Università Commerciale
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MSc. Finance/CLEFIN
2014/2015 Edition

Advanced Tools for Risk Management and Asset Pricing

June 2015 Exam for Attending Students

Time Allowed: 60 minutes

Family Name (Surname)	First Name	Student Number (Matr.)

Please answer all questions by choosing the most appropriate alternative(s) and/or by writing your answers in the spaces provided. You need to carefully justify and show your work in the case of "open" questions. There is only one correct answer(s) for each of the multiple choice questions: each selected alternative that is correct will be awarded one point; wrong answers will be penalized with minus 0.5 point. Correct answers not selected and questions that have been left blank will receive zero points. Only answers explicitly reported in the appropriate box will be considered. No other answers or indications pointing to potential answers will be taken into consideration. In the case of "open" questions, the maximum number of points is indicated.

Question 1. Which of the following statements is FALSE?

☐ (A) Both Ito and Stratonovich intergrals are martingales

☐ (B) Ito Integral is defined as $\int_0^T W_s(\omega)dW_s(\omega) = \frac{1}{2}W_t(\omega) - \frac{1}{2}dt$

☐ (C) Stratonovich Integral is defined as $\int_0^T W_s(\omega)dW_s(\omega) = \frac{1}{2}W_t(\omega)^2$

☑ (D) Standard chain rule is preserved for both Ito and Stratonovich Integrals

Question 2. Which of the following statements about Copula is FALSE?

☐ (A) D-dimensional copula function is d-increasing

☑ (B) $C(u_1, u_2) = max(u_1 + u_2, 0)$ is a copula function

☐ (C) $C(\boldsymbol{u}) = 0$ if $\boldsymbol{u} \in [0,1]^{\boldsymbol{d}}$ has at least one component $u_i = 0$

☐ (D) Copulas can be used in conjunction with marginal distribution functions to construct multivariate distribution functions

Question 3. Let X_1 and X_2 be two random variables with marginal F_1 and F_2 respectively. Then, which of the following statements is FALSE?

☐ (A) If for X_1 and X_2 $\lim_{q\to 1^-} P\left(X_2 > F_2^{\leftarrow}(q) | X_1 > F_1^{\leftarrow}(q)\right) = 0$, then X_1 and X_2 are asymptotically independent in the upper tail

☑ (B) If for X_1 and X_2 $\lim_{q\to 0^+} P\left(X_2 > F_2^{\leftarrow}(q) | X_1 > F_1^{\leftarrow}(q)\right) = 0$, then X_1 and X_2 are asymptotically independent in the lower tail

☐ (C) Tail dependence measures depend only on the copula of X_1, X_2

☐ (D) Measures of extremal dependence between a pair of random variables X_1 and X_2 depend only on the copula of X_1 and X_2

Question 4. Consider two standard normal random variables X_1, X_2 that are jointly normal with correlation ρ. Write the copula functions for the following values of ρ:

a) $\rho = 0$
b) $\rho = 1$
c) $\rho = 1/2$
d) Write the copula for the random vector (X_1, X_2^3) when $\rho = 1/2$

(Note: Each sub question is worth 1 point).

Answer.

As the variables are standard normal, if we call Φ the cdf of the standard normal we know that

$$\Phi(X_1) = U_1, \Phi(X_2) = U_2$$

are uniform random variables. The copula is then defined as the multivariate distribution of the uniforms,

$$C(u_1, u_2) = Q(U_1 \le u_1, U_2 \le u_2)$$

a) For $\rho = 0$ in two jointly standard Gaussians, we know this corresponds to having independent random variables X_1 and X_2. It follows that $\Phi(X_1) = U_1, \Phi(X_2) = U_2$ are also independent, in that they are deterministic transforms of the independent X_1 and X_2.
We have therefore the copula as

$$C(u_1, u_2) = Q(U_1 \le u_1, U_2 \le u_2) = by\ independence = Q(U_1 \le u_1)Q(U_2 \le u_2) = u_1 u_2$$

given that the uniform cdf is $Q(U \le u) = u$.

b) For $\rho = 1$ in two jointly standard Gaussians, we know this corresponds to having total dependent random variables X_1 and X_2, i.e. $X_1 = X_2$. It follows that $\Phi(X_1) = U_1, \Phi(X_2) = \Phi(X_1) = U_1$.
We have therefore the copula as

$$C(u_1, u_2) = Q(U_1 \le u_1, U_2 \le u_2) = Q(U_1 \le u_1\ and\ U_2 \le u_2)$$
$$= Q(U_1 \le min(u_1, u_2)) = min(u_1, u_2)$$

given that the uniform cdf is $Q(U \le u) = u$.

c) For $\rho = 1/2$ we cannot invoke a special calculation; we obtain just the Gaussian copula for correlation parameter ½, that cannot be written in closed form but only as an integral of the related bivariate density.

Question 5. Which of the following statements about base correlations is TRUE?

☐ (A) Typically, base correlation presents a smile
☐ (B) It depends on pairs of detachment points
☐ (C) It is inconsistent across the capital structure but consistent at the level of single tranche
☑ (D) It is inconsistent across the capital structure

Question 6. Which of the following statements about different approaches to price CDOs is FALSE?

☐ (A) The One Factor Gaussian Copula Approach allows for dimensionality reduction in the calculation of the joint default probability of *n* names
☑ (B) Dynamic(al) Loss Approaches allow to calculate single name sensitivities
☐ (C)Dynamic(al) Loss Approaches are able to capture the phenomenon of clustered (sector) defaults associated to masses in the far right tail of the loss distribution
☐ (D) The Implied Copula Approach by Hull and White is able to capture the phenomenon of clustered (sector) defaults associated to masses in the far right tail of the loss distribution

Question 7. Which of the following statements about Economic Capital in the Vasicek Portfolio model is FALSE?

☑ (A) It is reliable for any portfolio of loans
☐ (B) It depends on the confidence level
☐ (C) It depends on the first moment of the loss distribution
☐ (D) It is an asymptotic formula

Question 8. Which of the following statements about First Passage Time models is FALSE?

☐ (A) Default can occur at any time up to maturity
☐ (B) The Black Cox model does not allow for a flexible CDS calibration
☑ (C) AT1P models always produce reasonable results
☐ (D) Default is described through an endogenous process

Question 9. Considering the two termsheets below, which of the following statements is plausible:

Bonus Cap A	
Underlying	Fiat
Maturity	3 Years
Barrier	70%
Barrier type	American

Bonus Cap B	
Underlying	Fiat
Maturity	3 Years
Barrier	60%
Barrier type	American

☐ (A) Bonus Cap A has a Bonus equal to 114% and Bonus Cap B has a Bonus equal to 118%.
☑ (B) Bonus Cap A has a Bonus equal to 109% and Bonus Cap B has a Bonus equal to 105%.
☐ (C) Bonus Cap A has a Bonus equal to 107% and Bonus Cap B has a Bonus equal to 107%.
☐ (D) Bonus Cap A has a Bonus equal to 97% and Bonus Cap B has a Bonus equal to 94%.

Question 10. You are structuring an Equity Protection certificate with 100% capital protection; the Zero Coupon Bond costs Eur 96. An ATM call option on the FTSE MIB index costs 6 Euro. Because you would like your product to offer 100% participation to any potential appreciation of the underlying, which strategy of selection of an underlying *different* from the FTSE MIB index would you consider?

☐ (A) I shall not need any alternative selection of the underlying asset because I can already offer 100% participation to any potential appreciation of the underlying
☐ (B) I will be looking for an underlying asset with lower volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 6 to make a 100% protection possible
☐ (C) I will be looking for an underlying asset with higher volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible
☑ (D) I will be looking for an underlying asset with lower volatility and a higher dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 4 to make a 100% protection possible
☐ (E) None of the above

Question 11. A three-year Express investment certificate with coupons that grow over time in case the certificate is not auto-called, is replicated by:

☐ (A) Buying the underlying; buying a barrier option call down-and-out; selling a call with strike equal to the express strike; buying a series of digital/barrier puts of knock-out type with strike equal to the express strike, maturities equal to the liquidation dates, in number that increases according to the slope that one wants to impress to the coupon payment schedule

☑ (B) Buying the underlying; buying a barrier option put down-and-out; selling a call with strike equal to the express strike; buying a series of digital/barrier calls of knock-out type with strike equal to the express strike, maturities equal to the liquidation dates, in number that increases according to the slope that one wants to impress to the coupon payment schedule

☐ (C) Buying a zero-coupon bond; buying a barrier option call down-and-in; selling a call with strike equal to the express strike; buying a series of digital/barrier puts of knock-out type with strike equal to the express strike, maturities equal to the liquidation dates, in number that increases according to the slope that one wants to impress to the coupon payment schedule

☐ (D) None of the above

Question 12. Perform the following calculations and answer the related questions.

12a (1 point). Complete the following table concerning the P&L of a fixed, x3 fixed leverage certificate. Is the performance of the certificate the same as "3 times the index performance"? If not, why?

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-7.00%	100	
Day 2		13.00%		
Day 3		-3%		
Day 4		-8%		
Total				

Answer.

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-7.00%	100	-21%
Day 2	93	13.00%	79	39%
Day 3	105.09	-3%	109.81	-9%
Day 4	101.94	-8%	99.93	-24%
Total	92.00	-6.22%	75.94	-24.06%

-18.65% **-24.06%**

Clearly, in the presence of bearish markets, the leveraged certificate yields a performance that is lower (and considerably so, 541 basis points in only 4 days!) than the multiple applied to index (18.7%). This is due to the compounding effect in the presence of high volatility.

12b (0.5 points). Complete the following table concerning the P&L of a fixed, x3 leveraged certificate. Can you notice any difference vs. 12a? Where are these likely to come from?

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-1.75%	100	
Day 2		3.25%		
Day 3		-0.75%		
Day 4		-2.00%		
Total				

Answer.

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-1.75%	100	-5%
Day 2	98.25	3.25%	94.75	10%
Day 3	101.44	-0.75%	103.99	-2%
Day 4	100.68	-2.00%	101.65	-6%
Total	98.00	-1.33%	95.55	-4.45%
		-3.99%		**-4.45%**

Qualitatively, the result remains the same as in 12a but because there is much less volatility, the performances of the leveraged certificat and of 3xtimes the index are more similar. In fact, the absence of volatility derives from the fact that all the daily performances in this table are the same as in 12a, but divided by 4! However, 12a and 12b are not exactly comparable, as the index reaches 98.7 in this case and 93.8 in 12a.

12c (1 point). Complete the following table concerning the P&L of a fixed, x3 leveraged certificate. Can you notice any difference vs. 12a and 12b? Where are these likely to come from?

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-3.87%	100	
Day 2		3.01%		
Day 3		-3.00%		
Day 4		-2.37%		
Total				

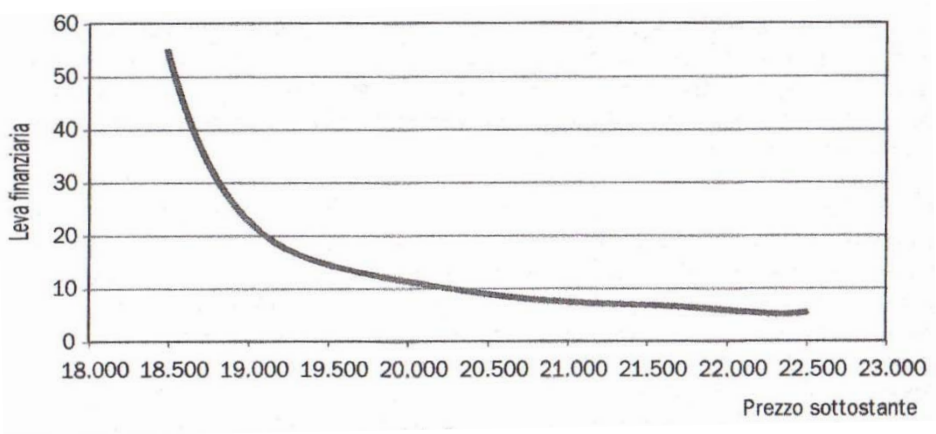
Answer.

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-3.87%	100	-12%
Day 2	96.13	3.01%	88.39	9%
Day 3	99.02	-3.00%	96.37	-9%
Day 4	96.05	-2.37%	87.70	-7%
Total	97.63	-6.22%	81.46	-18.54%
		-18.67%		**-18.54%**

Qualitatively, the result remains the same as in 12b: low volatility tames the compounding effect. Moreover, this case is directly comparable to 12a because the index reaches 93.8 in both cases.

12d (0.5 points) What alternative type of structured product (investment certificate) would allow you to escape the effects of compounding? What would be the costs/drawbacks of such a choice?

Answer. One could resort to dynamic leverage, turbo certificates. A Turbo implies dynamic leverage, i.e., a leverage ratio that is a function of the underlying price, given a fixed strike, see plot below for instance.



The problem with Turbos may often offer abysmal performances that are caused by these dynamic effects, which make them riskier than thought of. E.g., Camelia's book reports one example of a Turbo on the FTSE MIB with 10.9 leverage at issuance that, over time and in the face of a +12.5% by the index, makes a 117% return, i.e., 117/12.5 = 9.4 only. In the case of a -5.6% by the underlying, Turbo yields a loss of 70%, i.e., 70/5.6 =12.5.