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Network Models of Financial Contagion and Connectedness

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20254 – Advanced Quantitative Methods for Asset Pricing and Structuring

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Plan of the Talk

- Generalities and Motivation
- Using VAR Models Variance Decompositions to Measure Connectedness and Identify Return and Volatility Spillovers
- Cholesky vs. Generalized Variance Decompositions
- Relationship to Network Connectedness Models: Variance Decompositions, Adjacency, Nodes Degree, and Distance
- Relationship to Other Systemic Risk Measures: Marginal Expected Shortfall, CoVar, and Delta-CoVar
- In-Sample vs. Out-of-Sample Variance Decompositions: Does It Matter?
- Dealing with Large Scale Networks with LASSO/Net Models
- Open Research Questions

Relationship to Connectedness Network Measures

- In this case, we have

$$d_{ij}^{gH} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} (e_i' A_h \Sigma e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \Sigma A_h' e_i)} \qquad \tilde{d}_{ij}^{g} = \frac{d_{ij}^{g}}{\sum_{j=1}^{N} d_{ij}^{g}}$$

Scaled to sum to 1

- C^H depends on the set of variables $\boldsymbol{x}$ whose connectedness is to be examined, the predictive horizon H, the dynamics captured by $\mathbf{A}(h)$, and the approximating VAR(p) model, $C(\boldsymbol{x}, H, \mathbf{A}(h), M(\theta))$
- We may also see the connection table and all of its elements to vary over time, and write $C_t(x, H, A_t(h), M(\theta_t))$ to be estimated as

$$\widehat{C}_t(x, H, A_t(h), M(\hat{\theta}_t))$$

  - Varying H lets us break connectedness in "long-run", "short-run", etc.
  - In the limit as $H \to \infty$, we obtain an unconditional VD
  - Many choices are possible to allow for time-varying parameters
  - A simple scheme involves use of a rolling estimation window
- A network $\mathcal{N}$ is composed of N nodes and L links between nodes
- Distance s_{ij} between 2 nodes i and j is smallest number of links that must be traversed to go from i to j; $\mathcal{N}$ is connected if $s_{ij} \leq N-1, \forall i, j$

A Simple Example

- Consider a simple example with $N = 4$

	x_1	x_2	x_3	x_4	From Others
$\mathbf{x_1}$	96	1	2	1	4
$\mathbf{x_2}$	28	67	1	3	32
$\mathbf{x_3}$	14	14	70	1	29
$\mathbf{x_4}$	18	11	5	65	34
To Others	60	26	8	5	24.8

- The 12 off-diagonal entries in the **D** matrix measure pairwise directional connectedness
- The 3,2 entry of 14 means that shocks to x_2 are responsible for 14 percent of the H-step-ahead forecast error variance in x_3, $C_{3\leftarrow 2}$; in general $C_{i\leftarrow j} = d_{ij}$
- Note that in general $C_{i\leftarrow j} \neq C_{j\leftarrow i}$
- Sometimes we are interested in net pairwise directional connectedness; for example, for x_2 and x_3 we have $C_{23} = C_{3\leftarrow 2} - C_{2\leftarrow 3} = 14 - 1 = 13$
- The value of 29 in the third entry of the rightmost column, for example, means that x_2 receives 29 percent of its variation from others (x_1, x_3, x_4)
- There are 2*N* total directional connectedness measures, *N* "to others" and *N* "from others"

Relationship to Connectedness Network Measures

- A network is described by an *N*×*N* adjacency matrix A of 0s and 1s, A = [A_{ij}], where A_{ij} = 1 if nodes *i* & *j* are linked, A_{ij} = 0 otherwise
  - A is symmetric, because if *i* & *j* are connected, so too must be *j* and *i*
  - All network properties are embedded in A and any sensible connectedness measure is based on A
- Node's degree is its # of links to other nodes: $\delta_i = \sum_{j=1}^{N} A_{ij} = \sum_{j=1}^{N} A_{ji}$
- The degree distribution is the probability distribution of degrees across nodes
- The mean of the degree distribution (mean degree) has emerged as a benchmark measure of overall network connectedness
- The just-described adjacency matrix and degree distribution might more precisely be called "1-step", as the links are direct
- Even if *i* is not directly linked to *j*, *i* may be linked to *k*, and *k* to *j*, so that *i* and *j* are linked at a distance of 2 steps rather than one
  - Distinction btw. 1- and multi-step adjacency emphasizes distance
- Distance is a two-node property, in contrast to degree

Relationship to Connectedness Network Measures

- The **diameter of a network is the maximum distance between any two nodes**, $s_{max} = \max_{i,j} s_{ij}$
- Smaller network diameter $\Longrightarrow$ greater overall connectedness
  - A large-N approximation relates diameter, network mean degree, and network size in **Erdős–Rényi random networks**, $s_{max} \simeq \ln N/(\ln \mathrm{E}(\delta))$
  - In words, network diameter grows only as a function of $\ln N$
  - For N = 300, 000, 000 (the US population) and mean degree $\mathrm{E}(\delta)$ = 20, network diameter is still small ($s_{max} \approx 6$)
  - Erdős–Rényi random networks have the simplest imaginable probabilistic model of link formation: independent Bernoulli trials with fixed probability; hence the degree distribution is binomial
  - Erdős–Rényi networks have emerged as a canonical benchmark, but they are sometimes poor descriptions of real-world networks, due for example to strategic aspects of link formation such as clustering
  - Watts and Strogatz (1998) have shown that the "network diameter grows only as $\ln N$" approximation nevertheless holds in networks with small clusters of linked nodes with just a few long-range links

Relationship to Connectedness Network Measures

- Mathematical characterization of the **"small-world" phenomenon**, namely that diameters tend to be small even for huge networks
- It emphasizes in a precise way the importance of the mean degree as a measure of network connectedness, that encompasses diameter

- Variance decompositions define networks: **the variance decomposition matrix D is a special, restricted network adjacency matrix A**
  - The adjacency matrix *A* is not filled simply with 0–1 entries: the entries are weights, with some strong and others potentially weak
- **The VD links are directed, that is, the strength of the *ij* link is not necessarily the same as that of the *ji* link**, so the adjacency matrix is generally not symmetric
- There are constraints on the row sums of A: each row must sum to 1 because the entries are variance shares and $A_{ii} = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij}$
  - The diagonal elements of A are no longer 0
  - Node degrees are obtained by summing weights in [0, 1] and there are now "to-degrees" and "from-degrees", corresponding to row sums and column sums

Relationship to Measures of Systemic Risk

- The from-degree of node *i* is $\delta_i^{from} = \sum_{\substack{j=1 \\ j \neq i}}^{N} A_{ij}$
- The from-degree distribution is the distribution of from degrees across nodes; it is a univariate distribution with support on [0, 1]
- The to-degree of node *j* is $\delta_i^{to} = \sum_{\substack{i=1 \\ i \neq j}}^{N} A_{ij}$
- The total directional connectedness measures $C_{i \leftarrow \bullet}$ and $C_{\bullet \leftarrow j}$ are the from-degrees and to-degrees, respectively, associated with the nodes of the weighted directed network D

- Total connectedness C is the mean degree of the network D (to or from—it is the same either way)
- Jadbabaie, Lin, and Morse (2002) show that an overall measure of connectivity of a network ("algebraic connectivity") is given by the second smallest Laplacian eigenvalue
  - The smallest Laplacian eigenvalue is simply 0 by construction
- The Laplacian matrix is $L = K - A$, where K is a diagonal matrix containing the node degrees and A is the adjacency matrix

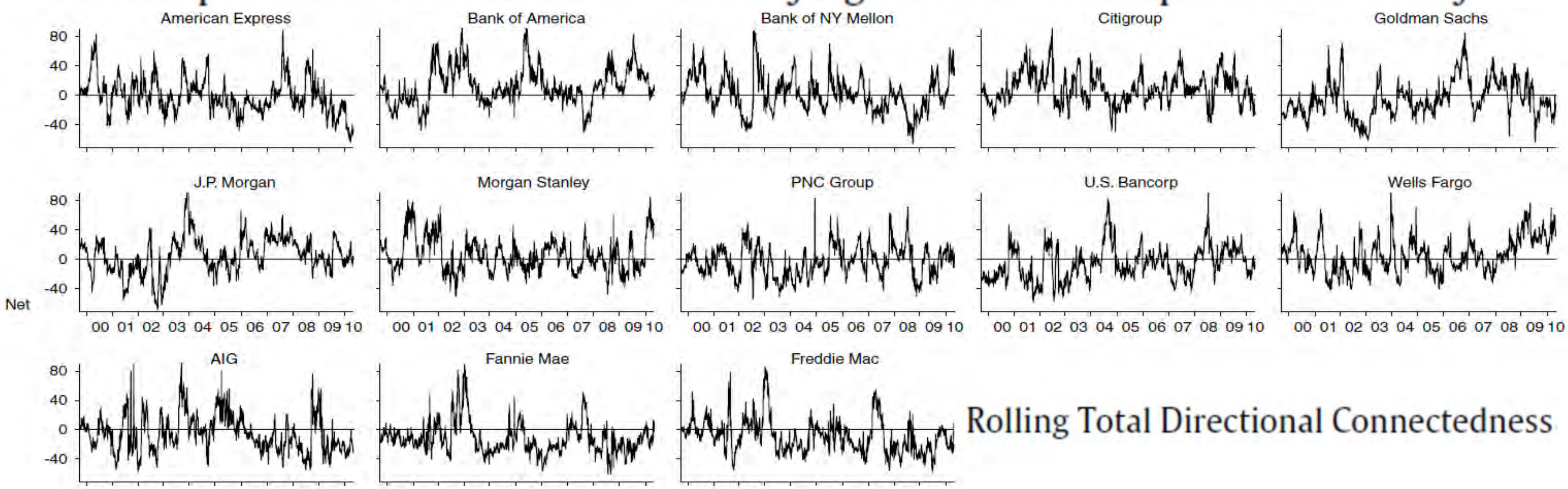
One Example from the US Financial System

- The larger the second smallest Laplacian eigenvalue, λ_2, the more difficult it is to separate a network into disconnected subnetworks by eliminating a few links
- High frequency analysis of financial institutions' connectedness seems to require high-frequency balance sheet and other information, which is generally unavailable
- Fortunately, stock returns and return volatilities are available, which reflect forward-looking assessments of many, often privately-informed, agents as regards the relevant connections

	AXP	BAC	BK	C	GS	JPM	MS	PNC	USB	WFC	AIG	FNM	FRE	**FROM**
AXP	20.0	8.5	7.1	10.3	5.8	9.8	8.8	5.1	8.0	7.8	3.2	2.6	3.0	80.0
BAC	8.3	19.1	6.0	10.6	5.8	8.0	7.4	6.1	7.1	9.2	4.2	3.5	4.6	80.9
BK	8.4	8.3	18.8	8.4	6.2	9.3	8.5	5.7	8.4	8.3	4.2	2.4	3.0	81.2
C	9.5	9.6	5.4	20.4	4.9	8.7	7.8	5.2	7.0	8.0	5.4	3.5	4.7	79.6
GS	8.2	8.6	6.8	7.6	22.1	8.8	13.3	4.0	6.0	7.6	2.4	1.9	2.6	77.9
JPM	10.2	8.6	7.1	10.6	6.2	18.8	9.5	5.2	7.8	7.3	3.6	2.5	2.6	81.2
MS	9.2	8.3	7.1	8.9	9.8	9.7	20.5	4.2	5.5	7.1	3.4	2.8	3.6	79.5
PNC	7.7	8.8	7.4	8.5	4.6	7.6	6.6	18.1	7.6	8.8	5.2	4.2	4.9	81.9
USB	9.3	9.9	7.6	9.9	5.7	8.7	6.4	5.4	20.1	8.5	4.3	1.6	2.7	79.9
WFC	8.3	10.2	6.5	9.8	6.2	7.6	7.1	5.9	7.3	18.0	3.8	3.8	5.3	82.0
AIG	5.3	7.3	4.9	8.8	2.6	5.2	4.9	6.2	6.0	5.6	27.5	6.6	9.0	72.5
FNM	4.2	5.4	2.5	6.0	2.3	3.5	3.8	5.5	1.9	6.8	6.5	29.6	22.0	70.4
FRE	4.3	6.3	2.9	6.5	2.6	3.3	4.1	5.2	2.9	7.3	7.4	17.6	29.6	70.4
TO	92.9	99.7	71.3	106.1	62.7	90.2	88.2	63.7	75.5	92.2	53.8	53.1	68.1	**78.3**
NET	13.0	18.8	−9.9	26.5	−15.2	8.9	8.7	−18.2	−4.4	10.2	−18.7	−17.4	−2.3	

One Example from the US Financial System

Rolling Total Connectedness. The rolling estimation window width is 100 days, and the predictive horizon for the underlying variance decomposition is 12 days.



Rolling Total Directional Connectedness

One Example from the US Financial System

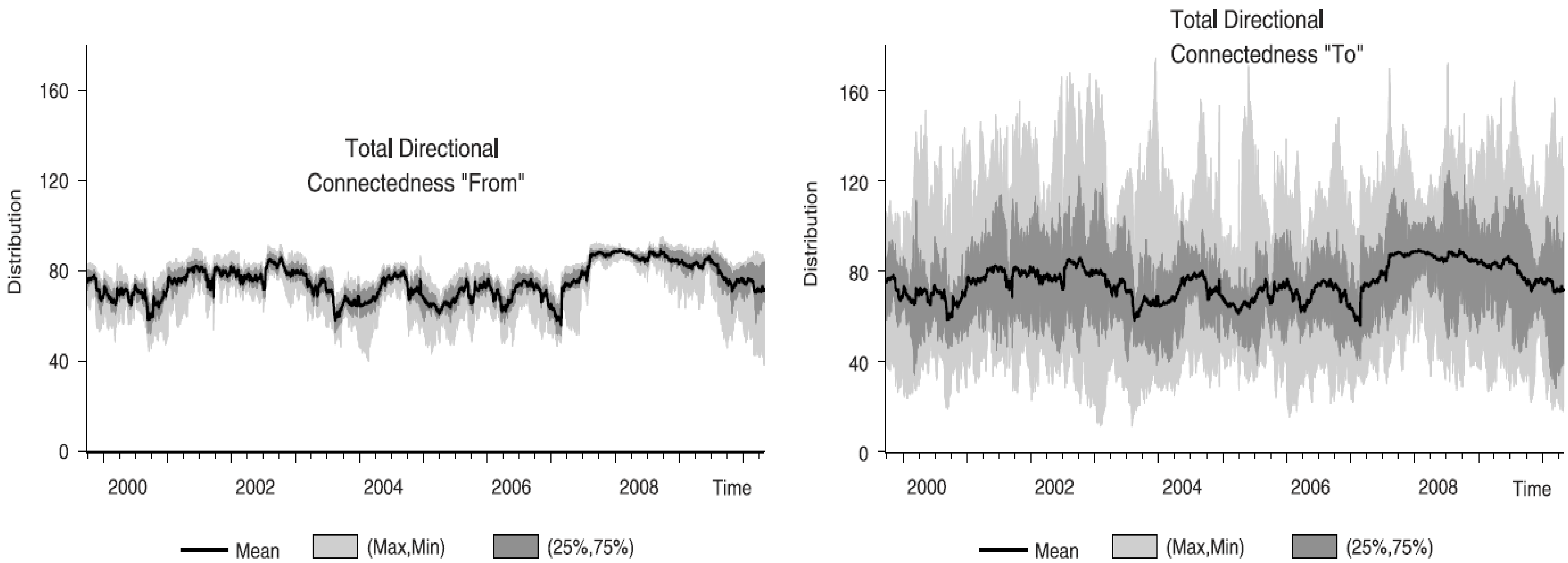


Fig. 4. Rolling Distribution of Total Directional Connectedness. We plot the time series of daily min, 25%, mean, 75%, and max of the distributions of "to" and "from" total directional connectedness. The rolling estimation window width is 100 days, and the predictive horizon for the underlying variance decomposition is 12 days.

- Temporal changes in the dispersion and skew of the "to" and "from" connectedness may contain useful information
- It appears that "from" connectedness gets not only more dispersed but also more left-skewed during crises, and simultaneously that "to" connectedness gets more right-skewed
- During crisis times relatively more than non-crisis times, a few firms receive very little volatility, and a few firms transmit very much

One Example from the US Financial System

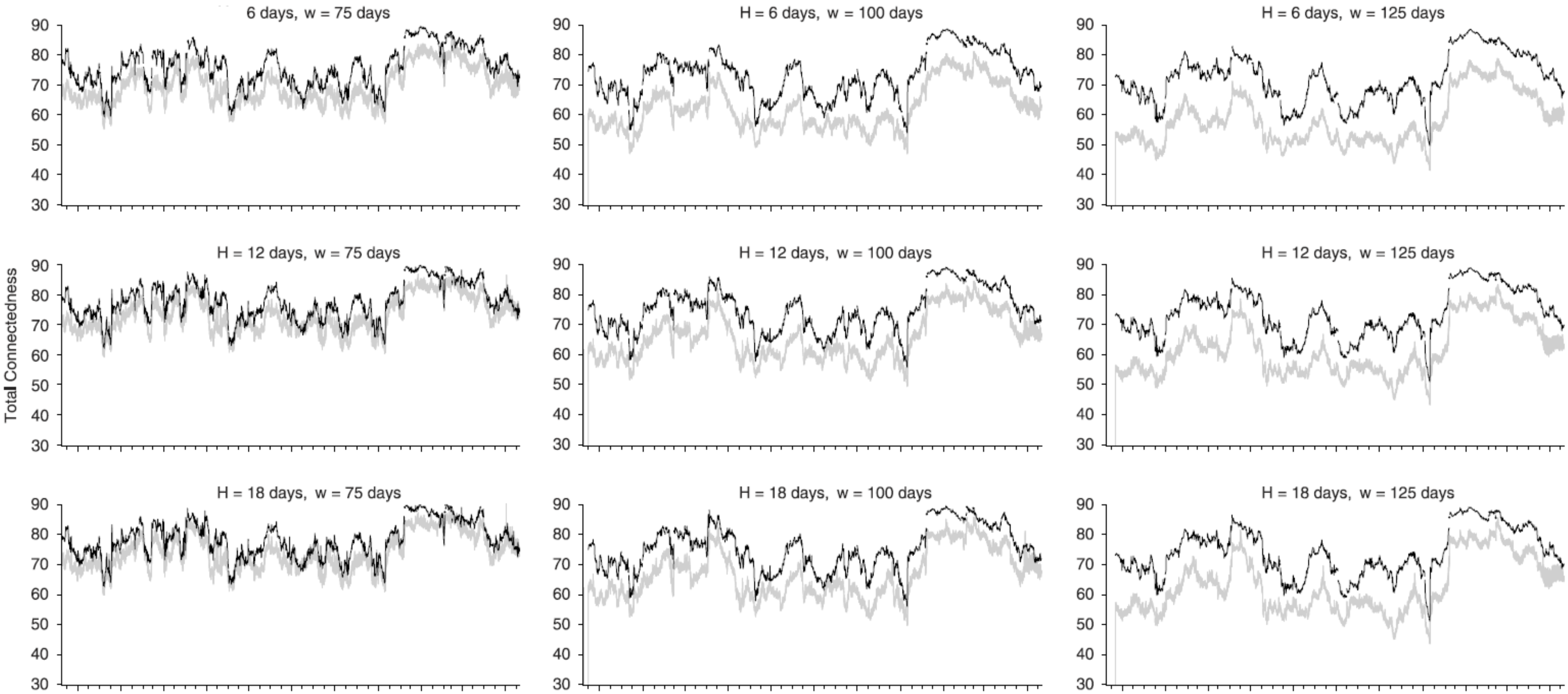
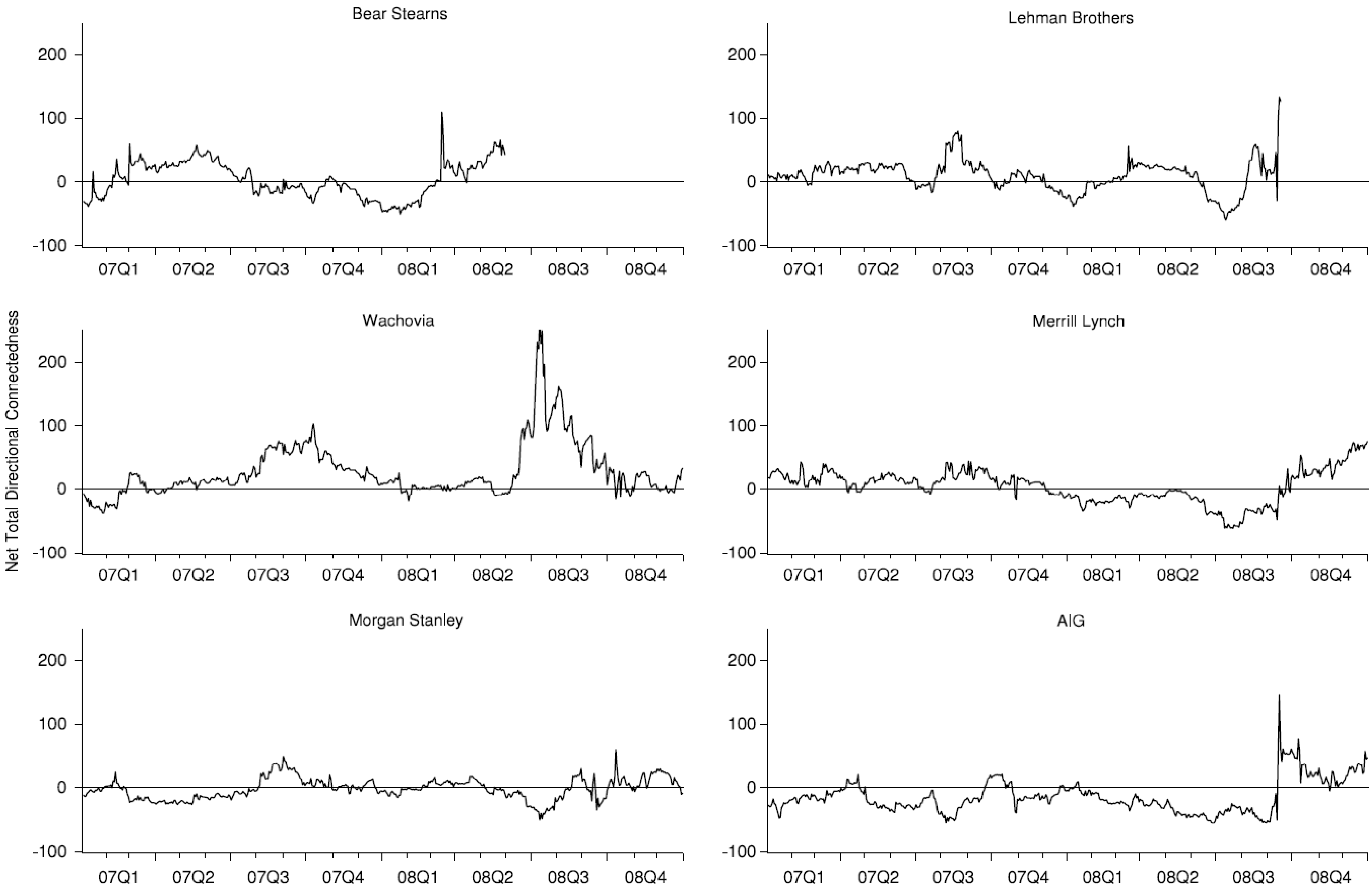


Fig. 5. Robustness of Total Connectedness. We explore estimation window widths w of 75, 100 and 125 days, predictive horizons H of 6, 12 and 18 days, and a variety of Cholesky orderings. In each subgraph, the solid line corresponds to our benchmark ordering, and the gray band corresponds to a [10%, 90%] interval based on 100 randomly-selected orderings.

- As the GVD treats each variable to be ordered as the first variable in the system, total connectedness obtained from Cholesky is the upper bound of the one obtained from the generalized identification
- Nevertheless, in all graphs the two series move in accordance

One Example from the US Financial System



One Example from the US Financial System

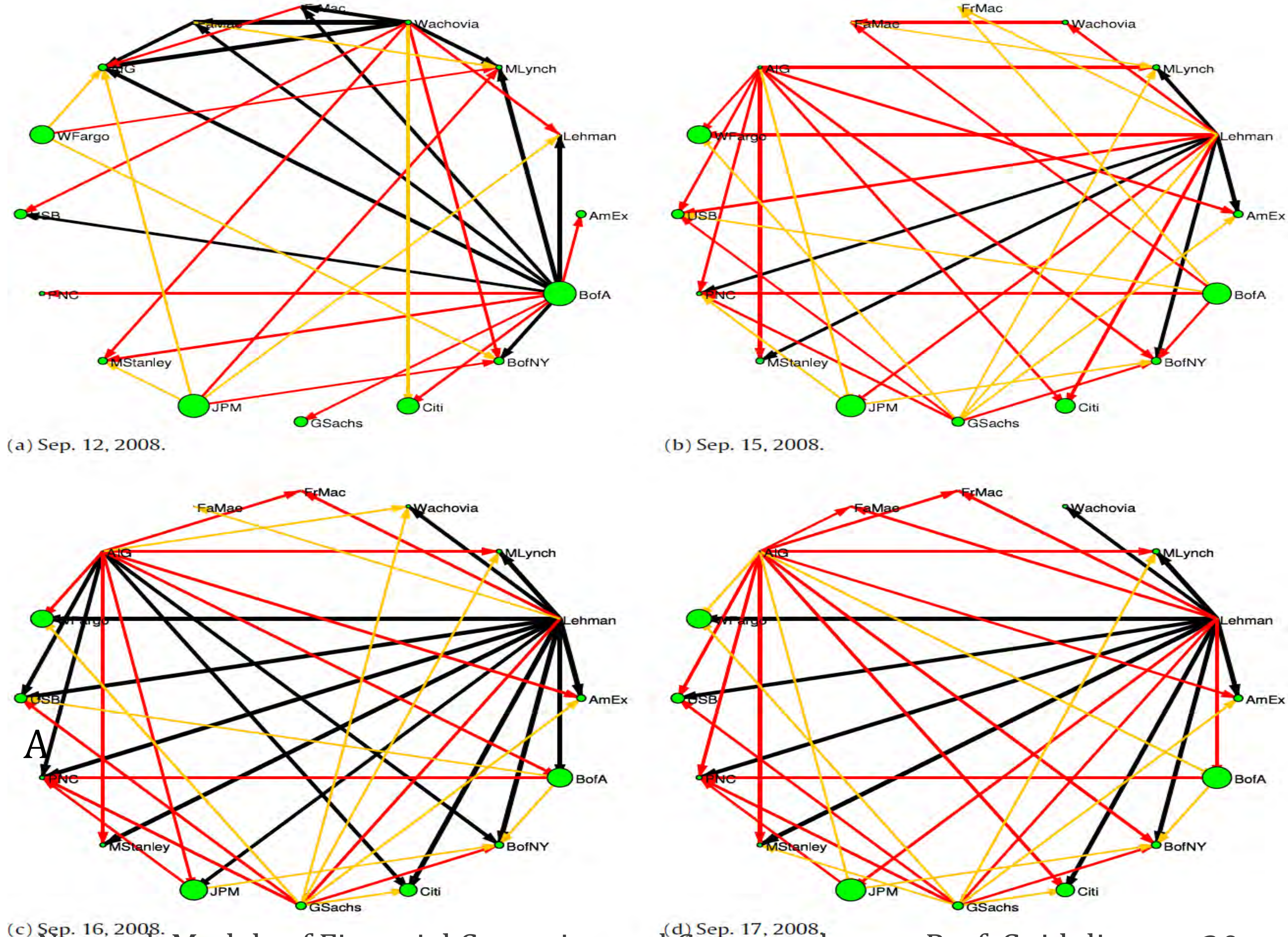


Fig. 7. Net Pairwise Directional Connectedness During the Lehman Bankruptcy. Notes: we show the most important directional connections among the pairs of sixteen bank stocks on each day. Black, red and orange links (black, gray and light gray when viewed in grayscale) correspond to the first, fifth and tenth percentiles of all net pairwise directional connections from June 1 to December 31, 2008. Node size indicates stock market capitalization. (For interpretation of the references to colour in this figure legend,

One Example from the International Banking System

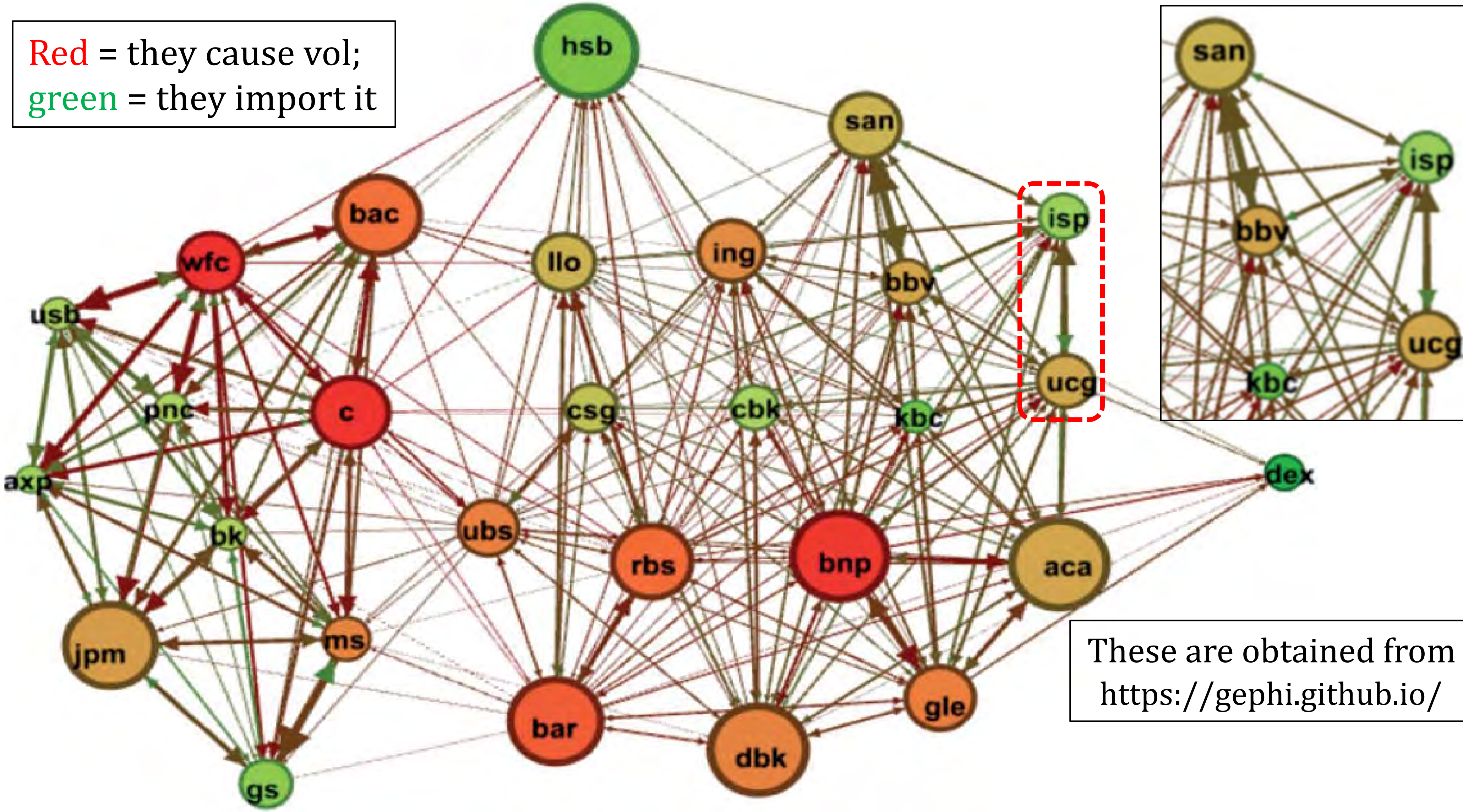
- Study daily stock return volatilities of 35 major FIs, 18 European and 17 United States, January 2004 to June 2014
- The European FIs are commercial banks, the U.S. sample includes 7 commercial banks, 2 investment banks, and one credit card company
- The vast majority of the included FIs, whether U.S. or European, are classified as Global Systemically Important Banks (G-SIBs) in the list announced by the Financial Stability Board on Nov. 1, 2012

			Market Cap.		Assets
Institution	Ticker	Country	12/29/06	5/30/14	3/31/14
Dexia	DEX	Belgium	31	0.1	473
KBC	KBC		45	25	339
Deutsche Bank	DBK	Germany	70	41	2254
Commerzbank	CBK		25	18	791
BNP Paribas	BNP	France	101	87	2,593
Societe Generale	GLE		79	46	1743
Credit Agricole	ACA		63	39	2139
Unicredit Group	UCG	Italy	91	51	1159
Intesa San Paolo	ISP		46	52	861
ING Bank	ING	Netherlands	98	54	1306
Bank Santander	SAN	Spain	117	121	1610
BBVA	BBVA		85	76	825
UBS	UBS	Switzerland	128	77	993
Credit Suisse Group	CSG		85	48	1111
HSBC	HSBA	UK	211	201	2758
Barclays	BARC		93	68	2272
Royal B. Scotland	RBS		123	36	1708
Lloyds Bank	LLOY		63	93	1405

- In the picture, node size is asset size + color indicates total directional connec “to others”; node location indicates avg. pairwise connected-ness; edge thickness is avg. pairwise directional connectedness

One Example from the International Banking System

Red = they cause vol;
green = they import it

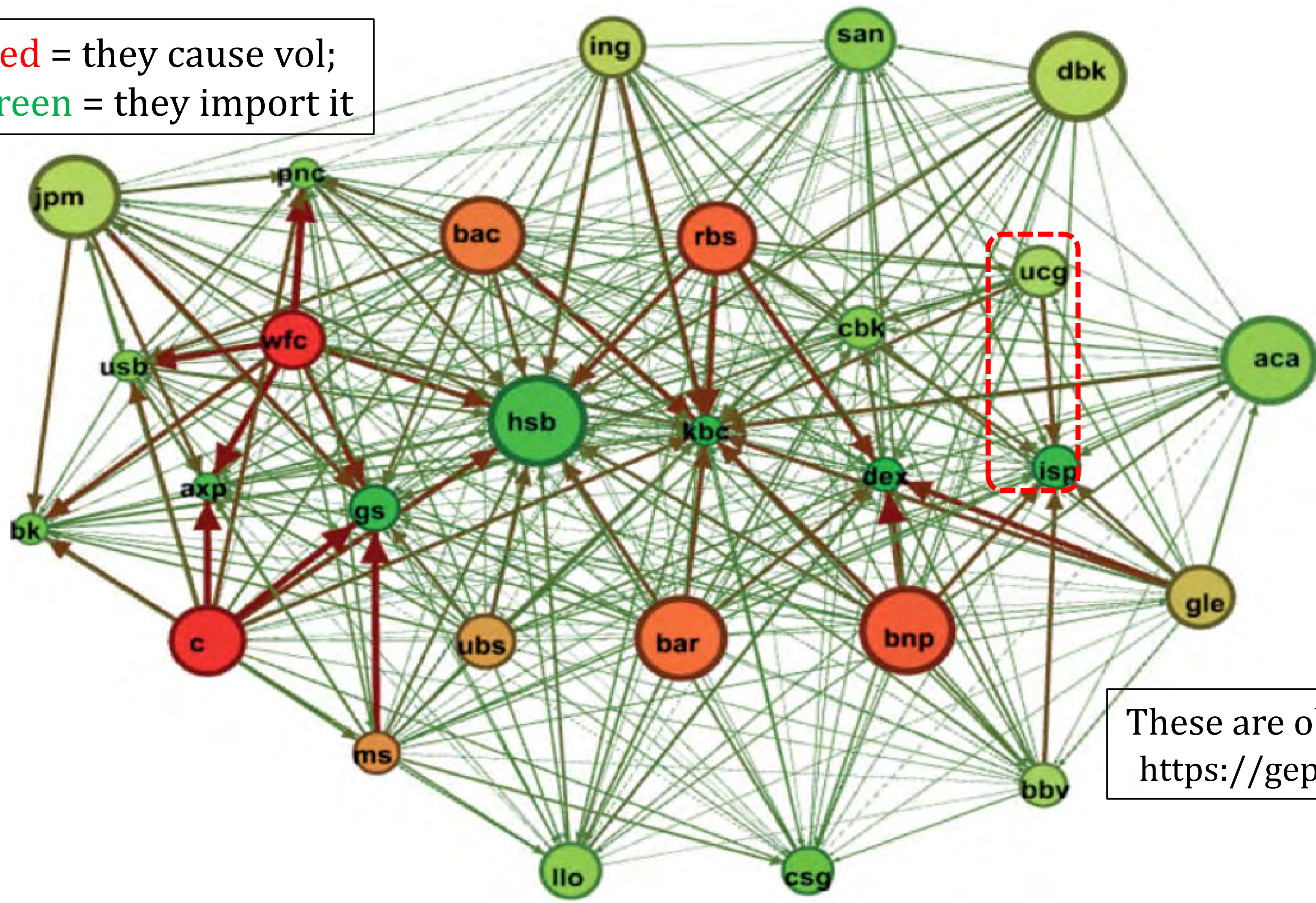


These are obtained from https://gephi.github.io/

- This is the full-sample static volatility connectedness (network) plot
- The color of each node indicates the size of total volatility connectedness "to" others (from red to brown, dark green, and light-green (weakest))

One Example from the International Banking System

Red = they cause vol;
green = they import it



These are obtained from https://gephi.github.io/

- This is the full-sample static net volatility connectedness (network) plot
- Isp is taken in the cross-fire and also hit by Ucg and Societé Genèrale

One Example from the International Banking System

- Aggregating by country, Italy and Belgium are the biggest importers of volatility, while the US and—surprisingly—France are the net exporters

	BEL	GER	FRA	ITA	NLD	SPA	UK	SWI	USA	FROM
Belgium (2)	69.9	12.5	24.7	14.1	7.2	13.4	25.6	8.0	24.7	130.1
Germany (2)	9.2	45.8	27.5	16.0	9.5	14.9	27.2	15.9	34.1	154.2
France (3)	14.3	24.3	82.0	26.6	13.9	28.4	40.5	22.2	47.8	218.0
Italy (2)	11.4	15.8	31.7	51.8	8.3	21.2	25.7	11.5	22.5	148.2
Netherlands (1)	4.5	8.6	14.1	7.8	15.2	8.7	15.1	7.6	18.3	84.8
Spain (2)	9.9	14.6	31.9	19.2	9.0	53.2	24.2	12.2	25.8	146.8
UK (4)	15.2	26.1	42.0	22.1	15.4	25.0	125.0	29.4	99.8	275.0
Switzerland (2)	5.1	15.9	22.7	11.0	8.1	13.5	26.9	44.5	52.2	155.5
United States (10)	12.1	38.1	48.4	20.3	19.4	24.2	93.2	55.3	689.1	310.9
TO	81.7	155.8	243.1	137.0	90.9	149.3	278.3	162.1	325.2	
FROM	130.1	154.2	218.0	148.2	84.8	146.8	275.0	155.5	310.9	
NET	–48.4	1.6	25.1	–11.1	6.1	2.5	3.3	6.6	14.2	**81.7**

- Italy receives/gives high vol. from/to France, more than US or Germany
- The highest pairwise connectedness measure is from the US to the UK; in return, the pairwise connectedness from the UK to US is ranked second
- Spain is a net transmitter of volatility shocks

One Example from the International Banking System

- From the beginning to May 30, 2008, there were 17 U.S. FIs (total 35), including AIG, Fannie Mae (FNM), Freddie Mac (FRE), Merrill Lynch (MER), Wachovia (WB), Lehman Brothers (LEH), and Bear Stearns (BSC)

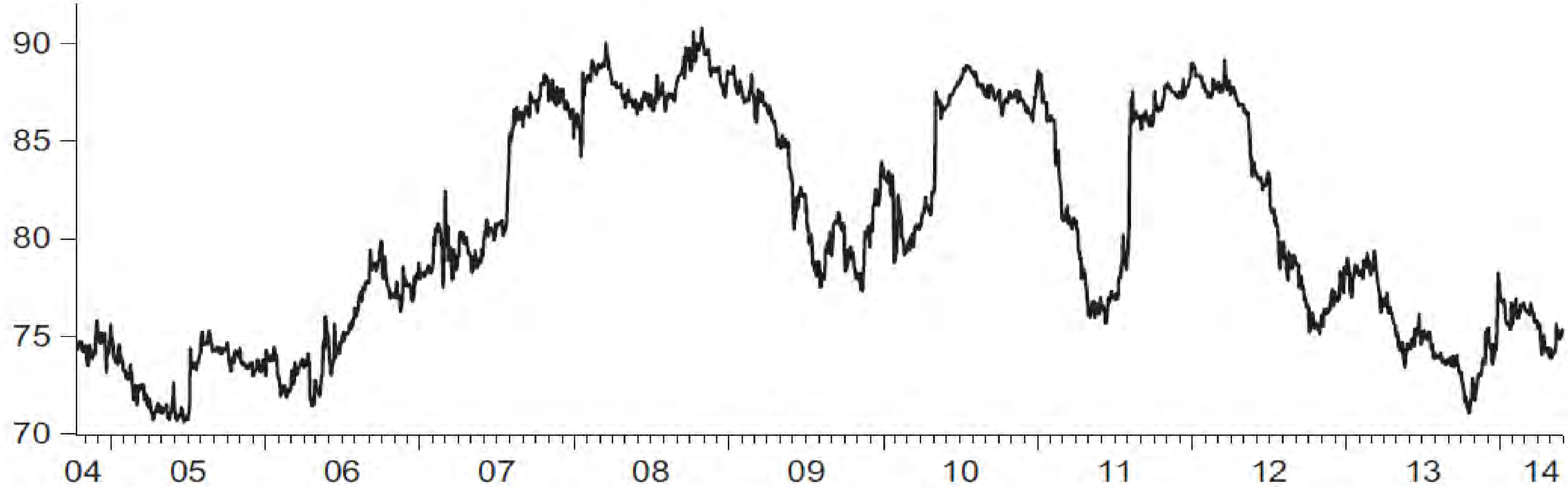
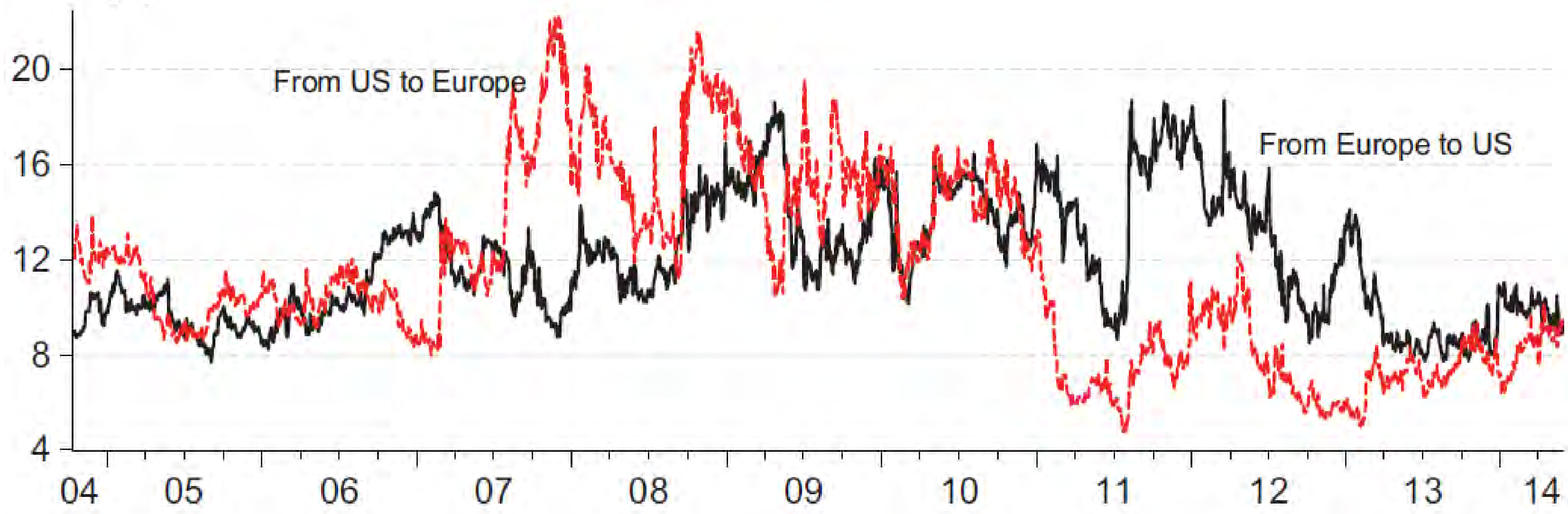
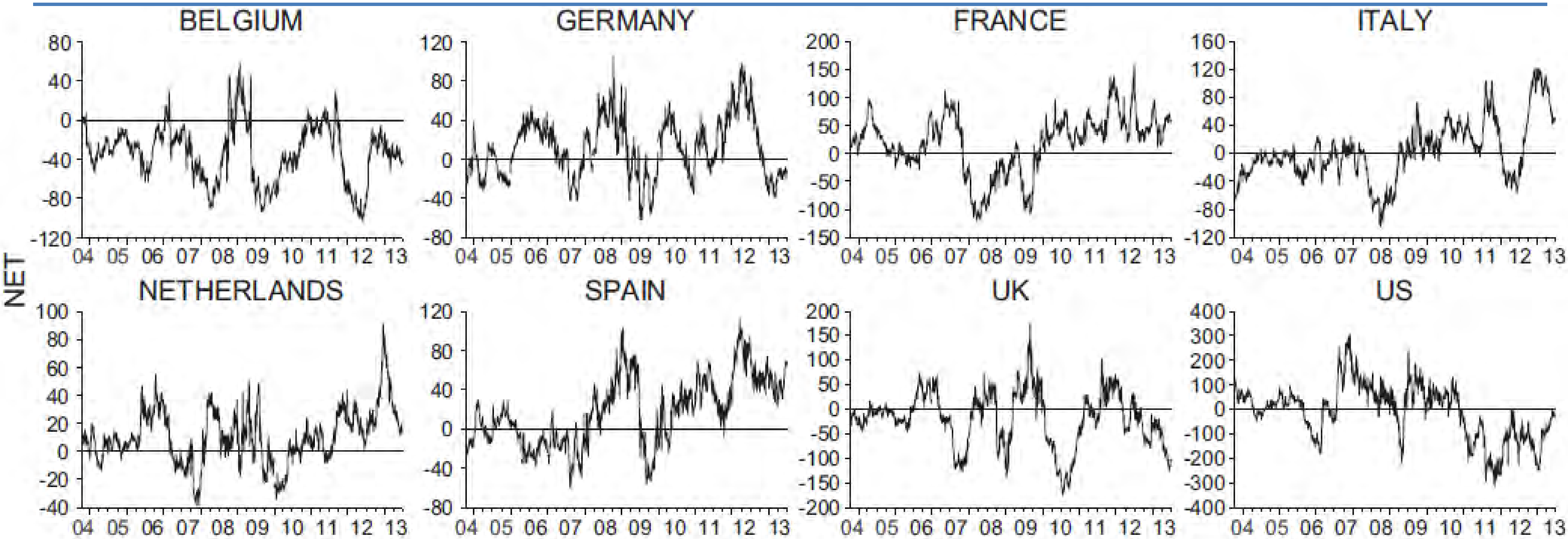


Figure 3 Dynamic total volatility connectedness with 200-day rolling-sample window. The predictive horizon for the underlying variance decomposition is 12 days.

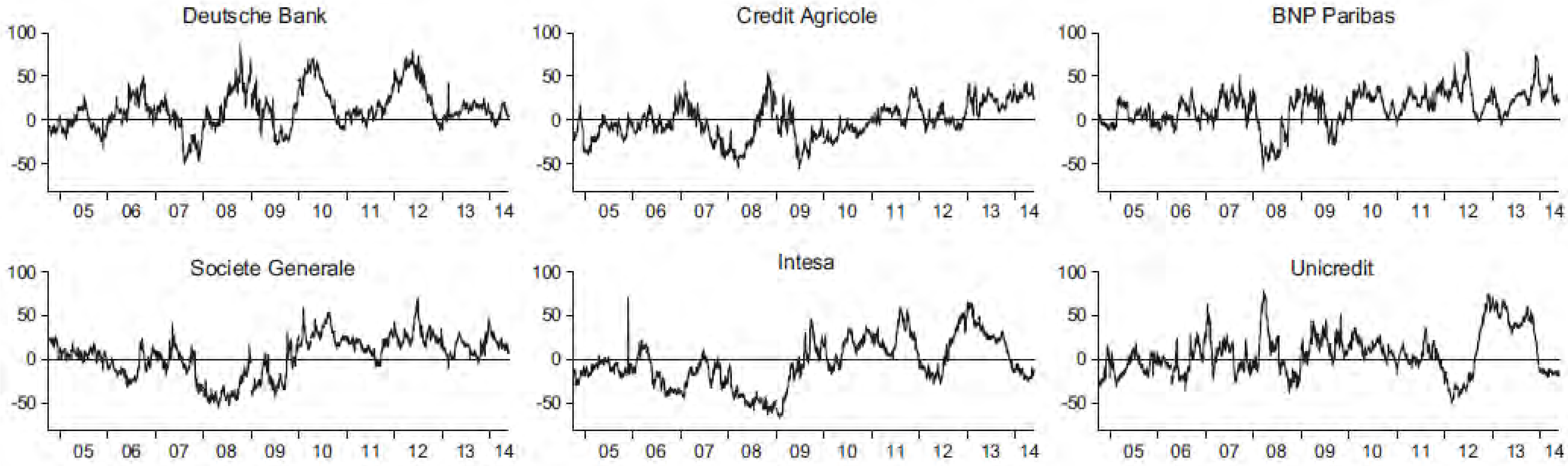


One Example from the International Banking System



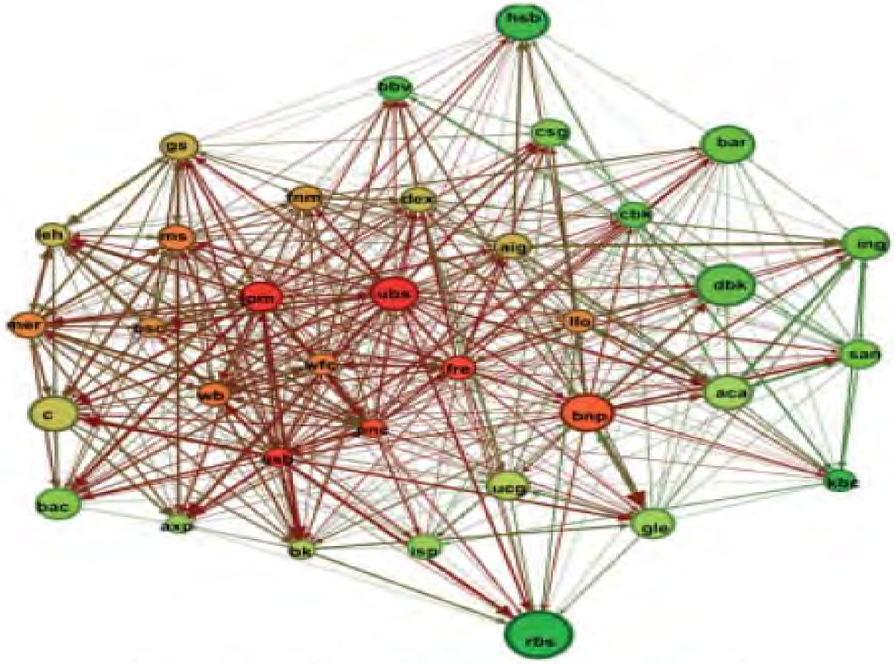
- High to- and positive net-connectedness of French FIs during the liquidity crisis of August 2007 show their troubles during this period
- On Aug. 9, 2007, BNP announced frozen redemptions from 3 hedge funds
- German FIs also suffered badly from the crisis
- Belgian FIs were in trouble following the bankruptcy of Lehman as both Fortis and Dexia were on the brink of collapse
- In the summer 2012, Italian FIs were hit by consecutive rating down-grades; net connectedness declined sharply in late 2011 only following the ECB's announcement of LTRO

The Connectedness of a Few European Banks

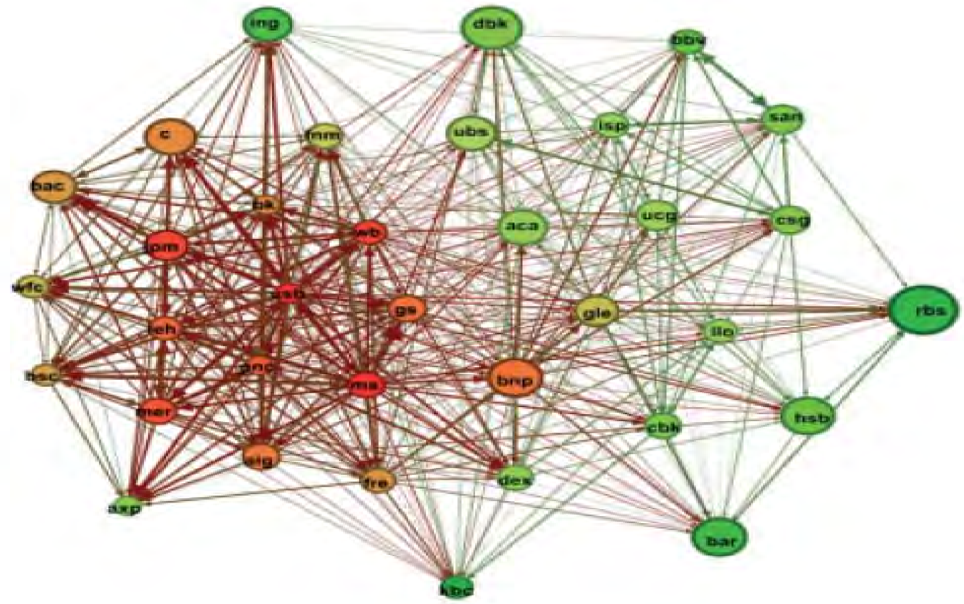


- The net-connectedness of Unicredit fluctuated substantially over time
- Following its attempts to raise capital at low prices and the political uncertainty before the Italian general elections increased net connectedness to 75% by December 2012, to come down in late 2013
- Many commentators were predicting the demise of Unicredit
- Intesa San Paolo's net-conn fluctuated more widely than Unicredit with a substantial negative net-connectedness in 2006 and during the crisis!
- With the European crisis, since 2010, its net-connectedness moved into positive territory and increased over time, reaching 60%, in 2011

The Financial Crisis Erupts

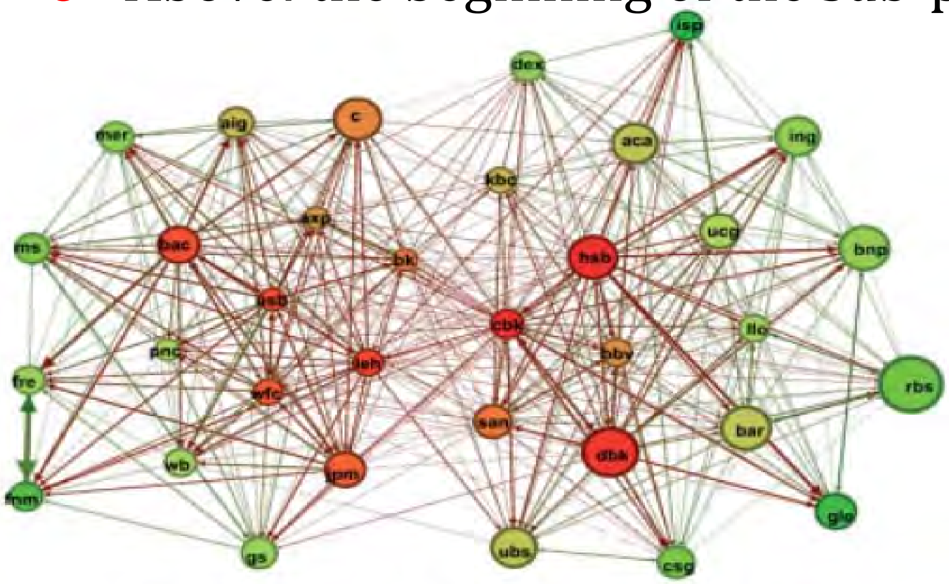


(c) August 1, 2007 (index=85.4%)

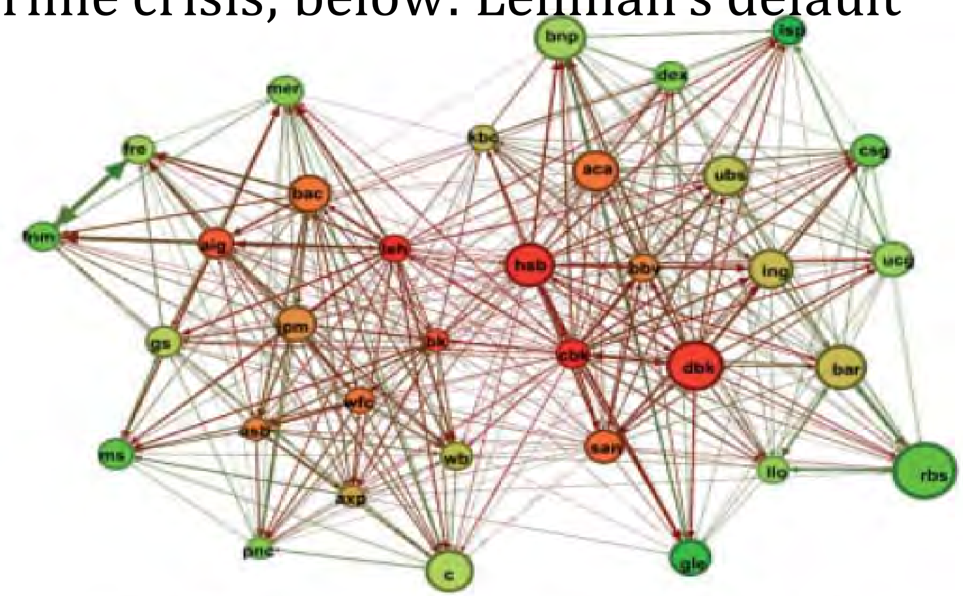


(d) October 22, 2007 (index=88.4%)

- Above: the beginning of the sub-prime crisis; below: Lehman’s default

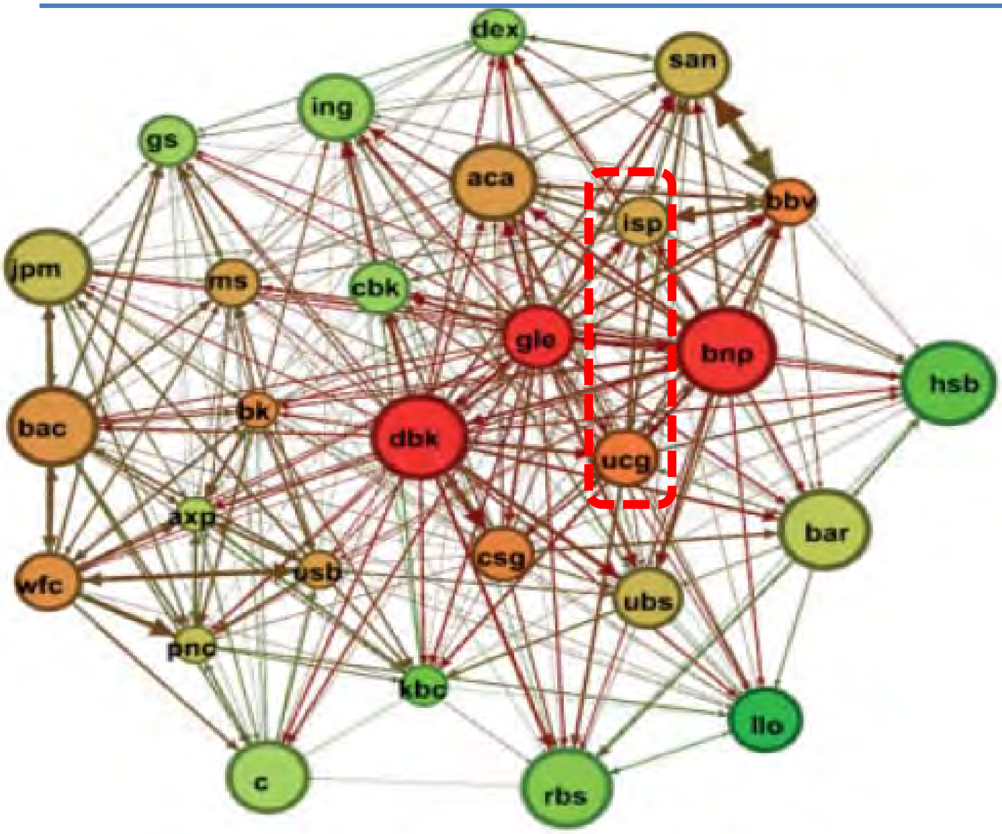


(a) September 1, 2008 (index=88.0%)

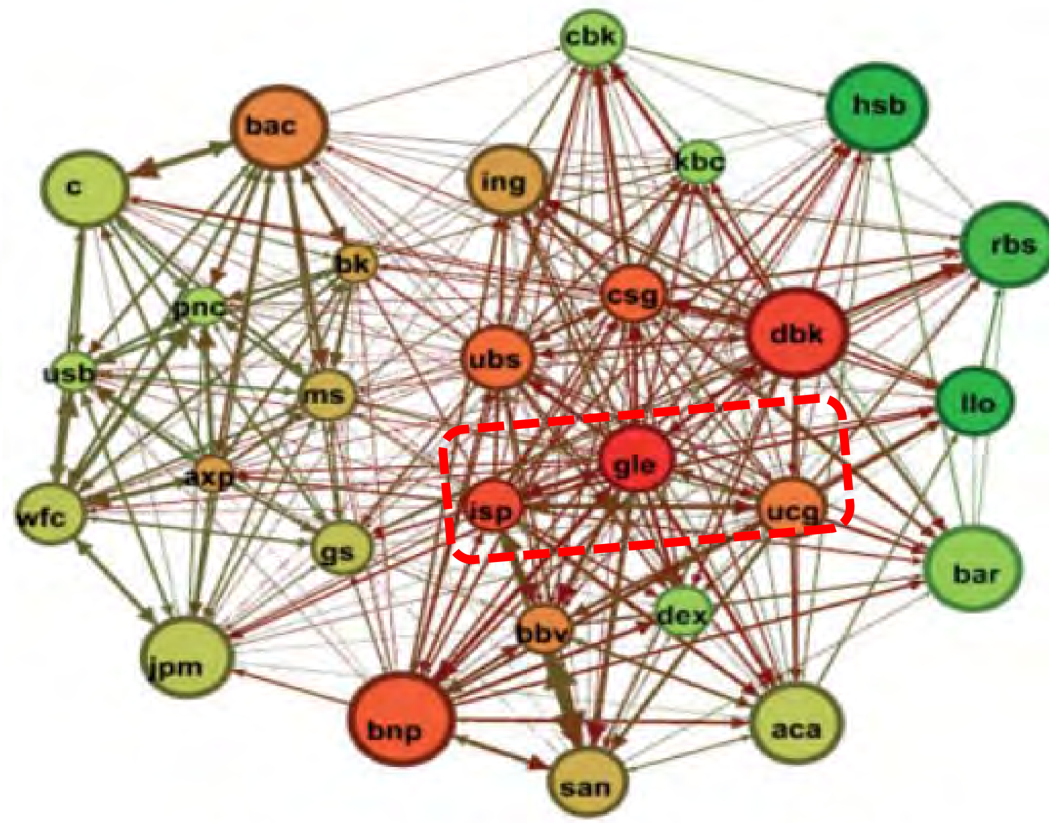


(b) September 12, 2008 (index=88.4%)

The European Sovereign Crisis Erupts



(c) May 6, 2010 (index=87.6%)



(d) July 19, 2010 (index=88.9%)

- 6 months after the public learned that the Greek sovereign debt stock was actually much higher than previously known, the EU finally announced a halfhearted financial rescue package to deal with the Greek sovereign debt crisis a few days after the German local elections in May 2010
- In July 2010, Moody's lowered Portugal's government bond ratings by two notches from AA2 to A1

Appendix: Algebra of VMA(∞) Representation

The in-sample variance decomposition method is based on the generalized impulse function of Pesaran and Shin (1998). Similarly to Diebold and Yilmaz (2014, 2016), the generalized error variance decomposition is preferred over a standard Cholesky-factor decomposition because it is independent from the ordering of the variables. Consider a vector stochastic process $\{\boldsymbol{x}_t\}$ of N random variables which follows a vector autoregressive model of order p,

$$\boldsymbol{x}_t = \sum_{i=1}^{p} \boldsymbol{A}_i \boldsymbol{x}_{t-i} + \boldsymbol{B}\boldsymbol{w}_t + \boldsymbol{\varepsilon}_t\,, \quad t = 1, 2, \ldots, T \qquad (*)$$

where $\boldsymbol{w}_t$ is a $q \times 1$ vector of deterministic variables, $\boldsymbol{A}_i$ and $\boldsymbol{B}$ are $N \times N$ and $N \times q$ coefficient matrices, and $\boldsymbol{\varepsilon}_t$ is a N-dimensional innovation process with $E(\varepsilon_t) = \boldsymbol{0}$, $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') = \boldsymbol{\Sigma}_\varepsilon^{IS}$ and $E(\boldsymbol{\varepsilon}_t | \boldsymbol{w}_t) = \boldsymbol{0}$ for $\forall t$, $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_{t'}') = \boldsymbol{O}$ for $t \neq t'$. Assuming that the process in (*) is covariance stationary, the VAR(p) model in (*) can be expressed as an infinite-order vector moving average process,

$$\boldsymbol{x}_t = \sum_{i=0}^{\infty} \boldsymbol{\Phi}_i \boldsymbol{\varepsilon}_{t-i} + \sum_{i=0}^{\infty} \boldsymbol{\Phi}_i \boldsymbol{B} \boldsymbol{w}_{t-i}\,, \quad t = 1, 2, \ldots, T,$$

Appendix: Algebra of VMA(∞) Representation

The sequence of coefficient matrices $\boldsymbol{\Phi}_i$ can be recursively calculated as:

$$\boldsymbol{\Phi}_i = 0 \ \ if \ \ i < 0, \qquad \boldsymbol{\Phi}_0 = \boldsymbol{I}_N, \qquad \boldsymbol{\Phi}_i = \sum_{l=1}^{p} \boldsymbol{A}_l \boldsymbol{\Phi}_{i-l}\,.$$

Therefore $\boldsymbol{\Phi}_1 = \boldsymbol{A}_1$, $\boldsymbol{\Phi}_2 = \boldsymbol{A}_1 \boldsymbol{A}_1$, etc.