Università Commerciale
Luigi Bocconi

MSc. Finance/CLEFIN
2017/2018 Edition

Advanced Quantitative Methods for Asset Pricing and Structuring

June 2018 Exam for Non Attending Students – Solutions

Time Allowed: 105 minutes

Family Name (Surname)	First Name	Student Number (Matr.)

Please answer all questions by choosing the most appropriate alternative(s) and/or by writing your answers in the spaces provided. You need to carefully justify and show your work in the case of "open" questions. There is only one correct answer(s) for each of the multiple choice questions. Correct answers not selected and questions that have been left blank will receive zero points. Only answers explicitly reported in the appropriate box will be considered. No other answers or indications pointing to potential answers will be taken into consideration. In the case of "open" questions, the maximum number of points is indicated.

Question 1 (1.5 pts). Which of the following statements about dependence is TRUE?

☐ (A) Linear correlation is a good measure of dependence across defaults

☐ (B) Dependence across default times is modeled directly by introducing dependence across standard normal variables

☑ (C) Dependence across default times is loaded into a copula function on uniform distributions

☐ (D) Dependence can be introduced across deterministic intensity processes

Question 2 (1.5 pts). Consider a standard CDS Index (e.g., the i-Traxx). Which of the following statements is TRUE?

☐ (A) The one factor Gaussian copula is parametrized by a matrix of 7750 pairwise correlation values

☐ (B) The copula is parametrized in terms of a unique pairwise correlation value

☐ (C) The Large Homogeneous Portfolio approach assumes that correlation is parametrized by a matrix of 125 pairwise values

☑ (D) None of the above

Question 3 (1.5 pts). The following table shows, at different times (columns 1 to 5), the values of four trades as well as the future exposures to the counterparty, with and without netting.

Trade ID	1	2	3	4	5
1	-2	0	4	-2	-6
2	0	3	-2	-3	0
3	3	-1	3	2	-6
4	-3	2	-4	-1	-3
Exposures					
No Netting	0	5	7	2	-15
Netting	-2	4	1	0	0

Which of the following statement is TRUE?

☐ (A) There are three mistakes
☐ (B) There are two mistakes in the "No Netting" exposures
☐ (C) There are two mistakes in the "Netting" exposures
☑ (D) None of the above

Question 4 (1.5 pts). Which of the following statements about counterparty risk is FALSE?

☐ (A) A collateral account is a contractual clause aimed at mitigating counterparty risk, while the contract is still alive
☐ (B) At any given future time, the PFE is given by a quantile of the counterparty exposure
☑ (C) A bank selling call options on its own stock is exposed to wrong way risk
☐ (D) Expected positive exposure (EPE) is the average EE in time up to a given future date

Question 5 (1.5 pts). Which of the following statements about CVA calculation is FALSE?

☐ (A) The CVA of an interest rate swap admits a closed form expression
☑ (B) The CVA of a portfolio of interest rate swaps admits a closed form expression
☐ (C) In a portfolio of interest rate swaps, as the swaps get closer to maturity, the value of the portfolio approaches zero
☐ (D) The exposure of a portfolio with netting agreements is a lower bound for the exposure of the same portfolio without them.

Question 6 (1.5 pts). Which of the following statements about implied correlations is TRUE?

☐ (A) Implied correlation cannot yield negative expected tranche losses
☐ (B) Typically, implied correlation depends on pairs of attachment points
☐ (C) Typically, compound correlation is represented by different models, even at the level of single tranche
☑ (D) Two tranches on the same pool (same maturity) yield different values of compound correlation

Question 7 (1.5 pts). Which of the following statements about diffusion processes is FALSE?

☐ (A) The Vasicek and the CIR processes are mean reverting
☑ (B) The Vasicek and the CIR processes (with the same parameters) have the same variance
☐ (C) The Vasicek process is a normally distributed
☐ (D) The CIR process is an affine model

Question 8 (1.5 pts). Which of the following statements about Reduced Form (Intensity) models is FALSE?

☐ (A) Default can described by means of a stochastic barrier
☐ (B) The probability of having more than one jump in an arbitrarily small time goes to zero faster than time
☑ (C) Time intervals between two jumps are distributed as a Poisson distribution
☐ (D) Stochastic intensity models do not allow to attain large levels of implied option volatilities for CDS rates

Question 9 (3 pts).
What is an inverted term structure of default intensities? Under which conditions can it occur? Make an explicit example.

Answer.
See Lecture "Reduce form (intensity) models" (slides 37-49).
An inverted term structure of default intensities is characterized by larger values of first short term intensities even though the shape is not necessarily monotonic. It occurs when a company is perceived to be riskier in the short term and the market quotes larger values for the CDS spread for the shortest maturities. Examples: Parmalat and Lehman Brothers under stressed conditions.

Question 10. **(1.5 pts)**. You are given the following data about the market of options on the Eurostoxx50 index:

Option type	
Put Option, Strike=100%, 1Y	2 Euro
Put Option, Strike=80%, 5Y	2.50 Euro
Put Option, Strike=100%, 5Y	4 Euro
Call Option, Strike=100%, 1Y	8 Euro
Call Option, Strike=90%, 5Y	6 Euro
Call Option, Strike=100%, 5Y	16 Euro

Also consider that the cost of a ZCB with expiry in 5 years is 96 Euros. Kaleb wants to structure a certificate with full capital protection that allows the investor to participate only to the positive performance of Eurostoxx50, that is, at maturity (5 year from now), the certificate will pay €100*(1+ P x Max [0, (Eurostoxx_Final-Eurostoxx_Initial)/Eurostoxx_Initial]). Considering an initial selling price of €100, what is the maximum participation rate P that the certificate can offer to the investors?

☐ (A) 50%
☐ (B)75%
☐ (C)100%
☑ (D)25%.

Answer.
Correct answer is D, because you have 4 euros to spend and you need to buy a 5Y ATM (100% strike) call on Eurostoxx50 index, which costs 16 Euros. Therefore, you can buy 4/16=0.25 call options for each certificate.

Question 11 (1.5 pts). The matrix below shows the pair-wise correlations between returns on three different stocks. For simplicity, assume the stocks are characterized by the same volatility and do not pay dividend. Which of the proposed combinations of stocks will allow the highest coupon for a Reverse Convertible certificate written on an **equally weighted** basket of **two only of these three stocks** as its underlying?

	Univeler	Trotter	Brambo
Univeler	1		
Trotter	0.7	1	
Brambo	0.2	0.5	1

☐ (A) Brambo and Univeler followed by Brambo and Trotter
☑ (B) Trotter and Univeler, followed by Brambo and Trotter
☐ (C) Brambo and Unilever, followed by Univeler and Trotter
☐ (D) Totter and Brambo

Answer.
Correct answer is B. In case of a Reverse Convertible we sell an ATM put option, therefore, in case of a linear basket, the higher the correlation, the more expensive the put option, the higher amount we can spend for the coupon.

Question 12 (1.5 pts). Martin, a structurer at Gordon Socks, every Friday produces a newsletter to show to the sales team a number of alternative Bonus Cap certificates, for immediate issuance (selling price will be €100). A piece of his last newsletter, concerning Bonus Cap on Fiat, is reported below. In the cells of the table Markus has reported the Bonus Amount (equal to the Cap Amount). Which of the following statements is the MOST LIKELY?

Barrier / Tenor	**1Y**
60%, American	117
70%, American	120
60%, European	115
70%, European	112

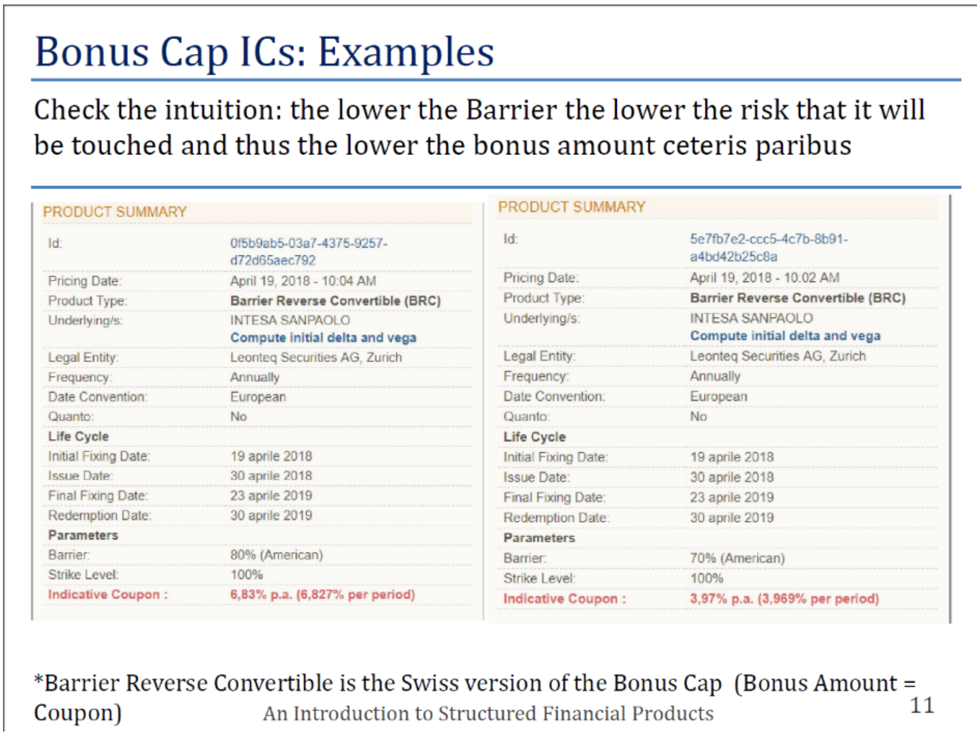
☑ (A) Markus has obviously done a mistake: indeed, we would expect the Bonus (Cap) Amount of a certificate to become lower when we lower the barrier, all else being equal

☐ (B) Markus has obviously done a mistake: indeed, we would expect the Bonus (Cap) Amount of a certificate with American Barrier to be lower than that of a certificate with European Barrier, all else being equal

☐ (C) Markus has obviously done a mistake: indeed, we would expect the Bonus (Cap) Amount of a certificate to become higher when we lower the barrier, all else being equal

☐ (D) Markus has obviously done a mistake: indeed, we would expect the Bonus (Cap) Amount of a certificate with American Barrier to be equal than that of a certificate with European Barrier, all else being equal

Answer.
A is the correct answer.



Question 13 (1.5 pts). You have been given the following data about three different indices that your boss would like you to use as alternative underlyings for an Equity Protection certificate (with full capital protection). Which of the following statement is the MOST LIKELY? (To answer this question, ignore any effect that may derive from different currency of denomination of the underlyings)

Underlying	*Volatility*	*Dividend yield*
Eurostoxx 50	22%	1.30%
S&P 500	17%	1.50%
Ftse Mib	25%	2.00%

☐ (A) You will probably be able to offer a higher participation rate on the Eurostoxx 50 than on Ftse Mib, because both the volatility and the dividend yield are higher for the Ftse Mib and this fact certainly increases the price of the call option on the Ftse Mib that you need to sell compared to the one on the Eurostoxx 50, all else being equal

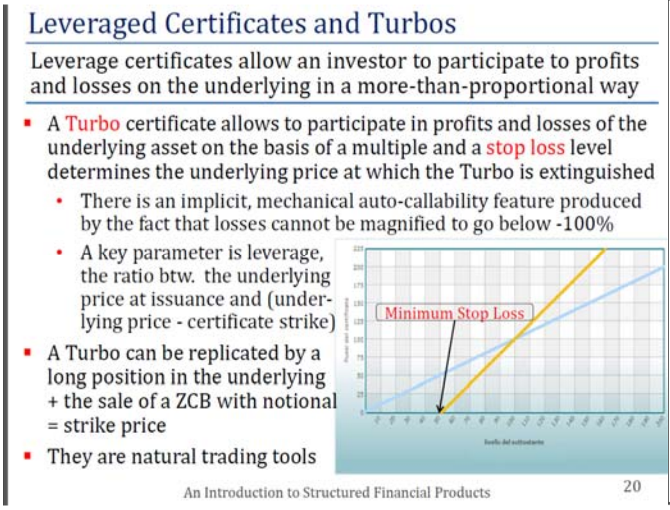
☐ (B) You will probably be able to offer a higher participation rate on S&P than on Eurostoxx 50, because both the volatility and the dividend yield are higher for the Ftse Mib and this fact certainly increases the price of the put option on the Ftse Mib that you need to sell compared to the one on the Eurostoxx 50, all else being equal

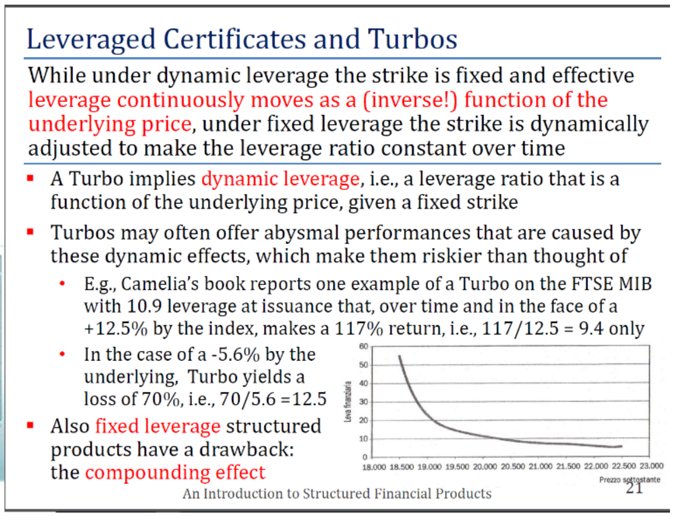
☑ (C) You will probably be able to offer a higher participation rate on the S&P 500 than on Eurostoxx 50, because the volatility of S&P 500 is lower, and its dividend yield is higher than the one of Eurostoxx 50 and this fact decreases the price of the call option on the S&P 500 that you need to buy compared to the one on the Eurostoxx 50, all else being equal

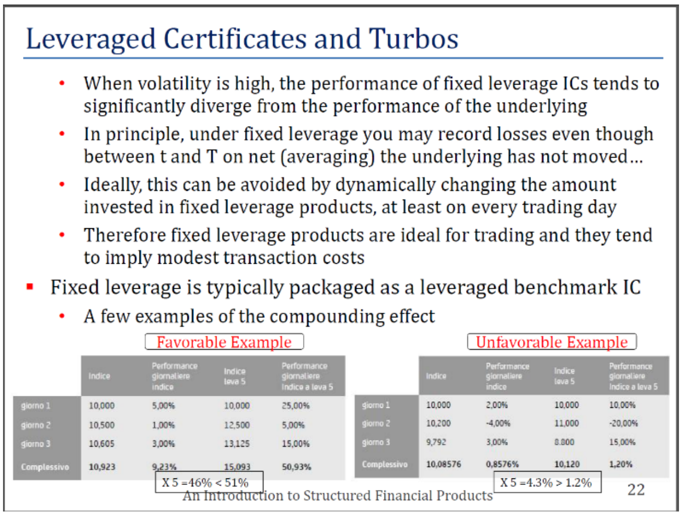
☐ (D) You will probably be able to offer a higher participation rate on the Ftse Mib than on S&P 500, because the volatility of Ftse Mib is higher than the volatility on S&P 500 and this fact decreases the price of the put option on the S&P 500 that you need to buy compared to the one on the Eurostoxx 50, all else being equal

Answer.
C is correct. The price of the call option that we need to buy to replicate an Equity Protection is lower when: a) volatility is lower; b) dividend yield is higher.

Question 14 (3 pts). Describe the characteristics of Turbo and Fixed Leverage certificates, making sure to clearly explain the main differences between dynamic and fixed leverage. Also discuss the major drawbacks of each of the two type of leverage certificates.







Question 15 (3 pts). Discuss the option decomposition of a Bonus Cap on Intesa San Paolo with a 70% American Barrier, a Bonus Amount (equal to the Cap Amount) of 120 Euros and a maturity of 1 year (hint: I would recommend that you start buying a zero-coupon bond). Also draw the payoff of the certificate and discuss the possible scenarios at maturity. How do you think that the Bonus Amount (equal to Cap Amount) of an otherwise similar Bonus certificate with European barrier will be compared to 120 Euros? Clearly justify your answer.

Debriefing

_Buy a ZCB for an amount 120 Euros (discounted, but you can also ignore this because of very low interest rates)

_Sell a D&I put with strike 120% and American barrier at 70% and tenor 1Y

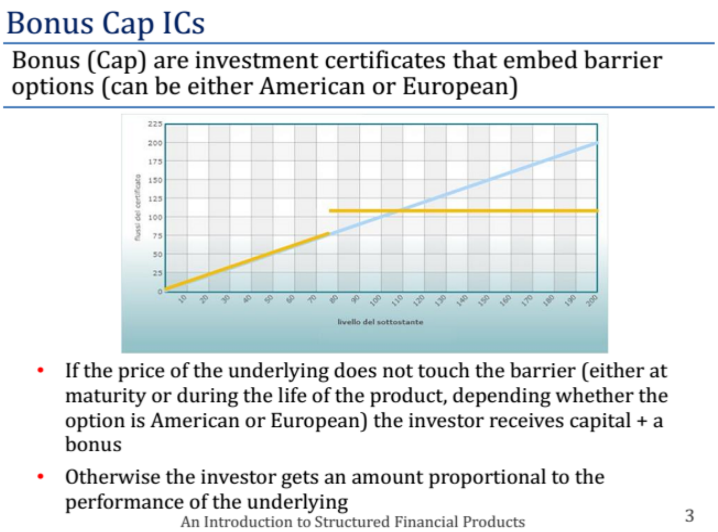
If the barrier is not triggered than the option does not come to life and we are left with 120 Euros coming from the ZCB, no matter what the price of the underlying is.

If the barrier is triggered, we have two possible outcomes at maturity:

_ if S<K, the option is exercised, and we have to pay K=120 Euros to the option seller. Therefore, the payoff is S (we are long the underlying that we receive from the put option buyer)

_ if S>K the option is out of the money and we are left with the 120 Euros of ZCB.

A similar certificate with European barrier would pay a lower Bonus. Intuition: the probability of ending up with a barrier breach is lower.



Question 16 (1.5 pts). With reference to the Monte Carlo approximation of the Rho of a structure with strike K and time-to-maturity τ, which of the following statements is the MOST LIKELY?

☐ (A) The formula to be applied is

$$\nu \equiv \lim_{h\to 0} \frac{O(r_t, \sigma_t + h, S_t; K, \tau) - O(r_t, \sigma_t - h, S_t; K, \tau)}{2h}$$

where $O(r_t, \sigma_t, S_t; K, \tau)$ is the structure price from some stochastic volatility and interest rate model with initial volatility σ_t and $\sigma_t > h$. However, the measurement error of the two prices at $\sigma_t + h$ and $\sigma_t - h$ can be large for Monte Carlo and result in an error of $O(1/(hN))$

☐ (B) The formula to be applied is

$$\nu \equiv \lim_{h\to 0} \frac{O(r_t + h, \sigma_t, S_t; K, \tau) - O(r_t, \sigma_t, S_t; K, \tau)}{2h}$$

where $O(r_t, \sigma_t, S_t; K, \tau)$ is the structure price from Black Scholes model with constant volatility σ_t and $\sigma_t > h$. However, the measurement error of the two prices at $\sigma_t + h$ and $\sigma_t - h$ can be large for Monte Carlo simulations and result in an error of $O(h/(N^{1/2}))$

☐ (C) It is impossible to approximate the Rho of a structure using Monte Carlo, because these can be just applied to the calculation of the delta of an option or basket of options

☑ (D) The formula to be applied is

$$\nu \equiv \lim_{h\to 0} \frac{O(r_t + h, \sigma_t, S_t; K, \tau) - O(r_t - h, \sigma_t, S_t; K, \tau)}{2h}$$

where $O(r_t, \sigma_t, S_t; K, \tau)$ is the structure price from some stochastic volatility model with initial interest rate r_t. However, the measurement error of the two prices at $r_t + h$ and $r_t - h$ can be large for Monte Carlo simulations and result in an error of $O(1/(hN^{1/2}))$

Question 17 (1.5 pts). In an expected power utility portfolio maximization problem (with coefficient of relative risk aversion $\gamma > 1$) in which the asset menu includes a risky asset, cash, and a derivative structure with generic payoff that depends on both the variance and the price of the underlying asset, which of the following statements is the MOST LIKELY?

☐ (A) The optimal demand of the derivative is non-zero if and only markets are complete, which means that the structure will be traded as a way to statically diversify risk, and empirically turns out to depend on the sign and the size of the equity risk premium; however, because any derivative with payoff that has non-zero Vega may complete the markets, its ex-ante economic value does not specifically depend on its payoff function

☑ (B) The optimal demand of the derivative is non-zero if and only markets are incomplete, which means that the structure will be traded as a way to hedge volatility risk, and empirically turns out to depend on the sign and the size of the volatility risk premium; however, because any derivative with payoff that has non-zero Vega may complete the markets, its ex- ante economic value does not specifically depend on its payoff function

☐ (C) The optimal demand of the derivative has one static and one dynamic, hedging component; while the former is always non-zero, the latter is non-zero if and only markets are incomplete, which means that the structure is traded as a way to hedge volatility risk

☐ (D) The optimal demand of the derivative is non-zero if and only markets are incomplete, which means that the structure is traded as a way to hedge volatility risk, and empirically turns out to depend on the sign and the size of the volatility risk premium; as we would expect, the ex-ante economic value of the derivative will depend on its payoff function

Question 18 (1.5 pts). Practitioners frequently use *equity return* correlations as proxies for asset value (A_t) and credit correlations, with corrections to reflect the fact that returns may be affected by factors unrelated to credit risk and are normally leveraged, with debt D. With reference to this fact, which of the following statements is the LEAST LIKELY?

☐ (A) This gives them the advantage that the data are sufficiently frequent and the applicable methods flexible enough to capture the fact that asset value and credit risk correlations are strongly time-varying

☐ (B) The most basic model that can be used to support this practice is Merton's model, according to which the fixed-maturity debt (D) of a firm may be priced as a European put option with strike that equals the value of the debt and equity capital as its underlying asset

☐ (C) The most basic model that can be used to support this practice is Merton's model, which implies a risk-neutral probability of default of

$$\Pr(A_{t+T} < D) = 1 - \Phi\left(d - \sigma_A\sqrt{T}\right) = \Phi\left(\sigma_A\sqrt{T} - d\right)$$

where A_t is the value of the assets of the firm, σ_A the per-period standard deviation of equity returns, D is the face value of debt, T its maturity, and

$$d = \frac{\ln(A_t/D) + \left(r_f + \sigma_A^2/2\right)T}{\sigma_A\sqrt{T}}$$

☑ (D) All of the above claims are likely

Question 19 (1.5 pts). With reference to the two formulas:

$$\hat{\phi}_t = \frac{\eta}{\gamma} - \frac{\xi\rho}{\gamma\sqrt{1-\rho^2}} - \left(\frac{\xi\rho}{\gamma\sigma\sqrt{1-\rho^2}} + H(T-t)\right)\frac{g_S}{g_V}S_t$$

$$\hat{\psi}_t = \left(\frac{\xi\rho}{\gamma\sigma\sqrt{1-\rho^2}} + H(T-t)\right)\frac{O_t}{g_V}.$$

which of the following is the MOST LIKELY statement?

☐ (A) The first equation is the optimal demand for a risky asset from a mean-variance maximization problem under stochastic volatility but no jumps, that adjusts the resulting hedging demand for the delta exposure that is purchased through the optimal demand of the derivative,

$$\frac{\xi\rho}{\gamma\sqrt{1-\rho^2}}$$

The second equation is the optimal demand of the derivative, that depends on its exposure to volatility risk, g_V (also known as vega).

☑ (B) The first equation is the optimal demand for a risky asset from an expected power utility maximization problem under stochastic volatility but no jumps, that adjusts the resulting hedging demand for the delta exposure that is purchased through the optimal demand of the derivative,

$$\left(\frac{\xi\rho}{\gamma\sigma\sqrt{1-\rho^2}} + H(T-t)\right)\frac{g_S}{g_V}S_t$$

The second equation is the optimal demand of the derivative, that depends on its exposure to volatility risk, g_V (also known as vega).

☐ (C) The first equation is the optimal demand for the derivative from an expected power utility maximization problem under stochastic volatility but no jumps, that adjusts the resulting hedging demand for the delta exposure that is purchased through the optimal demand of the derivative,

$$\left(\frac{\xi\rho}{\gamma\sigma\sqrt{1-\rho^2}} + H(T-t)\right)\frac{g_S}{g_V}S_t$$

The second equation is the optimal demand of the risky asset, that depends both its current no-arbitrage price S_t and its exposure to volatility risks, g_V (also known as vega).

☐ (D) The first equation is the optimal demand for a risky asset from an expected power utility maximization problem under a jumping process but constant volatility, that adjusts the resulting hedging demand for the delta exposure that is purchased through the optimal demand of the derivative,

$$\left(\frac{\xi\rho}{\gamma\sigma\sqrt{1-\rho^2}} + H(T-t)\right)\frac{g_S}{g_V}S_t$$

The second equation is the optimal demand of the derivative, that depends both its current no-arbitrage price O_t and its exposure to underlying price risk, g_V (also known as delta).