Demographics Trends and Stock Market Returns

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Outline

- Return Predictability and the dynamic dividend growth model
- Demographic Trends in the ddg model
- Long-run regressions and cointegration
- Out-of-sample projections
- Equity Premium Simulation up to 2050
- Related Research: The Term Structure of Stock Market Risk

The Dynamic Dividend Growth Model

Define the one-period holding return in the stock market as follows:

$$H_{t+1}^{s} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

dividing both sides by $\left(1 + H_{t+1}^{s}\right)$ and multiplying both sides by $\frac{P_t}{D_t}$ we have:

$$\frac{P_t}{D_t} = \frac{1}{\left(1 + H_{t+1}^{s}\right)} \frac{D_{t+1}}{D_t} \left(1 + \frac{P_{t+1}}{D_{t+1}}\right)$$

taking logs and with lowercase letters denoting logs of uppercase letters we have:

$$p_t - d_t = -r_{t+1}^{s} + \Delta d_{t+1} + \ln\left(1 + e^{p_{t+1} - d_{t+1}}\right)$$

Taking a first order Taylor expansion of the log term around the mean price-dividend $\overline{p-d}$ we have:

$$\ln\left(1 + e^{p_{t+1} - d_{t+1}}\right) = \ln\left(1 + e^{\overline{p-d}}\right) + \frac{e^{\overline{p-d}}}{1 + e^{\overline{p-d}}} \left(p_{t+1} - d_{t+1} - \overline{p-d}\right)$$

$$
\begin{aligned}
p_t - d_t &= \left(\overline{pd}\right) - r^s_{t+1} + \Delta d_{t+1} + \\
&\quad + \frac{P/D}{1+P/D}\left(p_{t+1} - d_{t+1} - \overline{pd}\right) \\
&= \left(\overline{pd}\right) - r^s_{t+1} + \Delta d_{t+1} + \rho\left(p_{t+1} - d_{t+1} - \overline{pd}\right) \\
\rho &= \frac{e^{\overline{p-d}}}{1+e^{\overline{p-d}}}
\end{aligned}
$$

So total stock market returns can be written as follows:

$$
r^s_{t+1} = \rho\left(pd_{t+1} - \overline{pd}\right) + \Delta d_{t+1} - \left(pd_t - \overline{pd}\right)
$$

The solution

Solving the dynamic dividend growth (DDG) forward

$$\left(pd_t - \overline{pd}\right) = E_t \sum_{j=1}^{m} \rho^{j-1} \left(\Delta d_{t+j}\right) - E_t \sum_{j=1}^{m} \rho^{j-1} \left(r^s_{t+j}\right) + \rho^m E_t \left[pd_{t+m+1} - \overline{pd}\right]$$

- The dynamic dividend growth is based on the assumption of stationarity of the (log) dividend-price ratio.
- Consistently with such an assumption, under the maintained hypothesis that stock market returns, and dividend-growth are covariance-stationary, Eq. (**??**) says that the log of the price-dividend ratio is stationary (the log of price and the log of dividend are cointegrated with a (-1,1) cointegrating vector), and that deviations of (log) prices from the common trend in (log) dividends summarize expectations of either stock market returns, or dividend growth or some combination of the two.

The Properties of the DDG model

- The model implies the possibility that **long-run** returns are predictable. So forecasting models for the stock market return should perform better the longer the forecasting horizon.
- The forecasting performance for stock market returns depends crucially on the forecasting performance for dividend growth. Note that in the case in which the dividend yield predicts expected dividend growth perfectly the proposition that returns are not predictable holds in the data. However, the empirical evidence available tells us that the dividend yield does not predict dividend growth (Cochrane 2006).

The Properties of the DDG model

- If other variables than the dividend yield are predictors of dividend growth, then the combination of these variables with the dividend yield delivers the best predicting model for the stock market (Lettau and Ludvigson 2005).
- Inflation illusion and the stock market (Cohn and Modigliani 1979, Campbell and Vuoltenahoo 2004)
- The validity of the linearization on which the model is based requires that the dividend yield fluctuates around a constant mean (around which the model is effectively linearized) (Lettau and Van Nieuwemburgh(2008), Boudouk et al.(2007)).

The Empirical Investigation of the dynamic dividend growth model

(i) $(p-d)_t$ is a very persistent time-series and forecasts stock market returns and excess returns over horizons of many years (Fama and French (1988), Campbell and Shiller (1988), Cochrane (2001, Ch. 20), and Cochrane(2007)).
(ii) $(p-d)_t$ does not have important long-horizon forecasting power for future discounted dividend-growth (Campbell (1991), Campbell, Lo and McKinlay(1997) and Cochrane(2001)).
(iii) the very high persistence of $(p-d)_t$ has led some researchers to question the evidence of its forecasting power for returns, especially at short-horizon. Careful statistical analysis that takes full account of the persistence in $(p-d)_t$ provides little evidence in favour of predictability (Nelson and Kim, 1993; Stambaugh, 1999; Ang and Bekaert, 2007; Valkanov, 2003; Goyal and Welch, 2003 and Goyal and Welch 2008). Structural breaks hava also been found (Neely and Weller(2000) and Paye and Timmermann(2006), Rapach and Wohar(2006)).

The Empirical Investigation of the Dynamic dividend growth model

(iv) More recently, Lettau and Ludvigson (2001, 2005) have found that dividend growth and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer's intertemporal budget constraint. *cay* and *cdy* are much less persistent time-series than $(p-d)_t$, they are predictors of dividend-growth and, when included in a predictive regression relating stock market returns to $(p-d)_t$, they swamp the significance of this variable.

Interpreting the Evidence

- (Lettau&Van Nieuwerburgh, 2008, LVN henceforth). LVN use a century of US data to show evidence on the breaks in the constant mean $\overline{(p-d)}$. As a matter of fact, the evidence from univariate test for non-stationarity and bivariate cointegration tests does not lead to the rejection of the null of the presence of a unit-root in $(p-d)_t$
- LVN identify two breaks in 1954 and 1991 via purely statistical methods. The nature of such breaks is not investigated.

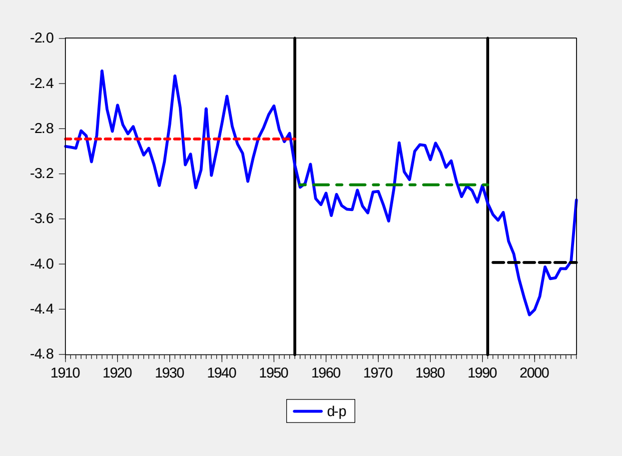

Figure 1. The time series of log dividend price ratio $(d_t - p_t)$. Annual data from 1909 to 2008.

Demographics in the DDG

- The GMQ model relates the slowly evolving mean in the log price-dividend is related to demographic trends.
- We take the GMQ model to the data via the conjecture that fluctuations in MY could capture a slowly evolving mean in $(p-d)_t$ within the dynamic dividend growth model.

A small "structural" model with demographics

$$
\begin{aligned}
\Delta d_{t+1} &= \varepsilon_{1,t+1} && (1)\\
dp_{t+1} &= \varphi_{22} dp_t + \varphi_{23} MY_{t+1} + \varepsilon_{2,t+1} && (2)\\
r^s_{t+1} &= \Delta d_{t+1} - \rho\left[dp_{t+1} - \overline{dp}_{t+1}\right] + \left[dp_t - \overline{dp}_t\right] + \varepsilon_{3,t+1} && (3)
\end{aligned}
$$

$$
\begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \sim \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \Sigma \right]
$$

Solving the model forward

We assume that the relevant linearization value for computing returns from time t to time $t+m$ is the conditional expectation of the dividend-yield for time $t+m$, given the information available at time t. By solving the model we then have

$$\sum_{j=1}^{m}\rho^{j-1}\left(r^{s}_{t+j}\right) = dp_t - \left[\varphi_{22}^{m} dp_t + \sum_{j=1}^{m}\varphi_{22}^{j-1}\varphi_{23} MY_{t+m+1-j}\right] + u_{t+m} \quad (4)$$

$$= \left(1-\varphi_{22}^{m}\right) dp_t - \sum_{j=1}^{m}\varphi_{22}^{j-1}\varphi_{23} MY_{t+m+1-j} + u_{t+m}$$

$$u_{t+m} = \sum_{j=1}^{m}\rho^{j-1}\left(\varepsilon_{1,t+j}+\varepsilon_{3,t+j}\right) - \rho^{m}\sum_{j=1}^{m}\varphi_{22}^{j-1}\varepsilon_{2,t+m+1-j}$$

Testing the GMQ Model

$$\sum_{j=1}^{\infty} \rho^{j-1} E_t[(h^s_{t+j} - \bar{h})] = \overline{(p-d)_t} - (p-d)_t + \sum_{j=1}^{\infty} \rho^{j-1} E_t[(\Delta d_{t+j} - \bar{d})]$$

$$\overline{(p-d)_t} = \beta_0 + \beta_1 MY_t + u_t$$

- long-run forecasting regressions
- cointegration

Long-Run Forecasting Regression

$$
\begin{aligned}
\sum_{j=1}^{k}(h^{s}_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
\sum_{j=1}^{k}(\Delta d_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
\sum_{j=1}^{k}(h^{s}_{t+j} - \Delta d_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
k &= 1, ..., 6
\end{aligned}
$$

Long-Run Forecasting Regression

Table 1.1: Long-horizon regressions (1910 -2008)

j-period regressions for real stock returns

$$\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

$\underset{(t-stat)}{\chi^2}$ $\underset{(0.000)}{15.63}$ horizon k in years

	1	2	3	4	5	6
β_1 (t-stat)	−0.27 (−4.000)	−0.28 (−6.287)	−0.23 (−6.563)	−0.21 (−9.077)	−0.19 (−9.352)	−0.16 (−7.824)
β_2 (t-stat)	0.30 (3.679)	0.32 (5.748)	0.26 (5.801)	0.24 (7.974)	0.21 (8.291)	0.17 (7.065)
β_3 (t-stat)	0.44 (4.065)	0.45 (5.875)	0.37 (6.509)	0.34 (7.627)	0.29 (7.865)	0.26 (7.710)
$\beta_1 = -\beta_2$ (t-stat / w.b.p-values)	−0.19 (−3.645 / 0.002)	−0.20 (−5.439 / 0.000)	−0.16 (−6.016 / 0.000)	−0.16 (−7.652 / 0.000)	−0.14 (−8.584 / 0.002)	−0.13 (−7.740 / 0.008)
$t/\sqrt{T}$ − test	{0.30**}	{0.49***}	{0.59***}	{0.75***}	{0.89***}	{0.95***}
$\beta_3 \mid (\beta_1 = -\beta_2)$ (t-stat)	0.42 (3.447)	0.44 (4.636)	0.38 (5.059)	0.36 (6.006)	0.33 (6.578)	0.29 (6.915)
$adjR^2$	0.09	0.25	0.33	0.44	0.52	0.54
$adjR^2(\beta_1 = -\beta_2)$	0.07	0.20	0.27	0.37	0.46	0.50
F-statistic	4.10	12.18	16.74	26.62	36.74	39.48

Testing MY against alternative models:

$$\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
$(\overline{d-p})^{LVN}_t$	β_3 (t-stat)	0.44 (4.149)	0.45 (6.138)	0.39 (7.242)	0.36 (9.009)	0.32 (10.383)	0.28 (10.449)
	β_4 (t-stat)	0.07 (0.665)	0.09 (1.128)	0.05 (0.665)	0.03 (0.626)	0.01 (0.140)	−0.03 (−0.636)
$(\overline{d-p})^{BMRW}_t$	β_3 (t-stat)	0.44 (2.850)	0.46 (4.849)	0.39 (6.229)	0.35 (8.561)	0.31 (10.286)	0.27 (10.925)
	β_4 (t-stat)	0.51 (3.602)	0.37 (2.945)	0.21 (2.443)	0.09 (1.496)	0.02 (0.407)	−0.03 (−1.103)

Long-Run Forecasting Regression

Table 1.2: Long-horizon regressions (1910-2008)

j-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 (t-stat)	2.66 (0.015)	horizon k in years				
	1	2	3	4	5	6
β_1 (t-stat)	0.19 (2.097)	0.10 (1.827)	0.05 (1.400)	0.03 (1.222)	0.02 (1.006)	0.01 (0.803)
β_2 (t-stat)	−0.23 (−2.196)	−0.13 (−1.996)	−0.07 (−1.585)	−0.04 (−1.441)	−0.03 (−1.308)	−0.02 (−1.153)
β_3 (t-stat)	−0.15 (−1.250)	−0.03 (−0.371)	0.04 (0.780)	0.07 (1.863)	0.08 (2.428)	0.08 (2.755)
$\beta_1 = -\beta_2$ (t-stat / w.b.p-values)	0.10 (1.580 / 0.240)	0.05 (1.118 / 0.433)	0.02 (0.638 / 0.353)	0.01 (0.298 / 0.171)	−0.00 (−0.018 / 0.096)	−0.00 (−0.234 / 0.106)
$\beta_3 \mid (\beta_1 = -\beta_2)$ (t-stat)	−0.12 (−1.118)	−0.03 (−0.366)	0.03 (0.714)	0.06 (1.700)	0.06 (2.067)	0.06 (2.134)
$adjR^2$	0.15	0.09	0.05	0.04	0.05	0.06
$adjR^2(\beta_1 = -\beta_2)$	0.06	0.03	0.02	0.02	0.03	0.03
F-statistic	6.87	4.05	2.73	2.52	2.66	3.04

Testing MY against alternative models:

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
$\overline{(d-p)}_t^{LVN}$	β_3 (t-stat)	−0.09 (−0.877)	−0.00 (−0.033)	0.05 (0.993)	0.07 (1.886)	0.08 (2.511)	0.09 (2.886)
	β_4 (t-stat)	−0.24 (−2.412)	−0.14 (−2.094)	−0.08 (−1.520)	−0.05 (−0.942)	−0.02 (−0.321)	0.00 (0.024)
$\overline{(d-p)}_t^{BMRR}$	β_3 (t-stat)	−0.18 (−1.304)	−0.05 (−0.697)	−0.00 (−0.001)	0.03 (0.772)	0.04 (1.752)	0.06 (2.509)
	β_4 (t-stat)	0.09 (0.879)	0.14 (1.981)	0.16 (2.122)	0.16 (2.206)	0.15 (2.977)	0.08 (2.304)

Long-Run Forecasting Regression

Table 1.3: Long-horizon regressions (1910-2008)

j-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(h^s_{t+j}-\Delta d_{t+j}) = \beta_0+\beta_1 p_t+\beta_2 d_t+\beta_3 MY_t+\varepsilon_{t,t+j}$$

χ^2 (t-stat): 27.94 (0.000) — horizon k in years

	1	2	3	4	5	6
β_1 (t-stat)	−0.46 (−6.159)	−0.39 (−8.285)	−0.28 (−9.393)	−0.24 (−10.344)	−0.20 (−14.944)	−0.17 (−12.523)
β_2 (t-stat)	0.53 (6.127)	0.45 (8.234)	0.33 (9.157)	0.28 (10.022)	0.24 (13.150)	0.20 (10.940)
β_3 (t-stat)	0.59 (4.631)	0.47 (5.313)	0.33 (5.297)	0.27 (5.697)	0.22 (6.202)	0.18 (6.361)
$\beta_1=-\beta_2$ (t-stat / w.b.p-values)	−0.28 (−4.961 / 0.001)	−0.24 (−6.603 / 0.000)	−0.18 (−7.345 / 0.000)	−0.16 (−8.516 / 0.000)	−0.14 (−13.217 / 0.000)	−0.12 (−12.976 / 0.002)
$t/\sqrt{T}-test$	{0.47***}	{0.67***}	{0.77***}	{0.91***}	{1.04***}	{1.12***}
$\beta_3 \mid (\beta_1=-\beta_2)$ (t-stat)	0.55 (3.610)	0.47 (4.206)	0.35 (4.204)	0.31 (4.646)	0.27 (5.160)	0.23 (5.406)
$adjR^2$	0.24	0.45	0.51	0.60	0.66	0.67
$adjR^2(\beta_1=-\beta_2)$	0.16	0.29	0.35	0.43	0.50	0.53
F-statistic	11.49	27.74	35.07	50.25	65.08	67.56

Testing MY against alternative models:

$$\sum_{j=1}^{k}(h^s_{t+j}-\Delta d_{t+j}) = \beta_0+\beta_1 p_t+\beta_2 d_t+\beta_3 MY_t+\beta_4 z_t+\varepsilon_{t,t+j}$$

$z_t=$		1	2	3	4	5	6
$(d-\overline{p})_t^{LVN}$	β_3 (t-stat)	0.53 (4.111)	0.45 (5.041)	0.34 (5.453)	0.29 (6.172)	0.24 (7.314)	0.19 (8.584)
	β_4 (t-stat)	0.31 (2.618)	0.23 (2.739)	0.13 (1.915)	0.08 (1.512)	0.02 (0.513)	−0.03 (−0.719)
$(d-\overline{p})_t^{BMRR}$	β_3 (t-stat)	0.62 (4.100)	0.51 (6.105)	0.39 (6.490)	0.32 (7.313)	0.27 (8.837)	0.21 (9.256)
	β_4 (t-stat)	0.42 (2.294)	0.22 (1.881)	0.05 (0.704)	−0.07 (−1.734)	−0.13 (−4.349)	−0.11 (−4.035)

20-Year horizon

A Summary:

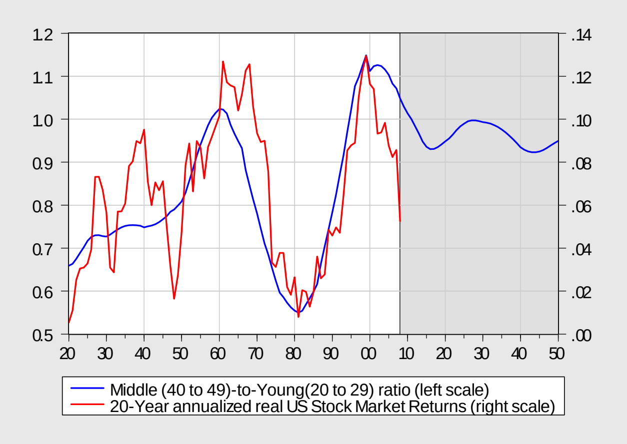

Figure 3: MY and 20-year annualized real US stock market returns

Table 2.1: Estimates from a cointegrated VAR (1911-2008)

Cointegrating vector	p_t	d_t	MY_t	C
β $(s.e.)$	-1.00	1.21 (0.035)	1.107 (0.25)	2.16
Error Correction Model	Δp_t	Δd_t	ΔMY_t	
α $(s.e.)$	0.29 (0.096)	-0.12 (0.046)	0.007 (0.007)	
$Adj.\ R^2$	0.126	0.43	0.63	
Cointegration Test	Trace	p-value	Max eigen	p-value
Hypothesized No of CE(s)				
None	29.68	0.05	22.86	0.028
At Most 1	6.82	0.59	6.75	0.51

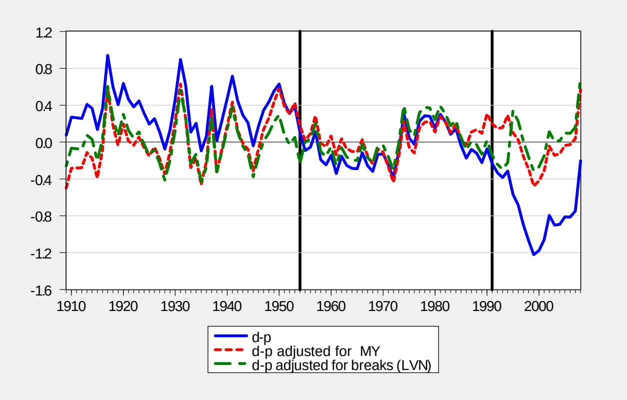

Figure 4.1: (d-p), (d-p) adjusted for breaks (LVN) and fluctuatins of (d-p) around a time-varying mean determined by *MY*

MY, CAY and CDY

Evaluating the effect of the inclusion of *cay* and *cdy* in the long-run forecasting regressions that also include MY_t

- is a parsimonious way of evaluating the model with MY_t against all financial ratios traditionally adopted to predict returns. *cay* and *cdy* dominate all the traditionally adopted financial ratios
- it would allow further investigation on the presence of a common component in dividend and stock market returns suggested by LL(2005) but not consistent with our findings in Table 1.3, that witness the significance of MY for predicting long-run returns and long-run returns adjusted for dividend growth.
- it could shed further light on the relative importance of *cay* and *cdy* and MY_t for predicting returns and dividend growth in the dynamic dividend growth model.

$$\sum_{j=1}^{k}\left(h^{s}_{t+j}\right)=\beta_0+\beta_1 p_t+\beta_2 d_t+\beta_3 MY_t+\beta_4 z_t+\varepsilon_{t,t+j}$$

horizon k in years

		1	2	3	4	5	6
β_1 $(t-stat)$	cay_t	-0.41 (-4.050)	-0.29 (-4.694)	-0.21 (-5.570)	-0.20 (-7.222)	-0.19 (-8.257)	-0.15 (-8.790)
	cdy_t	-0.51 (-6.487)	-0.40 (-6.442)	-0.29 (-6.840)	-0.24 (-8.592)	-0.21 (-11.549)	0.19 (-12.403)
β_2 $(t-stat)$	cay_t	0.50 (3.994)	0.36 (4.492)	0.25 (5.230)	0.24 (6.373)	0.22 (7.067)	0.17 (7.384)
	cdy_t	0.63 (6.210)	0.48 (6.134)	0.34 (6.287)	0.28 (7.143)	0.25 (9.064)	0.22 (9.832)
β_3 $(t-stat)$	cay_t	0.66 (4.194)	0.48 (5.077)	0.36 (6.327)	0.32 (8.823)	0.29 (10.967)	0.24 (13.000)
	cdy_t	0.781 (5.467)	0.57 (5.774)	0.42 (6.146)	0.35 (7.756)	0.31 (9.394)	0.26 (10.067)
β_4 $(t-stat)$	cay_t	2.06 (1.336)	2.41 (3.329)	1.94 (3.111)	0.87 (1.192)	0.71 (1.471)	1.15 (3.739)
	cdy_t	-0.51 (-0.718)	0.25 (0.490)	0.27 (1.102)	0.04 (0.225)	0.06 (0.498)	0.44 (3.246)
$adjR^2$	cay_t	0.35	0.61	0.70	0.75	0.82	0.87
	cdy_t	0.34	0.56	0.65	0.74	0.81	0.86

Predictive Performance:1-year horizon

	In-Sample				Out-of-Sample			
(k=1)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	3.03	1.64	12.92	16.17	-11.22	14.54	18.60	-17.43
dp_t^{LVN}	6.36	2.64	11.93	16.20	-5.25	13.58	18.09	-8.09
cdy_t	-1.47	0.48	12.03	14.00	-16.11	13.24	15.12	-19.97
dp_t^{DT}	19.48	4.37	10.08	15.32	11.20	10.91	16.27	4.68
dp_t^{DT} cdy_t	38.64	5.97 -0.32	8.71	10.97	28.61	9.83	11.86	14.36
H.M.	–	–	12.92	16.70	–	13.40	17.63	–

Predictive Performance:2-year horizon

(k= 2)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	5.46	1.70	15.72	20.71	-55.77	24.21	30.94	-4.01
dp_t^{LVN}	15.74	2.45	14.20	19.91	-11.72	19.99	26.20	-3.30
cdy_t	2.92	1.13	16.00	21.93	-55.19	20.04	27.45	-1.33
dp_t^{DT}	48.32	7.88	12.20	17.52	35.52	14.39	19.90	3.11
dp_t^{DT}	62.63	6.32	10.58	14.17	40.85	13.16	16.94	6.08
cdy_t		1.40						
H. M.	–	–	16.19	21.91	–	18.65	24.79	–

Predictive Performance:3-year horizon

k= 3	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	6.34	1.99	18.26	24.87	-88.56	33.98	43.59	1.32
dp_t^{LVN}	10.76	1.70	17.91	24.81	-25.89	28.40	35.61	-4.41
cdy_t	5.91	1.41	19.45	26.94	-41.43	26.64	33.70	-1.97
dp_t^{DT}	49.73	6.70	13.22	19.00	43.95	18.15	23.32	2.90
dp_t^{DT}	64.89	7.24	13.67	16.99	48.54	17.06	20.33	2.50
cdy_t		2.99						
H. M.	–	–	19.38	26.65	–	25.26	31.74	–

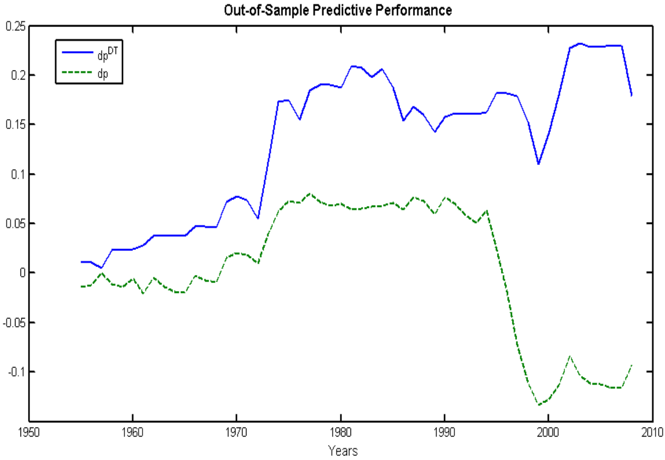

Figure 5: differences of cumulative RMSE of forecasts based on the historical prevaling mean and RMSE of forecasting models based on $(d-p)_t$ and on $(d-p)_t$ corrected for MY.

Long Run Projections

- We concentrate on 5-year excess returns and estimate the following model:

$$\sum_{j=1}^{5}(h^{s}_{t+j} - r_{f,t+H}) = c_1 + c_2\left(p_t - c_3 d_t - c_4 MY_t\right) + u_{1t} \quad (5)$$

$$\begin{bmatrix} \Delta p_{t+1} \\ \Delta d_{t+1} \end{bmatrix} = \begin{bmatrix} c_5 \\ c_{10} \end{bmatrix} + \begin{bmatrix} c_6 \\ c_{11} \end{bmatrix} \begin{bmatrix} 1 & -c_3 & -c_4 \end{bmatrix} \begin{bmatrix} p_t \\ d_t \\ MY_t \end{bmatrix} + \begin{bmatrix} c_7 & c_8 & c_9 \\ c_{12} & c_{13} & c_{14} \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta d_t \\ \Delta MY_t \end{bmatrix} + \begin{bmatrix} u_{2t} \\ u_{2t} \end{bmatrix},$$

Demographics and the Equity Premium

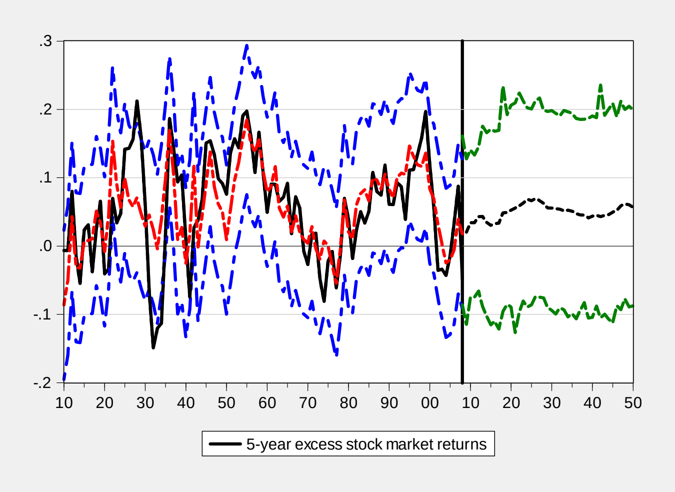

Figure 6: within sample and out-of-sample projections for 5-year stock market excess returns.

Conclusions

- The slowly evolving trend in the mean dividend/price is determined by a demographic variable, MY , the ratio of middle-age to young population. We have shown that MY captures well a slowly evolving component in the mean dividend/price ratio and it is strongly significant in long-horizon regressions for real stock market returns.
- The empirical results we have reported should be of special relevance to the strategic asset allocation literature, in which the log dividend-price ratio is often used in VAR models as a stationary variable capturing time-variation in the investment opportunity set, and as an input into the optimal asset allocation decision of a long-horizon investor. Allowing for the presence of MY in the VAR models used to estimate the time profile of returns and their volatility might cast new light on the hot debate on the safety of stock market investment for the long-run (see Campbell and Viceira (2002), Pastor and Stambaugh(2009)).

The TS of Stock Market Risk

- The fact that a slow moving variables determined by demographics has very little impact on predictability of stock market returns at high frequency but a sizeable and strongly significant impact at low frequency has some obvious consequences on the slope of stock market risk, defined as the conditional variance and covariance per period of asset returns.
- When demographic trends are used to model the slow moving fluctuations in the dividend-price ratio a natural decomposition of this variable into an high volatility "noise" component, reflecting high-frequency stock market fluctuations, and a low-volatility "information" component reflecting the slowly evolving long-run trend. The dominance of the "noise" component at high frequency and of the information component at low frequency should lead naturally to a positive relation between predictability of returns and forecasting horizon and to a negatively sloped term structure of risk.
- The VAR based approach to the TS of stock market risk underestimates this slope

The VAR approach

$$
\begin{aligned}
(z_t - E_z) &= \Phi_1 (z_{t-1} - E_z) + \nu_t \\
\nu_t &\sim \mathcal{N}(0, \Sigma_\nu)
\end{aligned}
$$

$$
\begin{aligned}
z_t &= \begin{bmatrix} r_t^s \\ d_t - p_t \end{bmatrix}, E_z = \begin{bmatrix} E_{r^s} \\ E_{d-p} \end{bmatrix} \\
\Phi_1 &= \begin{bmatrix} 0 & \varphi_{1,2} \\ 0 & \varphi_{2,2} \end{bmatrix} \\
\begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix} &\sim \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{matrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{matrix} \right]
\end{aligned}
$$

$$
\begin{aligned}
Var_t \left[(z_{t+1} + ... + z_{t+k}) \mid D_t \right] &= \Sigma_\nu + (I + \Phi_1)\Sigma_\nu (I + \Phi_1)' + \\
&\quad (I + \Phi_1 + \Phi_1^2)\Sigma_\nu (I + \Phi_1 + \Phi_1^2)' + ... \\
&\quad + (I + \Phi_1 + ... + \Phi_1^{k-1})\Sigma_\nu (I + \Phi_1 + ... +
\end{aligned}
$$

The VAR approach

This implies that in our simple bivariate example the term structure of stock market risk takes the form

$$\sigma_r^2(k) = \sigma_1^2 + 2\varphi_{1,2}\sigma_{1,2}\psi_1(k) + \varphi_{1,2}^2\sigma_{2,2}^2\psi_2(k)$$

where

$$\psi_1(k) = \frac{1}{k}\sum_{l=0}^{k-2}\sum_{i=0}^{l}\varphi_{2,2}^i \quad k > 1$$

$$\psi_2(k) = \frac{1}{k}\sum_{l=0}^{k-2}\left(\sum_{i=0}^{l}\varphi_{2,2}^i\right)^2 \quad k > 1$$

$$\psi_1(1) = \psi_2(1) = 0$$

The TS of Risk by direct regression

We measure the term structure of stock market risk by estimating the following "structural" system of eleven equations:

$$
\begin{aligned}
\frac{1}{\sqrt{m}}\sum_{j=1}^{m} r_{t+j}^{s} &= \delta_{0,m} + \frac{1}{\sqrt{m}}\left(1-\varphi_{22}^{m}\right) dp_t - \frac{\varphi_{23}}{\sqrt{m}}\left(\sum_{j=1}^{m}\varphi_{22}^{j-1} MY_{t+m+1-j}\right) + \\
m &= 1, \ldots, 10 \\
dp_{t+1} &= \varphi_{20} + \varphi_{22} dp_t + \varphi_{23} MY_{t+1} + \varepsilon_{2,t+1}
\end{aligned}
$$

An unrestricted version is also estimated to perform a test of the validity of the relevant restrictions:

$$
\begin{aligned}
\frac{1}{\sqrt{m}}\sum_{j=1}^{m} r_{t+j}^{s} &= \delta_{0,m} + \frac{\delta_{1m}}{\sqrt{m}} dp_t + \frac{\delta_{2m}}{\sqrt{m}}\left(\sum_{j=1}^{m}\varphi_{22}^{j-1} MY_{t+m+1-j}\right) + u_{t+m} \\
dp_{t+1} &= \varphi_{20} + \varphi_{22} dp_t + \varphi_{23} MY_{t+1} + \varepsilon_{2,t+1}
\end{aligned}
$$

Table 1: System Estimation (1910-2009)

$$dp_{t+1} = \varphi_{20} + \varphi_{22} dp_t + \varphi_{23} MY_{t-j} + \varepsilon_{2t+1}$$

$$\text{UM: } \frac{1}{\sqrt{m}} \sum_{j=1}^{m} r^s_{t+j} = \delta_{0m} + \frac{\delta_{1m}}{\sqrt{m}} dp_t + \frac{\delta_{2m}}{\sqrt{m}} \left(\sum_{j=1}^{m} \varphi_{22}^{j-1} MY_{t+m+1-j} \right) + u_{t+m} \quad m = 1,..,10$$

$$\text{RM: } \frac{1}{\sqrt{m}} \sum_{j=1}^{m} r^s_{t+j} = \delta_{0m} + \frac{1}{\sqrt{m}} \left(1 - \varphi_{22}^m\right) dp_t - \frac{\varphi_{23}}{\sqrt{m}} \left(\sum_{j=1}^{m} \varphi_{22}^{j-1} MY_{t+m+1-j} \right) + u_{t+m}$$

		horizon m in years									
UM		1	2	3	4	5	6	7	8	9	10
δ_{1m}		0.18	0.38	0.48	0.61	0.70	0.73	0.78	0.86	0.89	0.89
$(t-stat)$		(3.87)	(5.57)	(5.61)	(6.96)	(7.99)	(7.17)	(7.25)	(8.73)	(7.63)	(6.15)
$(GLS-PPT)$		(–)	(4.54)	(5.21)	(5.77)	(7.43)	(5.19)	(7.03)	(8.61)	(7.57)	(2.67)
δ_{2m}		0.41	0.52	0.56	0.64	0.69	0.70	0.74	0.78	0.81	0.83
$(t-stat)$		(3.23)	(3.69)	(3.85)	(4.28)	(4.55)	(4.69)	(4.78)	(4.93)	(4.94)	(5.01)
$(GLS-PPT)$		(–)	(3.85)	(3.50)	(3.87)	(4.21)	(4.10)	(4.06)	(4.10)	(4.19)	(4.84)
φ_{22}	0.61										
$(t-stat)$	(9.21)										
φ_{23}	−0.83										
$(t-stat)$	(−3.79)										
RM											
φ_{22}	0.76										
$(t-stat)$	(19.31)										
φ_{23}	−0.53										
$(t-stat)$	(4.41)										
χ^2_{12}	13.45 (0.34)	χ^2_{20}	17.19 (0.64)								
σ_{DepVar}		0.195	0.198	0.187	0.185	0.181	0.174	0.172	0.173	0.171	0.168
$\sigma_{u_{t+m}}$	UM	0.188	0.179	0.164	0.152	0.140	0.131	0.125	0.118	0.112	0.109
$\sigma_{u_{t+m}}$	RM	0.189	0.179	0.164	0.152	0.141	0.133	0.127	0.120	0.115	0.112
$adjR^2$	UM	–	0.18	0.24	0.33	0.41	0.44	0.48	0.54	0.57	0.58
$adjR^2$	RM	0.06	0.18	0.23	0.32	0.40	0.42	0.46	0.52	0.54	0.55

Table: The estimation is by GMM. GLS-PPT is the t-stat that explicitly accounts for the MA(m-1) errors structure as (see Pesaran, Pick and Timmermann (2010)). σ_{DepVar} is the annualized unconditional standard deviation, σ_{u} is the

The TS of Risk