Università Commerciale Luigi Bocconi

Mean-Variance Portfolio Choice in Excel

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Portfolio Management

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Overview

- Fundamentals: dealing with matrices in Excel
- Problem one: the efficient frontier with two risky assets (and no risk free asset)
- Problem two: asset allocation with many assets, including the risk-free one

Fundamentals: the use of matrices in Excel (1/4)

Matrix notation is essential to deal with multi-asset allocation problems in a compact and efficient way

We now review the most common and helpful functions that Excel provides to you to deal with matrices:

- Let's suppose that **A** is 2-by-2 matrix and that you want to perform the transpose (**A'**)
  - **TRANSPOSE (C5:D6)**

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4									
5		**A=**	5	8		**A'=**	5	7	
6			7	3			8	3	
7									
8									

REMEMBER, WHEN YOU DEAL WITH MATRICES YOU ALWAYS HAVE TO PRESS **CTRL + SHIFT + ENTER**

Fundamentals: the use of matrices in Excel (2/4)

- Let's suppose that **A** and **B** are two 2-by-2 matrix and that you want to perform their multiplication
  - **MMULT (C5:D6,C9:D10)**

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4									
5		**A=**	5	8		**A'=**	5	7	
6			7	3			8	3	
7									
8									
9		**B=**	9	5		**AB=**	181	57	
10			17	4			114	47	
11									
12									
13									

  - You need to select the result area G9:H10 (matrix is 2 x 2 so the result will be 2 x 2 too) and press CTRL + SHIFT + ENTER

Fundamentals: the use of matrices in Excel (3/4)

- Let’s suppose that **A** is a 3-by-3 matrix and that you want to invert it
  - **MINVERSE (C13:E15)**

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4							**TRANSPOSE OF MATRIX A**			
5		**A=**	5	8		**A'=**	5	7		
6			7	3			8	3		
7										
8							**MULTIPLICATION OF A AND B**			
9		**B=**	9	5		**AB=**	181	57		
10			17	4			114	47		
11										
12										
13		**C=**	8	5	11		**C^(-1)=**	-0.002409639	0.231325301	-0.040963855
14			6	2	1			-0.05060241	-0.142168675	0.139759036
15			9	11	5			0.115662651	-0.103614458	-0.03373494
16										
17										

  - You need to select the result area H13:J15 (matrix is 3 x 3 so the result will be 3 x 3 too) and press CTRL + SHIFT + ENTER

Fundamentals: the use of matrices in Excel (4/4)

- In addition, remember that the formula SUMPRODUCT allows you to multiply two vectors
  - **SUMPRODUCT (C18:C20, F18:F20)**

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4							**TRANSPOSE OF MATRIX A**			
5		**A=**	5	8		**A'=**	5	7		
6			7	3			8	3		
7										
8							**MULTIPLICATION OF A AND B**			
9		**B=**	9	5		**AB=**	181	57		
10			17	4			114	47		
11										
12									**INVERSE OF C**	
13			8	5	11			-0.002409639	0.231325301	-0.040963855
14		**C=**	6	2	1		**C^(-1)=**	-0.05060241	-0.142168675	0.139759036
15			9	11	5			0.115662651	-0.103614458	-0.03373494
16										
17										
18			3			6			**MULTIPLICATION OF D' AND E**	
19		**D=**	4		**E=**	7		86		
20			5			8				
21										

SUMPRODUCT MEANS:

3 x 6 + 4 x 7 + 5 x8

Problem one: the stock-bond asset allocation

- Let's suppose you can only invest in two assets:
  - a (US) stock index (here represented by the value-weighted CRSP index)
  - a (US) long-term (Treasury) bond index (here represented by the Ibbotson 10-year government bond index)
- You have available the monthly log-returns of the two indices
- First of all, you need to compute the statistics of the two series: the mean and the standard deviation of each series and the pair-wise correlation between them

	A	B	C	D
1				
2		**Mean**	**Variance**	**Weights**
3	**Equity**	9.06%		
4	**Tbond**	6.01%		
5				

If you recall log-returns properties (i.e. return over two periods is just the sum of the returns of each period) you can compute the annual mean return: it is simply equal to the monthly mean return multiplied by 12

B3 contains the formula => MEAN(Equity)*12 (where Equity is the name we gave to C2:C253, i.e. the monthly equity log-returns)

B4 contains the formula=> MEAN(Tbond)*12 (where bond are monthly bond log-returns)

Problem one: the stock-bond asset allocation

Similarly, the annual standard deviation of log-returns is obtained by multiplying by $\sqrt{12}$

C3 contains the formula => STDEV(Equity)*SQRT(12)

C4 contains the formula => STDEV(bond)*SQRT(12)

	A	B	C	D
1				
2		**Mean**	**Standard Deviation**	**Weights**
3	**Equity**	9.06%	15.57%	
4	**Tbond**	6.01%	7.047%	
5				
6				
7				
8				

Finally, we compute the correlation between the two series with the function "CORREL"=> CORREL(Equity, Tbond)

5				
6	CORRELATION MATRIX			
7		Equity	Tbond	
8	**Equity**	1	-0.18	
9	**Tbond**	-0.18	1	
10				
11				
12				
13				
14				
15				

Problem one: the stock-bond asset allocation

- Finally, we construct the variance-covariance matrix (let's call it **V**); as you know this is a symmetric matrix contains the variances of each asset in the main diagonal and the pair-wise covariances out of the main diagonal
- Recall that the formula for co- variance is: $\mathrm{Cov}(i,j)=\varrho\sigma_i\sigma_j$

In order to compute the variance-covariance matrix we can use COVAR formula of excel. HINT: you can use "INDIRECT" function plus the names that we have given to the data

	A	B	C	D
1	TASK 1: compute the statistics for stocks, bonds and for an equally weighted portf			
2				
3		Mean	Standard Deviation	equal weights
4	Equity	9.056%	15.568%	50%
5	Tbond	6.015%	7.047%	50%
6				
7	CORRELATION MATRIX			
8		Equity	Tbond	CORRELATION
9	Equity	1.00	-0.18	HAVE BEEN CO
10	Tbond	-0.18	1.00	CORRELATION
11				FORMULA, THE
12	VARCOV			FORMULA
13		Equity	Tbond	
14	Equity	0.024235338	-0.001921698	
15	Tbond	-0.001921698	0.004966527	
16				
17				
18				THE GLOBAL MI
19	Statistics equally weighted portfolio			ONE THAT HAS T
20	Portfolio Mean	7.54%		COMPUTE IT I N
21	Portfolio Variance			PORTFOLIO VAR
22	Standard Deviation			
23				

Why would you bother to do such a formula when you know that cell B13 is just the variance of equity returns (i.e. the square of cell C3)?

Excel makes your life **easier** when you deal with a LARGE amount of data (e.g. 5 assets imply a 5-by-5 **V** matrix!)
Now you can just drag and drop!

Problem one: the stock-bond asset allocation

- Now, let's suppose for a minute that we have an **equally weighted** portfolio and compute portfolio mean and variance (the two asset case is very simple and you do not necessarily need to use matrices… however we want to create a general set up that will be valid also when we add other assets)

PORTFOLIO MEAN

$$E(r_p) = w^T e$$

PORTFOLIO MEAN — WEIGHTS — MEAN RETURNS

SUMPRODUCT(B3:B4, D3:D4)

PORTFOLIO VARIANCE

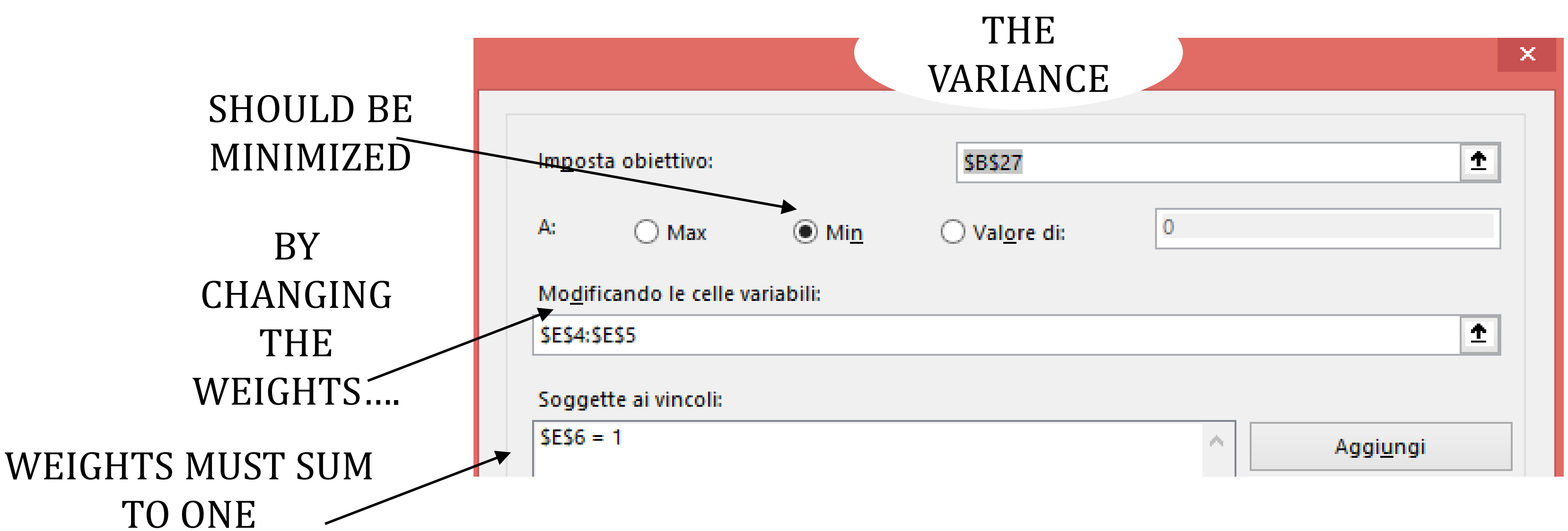
$$Var(r_p) = w^T V w$$

PORTFOLIO VARIANCE — WEIGHTS — VARIANCE – COVARIANCE MATRIX — WEIGHTS

MMULT(TRANSPOSE(B3:B4),MMULT(B14:C15, B3:B4))

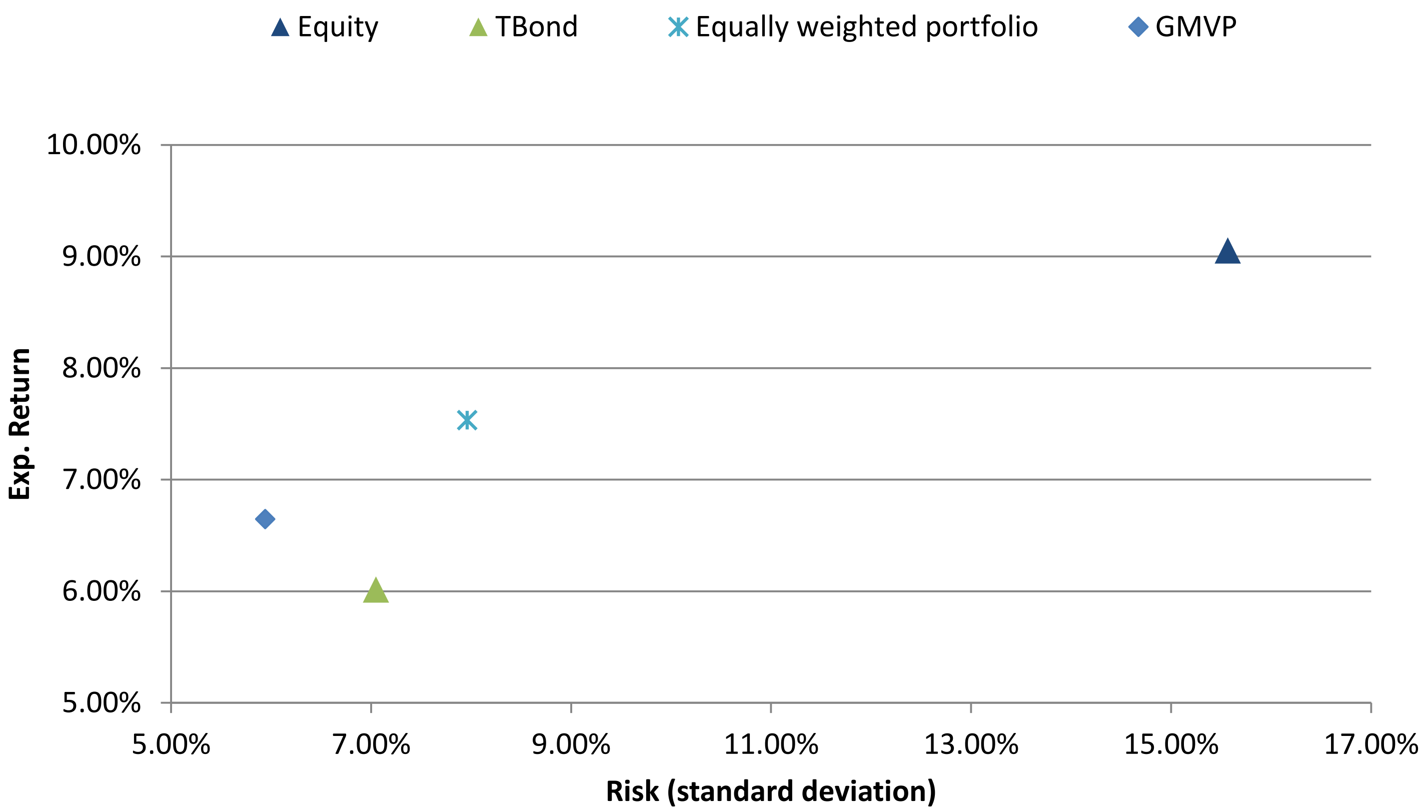
Problem one: the stock-bond asset allocation

- Now, we can compute also the Global Minimum Variance Portfolio, i.e., the portfolio with the minimum possible variance.
- This is an optimization problem that can be solved by using the **solver**
- To find the GMVP we ask to the solver to find the combination of weights that minimize the variance
- The only constraint is that the sum of weights should be equal to 100%



Problem one: the stock-bond asset allocation

- Notably, we can see from the picture that, as the "equity only" portfolio is below the GMVP, holding only equity is **NOT EFFICIENT**

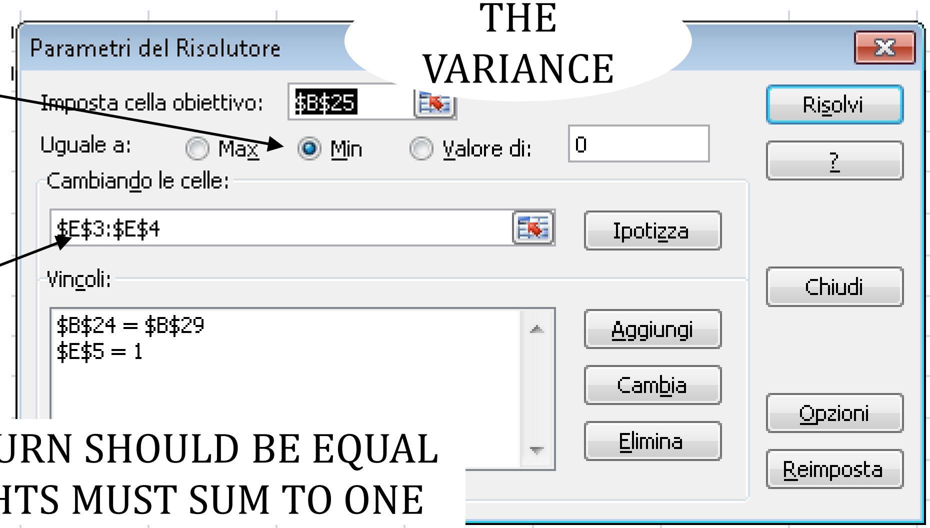


Problem one: the stock-bond asset allocation

- We can compute any point of the efficient frontier, using the **solver**
- Compared to what we did to find the GMVP, we ask to the solver to find the combination of weights that minimize the variance given a certain **target return**
- The only constraint is that the sum of weights should be equal to 100%
- If we want, we can also restrict the weights to be only positive (i.e., no-short selling allowed)

SHOULD BE MINIMIZED

BY CHANGING THE WEIGHTS....

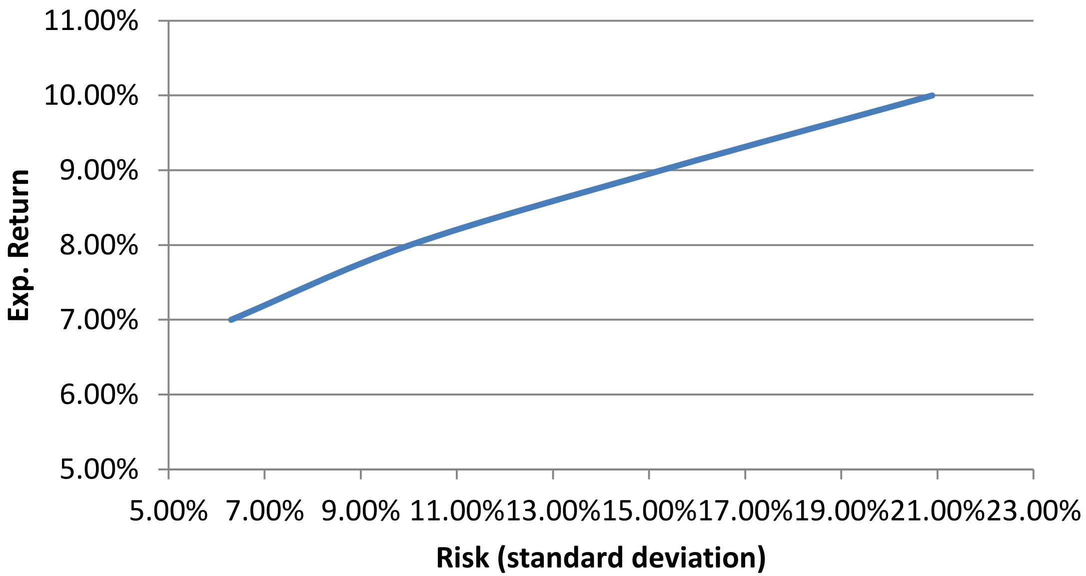


THE VARIANCE

GIVEN THAT EXPECTED RETURN SHOULD BE EQUAL TO THE TARGET AND WEIGHTS MUST SUM TO ONE

Problem one: the stock-bond asset allocation

- We can generate enough points on the efficient frontier such that we can draw (approximate by interpolation) with the excel scattered plot
- We start from the minimum-variance portfolio (as you know, it is non-sense to invest in anything that gives lower returns than the minimum-variance portfolio)
- We then generate other points on the frontier by setting higher target returns (than the return of the minimum variance portfolio)



Problem two: asset allocation with many assets

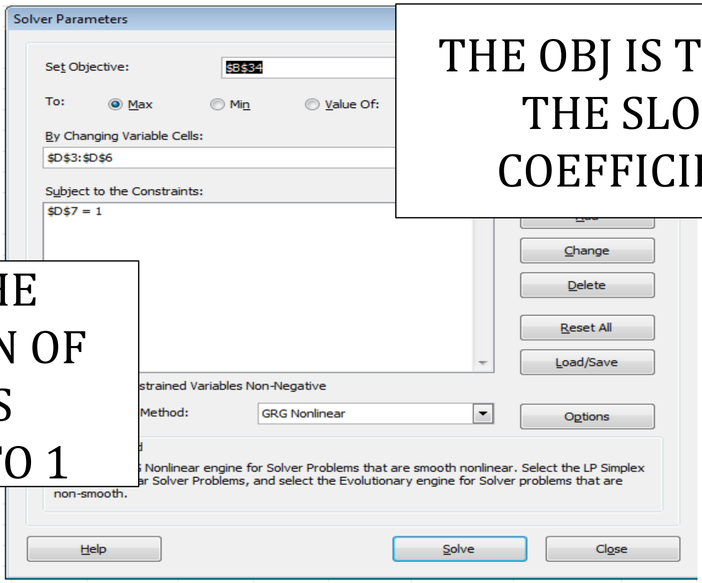
- We now consider a more general set up where:
  - we have 4 risky assets: equity, Treasury bonds, corporate bonds, and real estate
  - the investor can borrow and lend at the risk free rate (R_f)
  - we can consider lending at the riskless rate as investing in an asset with a safe outcome (e.g., T-bill) and borrowing at the riskless rate as selling such security short
  - therefore, we consider R_f equal to 2.64% (the average return of the T-bill)
  - by definition, the variance of the risk free asset is equal to zero
  - the formula for the expected return of a combination of a risky portfolio (A) and a risk-free asset is:

$$\bar{R}_C = R_F + \left(\frac{\bar{R}_A - R_F}{\sigma_A} \right) \sigma_C \quad \text{(CML)}$$

Problem two: the tangency portfolio

- As you already know, in this framework (with unlimited borrowing and lending at the risk free rate) we can split the allocation problem into two parts:
  - We now focus on determinating the tangency portfolio (G) => NO NEED TO KNOW INVESTOR'S RISK AVERSION COEFFICIENT
    - To solve this problem we need to maximize:

$$\tan\alpha = (R_A - R_f)/\sigma_A \quad \text{subject to} \quad \sum w_i = 1$$

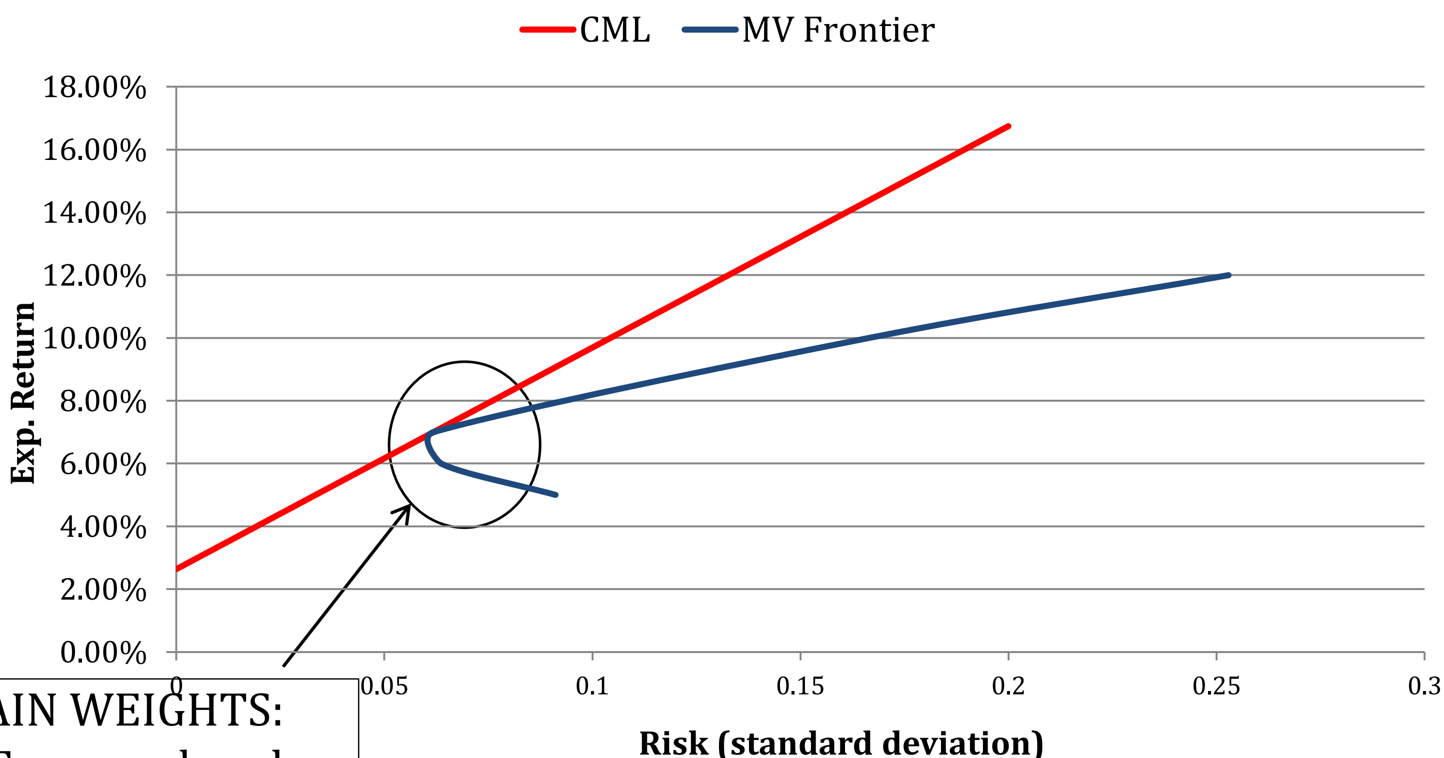


THE OBJ IS TO MAX THE SLOPE COEFFICIENT

UNDER THE ASSUMPTION OF WEIGHTS SUMMING TO 1

Problem two: (a) the tangency portfolio

- The tangency portfolio (or market portfolio) is unique, does not depend on the preferences of the investor



WE OBTAIN WEIGHTS:
- 0.71% Treasury bond
- -0.01% Corporate bond
- 0.25% Equity
- 0.06% Real Estate