Introduction to Counterparty Risk

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Outline

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Counterparty Risk: Definition

The **counterparty credit risk** is defined as the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default.
[Basel II, Annex 4, 2/A]

Counterparty risk is affected by:

- the OTC contract's **underlying volatility**
- the **correlation** between the underlying and default of the counterparty
- the **counterparty credit spreads volatility**

Counterparty Risk: Features

Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, the counterparty credit risk creates a **bilateral risk of loss**: the market value of the transaction can be positive or negative to either counterparty to the transaction.

[Basel II, Annex 4, 2/A]

- **Loans:** exposure at any future date is the outstanding balance, which is certain (without considering prepayments). Credit risk is **unilateral**
- **Derivatives:** exposure at any future date is determined by the market value at that date and is uncertain. Counterparty risk can be:
  - **unilateral:** one party (the investor) is considered default-free and only the exposure to the counterparty matters
  - **bilateral:** both parties are considered risky and face exposures depending on the value of the positions they hold against each other

OTC Derivatives

OTC derivatives are efficient and effective tools to **transfer financial risks** between market participants.

As a byproduct of such transfer:

- they **create credit risk** between the counterparties
- they **increase the connectedness** of the financial system

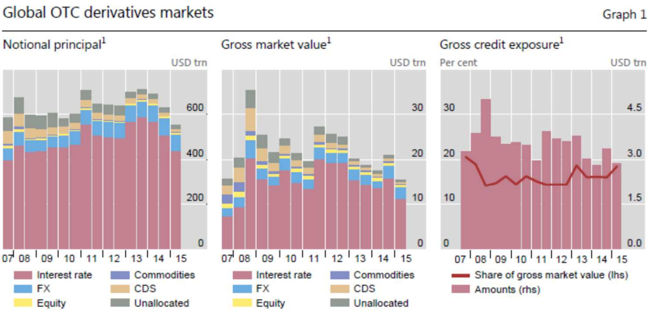
The **2008 financial crisis** showed that **counterparty-related losses** (e.g. changes in the credit spreads of the counterparties and changes in the market prices that drive the underlying derivative exposures) have been **much larger than default losses**.

OTC Market – BIS 2015

The volume of outstanding OTC derivatives has **grown exponentially over the past 30 years**. According to market surveys conducted by ISDA and BIS:

- **notional amounts** of outstanding interest rate and currency swaps went from **$866 billion in 1987** to $17.7 trillion in 1995, $99.8 trillion in 2002 and **more than $500 trillion in 2015**
- the **gross market value** of outstanding derivatives contracts – i.e. the cost of replacing all outstanding contracts at market prices prevailing on the reporting date – amounted to **$15.5 trillion** at the end June 2015.
- the **interest rate segment** accounts for the majority of OTC derivatives, with a notional amount of outstanding contracts of $435 trillion (**79%** of the global OTC markets)
- **FX derivatives** make up the second largest segment of the global OTC derivatives market with an OTC market share of **13%**, amounting to $75 trillion in terms of notional outstanding
- **central clearing** has become increasingly important in interest rate derivatives markets and credit default swap markets

OTC Market – Global Chart



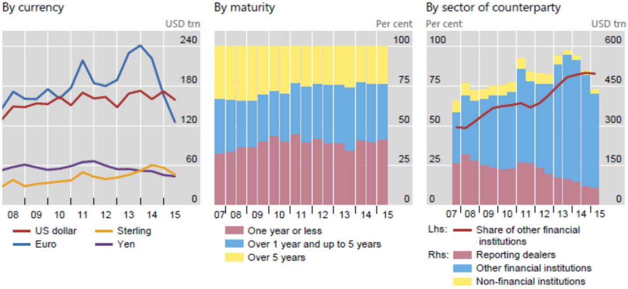
Further information on the BIS derivatives statistics is available at www.bis.org/statistics/derstats.htm.

[1] At half-year end (end-June and end-December). Amounts denominated in currencies other than the US dollar are converted to US dollars at the exchange rate prevailing on the reference date.

OTC Market – Interest Rate Derivatives Chart

OTC interest rate derivatives

Notional principal[1] Graph 3



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Counterparty Risk: Risk Management *vs* Pricing

Two approaches to counterparty risk:

- **counterparty risk management:**
  for internal purposes and for regulatory capital requirements, following **Basel II**
- **counterparty risk from a pricing point of view**:
  Credit Valuation Adjustment (CVA), when updating the price of instruments to account for possible default of the counterparty

However, **Basel III** has made the distinction less clear-cut.

Counterparty Risk Management

Counterparty risk is the risk that one bank faces in order to be able to lend money or invest towards a counterparty with relevant default risk.

The bank needs to **measure** that risk and **cover** for it by **setting capital aside**.

Credit VaR is calculated through the following steps:

1. the basic financial variables underlying the portfolio, including also defaults of the counterparties, are **simulated** under the **historical probability measure** $\mathbb{P}$, up to the risk horizon
2. at the risk horizon, in every simulated scenario of the basic financial variables, the portfolio is **priced**, eventually obtaining a number of scenarios for the portfolio value at the risk horizon. "Priced" means that discounted future cash flows of the portfolio after the risk horizon are averaged, conditional on each scenario at the risk horizon but under the **(pricing) risk neutral measure** $\mathbb{Q}$.

Counterparty Risk: Pricing

Pricing concerns updating the value of a specific instrument or portfolio, traded with a counterparty, by **adjusting the price** in order to take into account the risk of default of the counterparty.

The amount charged to the risky counterparty on the top of the default-free cost of the contract is known as **Credit Valuation Adjustment, or CVA**.

Since it is a price, it is computed entirely under the **(pricing) risk neutral measure** $\mathbb{Q}$.

Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly **two-thirds of losses** attributed to counterparty credit risk were due to **CVA losses** and only about one-third were due to actual defaults.

[Basel III, Press Release June 2011]

Exposures I

Counterparty exposure at any given future time is the larger between zero and the market value of the portfolio of derivative positions with a counterparty that would be lost if the counterparty were to default with zero recovery at that time.

Current exposure (CE) is the current value of the exposure to a counterparty.

Exposure at Default (EAD) is defined in terms of the exposure valued at the (random future) default time of the counterparty.

Exposures II

Potential future exposure (PFE) for a given date is the maximum of exposure at that date with a high degree of statistical confidence. For example, the 95% PFE is the level of potential exposure that is exceeded with only 5% probability. The curve of PFE in time is the potential exposure profile, up to the final maturity of the portfolio of trades with the counterparty.

The **maximum potential future exposure (MPFE)** represents the peak of PFE over the life of the portfolio. PFE and MPFE are used to determine **credit lines**.

Expected exposure (EE) is the average exposure on a future date. The curve of EE in time, as the future date varies, provides the expected exposure profile.

Expected positive exposure (EPE) is the average EE in time up to a given future date.

Outline

Credit Value Adjustment: Introduction

- **Credit Value Adjustment (CVA)** tries to measure the **expected loss** due to missing the remaining payments.
- CVA has become an integral part of **IAS 39** accounting rules and **Basel III** regulatory requirements.
- CVA is **defined as**:
  1. the **difference between the risk-free value and the risky value** of one or more trades or, alternatively,
  2. the **expected loss** arising from a future counterparty default.

CVA Definition 1

The Net Present Value of a derivative at time t is given by:

$$V(t) = \mathbb{E}_t[\Pi(t, T)]$$

where $\Pi(t, T)$ represents (the sum of) all discounted cash flows between times T and t. In the presence of counterparty risk, the sum of all discounted payoff terms between t and T is denoted by $\Pi^D(t, T)$.

CVA is defined, according to Canabarro and Duffie (2004), as the difference between the risk-free value and the risky value:

$$\boxed{\text{CVA} := \mathbb{E}_t[\Pi(t, T)] - \mathbb{E}_t[\Pi^D(t, T)]} \quad (1)$$

- **unilateral (asymmetric) CVA**, if only the default of the counterparty is considered
- **bilateral (symmetric CVA)**, if also the default of the investor is taken into account.

CVA Definition 1: Unilateral Case

We will show that, starting from definition (1) and accounting for all the cash-flows, unilateral CVA is given by:

$$\boxed{\text{CVA} = \text{L}_{\text{GD}}\, \mathbb{E}\left[\mathbb{1}_{\{\tau \leq T\}}\, D(0,\tau)\, (\mathbf{V}(\tau))^{+}\right]} \tag{2}$$

where:

- τ is the **default event**, as defined in the **Bilateral ISDA Master Agreement**
- $\mathbf{V}(\tau)$ is the (uncertain) **close-out amount**
- LGD is the expected loss given default, expressed as a percentage of the nominal close-out amount.

CVA Definition 1: Bilateral Case

Analogously, In the bilateral case, the risky value of the derivative takes into account both the default of the **counterparty C** and that of the **investor I**:

$$
\begin{aligned}
\mathbb{E}_t[\Pi^D(t,T)] = & \underbrace{V(t)}_{\text{risk-free}} \\
& - \underbrace{\mathrm{LGD}_C \, \mathbb{E}_t \left[\mathbb{1}_{\{\tau_C \le T, \tau_C \le \tau_I\}} D(t,\tau_C) \, (V(\tau_C))^+ \right]}_{\text{CVA}} \\
& + \underbrace{\mathrm{LGD}_I \, \mathbb{E}_t \left[\mathbb{1}_{\{\tau_I \le T, \tau_I \le \tau_C\}} D(t,\tau_I) \, (-V(\tau_I))^+ \right]}_{\text{DVA}}
\end{aligned}
\qquad (3)
$$

The formula is **symmetric**: the investor's DVA is equal to the counterparty's CVA, but an **investor cannot hedge its DVA spread risk** by selling CDS protection on itself.

CVA Definition 2: Expected Loss

- We consider here the **unilateral case**[1].
- CVA measures the risk of incurring losses on a portfolio of deals, upon default of the counterparty. The **loss** is material when the value of the portfolio at default, $V(\tau)$, is positive and default occurs before the maturity of the portfolio, i.e.:

$$\mathrm{Loss}(\tau) = \mathbb{1}_{\{\tau \leq T\}} \,\mathrm{L}_{\mathrm{GD}}\, (V(\tau))^+$$

- Unilateral CVA is defined as the **expected value of this loss**, discounted till evaluation time:

$$\boxed{\begin{aligned} \mathrm{CVA} &= \mathbb{E}[\, D(0,\tau)\, \mathrm{Loss}(\tau)] \\ &= \mathrm{L}_{\mathrm{GD}}\, \mathbb{E}\left[\mathbb{1}_{\{\tau \leq T\}}\, D(0,\tau)\, (V(\tau))^+ \right] \end{aligned}} \tag{4}$$

The resulting expression is in agreement with eq. (2).

[1] The extension to the bilateral case is straightforward.

2002 ISDA Master Agreement: Events of Default

Events of Default

- Failure to pay or deliver
- Breach of agreement; repudiation of agreement
- Default under specified transaction
- Bankruptcy ...

Termination Events

- Illegality
- Force majeure event
- Deferral of payments and deliveries during waiting period
- Tax events ...

Events which cause **Early Termination** of the Master Agreement are always **bilateral**.

Close-out Amount and Netting Set

- **Close-out Amount**

  When one of the two counterparties incurs in one of the events that causes the **Early Termination** of the contract, the non-defaulting party determines the amount of the **losses** or the **costs** of closing the position and replacing it with a new one with another counterparty (**substitution cost**).

- **Netting Set**

  The ISDA Master Agreement determines the possibility to **net out** all the positions with the defaulted counterparty.

  The **netting set P** is formed by p contracts p_i, whose individual value is $V_i(t)$. Each deal has weight $w_i = \pm 1$ if the investor is respectively receiver/payer:

$$V(t) = \sum_{i=1}^{p} w_i V_i(t)$$

Risk Free *vs* Substitution Close-out

- **Unilateral case:**
  The close-out amount is the net present value of the residual deal, calculated as a **risk-free** quantity, since the surviving counterparty is assumed to be default-free.

- **Bilateral case:**
  The close-out amount is the net present value of the residual deal, calculated by taking into account the **risk of default** of the survived party. It is called **substitution close-out** and can give rise to **contagion effects**.
  See Brigo and Morini (2010).

Positive Part of Close-out

The CVA formulae (2) and (3) depend on the **positive part of the close-out amount: $(\mathbf{V}(\tau))^+$.** Indeed, only the positive part of the close-out contributes to **counterparty risk.**

Consider the point of view of the **non-defaulting party**. If:

- $\mathbf{V}(\tau) < \mathbf{0}$
  the close-out amount is a **liability** and the non-defaulting party is due to pay it fully to the defaulted party
- $\mathbf{V}(\tau) \geq \mathbf{0}$
  the non-defaulting party is exposed to the risk that the defaulted party does not pay the close-out amount.

CVA Features

- CVA is a **credit hybrid**.
- The positive part of the close-out amount introduces an element of optionality in the payoff: i.e. a **call option with zero strike** on $V(\tau)$.
- Optionality renders the **payoff** under counterparty risk **model dependent**, even when the original payoff is model independent.

  Example: Interest Rate Swap (IRS)
  Without counterparty risk, the payoff is linear and model independent, requiring no dynamical model for the term structure (no volatility and correlation). In the presence of counterparty risk, the payoff transforms into a stream of swaptions, whose valuation requires an interest rate model.
- Optionality applies to the **whole netting set** with a given counterparty, making CVA valuation **computationally intensive**.

Unilateral CVA – Assumptions

We assume that:

- transactions are seen from the point of view of the **safe investor**, namely the company facing counterparty risk
- such investor is **default-free**

We denote by $\Pi^D(t, T)$ the sum of all discounted payoff terms between t and T, subject to counterparty default risk and by $\Pi(t, T)$ the analogous quantity when counterparty risk is not considered.

Unilateral CVA – Cash flows

Cash flows are given by:

1. if default comes after final maturity $\tau > T$, the original payoff:

$$+\mathbb{1}_{\{\tau > T\}}\,\Pi(t, T)$$

2. if default occurs before maturity $\tau < T$:
   1. the payments due before default:

   $$+\mathbb{1}_{\{\tau \le T\}}\,\Pi(t, \tau)$$

   2. the recovery of the residual net present value at default, if positive:

   $$+\mathbb{1}_{\{\tau \le T\}}\,\mathrm{Rec}\,D(t, \tau)\,\left(\mathbb{E}_\tau[\Pi(\tau, T)]\right)^+$$

   3. minus the total residual net present value at default, if negative:

   $$-\mathbb{1}_{\{\tau \le T\}}\,D(t, \tau)\,\left(-\mathbb{E}_\tau[\Pi(\tau, T)]\right)^+$$

Unilateral CVA – Formula

By summing up all terms, the total payoff subject to counterparty default risk becomes:

$$\begin{aligned}\Pi^D(t,T) &= \mathbb{1}_{\{\tau>T\}}\,\Pi(t,T) + \mathbb{1}_{\{\tau\le T\}}\,\Pi(t,\tau) \\ &+ \mathbb{1}_{\{\tau\le T\}}\,D(t,\tau)\left\{\text{Rec}\,\left(\mathbb{E}_\tau[\Pi(\tau,T)]\right)^+ + \left(\mathbb{E}_\tau[\Pi(\tau,T)]\right)^-\right\}\end{aligned} \tag{5}$$

Recalling the definition on slide 18

$$\text{CVA}_t := \mathbb{E}_t\left[\Pi(t,T) - \Pi^D(t,T)\right]$$

the CVA at time t turns out to be:

$$\boxed{\begin{aligned}\text{CVA}_t &= \text{LGD}\,\mathbb{E}_t\left[\mathbb{1}_{\{\tau\le T\}}\,D(t,\tau)\,\left(\mathbb{E}_\tau[\Pi(\tau,T)]\right)^+\right] \\ &= \text{LGD}\,\mathbb{E}_t\left[\mathbb{1}_{\{\tau\le T\}}\,D(t,\tau)\,\left(V(\tau)\right)^+\right]\end{aligned}} \tag{6}$$

Unilateral CVA – Proof I

Starting from the definition of CVA and expression (5) of the risky payoff, we derive formula (6), through the following steps:

1. we express the sum of all discounted payoff terms between t and τ, i.e. $\Pi(t,\tau)$, as a function of $\Pi(t,T)$ as follows:

$$\Pi(t,\tau) = \Pi(t,T) - D(t,\tau)\Pi(\tau,T)$$

2. Plugging this result into eq. (5) we get:

$$\begin{aligned}\Pi^D(t,T) &= \mathbb{1}_{\{\tau>T\}}\Pi(t,T) + \mathbb{1}_{\{\tau\le T\}}\Pi(t,T)\\ &\quad + \mathbb{1}_{\{\tau\le T\}} D(t,\tau)\left\{-\Pi(\tau,T) + \mathrm{Rec}\,(\mathbb{E}_\tau[\Pi(\tau,T)])^+ + (\mathbb{E}_\tau[\Pi(\tau,T)])^-\right\}\end{aligned}$$

and, using the fact that $\mathbb{1}_{\{\tau>T\}}\Pi(t,T) + \mathbb{1}_{\{\tau\le T\}}\Pi(t,T) = \Pi(t,T)$,

$$\begin{aligned}\Pi^D(t,T) &= \Pi(t,T)\\ &\quad + \mathbb{1}_{\{\tau\le T\}} D(t,\tau)\left\{-\Pi(\tau,T) + \mathrm{Rec}\,(\mathbb{E}_\tau[\Pi(\tau,T)])^+ + (\mathbb{E}_\tau[\Pi(\tau,T)])^-\right\}\end{aligned}$$

Unilateral CVA – Proof II

3. Recalling that $\text{CVA}_t := \mathbb{E}_t\left[\Pi(t,T) - \Pi^D(t,T)\right]$, we get:

$$\text{CVA}_t = -\mathbb{E}_t\left[\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\left\{-\Pi(\tau,T) + \text{Rec}\,(\mathbb{E}_\tau[\Pi(\tau,T)])^+ + (\mathbb{E}_\tau[\Pi(\tau,T)])^-\right\}\right]$$

4. We consider the first term in the sum $\mathbb{E}_t\left[-\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\,\Pi(\tau,T)\right]$.
Using the tower rule of expectations, i.e. $\mathbb{E}_t[\cdot] = \mathbb{E}_t[\mathbb{E}_\tau[\cdot]]$, with $\tau \ge t$ we have:

$$\begin{aligned}\mathbb{E}_t\left[-\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\,\Pi(\tau,T)\right] &= \mathbb{E}_t\left[-\mathbb{E}_\tau\left[\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\,\Pi(\tau,T)\right]\right] \\ &= \mathbb{E}_t\left[-\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\,\mathbb{E}_\tau\left[\Pi(\tau,T)\right]\right]\end{aligned}$$

5. Plugging this result in CVA_t, recalling the definition of close-out amount at default time, i.e. $V(\tau) := \mathbb{E}_\tau[\Pi(\tau,T)]$, and using the fact that $X = X^+ + X^-$, we obtain the final result, eq. (6):

$$\begin{aligned}\text{CVA}_t &= -\mathbb{E}_t\left[\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\left\{-(V(\tau))^+ - (V(\tau))^- + \text{Rec}\,(V(\tau))^+ + (V(\tau))^-\right\}\right] \\ &= -\mathbb{E}_t\left[\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\left\{-(V(\tau))^+ + \text{Rec}\,(V(\tau))^+\right\}\right] \\ &= \text{LGD}\,\mathbb{E}_t\left[\mathbb{1}_{\{\tau \le T\}}\, D(t,\tau)\,(V(\tau))^+\right]\end{aligned}$$

■

Outline

General Framework

We consider for simplicity the case of unilateral CVA.
The goal is to calculate CVA expressed as in formula (4), i.e.

$$\text{CVA} = \mathbb{E}[\, D(0,\tau)\, \text{Loss}(\tau)] = \text{LGD}\, \mathbb{E}\left[\mathbb{1}_{\{\tau \le T\}}\, D(0,\tau)\ (V(\tau))^{+}\right]$$

Closed-form formulae are only available:

- for certain **single deals**, i.e. in the absence of a netting set
- under the assumption of **independence** between counterparty risk, embedded in $\mathbb{1}_{\{\tau \le T\}}$ and the risks associated to the underlying exposure, embedded in $V(\tau)$

Wrong Way Risk (WWR)

In some cases, the assumption of independence leads to underestimate a significant source of potential loss. This is due to **Wrong Way Risk** or **Right Way Risk**.

ISDA definition of WWR

WWR is defined as the risk that occurs when "exposure to a counterparty is **adversely correlated** with the credit quality of that counterparty".

It arises when default risk and credit exposure **increase together**.

- **Specific WWR** arises due to counterparty specific factors: a rating downgrade, poor earnings or litigation.
- **General WWR** occurs when the trade position is affected by macroeconomic factors: interest rates, inflation, political tension in a particular region, etc...

WWR Examples

1. **Monoline insurers (e.g. Ambac and MBIA)**

   During the sub-prime crisis, the monolines specialized in guaranteeing mortgage-backed securities, when the mortgage market collapsed, saw their creditworthiness deteriorate and found themselves unable to pay all of the insurance claims. Almost all exposure mitigation from monoline insurance fell short due to the guarantors' increased probability of default under exactly the *same* conditions when insurance was most needed.

2. **Collateralized loan**

   Bank A enters into a collateralized loan with Bank B (the counterparty). The collateral that Bank B provides to A can be of different nature:

   - bonds issued by Bank B (**specific WWR**)
   - bonds issued by a different issuer belonging to a similar industry, or the same country or geographical region (**general WWR**). This kind of risk is both difficult to detect in the trading book, hard to measure and complex to resolve.

Right Way Risk (RWR)

Right way risk is the opposite of wrong way risk. It is the effect observed when the **exposure decreases as the default probability increases**, i.e. when there is a negative dependency between the two. The size of credit risk decreases as the counterparty approaches a potential default. RWR occurs when a company enters into transactions to **partially hedge an existing exposure**.

Examples:

- An airline usually protects itself against a rise in fuel prices by entering into long oil derivative contracts.
- A company would normally issue calls and not puts on its stock.

WWR and RWR are together referred to as **DWR (directional way risk)**.

Overview of Modeling Approaches

Goal: to **correlate** counterparty exposure with counterparty creditworthiness

1. Intensity (reduced form) models
2. Structural models
3. Cespedes *et al* (2010)
4. Hull and White (2012)
5. Basel II

Modeling Approaches

1. **Intensity (reduced form) models**
   - Default is described in terms of the **default time**, i.e. the first jump time of a Poisson/Cox process with (default) intensity $\lambda(t)$.
   - The stochastic process for $\lambda(t)$ is **correlated** to the stochastic process for the exposure.
   - Intensity models work well with exposures in the asset class of **interest rates**, **FX**, **credit**, **commodity**.
   - In the case of **equity**, **not enough correlation**. Structural models are more appropriate.
2. **Structural models**

   The stochastic process affecting the dynamics of the counterparty firm value is directly correlated to the stochastic process ruling the underlying of the contract. See Lecture 5 for a list o structural models and their calibration to the market.

Other Modeling Approaches

3. **Cespedes et al (2010)**

   Cespedes *et al* (2010) propose an **ordered scenario copula model**. Default events and exposures are driven by factor models, while a Gaussian copula is used to correlate exposure and credit events. The approach builds on existing exposure scenarios by a non-parametric sampling of exposure via the factor model.

4. **Hull and White (2012)**

   Hull and White (2012) model the hazard rate as a deterministic monotonic function of the value of the contract. Wrong-way (right-way) risk is obtained by making the hazard rate to be an increasing (decreasing) function of the contract's value.

Basel II

Basel II deals with wrong-way risk using the so-called **"alpha" multiplier** to increase the exposure, in the version of the model in which exposure and counterparty creditworthiness are assumed to be independent.

The effect is to **increase CVA by the alpha multiplier**.

The Basel II rules set alpha equal to **1.4** or allows banks to use their own models, with a floor for alpha of **1.2**, i.e.

- if a bank uses its own model, at minimum, the CVA has to be 20% higher than that given by the model where default and exposure are independent
- if a bank does not have its own model for wrong way risk it has to be 40% higher

Estimates of alpha reported by banks range from 1.07 to 1.10.

Monte Carlo Valuation I

- In the presence of a netting set, CVA can be calculated only through **multi-asset Monte Carlo** simulation, where financial instruments must be simulated until maturity. **Calibration** of the framework and Monte Carlo **simulation** with a sufficiently large number of scenarios is very **time consuming**.

- **Monte Carlo method** allows to **estimate the expected value** of a variable as the average of all its realizations across different **simulated scenarios** ω_k. In the case of **unilateral CVA**, starting from eq. (4), we get:

$$\boxed{\text{CVA} \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} D(0, \tau; \omega_k) \, \text{Loss}(\tau, X(\tau, \omega_k))} \tag{7}$$

where $\tau \equiv \tau(\omega_k)$ and we have made explicit the dependence of the loss variable Loss on the **risk drivers** affecting the cash flows, denoted compactly by $X(\tau, \omega_k)$.

Monte Carlo Valuation II

In order to simplify the calculation of:

$$\text{CVA} \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} D(0, \tau; \omega_k)\, \text{Loss}(\tau, X(\tau, \omega_k))$$

two approximations are commonly introduced:

- **Approximation 1**: Default bucketing
- **Approximation 2**: Default bucketing + independence
  This is equivalent to ignoring wrong/right way risk

Approximation 1: Default Bucketing

Assuming that default can be observed at discrete times $T_1, T_2, \ldots, T_b$, formula (7) can be simplified as:

$$\boxed{\mathrm{CVA} \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} \sum_{j=1}^{b} D(0, T_j; \omega_k)\, \mathrm{Loss}(T_j, X(T_j, \omega_k))} \tag{8}$$

where **defaults have been bucketed** but, a **joint model** for:

- the **default of the counterparty** $\mathbb{1}_{\{\tau(\omega_k) \in (T_{j-1}, T_j]\}}$ and
- the **value of counterparty exposure** $V(T_j) = \mathbb{E}_{T_j}[\Pi(T_j, T)]$

is still needed.

Approximation 1: Models I

- In order to simulate all the variables that affect the calculation of CVA, we need to define their **dynamics**. We follow two criteria:
  - **simplicity** of the models
  - **straightforward calibration** of the models to liquid market instruments

- Let $\mathbf{Z(t)} = \{\mathbf{X(t)}, \lambda_{\mathbf{C}}, \lambda_{\mathbf{I}}\}$ be the set of all processes underlying the calculation of CVA, where X denotes the risk drivers, λ_C and λ_I respectively the default of the counterparty and the default of the investor[2].
  Each process, under the **risk neutral measure** $\mathbb{Q}$, follows a dynamics

$$dZ_i(t) = (\ldots)dt + (\ldots)dW_i$$

  and is correlated to the others through $dW_i(t)\, dW_j(t) = \rho_{ij} dt$.

[2] In the following, we will consider the investor default free $\lambda_I = 0$, such that only the default of the counterparty matters $\lambda_C \equiv \lambda$.

Approximation 1: Models II

In the following, we will show as an illustrative example of underlying models only the case of an **exposure to interest rate risk**.

Analogously, models for other asset classes can be introduced and correlated to the model for the creditworthiness of the counterparty.

Approximation 1: Credit Model

- We consider an **intensity model**[3] for the credit spread
- If default of the counterparty were independent of the others sources of risk X, a **deterministic model** would be enough to bootstrap the probabilities of default of the counterparty
- When λ and X are dependent, a common choice is the **CIR model with jumps**:

$$\boxed{d\lambda(t) = \kappa(\mu - \lambda(t))dt + \nu\sqrt{\lambda(t)}dW_\lambda(t) + dJ^{\beta,F}(t)} \qquad (9)$$

where $dJ^{\beta,F}(t)$ represents the jump component and the model:

$$J^{\beta,F}(t) = \sum_{i=1}^{N(t)} Y_i \qquad N(t) \sim \text{Poisson}(\beta t) \qquad Y_i \; i.i.d. \sim F$$

$$F(x) = 1 - e^{-\gamma x} \qquad x \geq 0$$

- The model is **exactly calibrated** to CDS quotes, assuming deterministic interest rates

[3] For exposures to equity products, structural models are more appropriate

Approximation 1: Interest Rates Model

- A common choice is the **one-factor Hull and White** model, which can be equivalently[4] described by a **shifted short rate model**

$$\boxed{r^{\phi}(t) = r(t) + \phi(t)} \tag{10}$$

where $\phi(t)$ is a **deterministic shift extension** and the factor $r(t)$ evolves as a Vasicek:

$$dr(t) = k(\theta - r(t))dt + \sigma dW_r(t)$$

- The value of the **zero coupon bond** is given by:

$$P(t,T) = \Phi(t,T)\,\mathbb{E}\left[e^{-\int_t^T r(s)ds} \,\middle|\, \mathcal{F}_t\right] \quad \text{where} \quad \Phi(t,T) = e^{-\int_t^T \phi(s)ds}$$

- The model is **exactly calibrated to swaption prices and the discount curve**.

  In the multi-curve setting, the **deterministic shift** captures the differences between the curves of discounting (OIS) and forwarding (underlying instrument tenor)

[4] See Brigo and Mercurio, Chapter 3.3 and 3.8

Approximation 1: Correlation Default/Exposure

Finally, we introduce correlation between credit (9) and interest rates (10), as follows:

$$\boxed{\text{corr}(d\lambda(t), dr^{\phi}(t)) = \text{corr}(d\lambda(t), dr(t)) = \rho_{\lambda r} dt}$$

Approximation 2: Default Bucketing and Independence

Assuming that default of the counterparty is independent of its exposure (**absence of wrong-way risk**) eq. (8) can be further simplified according to:

$$
\begin{aligned}
\mathrm{CVA} &= \mathrm{LGD} \sum_{j=1}^{b} \mathbb{Q}(\tau \in (T_{j-1}, T_j])\, \mathbb{E}\left[D(0, T_j)\, (V(T_j))^{+}\right] \\
&= \mathrm{LGD} \sum_{j=1}^{b} \{\mathbb{Q}(\tau > (T_{j-1})) - \mathbb{Q}(\tau > T_j)\}\, \mathbb{E}\left[D(0, T_j)\, (V(T_j))^{+}\right] \\
&\approx \frac{\mathrm{LGD}}{N_{MC}} \sum_{j=1}^{b} \{\mathbb{Q}(\tau > (T_{j-1})) - \mathbb{Q}(\tau > T_j)\} \sum_{k=1}^{N_{MC}} D(0, T_j; \omega_k)\, [V(T_j, X(T_j, \omega_k))]^{+}
\end{aligned}
\tag{11}
$$

Defaults are bucketed and only **survival probabilities** are needed (no default model). An **option model** for counterparty exposure $V(t)$ is still needed.

Outline

Case Study 1: Single Interest Rate Swap (IRS)

A (Payer) Interest Rate Swap is a contract that exchanges payments between two legs, starting from a future time instant.

The fixed leg pays out the amount $N\,\alpha_i^F\,K$, with a fixed interest rate K, a nominal value N and a year fraction α_i^F between T_{i-1}^F and T_i^F, while the floating leg pays out the amount $N\,\alpha_i^f\,L(T_{i-1},T_i)$, where

$$L(T_{i-1},T_i) = \frac{1}{\alpha_i^f}\left[\frac{1}{P^{fwd}(T_{i-1},T_i)} - 1\right]$$

is the forward rate resetting at T_{i-1}^f and paying at T_{i-1}^f.

Schematically:

Fixed	$\rightarrow$	fixed rate $N\,\alpha_i^F\,K$ at $T_{a+1}^F,\ldots,T_b^F$	$\rightarrow$	Floating
Leg	$\leftarrow$	floating rate $N\,\alpha_i^f\,L(T_{i-1},T_i)$ at $T_{a+1}^f,\ldots,T_b^f$	$\leftarrow$	Leg

Case Study 1: Risk Drivers

We consider a EUR denominated IRS, where the fixed leg pays annually and the floating leg semi-annually.

Risk drivers for a EUR denominated IRS

1. EUR discounting curve (OIS – Overnight Indexed Swap)
2. EUR forwarding curve (6 months Euribor)
3. EUR interest rate volatility
4. CDS spread of counterparty C

Case Study 1: IRS under Independece

$$\mathrm{IRS}^D(t,K) = \mathrm{IRS}(t,K) - \mathrm{LGD} \sum_{j=a+1}^{b-1} \mathbb{Q}(\tau \in (T_{j-1}, T_j])\, \mathrm{SWAPTION}_{j,b}(t; K, S_{j,b}(t), \sigma_{j,b}) \quad (12)$$

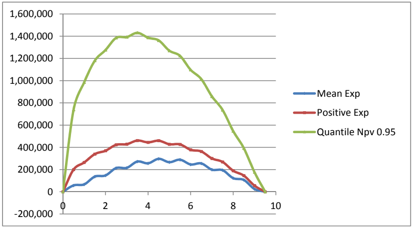
Counterparty model for credit spread

Survival probabilities are bootstrapped from CDS quotes, under the assumption of deterministic interest rates.

Counterparty exposure model

One-factor Hull and White (short rate) model for $r(t)$, calibrated to swaption quotes and zero curve data.

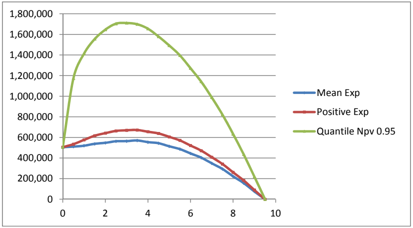
Case Study 1: Exposures Results

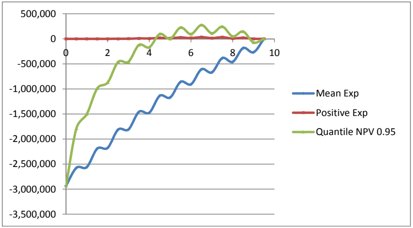


10 year payer swap
Notional $N = 10$ Million

From left to right:

- ATM: $K = 0.6\%$
- ITM: $K = 0.1\%$
- OTM: $K = 3.5\%$





Case Study 2: Portfolio of IRS

- 5 counterparties
- 30 IRS (payer, receiver), not all deals belonging to given netting sets

Deal ID	Cpty ID	Netting ID	Principal
1	5	5	813450
2	5	NaN	441321
3	1	NaN	629468
...	...	...	...
9	5	5	918177
...	...	...	...

- IRS maturities: from 1 to 7 years
- CDS market quotes in bps for each counterparty, for different maturities

Maturity	Cpty 1	Cpty 2	Cpty 3	Cpty 4	Cpty 5
1y	140	85	115	170	140
2y	185	120	150	205	175
3y	215	170	195	245	210
4y	275	215	240	285	265
5y	340	255	290	320	310

Case Study 2: Mark to Market Swap Prices

MtM swap prices are computed at each future simulation date and for each scenario

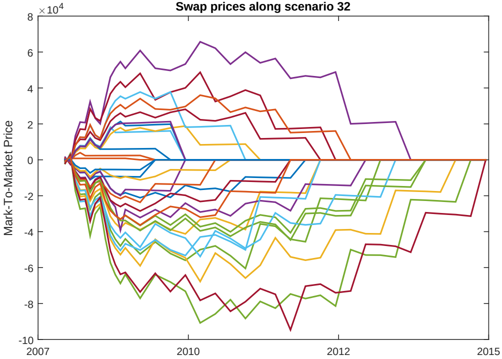
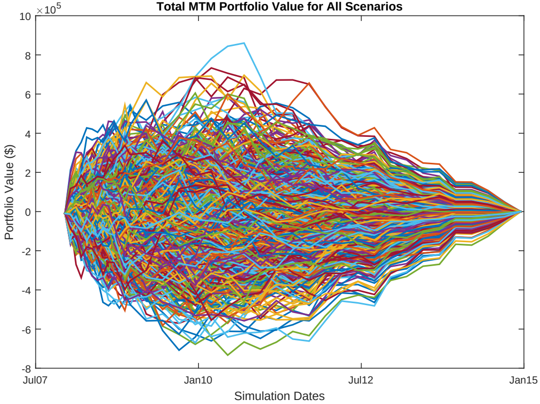


Figure: MtM of all IRS in the portfolio, for scenario 32.

Case Study 2: Simulated Portfolio Values

The total portfolio value is computed at each simulation date, for each scenario. As the swaps get closer to maturity, their values begin to approach zero since the aggregate value of all remaining cash flows decreases after each cash flow date.



Case Study 2: Counterparty Exposures I

- The **exposure** of a particular contract i at time t is given by:

$$E_i(t) = \max\{V_i(t), 0\} = (V_i(t))^+$$

- The **exposure to a counterparty** is the sum of the individual contract exposures:

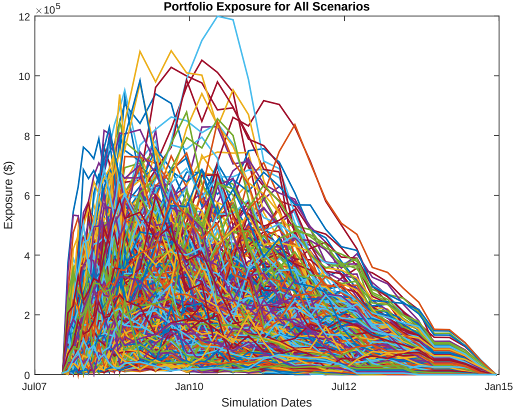
$$E_{cpty}(t) = \sum E_i(t) = \sum \max\{V_i(t), 0\} = \sum (V_i(t))^+$$

- In the presence of **netting agreements**, contracts are aggregated together and can offset each other. The total exposure of all contracts in a netting agreement is:

$$E_{netting}(t) = \max\left\{\sum V_i(t), 0\right\} = \left(\sum V_i(t)\right)^+$$

Case Study 2: Counterparty Exposures II

Exposure of the **entire portfolio**, at each simulation date and for each scenario.

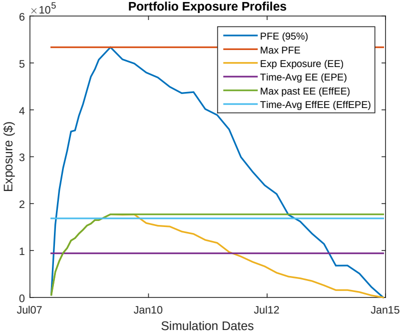


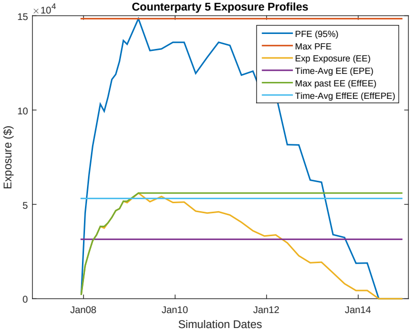
Case Study 2: Exposure Profiles I

(Non-discounted) exposure profiles:

- **PFE – Potential Future Exposure**
  A high percentile (95%) of the distribution of exposures at any given future date
- **MPFE – Maximum Potential Future Exposure**
  The maximum PFE across all dates
- **EE – Expected Exposure**
  The mean (average) of the distribution of exposures at each date
- **EPE – Expected Positive Exposure**
  Weighted average over time of the expected exposure
- **EffEE – Effective Expected Exposure**
  The maximum expected exposure up to time t
- **EffEPE – Effective Expected Positive Exposure**
  Weighted average over time of the effective expected exposure

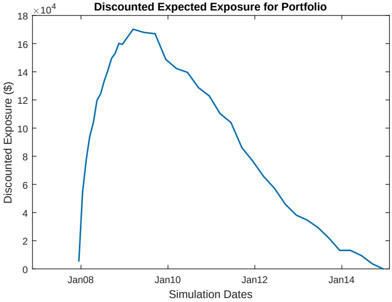
Case Study 2: Exposure Profiles II

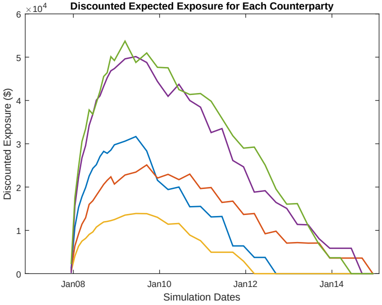




Case Study 2: Discounted Exposures

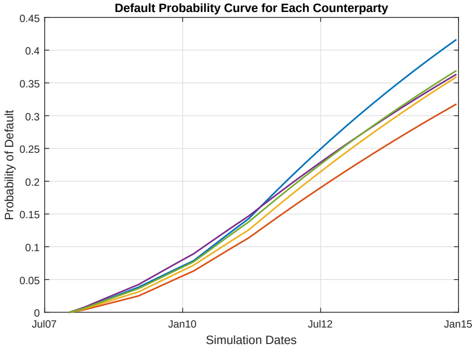
Discounted expected exposures are computed by using the discount factors obtained from a Hull and White simulation.





Case Study 2: Probabilities of Default

The default probability of a given counterparty is implied from the current market spreads of the counterparty's CDS (see Table in slide 55) at each simulation date, through a bootstrap procedure.



Case Study 2: CVA Computation

CVA is calculated through formula (11), which we write here in a simplified form:

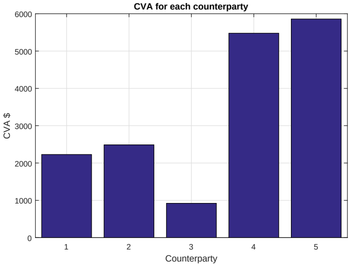
$$\text{CVA} \approx (1 - \text{Rec}) \sum_{j=1}^{b} \left[\text{PD}(t_j) - \text{PD}(t_{j-1})\right] \text{discEE}(t_j)$$

where:

- exposures are assumed to be independent of default (no wrong-way risk) and have been obtained using risk-neutral probabilities
- $\text{discEE}(t)$ is the discounted expected exposure at time t
- $\text{PD}(t) = 1 - \mathbb{Q}(\tau > t)$ is the default probability
- $\text{Rec} = 1 - \text{LGD}$ is the recovery, and for this example it has been assumed equal to 40%

Case Study 2: CVA Results

Counterparty	CVA ($)
1	2228.36
2	2487.60
3	920.39
4	5478.50
5	5859.30



Outline

Mitigating Counterparty Exposure

Mitigation of counterparty exposure can be achieved through[5]:

- **netting agreements**
- **collateralization**

[5] See for example Ballotta *et al.*

Netting

In presence of **multiple trades** with a counterparty, **netting agreements** allow, in the event of default of one of the counterparties, to aggregate the transactions before settling claims.

- In the absence of netting, the exposure is:

$$E(\tau) = \sum_{i=1}^{n} w_i E_i(\tau) = \sum_{i=1}^{n} w_i (V_i(\tau))^+$$

  where n is the number of contracts, w_i are the asset quantities, and E_i the corresponding exposures.

- A netting agreement is a **legally binding contract** between two counterparties based on which, in the event of default, the exposure results in:

$$\boxed{E_{netting}(\tau) = \left(\sum_{i=1}^{n} w_i V_i(\tau)\right)^+}$$

Example 1

Assumptions

Two counterparties, a bank B and a counterparty C, such that:

- C holds a currency option written by B with a market value of 50
- B has an IRS with C, having a marked to market value in favor of B of 80

Exposures

- The exposure of the bank B to the counterparty C is 80
- The exposure of the counterparty C to the bank B is 50
- The exposure of the bank B to the counterparty C, **with netting**, is 30

Example 2

The following table shows, at different times, the values of five trades as well as the future exposures to the counterparty, **with and without netting**.

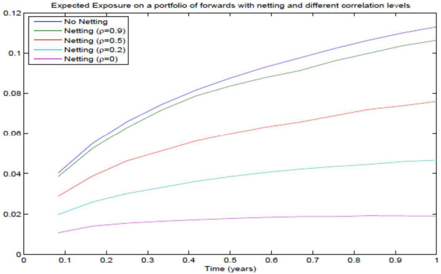
	Time (Months)				
Trade ID	1	2	3	4	5
1	10	-7	8	-6	-2
2	9	0	4	-2	2
3	7	7	5	10	-8
4	-7	-6	3	-6	-6
5	-5	-5	3	6	-6
	Exposures ($)				
No Netting	26	7	23	16	2
Netting	14	0	23	2	0

Table: Source Ballotta *et al* (forthcoming).

Example 3

Consider a portfolio of 10 homogeneous assets (long forward contracts) with IVol $= 38\%$, marginal default probability 0.025%, risk free rate $r = 3\%$, dividend yield $q = 1\%$ and the same cross-correlation levels.

The simulated CVA of such portfolio shows that **the larger the cross correlation** among assets, **the smaller the benefit** of the netting clause.



	CVA	CVA Reduction
No Netting	0.01518	
$\rho = 0.9$	0.01449	4.59%
$\rho = 0.5$	0.01055	30.48%
$\rho = 0.2$	0.00649	57.23%
$\rho = 0$	0.00304	80.00%

Figure and Table: Source Ballotta *et al* (forthcoming).

Collateral Definition

Collateralization is one of the most important techniques of mitigation of counterparty risk.

A **collateral account** is a contractual clause aimed at reducing potential losses incurred by investors in case of the default of the counterparty, while **the contract is still alive**.

Collateral ... in Theory

Consider the bank/investor B and the counterparty C.
Let $C(t)$ be the (cash) **collateral amount** posted by C to B, at time t.

- B has no exposure to the contract up to the collateral amount, while its losses are reduced by the collateral amount whenever the exposure exceeds it. The **collateralized exposure** $E_C(t)$ is defined as:

$$\boxed{E_C(t) = (E(t) - C(t))^+}$$

- Equivalently:

$$E_C(t) = E(t) - \left[C(t) - (C(t) - E(t))^+\right]$$

- The posting of collateral allows a mitigation of the exposure in favor of the part receiving it. This **mitigation** is positive and equal to the amount:

$$\boxed{C(t) - (C(t) - E(t))^+}$$

Collateral ... in Practice I

The **actual amount of the collateral** available at time t depends on the contractual agreement between the parties, specified in terms of:

1. **Posting threshold $H > 0$**

   i.e. the threshold which triggers the posting of collateral: below the threshold no collateral is posted.

   The underlying commercial reason for a threshold is that often parties are willing to take a certain amount of **credit risk** (equal to the threshold) before requiring collateral to cover any additional risk

2. **Margin period δ**

   i.e. the interval at which margin is monitored and called for:

$$C(t) = [E(t - \delta) - H]^+$$

   In case of default at time τ, the last call occurs at $(\tau - \delta)$.

   Most collateral agreements require **daily calculations**; however, in order to reduce operational requirements, weekly or monthly calculations can be agreed on, which result in increased credit risk

Collateral ... in Practice II

3. **Minimum transfer amount MTA**

   i.e. the amount below which no margin transfer is made. The collateral is set to zero if less than MTA:

   $$\boxed{C(t) = [E(t-\delta) - H]^{+} \; \mathbb{1}_{E(t-\delta)-H>\text{MTA}}}$$

   The presence of the MTA avoids the operational costs of small transactions and contributes to reduce the frequency of collateral exchanges

4. **Downgrade triggers**

   Sometimes, the threshold and the MTA vary during the lifetime of the contract if the parties agree on the inclusion of downgrade triggers, also known as **rating-based collateral calls**. These clauses force a firm to post more collateral to its counterparty, if it is downgraded below a certain level

Example 1: Downgrade Triggers

AIG (2008)

Soon after the collapse of Lehman Brothers, on September 16, 2008 AIG's credit rating was downgraded and it was required to post 15$ billion in collateral with its trading counterparties, leading to a liquidity crisis that essentially bankrupted all of AIG. AIG could not collect the required funds on such a short notice.

Citigroup, MS and RBS (2012)

Similarly, in June 2012 Moody's downgraded three major derivatives dealers (Citigroup, Morgan Stanley and Royal Bank of Scotland) below the crucial single A threshold, which has led to collateral calls from counterparties.

Example 2

The following table provides the exposure of a bilateral contract at different dates and under **different assumptions on the threshold H and the MTA**:

Time (Months)	0	2	4	6	8	10	12
$E(t)$	0	3	12	19	25	26	0
$C(t)$ $(H = 0,\ MTA = 0)$	0	0	3	12	19	25	26
$E_C(t)$	0	3	9	7	6	1	0
$C(t)$ $(H = 1,\ MTA = 0)$	0	0	2	11	18	24	25
$E_C(t)$	0	3	10	8	7	2	0
$C(t)$ $(H = 1,\ MTA = 2)$	0	0	0	11	18	24	25
$E_C(t)$	0	3	12	8	7	2	0

Table: Source Ballotta *et al* (forthcoming).

In general, **the larger the threshold, the less effective the collateral protection** and the longer the margining period, the higher the risk of upward movements in the value of the contract, and ultimately in the CVA.

Gap Risk I

Collateralization is **not able to fully eliminate** counterparty risk.

Sudden movements in the market between two margining dates can increase **both** the exposure and the probability of the relevant default event.
This originates gap risk.

Gap Risk is the residual (counterparty) risk which remains because:

- the **threshold** is non-zero
- the **margin period of risk** is the finite time needed to:
    1. initiate margin call
    2. allow the counterparty to post additional collateral
    3. liquidate the position and rehedge
- the **market can jump** (crash) within this time

Gap Risk II

In practice, gap risk is the risk that the corporate defaults, the bank survives and the contract moves in the money, given that, at the last margining date, the counterparty was solvent and the exposure out-of-the money.

In the presence of downgrade triggers, the counterparty may have downgrade triggers with many other banks thus its downgrade may lead to simultaneous sell-offs ("crowded market") leading to falling market (gap risk).

"Unravelling Gap Risk at Deutsche Bank", FT 26/05/2015

- In **2005** Deutsche bought \$100bn of insurance from Canadian pension funds against the possibility of default by some of the safest companies. Since both sides estimated extremely unlikely the simultaneous default of all companies, they agreed that the pension funds put up a **small amount of collateral**, initially 9% of the \$100bn.
- **During the 2008 crisis**, the increased risk of the companies going bankrupt made the trades become more valuable to Deutsche, **increasing their value** from \$2.63bn to \$10.65bn.
- However, the crisis did also increase the chance that pension funds would not be able to live up to their end of the bargain. With such a small percentage of collateral, the pension funds could ultimately decide it was better to **walk away from the trades**.
- The bank was supposed to account for this risk, known as the **gap risk**. The bank used five different methods to calculate gap risk, but instead of increasing the risk, each of the methods reduced it. In the end, the bank reduced its gap risk from \$200m to zero, though potentially, according to independent estimates by the SEC, Goldman Sachs and some ex-employees, it would have had to be around to \$12bn.

Deutsche was **taking the upside** of the trades, but **not the downside**.

Re-hypothecation

Banks can use collateral not only as a way of reducing credit risk, but also as a way of funding, through re-hypothecation, i.e. the practice of reusing, selling or lending assets which have been received as collateral.

According to a survey on margin published by ISDA in April 2010, 82% of large dealers reported re-hypothecating collateral received in connection with OTC derivatives transactions (Risk Magazine, October 2010).

If collateral is segregated and not available for re-hypothecation, banks have to assume that they need to raise funding to meet the cashflows over the life of the trade using their own internal funding curves (Risk Magazine, September 2010).

Collateral and Corporates

Very few corporates post collateral because they do not have enough liquid assets for the purpose. In addition, for a corporate the operational complexity associated with collateralization (negotiating a legal document, monitoring exposures, making cash transfers, etc.) may significantly increase the cost and resource requirements.

As a result, hedging with derivative can become so expensive that corporates will choose to accept higher levels of exposure instead (Risk Magazine, October 2011).

"The airline's Cologne-based [Lufthansa] head of finance, Roland Kern, expects its earnings to become more volatile - not because of unpredictable passenger numbers, interest rates or jet fuel prices, but because it does not post collateral in its derivatives transactions."

Outline

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