Università Commerciale Luigi Bocconi

Lecture 4: Mean-Variance Portfolio Choice in Excel

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Portfolio Management

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Overview

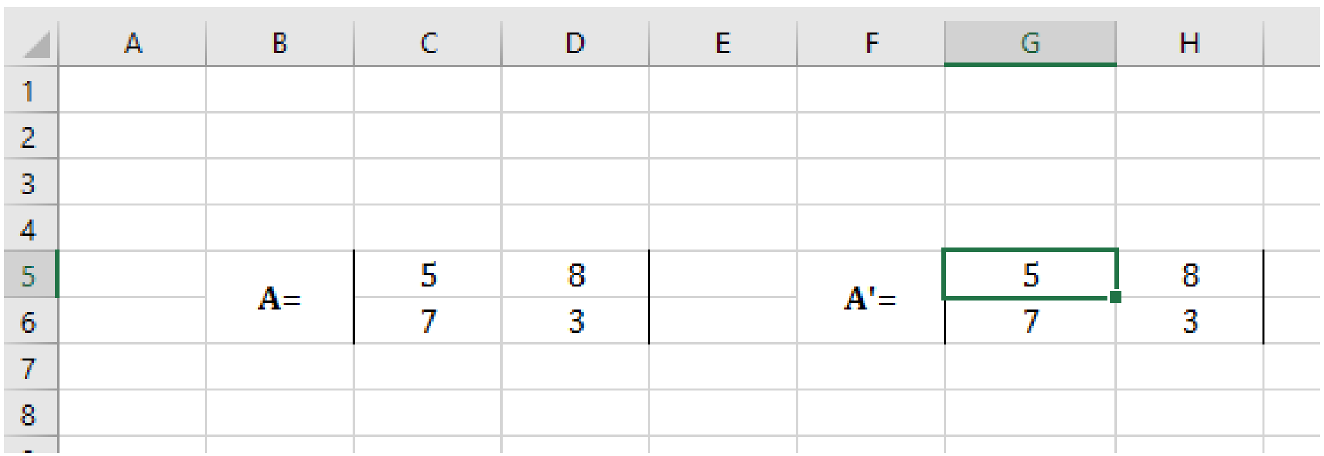
- Fundamentals: dealing with matrices in Excel
- Problem one: the efficient frontier with two risky assets (and no risk free asset)
- Problem two: asset allocation with many assets
- Problem two: tangency portfolio

Fundamentals: the use of matrices in Excel (1/4)

Matrix notation is essential to deal with multi-asset allocation problems in a compact and efficient way

We now review the most common and helpful functions that Excel provides to you to deal with matrices:

- Let's suppose that **A** is 2-by-2 matrix and that you want to perform the transpose (**A'**)
  - **TRANSPOSE (C5:D6)**



REMEMBER, WHEN YOU DEAL WITH MATRICES YOU ALWAYS HAVE TO PRESS **CTRL + SHIFT + ENTER**

Fundamentals: the use of matrices in Excel (2/4)

- Let's suppose that **A** and **B** are two 2-by-2 matrix and that you want to perform their multiplication
  - **MMULT (C5:D6,C9:D10)**

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4									
5		**A=**	5	8		**AB=**	181	57	
6			7	3			114	47	
7									
8									
9		**B=**	9	5					
10			17	4					
11									

  - You need to select the result area G5:H6 (matrix is 2 x 2 so the result will be 2 x 2 too) and press CTRL + SHIFT + ENTER

Fundamentals: the use of matrices in Excel (3/4)

- Let's suppose that **A** is a 3-by-3 matrix and that you want to invert it
  - **MINVERSE (C5:E7)**

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5			8	5	11			-0,00241	0,231325	-0,040964
6		**A=**	6	2	1		**A^(-1)=**	-0,050602	-0,142169	0,139759
7			9	11	5			0,115663	-0,103614	-0,033735
8										

  - You need to select the result area H5:J7 (matrix is 3 x 3 so the result will be 3 x 3 too) and press CTRL + SHIFT + ENTER

Fundamentals: the use of matrices in Excel (4/4)

- In addition, remember that the formula SUMPRODUCT allows you to multiply two vectors
  - **SUMPRODUCT (I11:I13, K11:K13)**

	H	I	J	K
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11	A=	3	B=	6
12		4		7
13		5		8
14				
15				86
16				

SUMPRODUCT MEANS:
3 x 6 + 4 x 7 + 5 x8

Problem one: the stock-bond asset allocation

- Let's suppose you can only invest in two assets:
  - a (US) stock index (here represented by the value-weighted CRSP index)
  - a (US) long-term (Treasury) bond index (here represented by the Ibbotson 10-year government bond index)
- You have available the monthly log-returns of the two indices
- First of all, you need to compute the statistics of the two series: the mean and the standard deviation of each series and the pair-wise correlation between them

	A	B	C	D
1				
2		**Mean**	**Variance**	**Weights**
3	**Equity**	9.06%		
4	**Tbond**	6.01%		
5				

If you recall log-returns properties (i.e. return over two periods is just the sum of the returns of each period) you can compute the annual mean return: it is simply equal to the monthly mean return multiplied by 12

B3 contains the formula => MEAN(Equity)*12 (where Equity is the name we gave to C2:C253, i.e. the monthly equity log-returns)

B4 contains the formula=> MEAN(Tbond)*12 (where Tbond are monthly bond log-returns)

Problem one: the stock-bond asset allocation

Similarly, the annual standard deviation of log-returns is obtained by multiplying by $\sqrt{12}$

C3 contains the formula => STDEV(Equity)*SQRT(12)

C4 contains the formula => STDEV(Tbond)*SQRT(12)

	A	B	C	D
1				
2		**Mean**	**Standard Deviation**	**Weights**
3	**Equity**	9.06%	15.57%	
4	**Tbond**	6.01%	7.047%	
5				
6				
7				
8				

Finally, we compute the correlation between the two series with the function "CORREL"=> CORREL(Equity, Tbond)

5				
6	CORRELATION MATRIX			
7		Equity	Tbond	
8	**Equity**	1	-0.18	
9	**Tbond**	-0.18	1	
10				
11				
12				
13				
14				
15				

Problem one: the stock-bond asset allocation

- Finally, we construct the variance-covariance matrix (let's call it **V**); as you know this is a symmetric matrix contains the variances of each asset in the main diagonal and the pair-wise covariances out of the main diagonal
- Recall that the formula for co- variance is: $\text{Cov}(i,j) = \varrho \sigma_i \sigma_j$

	A	B	C	D
1				
2		Mean	Standard Deviation	Weights
3	Equity	9.06%	15.57%	
4	Tbond	6.01%	7.047%	
5				
6	CORRELATION MATRIX			
7		Equity	Tbond	
8	Equity	1	-0.18	
9	Tbond	-0.18	1	
10				
11	VARCOV			
12		Equity	Tbond	
13	Equity	0.024235338	-0.001921698	
14	Tbond	-0.001921698	0.004966527	
15				
16				
17				
18				
19				
20				
21				

Cell B13 contains the formula:
B8*VLOOKUP($A13, $A$2:$C$4, 3, FALSE)*VLOOKUP(B$12, $A$2:$C$4, 3, FALSE)

Why would you bother to do such a formula when you know that cell B13 is just the variance of equity returns (i.e. the square of cell C3)?

Excel makes your life **easier** when you deal with a LARGE amount of data (e.g. 5 assets imply a 5-by-5 **V** matrix!)
Now you can just drag and drop!

Problem one: the stock-bond asset allocation

- Now, let's suppose for a minute that we have an **equally weighted** portfolio and compute portfolio mean and variance (the two asset case is very simple and you do not necessarily need to use matrices… however we want to create a general set up that will be valid also when we add other assets)

PORTFOLIO MEAN

$$E(r_p) = w^T e$$

PORTFOLIO MEAN: $E(r_p)$; WEIGHTS: w^T; MEAN RETURNS: e

SUMPRODUCT(B3:B4, D3:D4)

PORTFOLIO VARIANCE

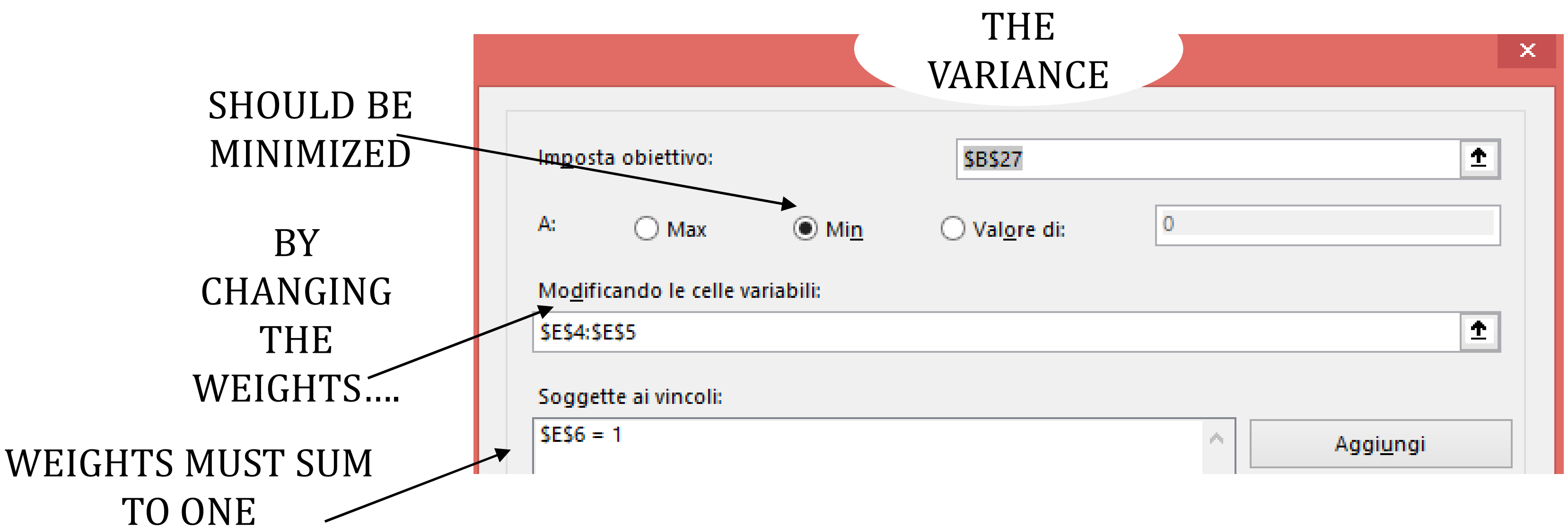
$$Var(r_p) = w^T V w$$

PORTFOLIO VARIANCE: $Var(r_p)$; WEIGHTS: w^T; VARIANCE – COVARIANCE MATRIX: V; WEIGHTS: w

MMULT(TRANSPOSE(B3:B4),MMULT (B13:C14, B3:B4))

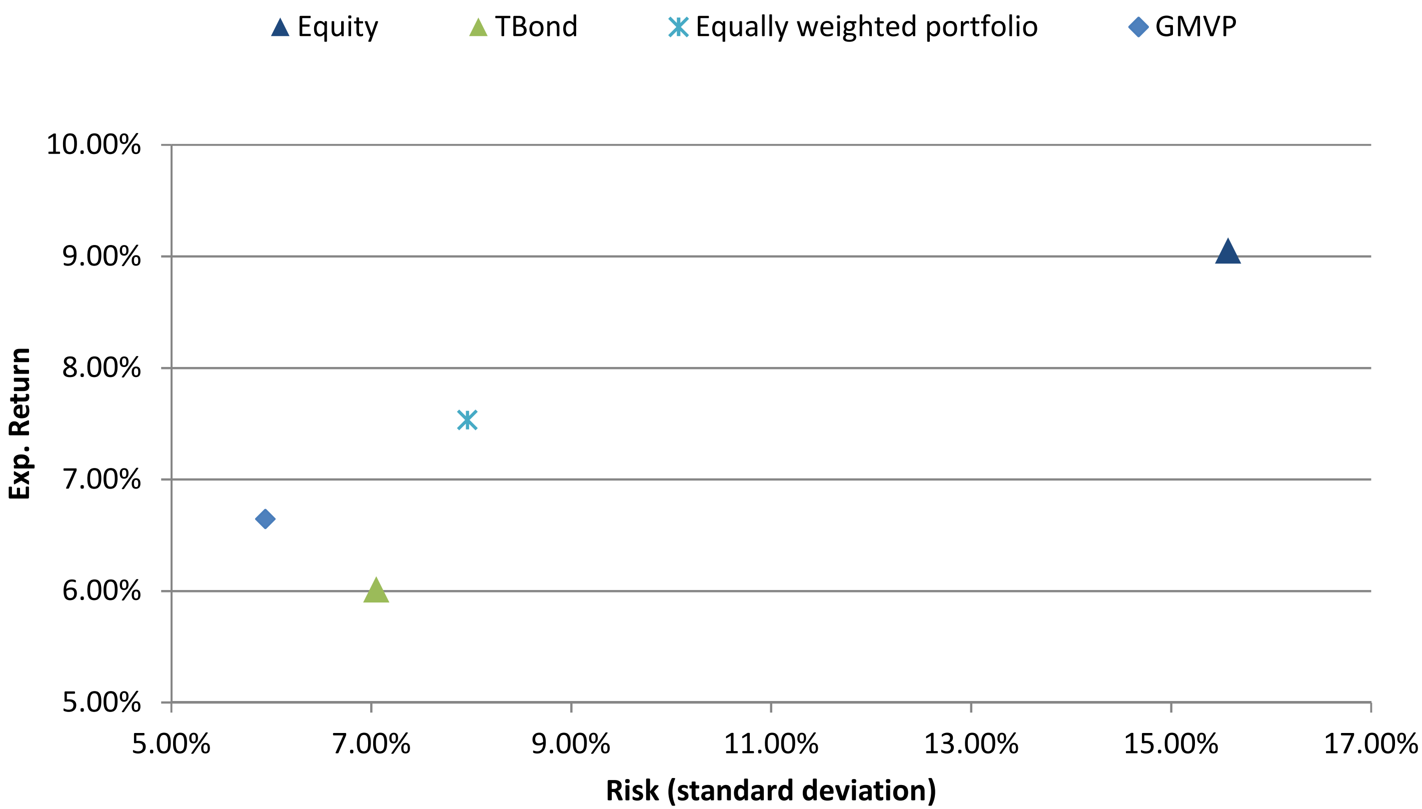
Problem one: the stock-bond asset allocation

- Now, we can compute also the Global Minimum Variance Portfolio, i.e., the portfolio with the minimum possible variance.
- This is an optimization problem that can be solved by using the **solver**
- To find the GMVP we ask to the solver to find the combination of weights that minimize the variance
- The only constraint is that the sum of weights should be equal to 100%



Problem one: the stock-bond asset allocation

- Notably, we can see from the picture that, as the “equity only” portfolio is below the GMVP, holding only equity is **NOT EFFICIENT**

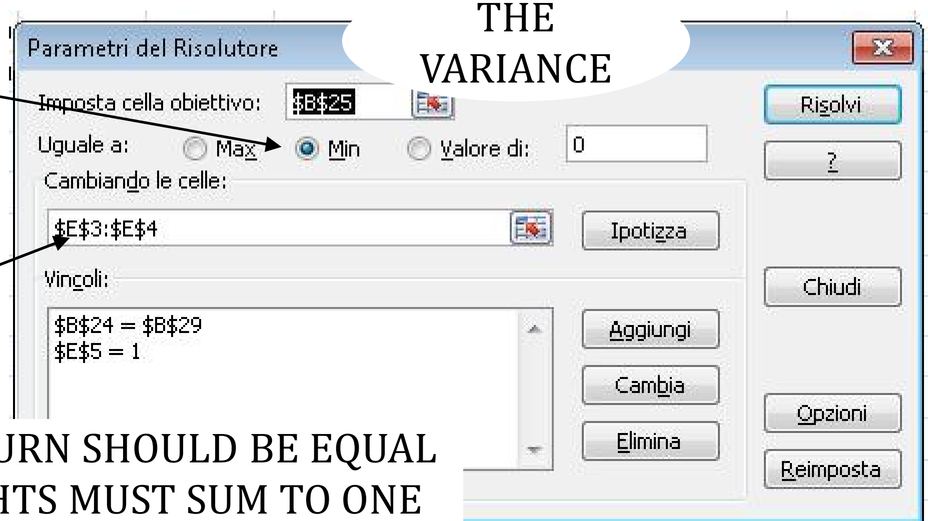


Problem one: the stock-bond asset allocation

- We can compute any point of the efficient frontier, using the **solver**
- Compared to what we did to find the GMVP, we ask to the solver to find the combination of weights that minimize the variance given a certain **target return**
- The only constraint is that the sum of weights should be equal to 100%
- If we want, we can also restrict the weights to be only positive (i.e., no-short selling allowed)

SHOULD BE MINIMIZED

BY CHANGING THE WEIGHTS....

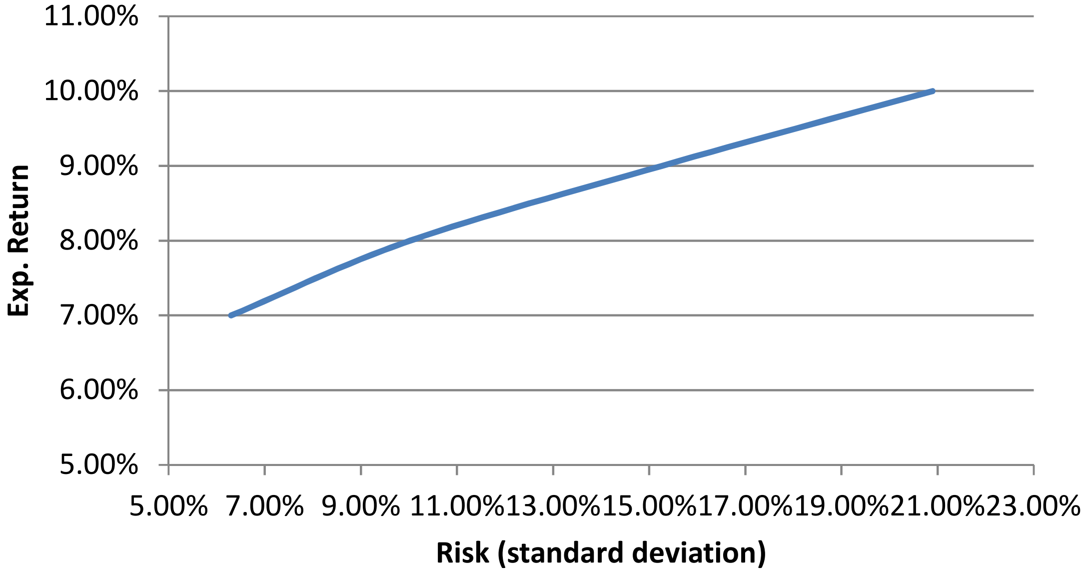


THE VARIANCE

GIVEN THAT EXPECTED RETURN SHOULD BE EQUAL TO THE TARGET AND WEIGHTS MUST SUM TO ONE

Problem one: the stock-bond asset allocation

- We can generate enough points on the efficient frontier such that we can draw (approximate by interpolation) with the excel scattered plot
- We start from the minimum-variance portfolio (as you know, it is non-sense to invest in anything that gives lower returns than the minimum-variance portfolio)
- We then generate other points on the frontier by setting higher target returns (than the return of the minimum variance portfolio)



Problem one: the stock-bond asset allocation

- There is also an analytical solution to identify points on the efficient frontier
- We can easy replicate it into excel: as discussed at the beginning of the lecture excel can invert the matrices for you (with the formula MINVERSE)
- **STEP 1:** we create the vector **u** (vector of ones); vector **e** is the vector of asset means that we already computed before
- **STEP 2:** we create the inverse of the variance-covariance matrix
- **STEP 3**: we compute the four quantities A, B, C and D

$$A = \mathbf{u}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{e} \quad B = \mathbf{e}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{e} \quad C = \mathbf{u}^{\mathrm{T}}\mathbf{V}^{-1}\mathbf{u} \quad D = BC - A^2$$

- To make life easier we firstly compute:

$$\mathbf{l} = \mathbf{V}^{-1}\mathbf{e} \qquad \mathbf{m} = \mathbf{V}^{-1}\mathbf{u}$$

Problem one: the stock-bond asset allocation

	A	B	C	D	E	F	G
1							
2		**Mean (e)**	**Standard Deviation**	**U**			
3	**Equity**	9.06%	15.57%	1.00			
4	**Tbond**	6.01%	7.047%	1.00			
5							
6	**CORRELATION MATRIX**				**INVERSE OF THE VARCOV MATRIX**		
7		**Equity**	**Tbond**				
8	**Equity**	1	-0.18		1.03165203	0.180703849	
9	**Tbond**	-0.18	1		0.180703849	1.03165203	
10							
11	**VARCOV**						
12		**Equity**	**Tbond**		**m**	**l**	
13	**Equity**	0.024235338	-0.001921698		1.21235588	0.104298403	
14	**Tbond**	-0.001921698	0.004966527		1.21235588	0.078418638	
15							

- Now we can simply compute the four quantities as:

$$A = \mathbf{u}^T\mathbf{l} \quad B = \mathbf{e}^T\mathbf{l} \quad C = \mathbf{u}^T\mathbf{m} \quad D = BC - A^2$$

- Note that A, B, C, D are scalar, so you do not need to select the area when you use MMULT this time

Problem one: the stock-bond asset allocation

- Following HL we compute **g** and **h**, the portfolios with 0% return (and minimum variance) and 100% return (and minimum variance), from which you can generate the rest of the frontier

$$\mathbf{g} = [B\mathbf{m} - A\mathbf{l}]/D \quad \mathbf{h} = [C\mathbf{l} - A\mathbf{m}]/D$$

- When you input the formula in excel, remember that you are dealing with arrays! You have to select the area where you are computing vectors **g** and **h** (in our case 2x1 vectors) and press CTRL + SHIFT + ENTER

- Now, if you want to retrieve the efficient portfolio with target return 7.5% you just have to compute **g+ h*7.5%**

Problem one: the stock-bond asset allocation

- Do you remember these weights?
- They are the same weights we have found using the solver
- Yet, HL can ONLY be used when there are NO CONSTRAINS
- Solver can always be used

	A	B	C	D	E	F	G
7		Equity	Tbond				
8	Equity	1	-0.18		1.03165203	0.180703849	
9	Tbond	-0.18	1		0.180703849	1.03165203	
10							
11	VARCOV						
12		Equity	Tbond		m	l	
13	Equity	0.024235338	-0.001921698		1.21235588	0.104298403	
14	Tbond	-0.001921698	0.004966527		1.21235588	0.078418638	
15							
16							
17				g	h		
18	A	0.18		-1.977774124	32.88083048		
19	B	0.01		2.977774124	-32.88083048		
20	C	2.42					
21	D	0.00					
22							
23				weights for target portfolio			
24	T	7.500%		0.488288161			
25				0.511711839			
26							

Problem two: asset allocation with many assets

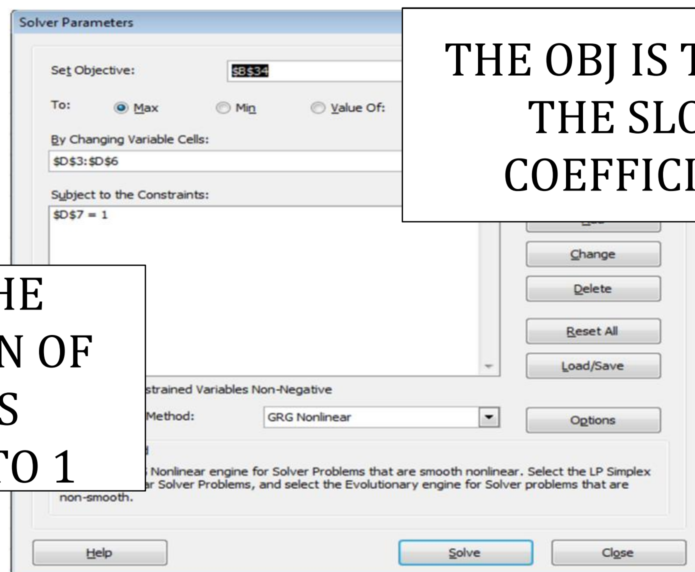
- We now consider a more general set up where:
  - we have 4 risky assets: equity, Treasury bonds, corporate bonds, and real estate
  - the investor can borrow and lend at the risk free rate (R_f)
  - we can consider lending at the riskless rate as investing in an asset with a safe outcome (e.g., T-bill) and borrowing at the riskless rate as selling such security short
  - therefore, we consider R_f equal to 2.64% (the average return of the T-bill)
  - by definition, the variance of the risk free asset is equal to zero
  - the formula for the expected return of a combination of a risky portfolio (A) and a risk-free asset is:

$$\bar{R}_C = R_F + \left(\frac{\bar{R}_A - R_F}{\sigma_A} \right) \sigma_C \quad \text{(CML)}$$

Problem two: the tangency portfolio

- As you already know, in this framework (with unlimited borrowing and lending at the risk free rate) we can split the allocation problem into two parts:
  - We now focus on determinating the tangency portfolio (G) => NO NEED TO KNOW INVESTOR'S RISK AVERSION COEFFICIENT
    - To solve this problem we need to maximize:

$$\tan\alpha = (R_A - R_f)/\sigma_A \quad \text{subject to} \quad \sum w_i = 1$$

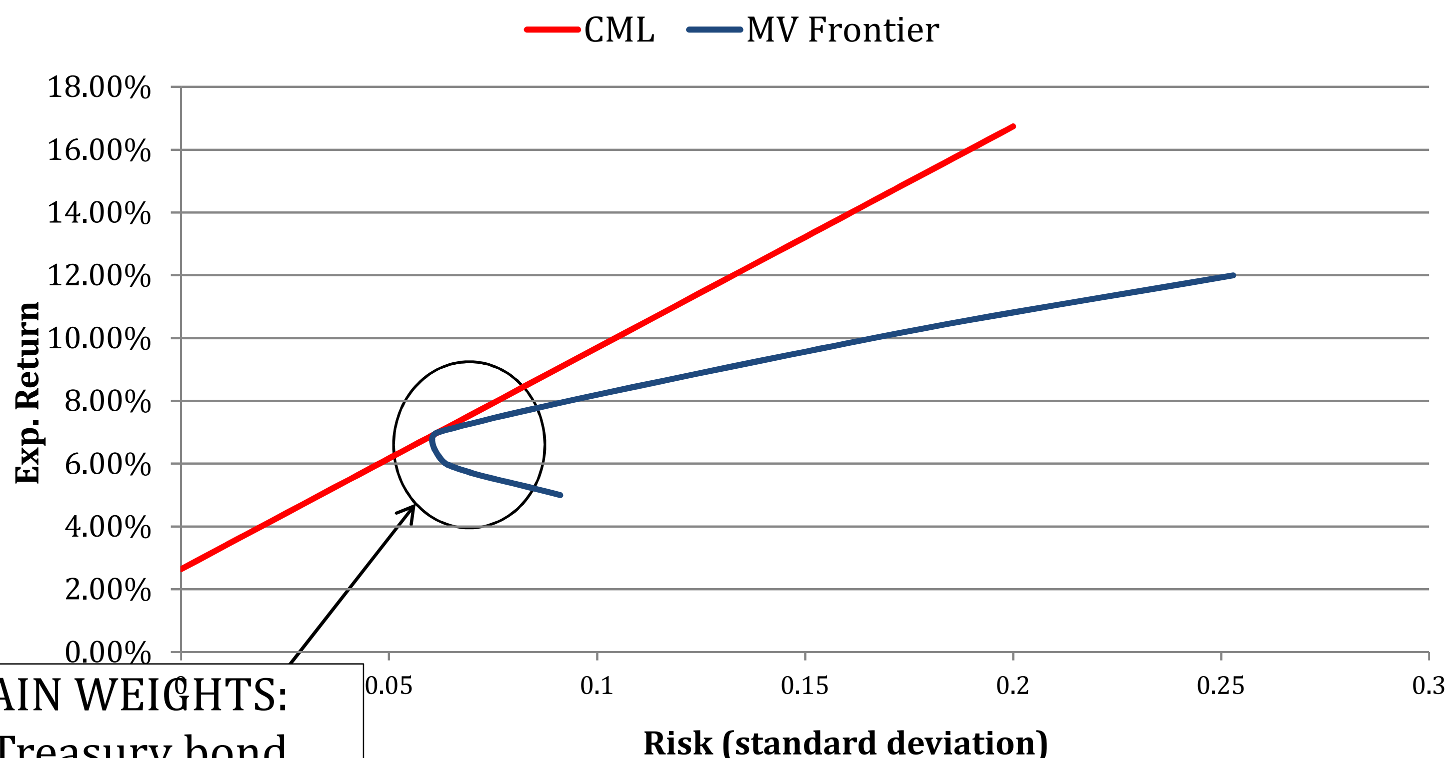


THE OBJ IS TO MAX THE SLOPE COEFFICIENT

UNDER THE ASSUMPTION OF WEIGHTS SUMMING TO 1

Problem two: (a) the tangency portfolio

- The tangency portfolio (or market portfolio) is unique, does not depend on the preferences of the investor



WE OBTAIN WEIGHTS:
- 0.71% Treasury bond
- -0.01% Corporate bond
- 0.25% Equity
- 0.06% Real Estate