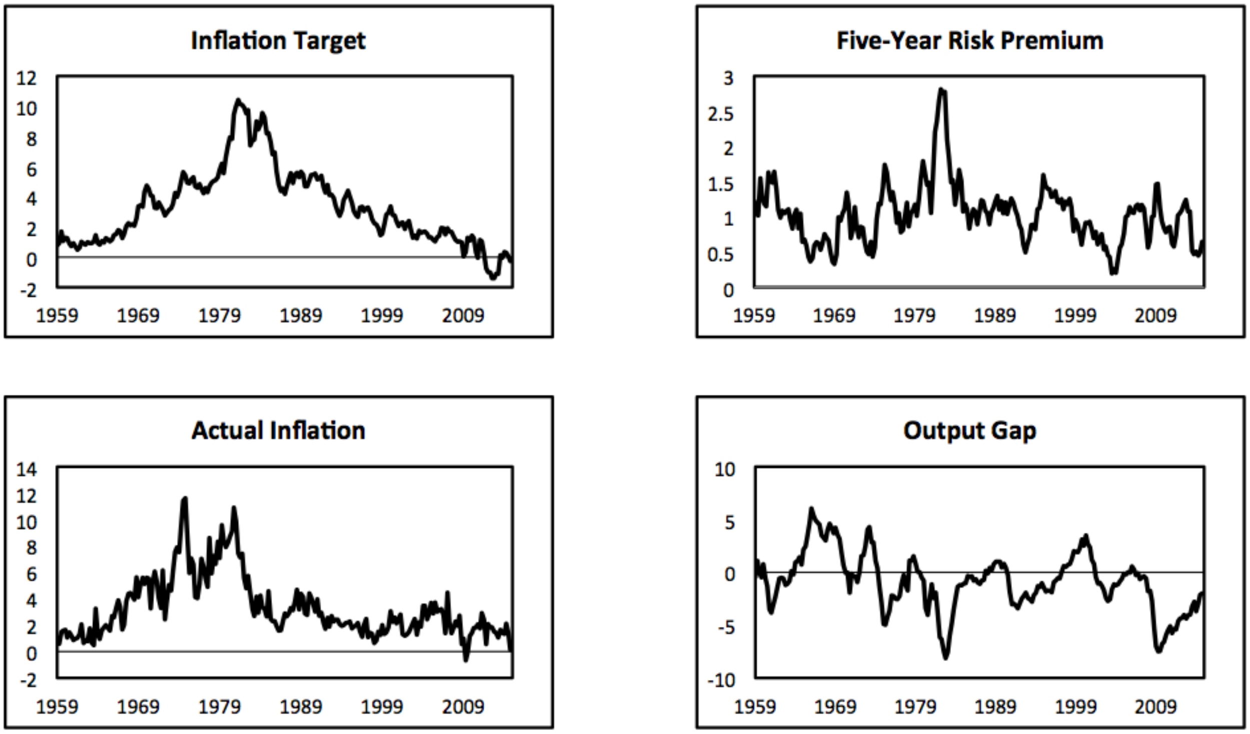
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation output shock ε_y. The inflation and interest rates are in annualized terms.

Figure 6. Smoothed Estimates of the Inflation Target and the Five-Year Bond Risk Premium



The left-hand column compares smoothed estimates of the inflation target from the model to the actual series for the inflation rate, when both measures are expressed in annualized terms. The right-hand column compares the smoothed estimates of the risk premium in the five-year bond yield to the actual series for the output gap, when the risk premium is expressed in annualized terms and the output gap in percentage points.

Figure 7. Smoothed Estimates of the Inflation Target and the Five-Year Bond Risk Premium, Extended Sample Period: 1959:1 - 2014:4



The left-hand column compares smoothed estimates of the inflation target from the model to the actual series for the inflation rate, when both measures are expressed in annualized terms. The right-hand column compares the smoothed estimates of the risk premium in the five-year bond yield to the actual series for the output gap, when the risk premium is expressed in annualized terms and the output gap in percentage points.

7 Appendix

7.1 Putting the Macroeconomic Model in Matrix Form

Rewriting the individual equations (1)-(5) in matrix form as (6) involves stacking them together, first, as

$$P_0 X_t = \mu_0 + P_1 X_{t-1} + \Sigma_0 \varepsilon_t,$$

where the state and innovation vectors X_t and ε_t are as defined in the text and where the matrices P_0, P_1, and Σ_0 and the vector μ_0 are given by

$$P_0 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & -(1-\rho_r)\rho_\pi & -(1-\rho_r)\rho_y & 0 & -(1-\rho_r)\rho_v \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & \rho_\tau & 0 \\ \rho_r & 0 & 0 & 0 & 0 \\ \rho_{\pi r} & \rho_{\pi\pi} & \rho_{\pi y} & 0 & \rho_{\pi v} \\ \rho_{yr} & \rho_{y\pi} & \rho_{yy} & 0 & \rho_{yv} \\ 0 & 0 & 0 & 0 & \rho_{vv} \end{bmatrix},$$

$$\Sigma_0 = \begin{bmatrix} 0 & 0 & 0 & \sigma_\tau & 0 \\ \sigma_r & 0 & 0 & 0 & 0 \\ 0 & \sigma_\pi & 0 & \sigma_{\pi\tau}\sigma_\tau & 0 \\ 0 & \sigma_{y\pi}\sigma_\pi & \sigma_y & \sigma_{y\tau}\sigma_\tau & 0 \\ \sigma_{vr}\sigma_r & \sigma_{v\pi}\sigma_\pi & \sigma_{vy}\sigma_y & \sigma_{v\tau}\sigma_\tau & \sigma_v \end{bmatrix},$$

and

$$\mu_0 = \begin{bmatrix} (1-\rho_\tau)\tau \\ (1-\rho_r)(g^r - \rho_y g^y) \\ -\rho_{\pi r} g^r - \rho_{\pi y} g^y \\ (1-\rho_{yy}) g^y - \rho_{yr} g^r \\ 0 \end{bmatrix}.$$

Multiplying this equation through by P_0^{-1} and defining $\mu = P_0^{-1}\mu_0$, $P = P_0^{-1}P_1$, and $\Sigma = P_0^{-1}\Sigma_0$ then leads directly to (6).

7.2 Deriving the Bond Pricing Equations

To derive the bond pricing equations (14)-(15), start by setting $n = 0$ and substitute (8) and $p_{t+1}^0 = 0$ (the price of at $t+1$ of a claim to a dollar at $t+1$ equals one dollar, hence zero after taking logs) into the right-hand side of (13) to obtain

$$\exp(p_t^1) = E_t[\exp(m_{t+1})] = \exp(-r_t).$$

Hence, consistency between (7) and (12) requires that

$$\bar{A}_1 + \bar{B}_1' X_t = -\delta' X_t$$

or, equivalently, that $\bar{A}_1 = 0$ and $\bar{B}_1' = -\delta'$ as noted in the text.

Next, for an arbitrary value of $n = 1, 2, 3, \ldots$, substitute (7), (8), and (12) into the right-hand side of (13) to obtain

$$\begin{aligned}
\exp(p_t^{n+1}) &= E_t\left[\exp\left(-\delta' X_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\exp\left(\bar{A}_n + \bar{B}_n' X_{t+1}\right)\right] \\
&= \exp\left(-\delta' X_t - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_n\right) E_t\left[\exp\left(-\lambda_t'\varepsilon_{t+1} + \bar{B}_n' X_{t+1}\right)\right] \\
&= \exp\left(-\delta' X_t - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_n\right) E_t\left\{\exp\left[-\lambda_t'\varepsilon_{t+1} + \bar{B}_n'\left(\mu + P X_t + \Sigma\varepsilon_{t+1}\right)\right]\right\} \\
&= \exp\left[\bar{A}_n + \bar{B}_n'\mu + \left(\bar{B}_n' P - \delta'\right) X_t - \frac{1}{2}\lambda_t'\lambda_t\right] E_t\left\{\exp\left[-(\lambda_t' - \bar{B}_n'\Sigma)\varepsilon_{t+1}\right]\right\} \\
&= \exp\left[\bar{A}_n + \bar{B}_n'\mu + \left(\bar{B}_n' P - \delta'\right) X_t - \frac{1}{2}\lambda_t'\lambda_t\right] \\
&\quad \times \exp\left[\frac{1}{2}\lambda_t'\lambda_t - \bar{B}_n'\Sigma\left(\lambda + \Lambda X_t\right) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n\right] \\
&= \exp\left[\bar{A}_n + \bar{B}_n'\left(\mu - \Sigma\lambda\right) + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n + \left(\bar{B}_n' P - \bar{B}_n'\Sigma\Lambda - \delta'\right) X_t\right],
\end{aligned}$$

where the second equality simply moves objects that are known at t outside of the conditional expectation, the third equality uses the law of motion (6) for the state vector, the fourth equality again moves objects that are known outside the expectation, the fifth equality uses the normality of ε_{t+1} to compute the expectation of the exponential function involving the vector of shocks, and the sixth equality simplifies the resulting expression. Matching coefficients after (12) is substituted into the left-hand side of this equation then implies that the difference equations (14) and (15) must hold as well.

7.3 Matching Average Long-Term Bond Yields

To see how the five elements of the vector λ in (9) and (10) can be calibrated to match the average yields on one through five-year discount bonds, multiply both sides of (15) by $\bar{\mu} = (1 - P)^{-1}\mu$, which by (6) keeps track of the steady-state values of the variables in the state vector X_t, and add the results to (14), after replacing μ with $(I - P)\bar{\mu}$, to obtain

$$\bar{A}_{n+1} + \bar{B}_{n+1}'\bar{\mu} = \bar{A}_n + \bar{B}_n'\bar{\mu} - \bar{B}_n'\Sigma\lambda \; + \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n - \bar{B}_n'\Sigma\Lambda\bar{\mu} - \delta'\bar{\mu}.$$

From (16), each term of the form $\bar{A}_n + \bar{B}_n'\bar{\mu}$ that enters into this difference equation equals $-ny^n$, where y^n denotes the steady-state yield on an n-period discount bond. Hence, the difference equation can be written more compactly as

$$(n+1)y^{n+1} = ny^n + z_n + \bar{B}_n'\Sigma\lambda,$$

where

$$z_n = \bar{B}_n'\Sigma\Lambda\bar{\mu} + \delta'\bar{\mu} - \frac{1}{2}\bar{B}_n'\Sigma\Sigma'\bar{B}_n.$$

Solving this difference equation forward starting from $y_1 = r$, where r is the steady-state value of the short-term nominal interest rate, yields

$$y^n = \frac{1}{n}\left(r + \sum_{j=1}^{n-1} z_j\right) + \left(\frac{1}{n}\sum_{j=1}^{n-1}\bar{B}_n'\Sigma\right)\lambda$$

for all $n = 2, 3, 4, \ldots$.

The average yield on an n-period bond can be used to measure y^n, the average values of the macroeconomic variables can be used to measure r and the elements of $\bar{\mu}$, and the estimated values of the parameters governing the model's dynamics can be used to compute z_n and $\bar{B}_n'\Sigma$ for all $n = 2, 3, 4, \ldots$. Since observations on yields at five longer maturities are used in the estimation, five versions of this last equation, with $n = 4$, $n = 8$, $n = 12$, $n = 16$, and $n = 20$ can be stacked into a vector, and the 5×5 matrix formed from the partial sums of the $\bar{B}_n'\Sigma$ terms on the right-hand side inverted, so as to solve uniquely for the elements of the 5×1 vector λ that both accurately demean the data and allow the model to match the average slope of the yield curve.

7.4 Constructing the State Space Model

The matrix U from the observation equation (20) in the state space model is given by

$$U = \begin{bmatrix} U_r \\ U_\pi \\ U_y \\ B_4' \\ B_8' \\ B_{12}' \\ B_{16}' \\ B_{20}' \end{bmatrix},$$

where the first three rows, with

$$U_r = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$U_\pi = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix},$$

and

$$U_y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

recover the observable short-term interest rate, inflation, and output gap variables from the state vector and the remaining rows are determined by the bond-pricing recursion (15) and therefore reflect the cross-equation restrictions imposed by the affine term structure model.

The form of the matrix V,

$$V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sigma_4 & 0 & 0 \\ 0 & \sigma_8 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{16} \\ 0 & 0 & 0 \end{bmatrix},$$

reflects the assumption that the one, two, and four-year bond yields are measured with errors, having standard deviations $\sigma_4 > 0$, $\sigma_8 > 0$, and $\sigma_{16} > 0$.

7.5 Extending the Sample Period

Table 4 displays maximum likelihood estimates of and standard errors for the model's 31 parameters obtained when the sample period is extended to run through 2014:4; in general, these estimates are similar to those reported in table 1 for the sample period that ends in 2007:4. Likewise, the forecast error variance decompositions in tables 5 and 6 and the impulse response functions in figures 8-12 for the extended sample period mostly resemble those from tables 2 and 3 and figures 1-5 for the period ending in 2007:4. One exception is that the shock to the inflation target appears somewhat more important when the model is estimated with the longer sample reflecting, perhaps, the model's attempt, shown most clearly in figure 7, to attribute low long-term bond yields to a negative inflation target beginning in 2011.

Table 4. Maximum Likelihood Estimates and Standard Errors

Extended Sample Period: 1959:1 - 2014:4

Parameter	ML Estimate	Std Error
ρ_r	0.6705	0.0392
ρ_π	0.1897	0.0796
ρ_y	0.1279	0.0205
ρ_v	0.0709	0.0164
$\rho_{\pi r}$	0.1994	0.1009
$\rho_{\pi\pi}$	0.8313	0.0431
$\rho_{\pi y}$	−0.0126	0.0146
$\rho_{\pi v}$	−0.0318	0.0131
ρ_{yr}	−1.1288	0.3716
$\rho_{y\pi}$	−0.0369	0.1551
ρ_{yy}	1.0450	0.0522
ρ_{yv}	0.0436	0.0411
ρ_{vv}	0.8847	0.0336
σ_τ	0.0012	0.0001
σ_r	0.0013	0.0001
σ_π	0.0025	0.0001
σ_y	0.0078	0.0004
$\sigma_{\pi\tau}$	−0.2578	0.1901
$\sigma_{y\pi}$	−0.1670	0.2241
$\sigma_{y\tau}$	0.7065	0.5886
σ_{vr}	4.3681	1.5903
$\sigma_{v\pi}$	−0.9491	0.6190
σ_{vy}	−0.6878	0.2097
$\sigma_{v\tau}$	−1.2116	1.3240
Λ_r	−9.6260	4.1114
Λ_π	−18.8791	9.3501
Λ_y	−19.7252	14.2870
Λ_τ	−1.7198	3.9688
σ_4	0.0004	0.0000
σ_8	0.0002	0.0000
σ_{16}	0.0001	0.0000

Note: The table reports the maximum likelihood estimate and bootstrapped standard error of each parameter listed.

Table 5. Forecast Error Variance Decompositions

Extended Sample Period: 1959:1 - 2014:4

Output Gap

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	0.0	0.0	1.1	0.3	98.7
4	6.4	0.0	1.1	1.4	91.1
12	20.4	0.2	1.3	6.9	71.2
20	23.0	0.4	1.3	10.2	65.1
∞	22.9	0.5	1.3	10.8	64.5

Inflation

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	0.0	0.0	10.9	89.1	0.0
4	0.2	2.6	17.4	78.9	0.8
12	0.4	7.1	31.6	54.8	6.2
20	1.3	7.2	41.9	41.1	8.6
∞	0.2	0.7	94.6	3.6	0.9

Short-Term Interest Rate

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	57.7	1.7	39.2	0.2	1.3
4	35.4	5.0	52.3	0.4	6.9
12	15.7	3.9	66.7	0.6	13.1
20	11.8	2.6	74.0	1.3	10.3
∞	0.7	0.1	98.5	0.1	0.6

Bond Risk Premia

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output
1	18.0	60.1	1.2	3.3	17.4
4	18.0	60.1	1.2	3.3	17.4
12	18.0	60.1	1.2	3.3	17.4
20	18.0	60.1	1.2	3.3	17.4
∞	18.0	60.1	1.2	3.3	17.4

Note: Each row decomposes the forecast error variance in the indicated variable at the indicated horizon into percentages attributable to each of the model's five shocks.

Table 6. Forecast Error Variance Decompositions

Extended Sample Period: 1959:1 - 2014:4

One-Year Bond Rate

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	36.4	13.3	43.1	0.0	2.2	5.0
4	20.2	15.3	55.8	0.0	7.0	1.6
12	9.1	9.0	70.0	1.0	10.3	0.6
20	7.0	6.0	77.3	1.6	7.6	0.4
∞	0.4	0.3	98.8	0.1	0.4	0.0

Two-Year Bond Rate

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	19.7	24.4	50.5	0.2	2.9	2.3
4	9.5	22.0	61.2	0.4	6.2	0.7
12	4.6	12.0	74.4	1.8	7.0	0.3
20	3.6	7.9	81.3	2.1	4.9	0.2
∞	0.2	0.4	99.1	0.1	0.2	0.0

Three-Year Bond Rate

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	10.6	25.9	59.4	0.9	3.1	0.0
4	4.5	21.2	67.9	1.3	5.1	0.0
12	2.5	11.1	79.3	2.5	4.7	0.0
20	2.0	7.3	85.2	2.4	3.1	0.0
∞	0.1	0.3	99.3	0.1	0.2	0.0

Four-Year Bond Rate

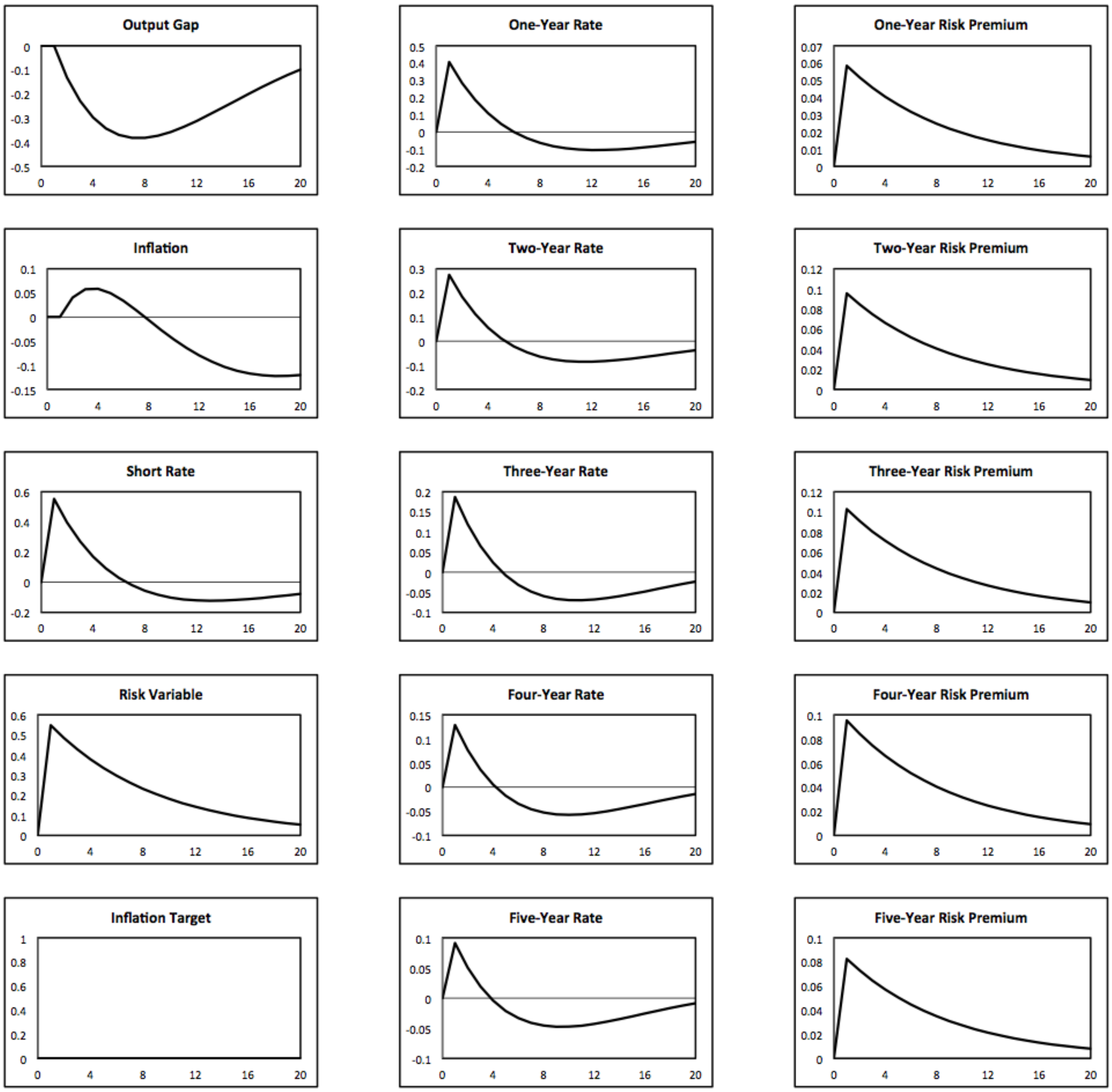
Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	5.6	21.8	67.0	1.6	2.9	1.1
4	2.2	17.2	74.3	2.0	3.9	0.3
12	1.4	8.9	83.8	2.7	3.1	0.1
20	1.1	5.8	88.6	2.4	2.0	0.1
∞	0.1	0.3	99.5	0.1	0.1	0.0

Five-Year Bond Rate

Quarters Ahead	Monetary Policy	Risk Premium	Inflation Target	Inflation	Output	Measurement Error
1	3.1	17.2	75.2	2.1	2.5	0.0
4	1.1	13.1	80.5	2.3	3.0	0.0
12	0.9	6.7	87.8	2.6	2.1	0.0
20	0.7	4.3	91.5	2.1	1.3	0.0
∞	0.0	0.2	99.6	0.1	0.1	0.0

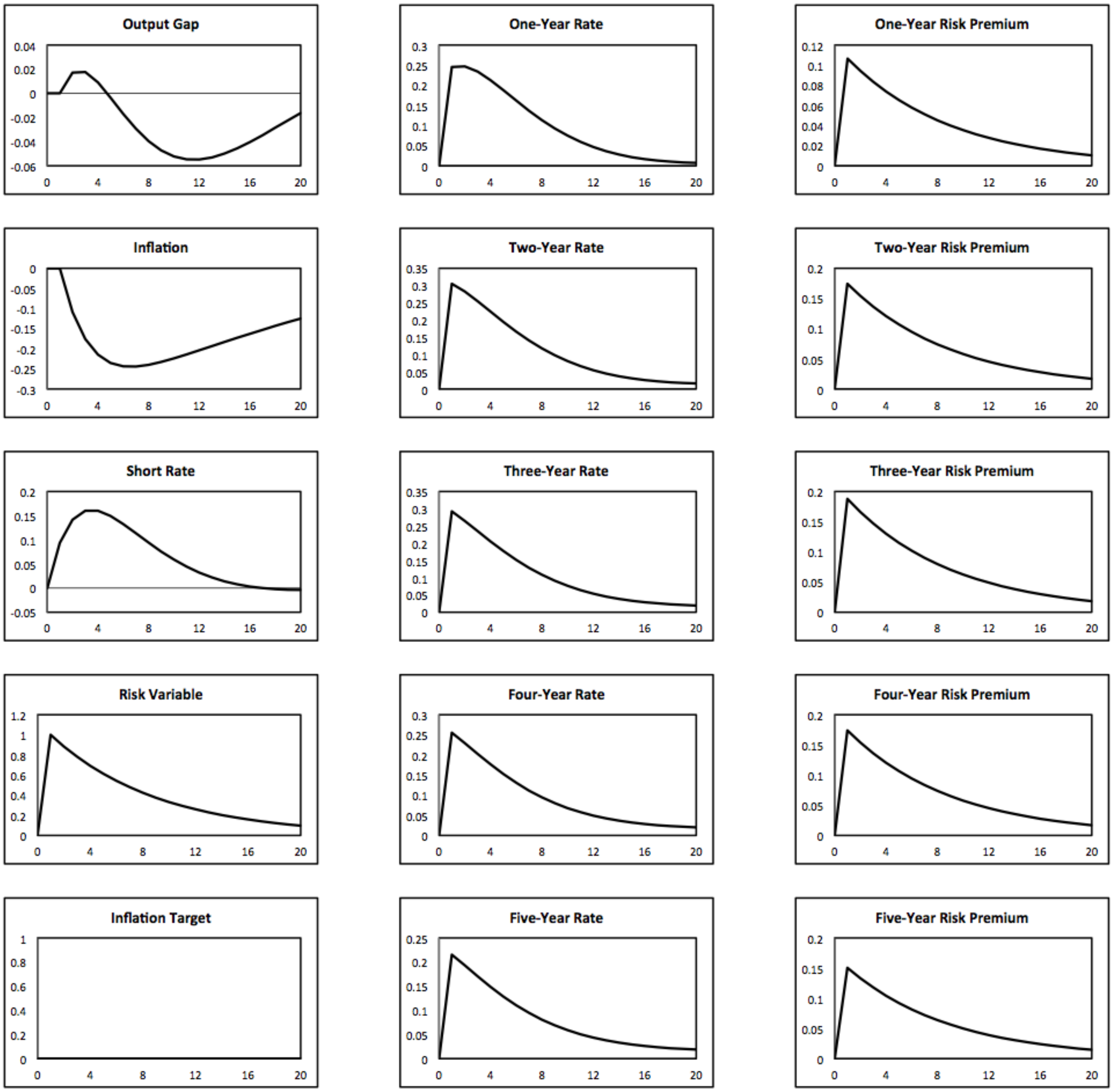
Note: Each row decomposes the forecast error variance in the indicated bond yield at the indicated horizon into percentages attributable to each of the model's five shocks and measurement error.

Figure 8. Impulse Responses to a Monetary Policy Shock, Extended Sample Period: 1959:1 - 2014:4



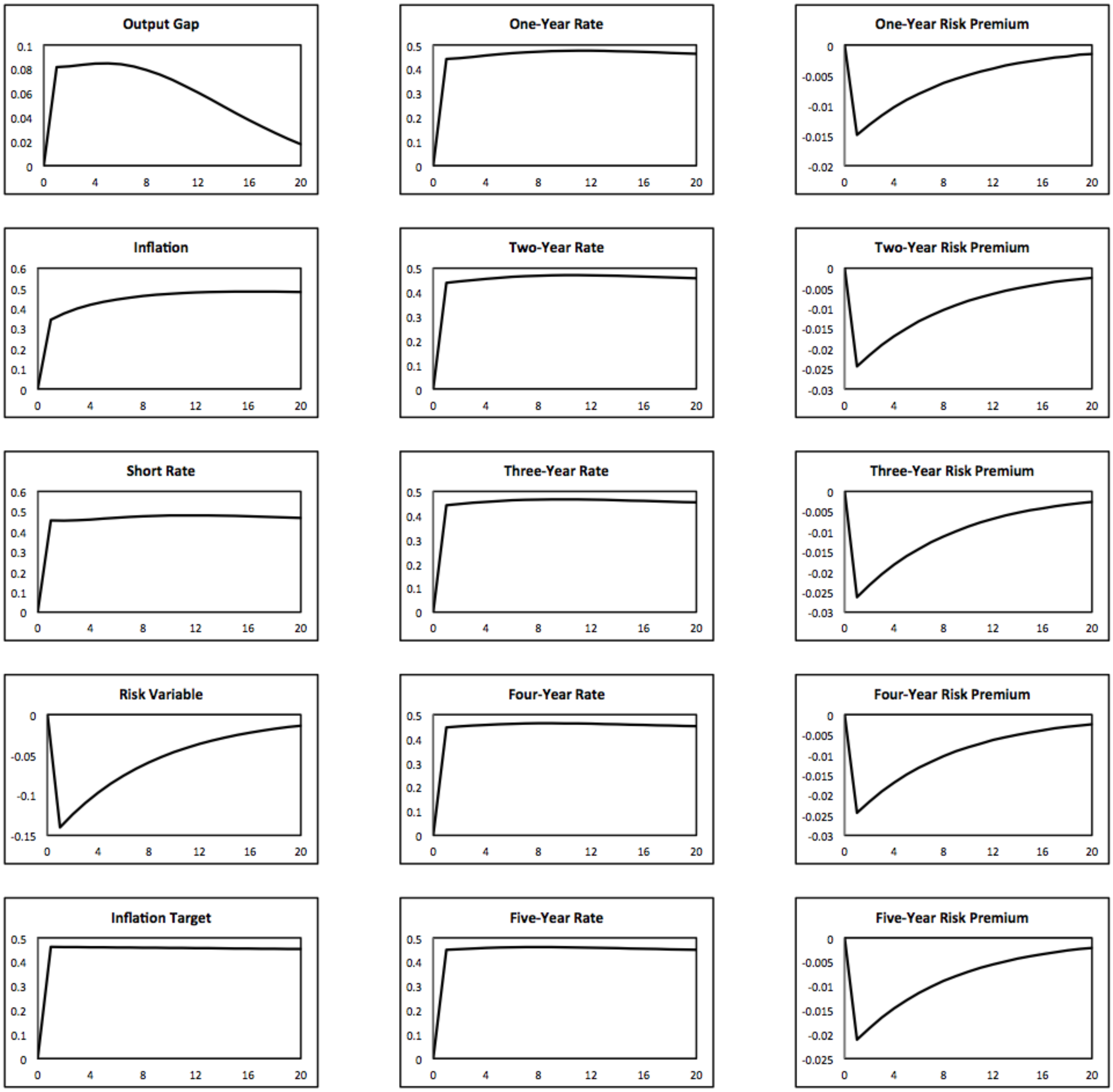
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation monetary policy shock ε_r. The inflation and interest rates are in annualized terms.

Figure 9. Impulse Responses to a Risk Premium Shock, Extended Sample Period: 1959:1 - 2014:4



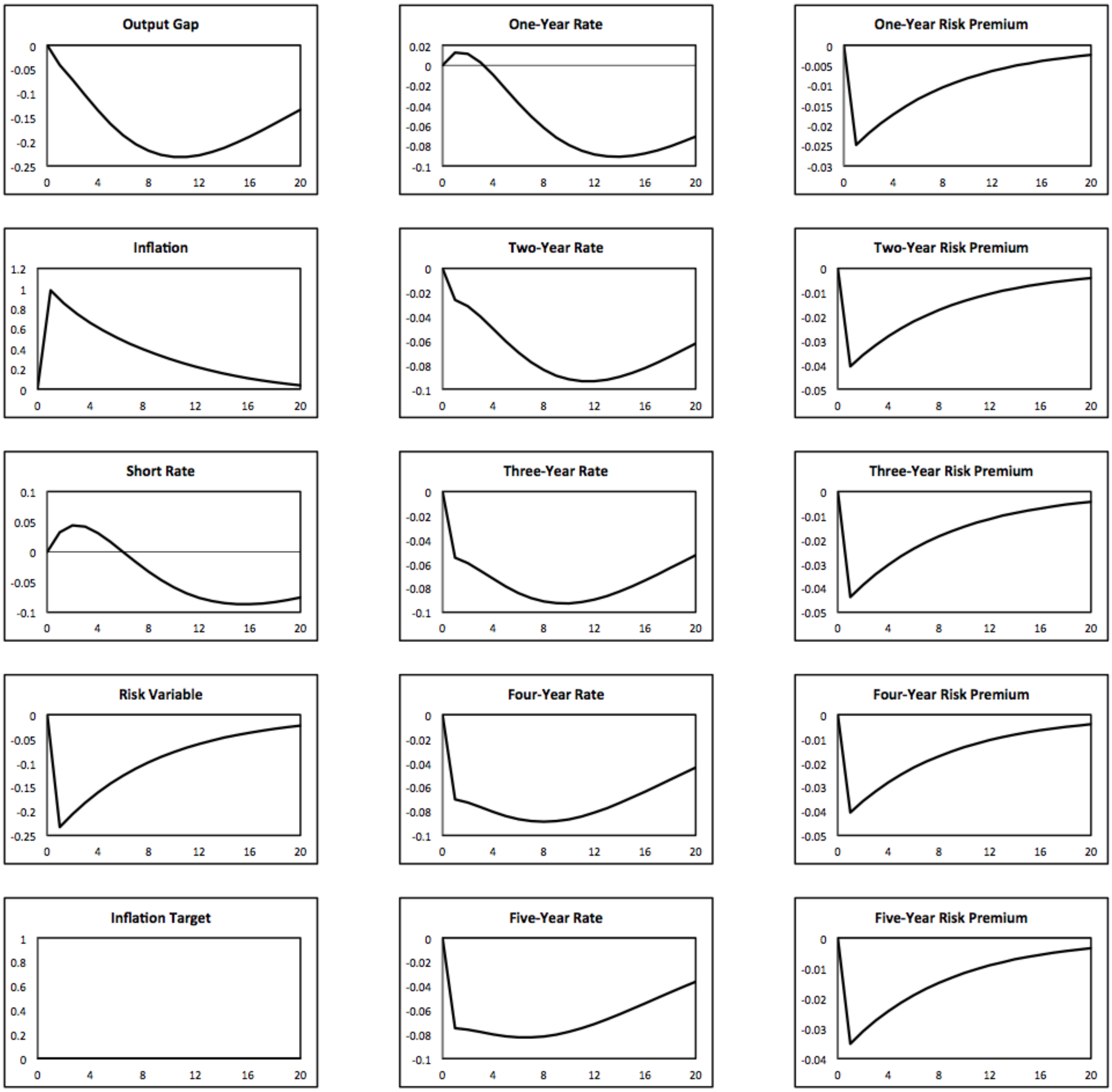
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation risk premium shock ε_v. The inflation and interest rates are in annualized terms.

Figure 10. Impulse Responses to an Inflation Target Shock, Extended Sample Period: 1959:1 - 2014:4



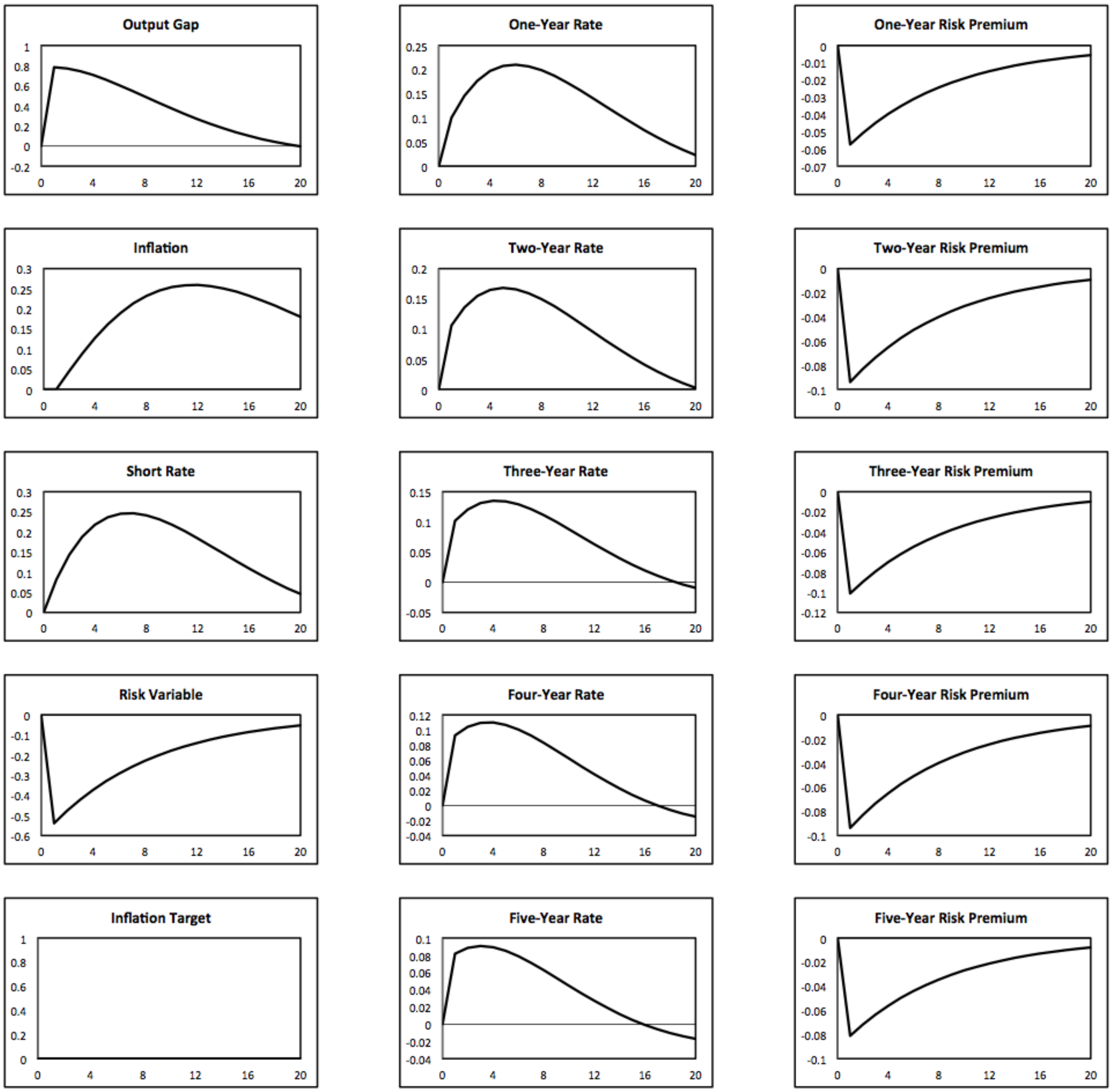
Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation inflation target shock ε_{τ}. The inflation and interest rates are in annualized terms.

Figure 11. Impulse Responses to an Inflation Shock, Extended Sample Period: 1959:1 - 2014:4



Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation inflation shock ε_π. The inflation and interest rates are in annualized terms.

Figure 12. Impulse Responses to an Output Shock, Extended Sample Period: 1959:1 - 2014:4



Each panel shows the percentage-point response of one of the model's variables to a one-standard-deviation output shock ε_y. The inflation and interest rates are in annualized terms.