Università Commerciale
Luigi Bocconi

Supplement to Lecture 3

Prof. Massimo Guidolin

20192– Financial Econometrics

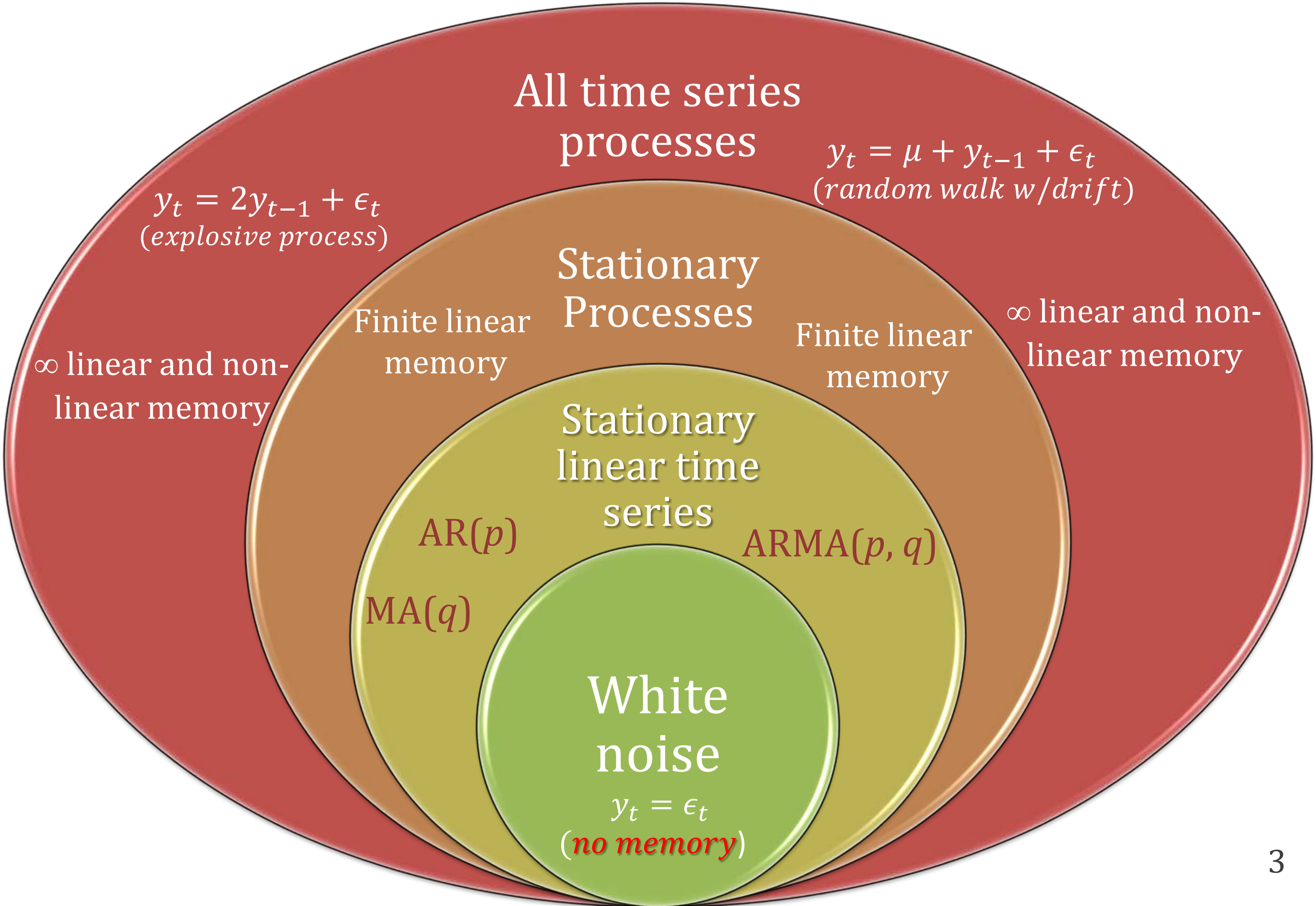
Winter/Spring 2018

Overview

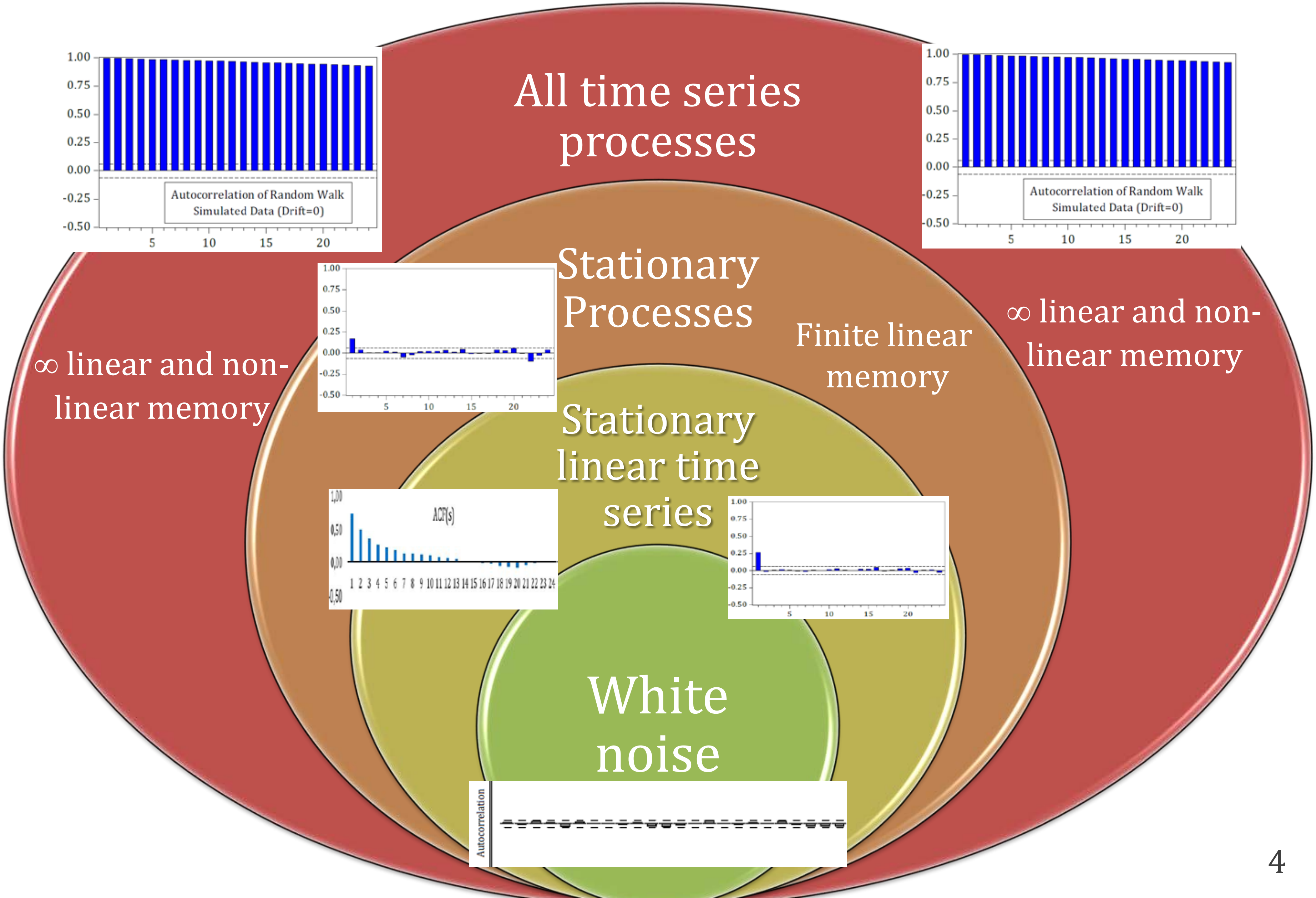
- Stationarity, non-stationarity, and white noise processes

- Invertibility of MA(q) processes

Different Memory Strengths



Different Autocorrelation Functions



From MA Processes to AR Processes

- In the lectures MA processes have been introduced as intuitively related to the practice of technical analysis—hence their name
- In what sense $y_t = \mu + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$ is a moving average of the data, the y's?
  - Pay attention to the algebra for a simple MA(1):
  - $y_t = \mu + \theta\epsilon_{t-1} + \epsilon_t = \mu + \theta[y_{t-1} - \mu - \theta\epsilon_{t-2}] + \epsilon_t$
  - The term in squared bracket comes from $y_{t-1} = \mu + \theta\epsilon_{t-2} + \epsilon_{t-1}$
  - Therefore $y_t = (1-\theta)\mu + \theta y_{t-1} - \theta^2\epsilon_{t-2} + \epsilon_t$ which is ARMA(1,2) with a missing coefficient for the ϵ_{t-1} term
  - Next, $y_t = (1-\theta)\mu + \theta y_{t-1} - \theta^2[y_{t-2} - \mu - \theta\epsilon_{t-3}] + \epsilon_t$ $= (1-\theta+\theta^2)\mu + \theta y_{t-1} - \theta^2 y_{t-2} + \theta^3\epsilon_{t-3} + \epsilon_t$, ARMA(2,3) with a missing coefficient for the ϵ_{t-1} and ϵ_{t-2} terms
  - Recursively proceeding in this way, we have that the original MA(1) may be written as an ARMA(p, p+1) as $p \longrightarrow \infty$

    $y_{t-1} = (1-\theta+\theta^2-\theta^3+\cdots)\mu + \theta y_{t-1} - \theta^2 y_{t-2} + \theta^3 y_{t-3} + \cdots + \epsilon_t$
    an AR(∞) model with coefficients $\phi_j = \theta^j$ and $\phi_0 = \sum_{j=0}^{\infty}(-\theta)^j$

From MA Processes to AR Processes

- Therefore a MA(1) process is indeed a moving average of past data,

$$y_t = \sum_{j=0}^{\infty} (-\theta)^j + \sum_{j=1}^{\infty} (-\theta)^j y_{t-j} + \epsilon_t$$

  but an infinite one with decaying weights
- Such objects are also called exponential moving averages
- Are we guaranteed that such an AR(∞) makes sense (technically, we say that the corresponding difference equation has a solution)?
- Visibly, this depends on whether the sequence $-\theta, \theta^2, -\theta^3, \theta^4, \ldots$ and convergence will be related, for instance to whether $|\theta| < 1$
- Such conditions are related to the invertibility of the process:

Definition **(Invertibility)** An invertible MA(q) model can be expressed as an $AR(\infty)$:

$$y_t = \sum_{i=1}^{\infty} \phi_i L^i y_{t-i} + u_t .$$

A MA(q) is invertible when the magnitude of all the roots of the MA polynomial exceeds the one.