Università Commerciale
Luigi Bocconi

# Lecture 5: Utility Based Portfolio Choice in Excel

Dr. Manuela Pedio

Portfolio Management

Spring 2019

# Overview

- Portfolio allocation with two assets…
- …. and different utility functions:
  - Bootstrapping (re-sampling, nonparametric methods)

# Portfolio asset allocation with two asset: Bootstrap (Step 1)

What is bootstrap about?

- We "roll the dice" and randomly select observations from an empirical distribution, i.e., our historical series
- We repeat the exercise a "*reasonably*" high number of times (e.g., 10,000)

- Our "dice" is the function RANDBETWEEN in Excel
  - This function extracts a random number between two numbers of our choice (in this case 1 and 252, where 252 is the total number of historical observations that we have)

# Portfolio asset allocation with two asset: Bootstrap (Step 2)

- For each random number that we extract, we select the corresponding empirical observation (through VLOOKUP function)
- We then **CUT & PASTE** the values we obtained: Excel continuously generate new random numbers every time we perform such (or any) operation
- We use these values to compute the wealth associated to each of the simulated values, for now assuming that the portfolio is equally weighted:

$$1 \times ((1+R_a) \times \omega_a + (1+R_b) \times \omega_b)$$

| We are assuming an initial wealth of 1 EUR | $R_a$ and $R_b$ are the simulated returns | Weights for now are 50% and 50% |
|---|---|---|

# Portfolio asset allocation with two asset: Bootstrap (Step 3)

- We compute the utility associated to each level of wealth considering three different utility functions:

1 **Negative exponential utility**

$$U(W) = 1 - e^{-\theta W} \quad \text{with } \theta > 0$$

2 **Power utility**

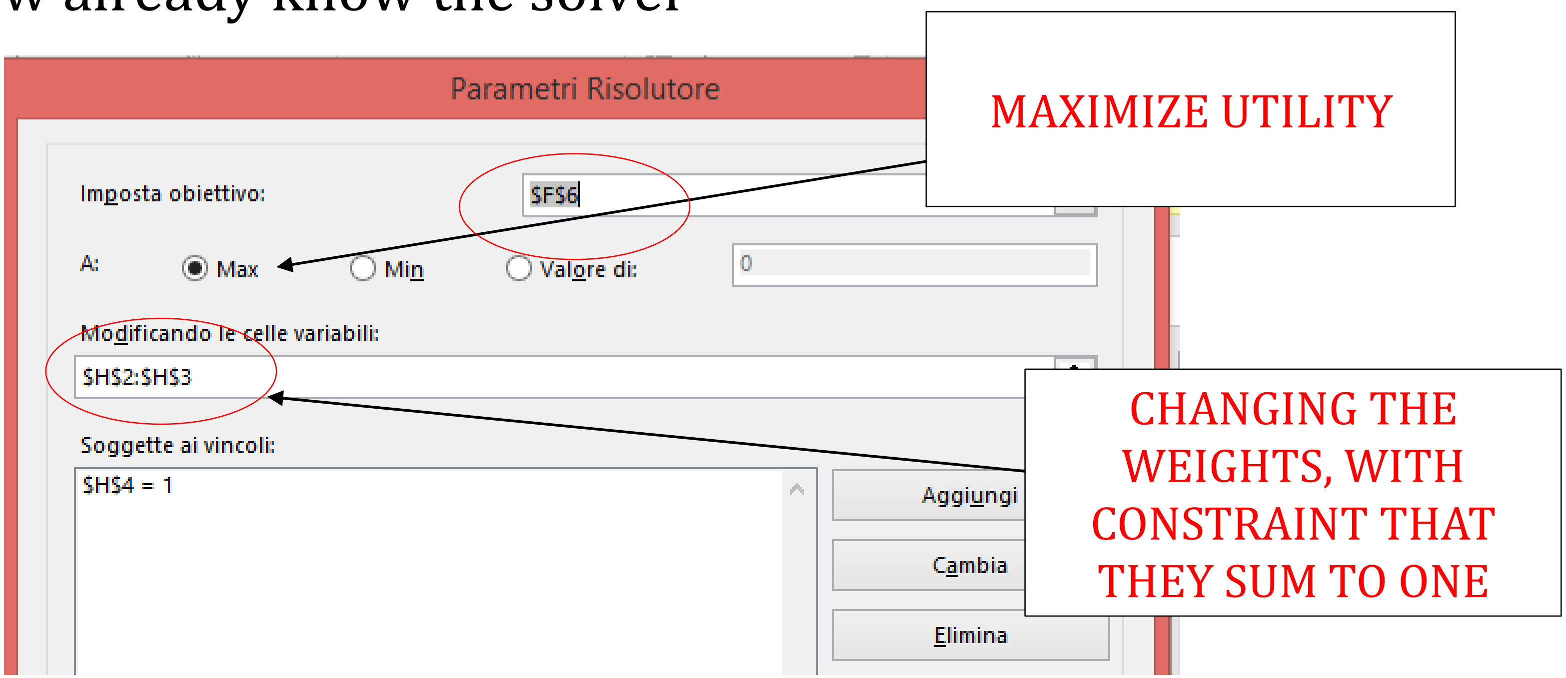
$$U(W) = \begin{cases} \frac{W^{1-\gamma}}{1-\gamma} & \gamma > 0, \gamma \neq 1 \\ \ln W & \gamma = 1 \end{cases}$$

3 **Quadratic**

$$U(W) = W - \frac{1}{2}\kappa W^2 \quad \text{with } \kappa > 0$$

# Portfolio asset allocation with two asset: Bootstrap (Step 4)

- Now we average the utility for all the simulated levels of wealth and we are ready for the last part of the exercise: maximize the expected utility
- This works because averaging across randomly drawn returns exploits (some) law of large numbers
- We now already know the solver



# Portfolio asset allocation with two asset: Bootstrap (Step 4)

- For the power utility function case, we need to be careful, as we need to impose our weights to be higher than 0 and lower than 1
- Otherwise the function will be maximized by W going to zero from the left, a small negative number that becomes increasingly large
- The reason is that a number divided by zero goes to infinity

$$U(W) = \begin{cases} \frac{W^{1-\gamma}}{1-\gamma} & \gamma > 0, \gamma \neq 1 \\ lnW & \gamma = 1 \end{cases}$$

$$T_{exp}(W) = \frac{1}{\theta} \qquad T_{power}(W) = \frac{1}{\gamma}W \qquad T_{quadr}(W) = \frac{1}{\kappa} - W$$

# Portfolio asset allocation with two asset: Bootstrap (Step 4)

- You can now play around, and see how do weights change when we change the parameters theta, gamma and kappa

- Risk tolerance :

$$T_{exp}(W) = \frac{1}{\theta} \qquad T_{power}(W) = \frac{1}{\gamma} W \qquad T_{quadr}(W) = \frac{1}{\kappa} - W$$

| | | |
|---|---|---|
| DECREASE THETA TO INCREASE RISK TOLERANCE | DECREASE GAMMA TO INCREASE RISK TOLERANCE | DECREASE KAPPA TO INCREASE RISK TOLERANCE |