Università Commerciale
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MSc. Finance/CLEFIN
2014/2015 Edition

Advanced Tools for Risk Management and Asset Pricing

June 2015 Exam for Non-Attending Students – Solutions

Time Allowed: 120 minutes

Family Name (Surname)	First Name	Student Number (Matr.)

Please answer all questions by choosing the most appropriate alternative and/or by writing your answers in the spaces provided. You need to carefully justify and show your work in the case of "open" questions. There is only one correct answer(s) for each of the multiple choice questions: each selected alternative that is correct will be awarded one point. Only answers explicitly reported in the appropriate box will be considered. No other answers or indications pointing to potential answers will be taken into consideration. In the case of "open" questions, the maximum number of points is indicated.

Question 1. Which of the following statements is FALSE?

☐ (A) Both Ito and Stratonovich intergrals are martingales
☐ (B) Ito Integral is defined as $\int_0^T W_s(\omega)dW_s(\omega) = \frac{1}{2}W_t(\omega) - \frac{1}{2}dt$
☐ (C) Stratonovich Integral is defined as $\int_0^T W_s(\omega)dW_s(\omega) = \frac{1}{2}W_t(\omega)^2$
☑ (D) Standard chain rule is preserved for both Ito and Stratonovich Integrals

Question 2. Which of the following statements about Copula is FALSE?

☐ (A) D-dimensional copula function is d-increasing
☑ (B) $C(u_1, u_2) = max(u_1 + u_2, 0)$ is a copula function
☐ (C) $C(\boldsymbol{u}) = 0$ if $\boldsymbol{u} \in [0,1]^{\boldsymbol{d}}$ has at least one component $u_i = 0$
☐ (D) Copulas can be used in conjunction with marginal distribution functions to construct multivariate distribution functions

Question 3. Let X_1 and X_2 be two random variables with marginal F_1 and F_2 respectively. Then, which of the following statements is FALSE?

☐ (A) If for X_1 and X_2 $\lim_{q\to 1^-} P\left(X_2 > F_2^{\leftarrow}(q) | X_1 > F_1^{\leftarrow}(q)\right) = 0$, then X_1 and X_2 are asymptotically independent in the upper tail

☑ (B) If for X_1 and X_2 $\lim_{q\to 0^+} P\left(X_2 > F_2^{\leftarrow}(q) | X_1 > F_1^{\leftarrow}(q)\right) = 0$, then X_1 and X_2 are asymptotically independent in the lower tail

☐ (C) Tail dependence measures depend only on the copula of X_1, X_2

☐ (D) Measures of extremal dependence between a pair of random variables X_1 and X_2 depend only on the copula of X_1 and X_2

Question 4. Which of the following statements about Copulas is TRUE?

☐ (A) Gaussian copula is asymptotically independent in both tails

☐ (B) Gumbel copula has upper tail dependence

☐ (C) Claiton copula has lower tail dependence

☑ (D) All of the above it TRUE

Question 5. Which of the following statements about Rank Correlations is FALSE?

☐ (A) To compute rank correlations one needs to know the ordering of the sample for each variable

☐ (B) Rank correlations are invariant under strictly increasing transformations

☑ (C) Rank correlations take a value of -1 when the variables are comonotonic and the value 1 when the variables are countermonotonic

☐ (D) The sample version of Spearman's Rho can be simply expressed as:

$$\rho(X_1, X_2) = \rho(R_1, R_2)$$

where R_1 and R_2 are rank variables

Question 6.

1. Write down a definition of Copula and its properties.
2. Consider two standard normal random variables X_1, X_2 that are jointly normal with correlation ρ. Write the copula functions for the following values of ρ:
   a) $\rho = 0$
   b) $\rho = 1$
   c) $\rho = 1/2$
   d) Write the copula for the random vector (X_1, X_2^3) when $\rho = 1/2$

Answer.

As the variables are standard normal, if we call Φ the cdf of the standard normal we know that

$$\Phi(X_1) = U_1, \Phi(X_2) = U_2$$

are uniform random variables. The copula is then defined as the multivariate distribution of the uniforms,

$$C(u_1, u_2) = Q(U_1 \leq u_1, U_2 \leq u_2)$$

a) For $\rho = 0$ in two jointly standard Gaussians, we know this corresponds to having independent random variables X_1 and X_2. It follows that $\Phi(X_1) = U_1, \Phi(X_2) = U_2$ are also independent, in that they are deterministic transforms of the independent X_1 and X_2.
We have therefore the copula as

$C(u_1, u_2) = Q(U_1 \leq u_1, U_2 \leq u_2) = by\ independence = Q(U_1 \leq u_1)Q(U_2 \leq u_2) = u_1 u_2$ given that the uniform cdf is $Q(U \leq u) = u$.

b) For $\rho = 1$ in two jointly standard Gaussians, we know this corresponds to having total dependent random variables X_1 and X_2, i.e. $X_1 = X_2$. It follows that $\Phi(X_1) = U_1, \Phi(X_2) = \Phi(X_1) = U_1$.
We have therefore the copula as

$$C(u_1, u_2) = Q(U_1 \leq u_1, U_2 \leq u_2) = Q(U_1 \leq u_1\ and\ U_2 \leq u_2)$$
$$= Q(U_1 \leq min(u_1, u_2)) = min(u_1, u_2)$$

given that the uniform cdf is $Q(U \leq u) = u$.

c) For $\rho = 1/2$ we cannot invoke a special calculation; we obtain just the Gaussian copula for correlation parameter ½, that cannot be written in closed form but only as an integral of the related bivariate density.

d) We know that the copula is invariant for transformations that preserve information, i.e. invertible. Since

$$(X_1, X_2) \rightarrow (X_1, X_2^3)$$

Is invertible, with inverse

$$(Y_1, (Y_2)^{1/3}) \leftarrow (Y_1, Y_2)$$

we have that the copula is the same as in point c) above.

Question 7. Which of the following statements about base correlations is TRUE?

☐ (A) Typically, base correlation presents a smile
☐ (B) It depends on pairs of detachment points
☐ (C) It is inconsistent across the capital structure but consistent at the level of single tranche
☑ (D) It is inconsistent across the capital structure

Question 8. Which of the following statements about different approaches to price CDOs is FALSE?

☐ (A) The One Factor Gaussian Copula Approach allows for dimensionality reduction in the calculation of the joint default probability of *n* names
☑ (B) Dynamic(al) Loss Approaches allow to calculate single name sensitivities
☐ (C)Dynamic(al) Loss Approaches are able to capture the phenomenon of clustered (sector) defaults associated to masses in the far right tail of the loss distribution
☐ (D) The Implied Copula Approach by Hull and White is able to capture the phenomenon of clustered (sector) defaults associated to masses in the far right tail of the loss distribution

Question 9. Which of the following statements about Economic Capital in the Vasicek Portfolio model is FALSE?

☑ (A) It is reliable for any portfolio of loans
☐ (B) It depends on the confidence level
☐ (C) It depends on the first moment of the loss distribution
☐ (D) It is an asymptotic formula

Question 10. Which of the following statements about First Passage Time models is FALSE?

☐ (A) Default can occur at any time up to maturity
☐ (B) The Black Cox model does not allow for a flexible CDS calibration
☑ (C) AT1P models always produce reasonable results
☐ (D) Default is described through an endogenous process

Question 11. Which of the following statements about compound correlations is FALSE?

☐ (A) Typically, compound correlation presents a smile
☐ (B) It depends on pairs of detachment points
☑ (C) It is inconsistent at the level of single tranche
☐ (D) It is inconsistent across the capital structure

Question 12. Which of the following statements about time homogeneous Poisson processes is FALSE?

☐ (A) The probability of having no jumps up to a certain time is an exponential function of minus that time
☐ (B) The probability of having more than one jump in an arbitrarily small time goes to zero faster than time
☑ (C) The probability of having one jump in an arbitrarily small time is exactly the constant appearing in the exponential function
☐ (D) A time homogeneous Poisson process is a unit-jump increasing, right continuous process, with stationary independent increments and zero initial value

Question 13. Which of the following statements about Cox processes is FALSE?

☐ (A) The first jump time, transformed through its cumulated intensity $\Lambda(\tau)$, is an exponential random variable independent of the default free information
☑ (B) The survival probability is given by: $P(\tau > T) = e^{-\Lambda(T)}$
☐ (C) Default is described by an exogenous jump process
☐ (D) Stochastic intensity models do not allow to obtain large levels of option implied volatilities for CDS rates

Question 14. Which of the following statements about credit modeling in a multi-factor set up is FALSE?

☐ (A) Sector concentration risk affects also the zero order term in the quantile expansion
☐ (B) The conditional asset correlation takes into account the effects of both sector concentration and name concentration
☑ (C) Name concentration risk cannot be diversified away
☐ (D) In the case of a large enough, fine-grained, portfolio, losses are primarily driven by the systematic factors

Question 15. Which of the following statements about the Sensitivity Based Approach (SBA) is FALSE?

☐ (A) Risk factors and sensitivities are defined by the regulator and must be computed by banks accordingly
☐ (B) Sensitivities are used as inputs into aggregation formulae which are intended to recognize hedging and diversification benefits of positions in different risk factors within an asset class
☐ (C) Vega risk and curvature risk do not apply to instruments that are not subject to optionality
☑ (D) Vega risk is applied to options in the calculation of both SBA for capital and SBA for margin

Question 16. Which of the following statements about Mapping methods for bespoke portfolios is FALSE?

☐ (A) Mapping consists in associating to the selected bespoken tranche an equivalent base tranche on a standard (index) portfolio
☐ (B) The correlation used to price the bespoke tranche is taken to be the correlation at the equivalent standard strike
☐ (C) In the ATM method the invariant measure of risk in a tranche is the strike as a fraction of its expected loss
☑ (D) The ATM method works well when taking into account the portfolio dispersion

Question 17. Consider the following formula:

$$Q(\tau_1 < T, \dots, \tau_n < T) = \int \left[\prod_{i=1}^{n} N\left(\frac{N^{-1}\left(1 - e^{-\Gamma_i(T)}\right) - \sqrt{\rho_i}\ y}{\sqrt{1-\rho_i}} \right) \right] \varphi(y) dy$$

1. What does it represent?
2. Under which assumptions has been derived?
3. Explain the meaning of the following quantities:
   - $N\left(\frac{N^{-1}(1-e^{-\Gamma_i(T)})-\sqrt{\rho_i}\ y}{\sqrt{1-\rho_i}}\right)$
   - $\prod_{i=1}^{n} N\left(\frac{N^{-1}(1-e^{-\Gamma_i(T)})-\sqrt{\rho_i}\ y}{\sqrt{1-\rho_i}}\right)$
   - $\varphi(y)$
   - ρ_i
   - $\Gamma_i(T)$

Answer. See Lecture 10 "(Basic) Multi-Name Credit Derivatives" (slides 41-46/68):

1. The formula represents the unconditional joint default probability of *n* names under the One-Factor Gaussian Copula approach
2. Assumptions:

- Dependence across names captured by Gaussian Copula
- One-factor approach i.e.

$$X_i = \sqrt{\rho_i}\, Y + \sqrt{1-\rho_i}\, \xi_i$$

(see slide 43/68)

- Deterministic hazard rates (intensities)

3. Meanings:
   - $N\left(\frac{N^{-1}(1-e^{-\Gamma_i(T)})-\sqrt{\rho_i}\, y}{\sqrt{1-\rho_i}}\right)$ = probability of default of name *i* conditional on the systematic factor *Y*
   - $\prod_{i=1}^{n} N\left(\frac{N^{-1}(1-e^{-\Gamma_i(T)})-\sqrt{\rho_i}\, y}{\sqrt{1-\rho_i}}\right)$ = joint default probability of *n* names, conditional on the systematic factor *Y* (single names, conditionally on *Y*, are independent)
   - $\varphi(y)$ = probability density of *Y* (standard Gaussian variable)
   - ρ_i = correlation of name *i* to the systematic factor *Y*
   - $\Gamma_i(T)$ = cumulated hazard rate of name *i* (deterministic)

Question 18. In the framework of the KMV model describe the process through which the default probability is derived (max 20 lines)

Answer. See Lecture 9 "Structural Models" (slides 27-32/65)

Question 19. Considering the two termsheets below, which of the following statements is plausible:

Bonus Cap A	
Underlying	Fiat
Maturity	3 Years
Barrier	70%
Barrier type	American

Bonus Cap B	
Underlying	Fiat
Maturity	3 Years
Barrier	60%
Barrier type	American

☐ (A) Bonus Cap A has a Bonus equal to 114% and Bonus Cap B has a Bonus equal to 118%
☑ (B) Bonus Cap A has a Bonus equal to 109% and Bonus Cap B has a Bonus equal to 105%
☐ (C) Bonus Cap A has a Bonus equal to 107% and Bonus Cap B has a Bonus equal to 107%
☐ (D) Bonus Cap A has a Bonus equal to 97% and Bonus Cap B has a Bonus equal to 94%

Question 20. You are structuring an Equity Protection certificate with 100% capital protection; the Zero Coupon Bond costs Eur 96. An ATM call option on the FTSE MIB index costs 6 Euro. Because you would like your product to offer 100% participation to any potential appreciation of the underlying, which strategy of selection of an underlying *different* from the FTSE MIB index would you consider?

☐ (A) I shall not need any alternative selection of the underlying asset because I can already offer 100% participation to any potential appreciation of the underlying
☐ (B) I will be looking for an underlying asset with lower volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 6 to make a 100% protection possible
☐ (C) I will be looking for an underlying asset with higher volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible
☑ (D) I will be looking for an underlying asset with lower volatility and a higher dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 4 to make a 100% protection possible
☐ (E) None of the above

Question 21. A three-year Express investment certificate with coupons that grow over time in case the certificate is not auto-called, is replicated by:

☐ (A) Buying the underlying; buying a barrier option call down-and-out; selling a call with strike equal to the express strike; buying a series of digital/barrier puts of knock-out type with strike equal to the express strike, maturities equal to the liquidation dates, in number that increases according to the slope that one wants to impress to the coupon payment schedule
☑ (B) Buying the underlying; buying a barrier option put down-and-out; selling a call with strike equal to the express strike; buying a series of digital/barrier calls of knock-out type with strike equal to the express strike, maturities equal to the liquidation dates, in number that increases according to the slope that one wants to impress to the coupon payment schedule
☐ (C) Buying a zero-coupon bond; buying a barrier option call down-and-in; selling a call with strike equal to the express strike; buying a series of digital/barrier puts of knock-out type with strike equal to the express strike, maturities equal to the liquidation dates, in number that increases according to the slope that one wants to impress to the coupon payment schedule
☐ (D) None of the above

Question 22. During 2014, in Italy the four categories of investment certificates that have raised the most interest in terms of total amounts issued and placed are, in the order:

- ☑ (A) Equity protection; express; bonus; credit linked
- ☐ (B) Equity protection; double win; outperformance; credit linked
- ☐ (C) Bonus; express; equity protection; credit linked
- ☐ (D) None of the above

Question 23. Consider the pricing of multi-underlyings Bonus investment certificates. You are considering three potential features: (i) a Bonus written on a linear basket of stocks, i.e. whose payoff depends on the portfolio returns; (ii) a Bonus written on a basket of stocks under the "Worst Of" feature; (iii) a Bonus written on a basket of stocks under the "Best Of" feature. All else being equal, which certificate will have the higher bonus amount?

- ☐ (A) The Bonus written on a linear basket of stocks
- ☐ (B) The Bonus written on a basket of stocks under the "Best Of" feature
- ☑ (C) The Bonus written on a basket of stocks under the "Worst Of" feature
- ☐ (D) None of the above

Question 24. Perform the following calculations and answer the related questions.

24a (1 point). Complete the following table concerning the P&L of a fixed, x3 fixed leverage certificate. Is the performance of the certificate the same as "3 times the index performance"? If not, why?

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-7.00%	100	
Day 2		13.00%		
Day 3		-3%		
Day 4		-8%		
Total				

Answer.

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-7.00%	100	-21%
Day 2	93	13.00%	79	39%
Day 3	105.09	-3%	109.81	-9%
Day 4	101.94	-8%	99.93	-24%
Total	92.00	-6.22%	75.94	-24.06%

-18.65% **-24.06%**

Clearly, in the presence of bearish markets, the leveraged certificate yields a performance that is lower (and considerably so, 541 basis points in only 4 days!) than the multiple applied to index (18.7%). This is due to the compounding effect in the presence of high volatility.

24b (0.5 points). Complete the following table concerning the P&L of a fixed, x3 leveraged certificate. Can you notice any difference vs. 24a? Where are these likely to come from?

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-1.75%	100	
Day 2		3.25%		
Day 3		-0.75%		
Day 4		-2.00%		
Total				

Answer.

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-1.75%	100	-5%
Day 2	98.25	3.25%	94.75	10%
Day 3	101.44	-0.75%	103.99	-2%
Day 4	100.68	-2.00%	101.65	-6%
Total	98.00	-1.33%	95.55	-4.45%
		-3.99%		**-4.45%**

Qualitatively, the result remains the same as in 24a but because there is much less volatility, the performances of the leveraged certificat and of 3xtimes the index are more similar. In fact, the absence of volatility derives from the fact that all the daily performances in this table are the same as in 24a, but divided by 4! However, 24a and 24b are not exactly comparable, as the index reaches 98.7 in this case and 93.8 in 24a.

24c (1 point). Complete the following table concerning the P&L of a fixed, x3 leveraged certificate. Can you notice any difference vs. 24a and 24b? Where are these likely to come from?

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-3.87%	100	
Day 2		3.01%		
Day 3		-3.00%		
Day 4		-2.37%		
Total				

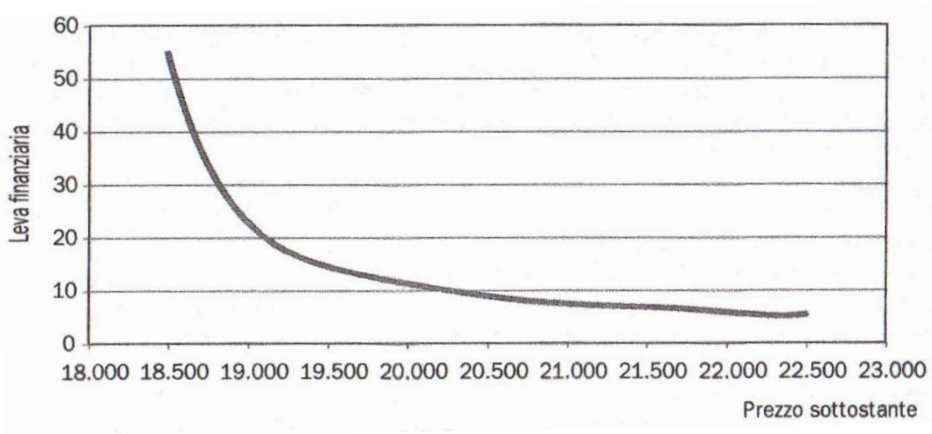
Answer.

	Underlying	Daily Performance	x3 Leveraged Index	Daily peformance of x3 Leveraged Index
Day 1	100	-3.87%	100	-12%
Day 2	96.13	3.01%	88.39	9%
Day 3	99.02	-3.00%	96.37	-9%
Day 4	96.05	-2.37%	87.70	-7%
Total	97.63	-6.22%	81.46	-18.54%
		-18.67%		**-18.54%**

Qualitatively, the result remains the same as in 24b: low volatility tames the compounding effect. Moreover, this case is directly comparable to 24a because the index reaches 93.8 in both cases.

24d (0.5 points) What alternative type of structured product (investment certificate) would allow you to escape the effects of compounding? What would be the costs/drawbacks of such a choice?

Answer. One could resort to dynamic leverage, turbo certificates. A Turbo implies dynamic leverage, i.e., a leverage ratio that is a function of the underlying price, given a fixed strike, see plot below for instance.



The problem with Turbos may often offer abysmal performances that are caused by these dynamic effects, which make them riskier than thought of. E.g., Camelia's book reports one example of a Turbo on the FTSE MIB with 10.9 leverage at issuance that, over time and in the face of a +12.5% by the index, makes a 117% return, i.e., 117/12.5 = 9.4 only. In the case of a -5.6% by the underlying, Turbo yields a loss of 70%, i.e., 70/5.6 =12.5.