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An Introduction to Structured Financial Products

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20541– Advanced Quantitative Methods for Asset
Pricing and Structuring

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Outline and objectives

- The Nature of Investment Certificates
- Market statistics for investment certificates
- Key ideas of structuring: from linear to non-linear payoffs
- Exotic options: digital, Asian, and barriers
- Reverse convertibles
- Bonus Cap certificates, certificates with (limited) capital protection
- The autocallability feature: Express certificates
- Certificates without capital protection: Benchmarks, Outperformance, and Discounts
- Leverage and Turbo certificates

The nature of investment certificates

Structured products are financial instruments that allow one to build highly customized risk-return profiles

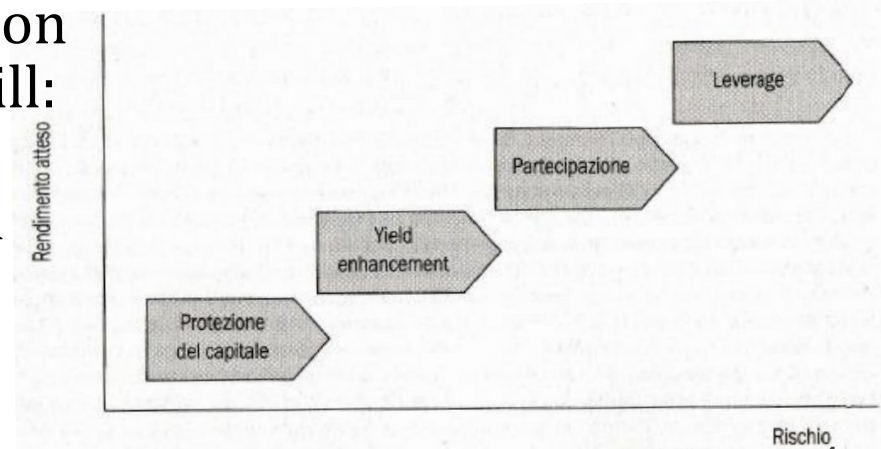
- Investment certificates (ICs) are structured products that are (portfolios of exotic) derivatives that have been **securitized** and that usually trade on regulated markets (e.g., SEDEX or EuroTLX in Italy, Frankfurt in Germany, etc.)
- They are best classified as vehicles to **passively** implement **alternative investment** choices
- This derives from a number of their features:
 - Wide variety of available risk/return profiles that make them alternative to traditional, linear payoffs
 - They are sometimes written on alternative asset classes (real estate, commodities, currencies, etc.)
 - Ability to neutralize the risk profiles from traditional asset classes
 - They often provide considerable **leverage**

The nature of investment certificates

ICs expose to market, liquidity, and issuing counterparty risks

ICs may provide equity protection, yield enhancement, diversification, and leverage

- ICs expose to three types of risk:
 - **Market risk**, due to variation in the price of the derivatives that enter the IC and reflecting interest rate, forex, underlying price, etc. risks
 - **Liquidity risk**, although the presence of dedicated market makers within official (electronic) trading venues tends to moderate it
 - **Issuing counterparty risk**, as IC are structured/assembled by private names with a specific merit of credit, unless collateral has been posted (e.g., Dws Go Safe, by DB)
- Four main kinds of ICs based on the customer's need they fulfill:
 - **Capital protection** (e.g., Equity Protection and Butterflies), total or partial, in spite of the limited participation to gains or losses by the underlying asset



The nature of investment certificates

- **Yield enhancement/stable market strategies** (e.g., Bonus, Cash Collect, and Express), providing high coupons on (seemingly) fixed income products through a (limited) participation to downside risk
- **Participation/diversification**, providing direct (linear, as in the case of Benchmarks or magnified to the upside as in the case of Outperformance ICs) exposures to large indices of (usually alternative, sophisticated) asset classes
- **Leverage** (e.g., Turbo and Short) to provide exposure with leverage, to the up- or the down-side
- ICs are related to, but different from **covered warrants**, that are European plain vanilla calls and puts that have been securitized and require no marginal activity
 - Some ICs can be seen as covered warrants with a zero strike which makes them “very deep” in-the-money instruments the exercise of which remains profitable until the underlying is worthless
- They are also different from structured bonds, in which the principal must be refunded at maturity to avoid default

Market Statistics for ICs

ICs have appeared in Germany and Switzerland in the 1990s and the corresponding markets have been rapidly growing in terms of turnover and listings also in Italy, France, and the Netherlands

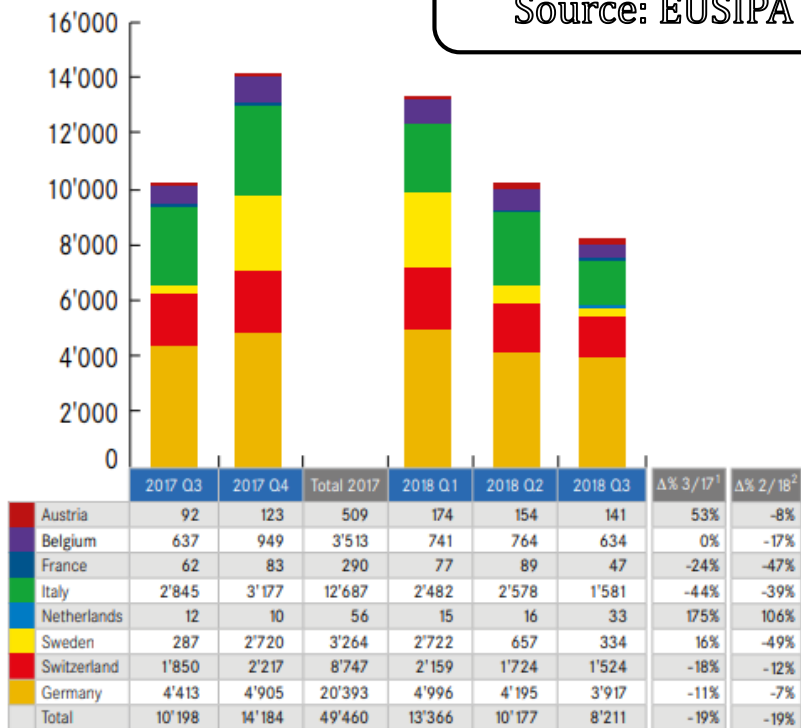
- In a IC, refund may be partial or even zero (covered warrant) without any default occurring
- However also structured bonds contain derivative components that will be priced in an economic sense in ways similar to ICs
- The market has recorded an impressive growth rate over time
 - The first IC has been issued in Germany, in 1989
 - The Swiss and German markets have been growing steadily over time since the 1990s
 - Also the Italian, French, and Dutch markets have recorded a boom in the 00s, while growth has been more difficult in the UK and Spain
 - ICs are currently listed in at least 14 European exchanges even though Stuttgart, Frankfurt, Zurich, Milan, and NYSE Euronext are main ones
 - The German market remains the largest and is propelled by individual investors

Market Statistics for ICs: Exchange Turnover

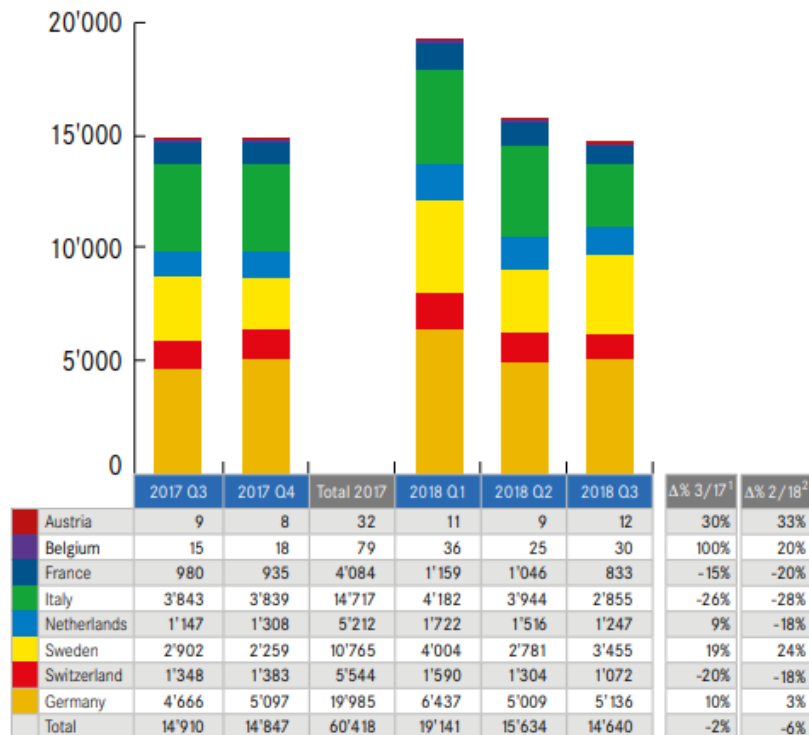
While the volume of investment certificates has decreased in 2018 vs. 2017, leveraged ICs are still strongly growing everywhere; Germany, Switzerland and Italy have the lion's share

2018 vs. 2017
Source: EUSIPA

Investment Products
in Mio. EUR

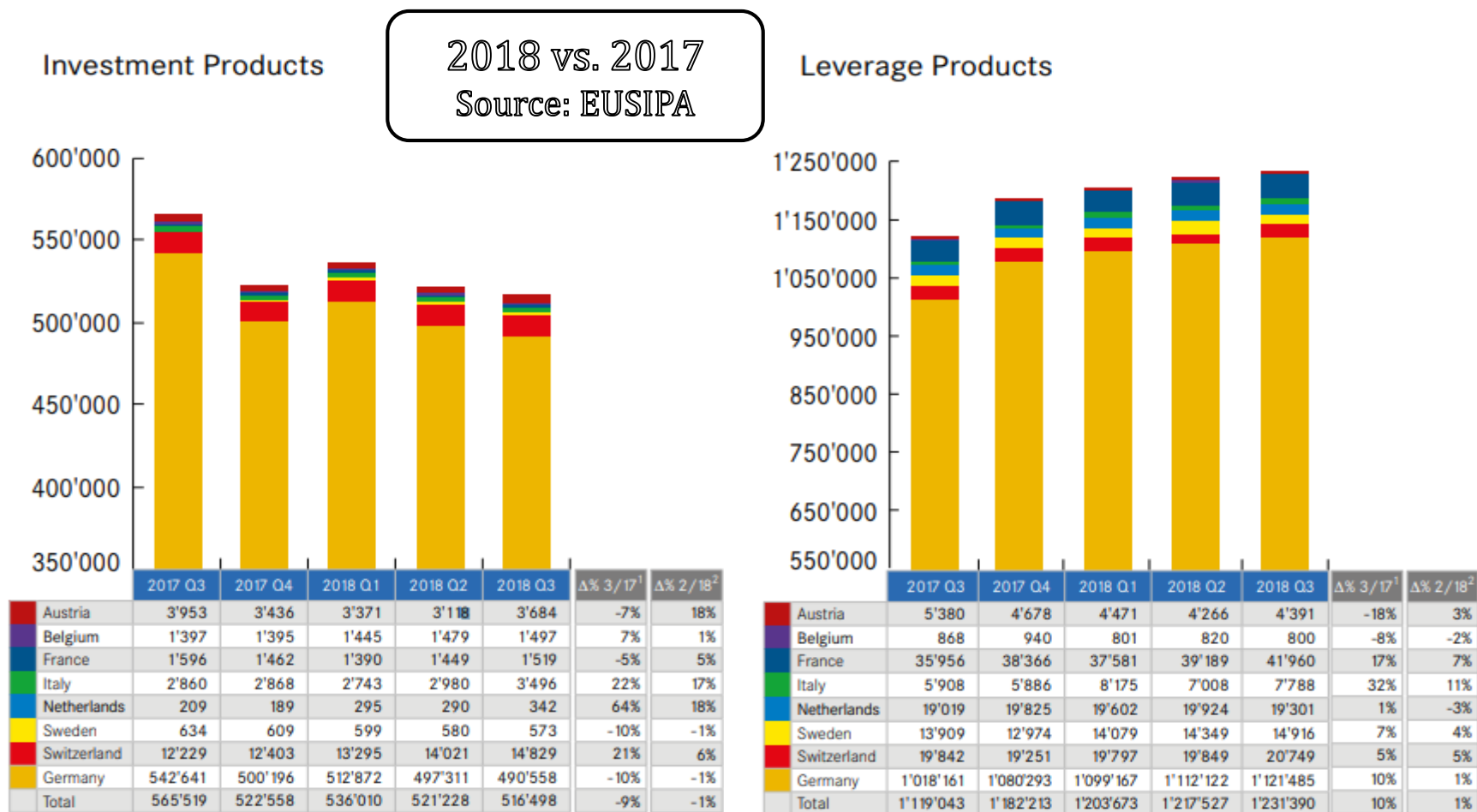


Leverage Products
in Mio. EUR



Market Statistics for ICs: Number of Listed Products

The graph shows the number of **outstanding products** in each quarter; Germany always has the greatest stock: cost associated with issuance and listing very limited in Germany

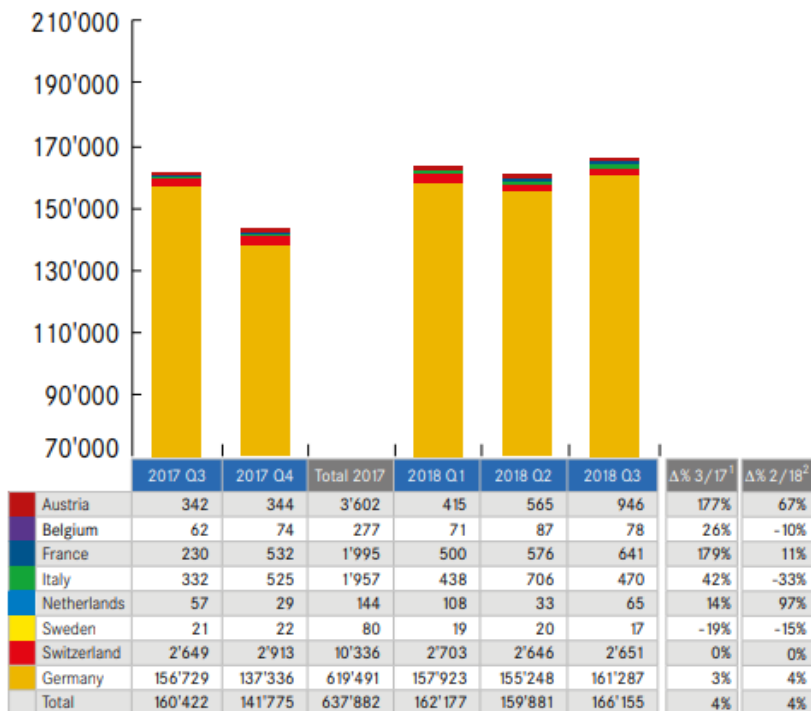


Market Statistics for ICs: Number of Listed Products

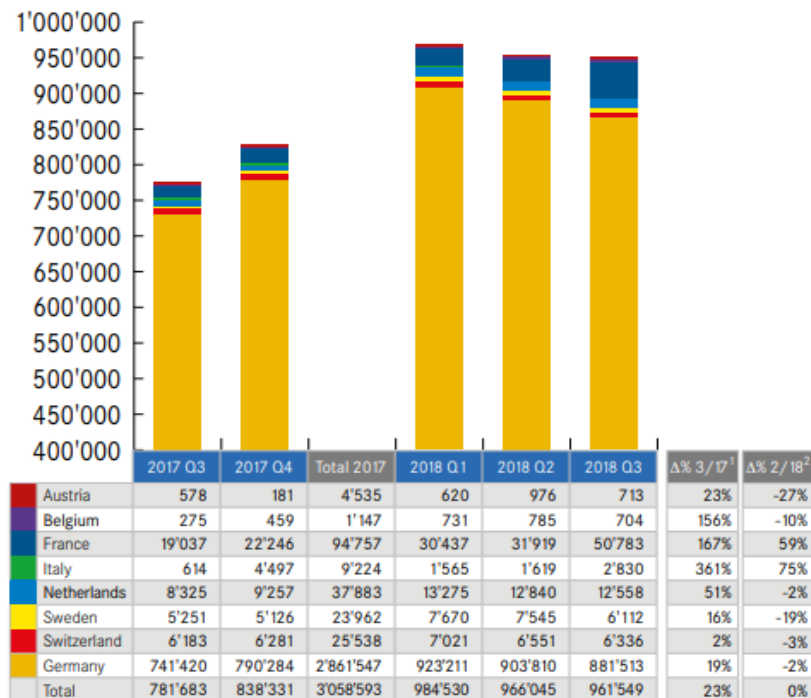
This graph shows the number of **newly issued** products in each quarter; the differential btw. turnover and listing stats implies differences in the way ICs are used in different countries

2018 vs. 2017
Source: EUSIPA

Investment Products



Leverage Products



Key European Venues

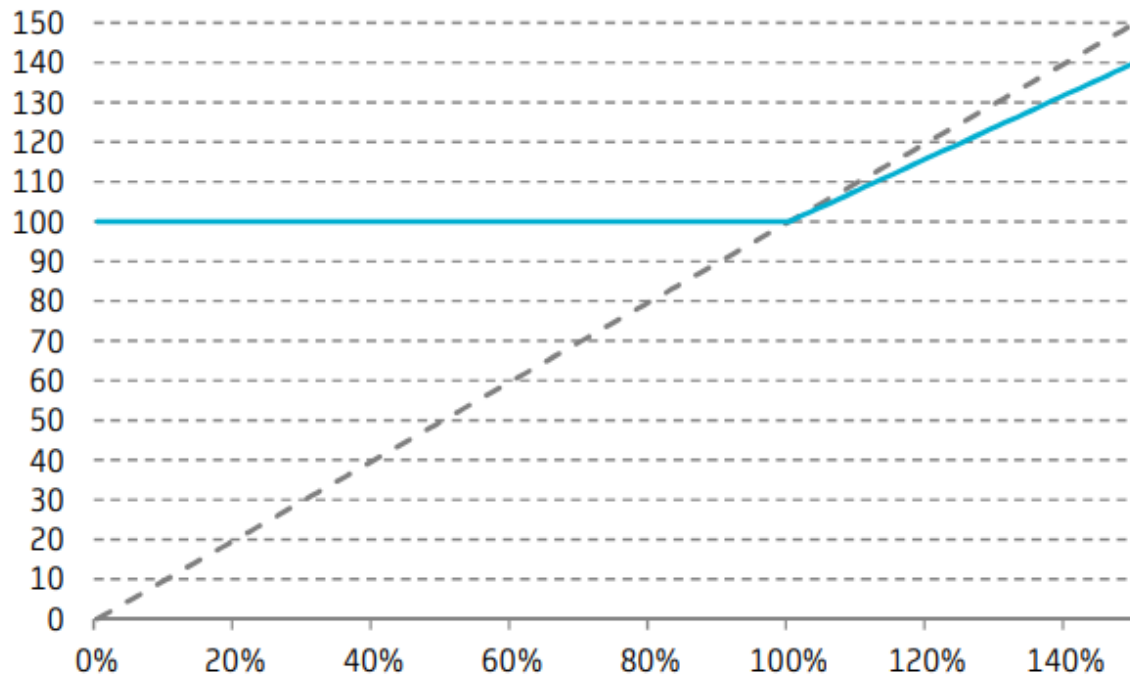
Most markets are officially regulated and not just self regulated
 Order-driven and continuous trading are the dominant microstructural set ups; market makers provide liquidity

	SeDeX	Euwax	Scoach Francoforte	Scoach Svizzera	NYSE Euronext
<i>Status normativo</i>	Regolamentato	Non regolamentato	Non regolamentato	Regolamentato	Regolamentato
<i>Autorità di vigilanza</i>	Consob	BaFin	BaFin	FINMA	AMF (F), AFM (NL), CMVM (P), CBFA (B)
<i>Sistema esecuzione ordini</i>	Order driven con specialista	Ibrido con elementi di brokered market, quote driven, order driven	Ibrido con elementi di brokered market, quote driven, order driven	Order driven con specialista	Order driven con specialista
<i>Modalità di negoziazione principale</i>	Negoziazione continua	Aste continue	Aste continue	Negoziazione continua	Negoziazione continua con request for execution
<i>Piattaforma di trading</i>	TradElect	Stuttgart trading system	XETRA	SWXess ^a	UTP (Universal Trading Platform)
<i>Primi tre emittenti^b</i>	1. Société Générale 2. UniCredit 3. BNP Paribas	1. Deutsche Bank 2. Commerzbank 3. Royal Bank of Scotland	1. Deutsche Bank 2. Dresdner Bank 3. Commerzbank	1. Vontobel 2. ZKB 3. UBS	1. Royal Bank of Scotland 2. BNP Paribas 3. Société Générale

Equity Protection Investment Certificates

The simplest possible payoff is the equity protection, that is basically a bond that eventually pays out the positive performance of the underlying

IMPORTO DI RIMBORSO A SCADENZA (valori in Eur)



Equal to:

- a long position in a Zero Coupon Bond (ZCB)
- a long position in a **call ATM**

Sottostante in % rispetto al valore iniziale

--- Sottostante

Equity Protection Investment Certificates

Problem: what if we try to build a 100% protected certificate (e.g., tenor 5Y) in the current rate environment?

Swap [EUR]	
Maturity	Price
1 Yr	-0.21
2 Yr	-0.15
3 Yr	-0.07
4 Yr	0.03
5 Yr	0.14
6 Yr	0.25
7 Yr	0.36
8 Yr	0.46
9 Yr	0.57
10 Yr	0.67

$100 \text{ €} / (1.0014)^5 = 99.3 \text{ €}$
Less than 2 EUR to be spent for the option (non considering fees!!)

Is that correct to discount at the risk free rate? Well, depends, a lot of debate in the industry on the topic (in principle you should include the credit spread of the issuer, but even considering that, difficult to buy ATM call on whatever)

Equity Protection Investment Certificates

Let's try with some numbers... How do we replicate a 5-year bond that pays the principal plus (some of) the performance (if positive) of the Euro STOXX 50 at maturity ?

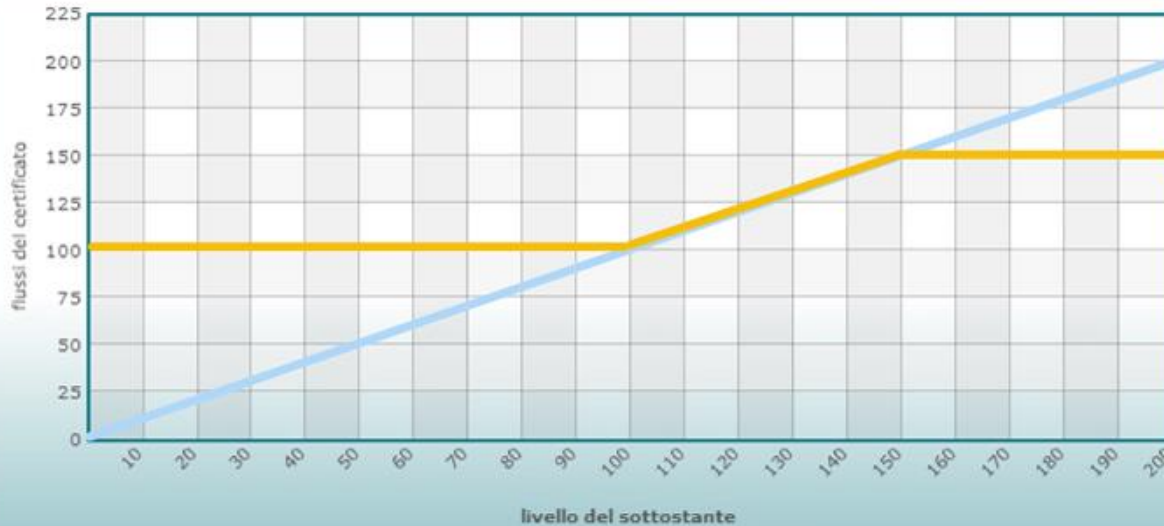
- Assuming that we also consider a credit spread (say, 50 bps per year), the discounted value of the bond component is approximately € 96.80 → this will guaranteed the repayment of the investors' capital at maturity
- You have € 3.20 (3.20% of the notional) to spend in the ATM option in order to give participation to the upside
- Would you afford to buy a call option on the entire notional to get the performance of Euro STOXX 50 at maturity? If the cost of the option is 6.40% you would only afford to pay 50% of the performance of the Euro STOXX 50 at maturity:

$$100 \times [1 + 0.5 \times \max \{ S_5 / S_0 - 1; 0 \}]$$

Underlying price

Equity Protection Caps (Collar ICs)

Alternative to reduce the cost of the structure: cap the participation to the upside by adding the sale of a call out of the money increase interim participation rate



Equal to:

- a long position in a ZCB
- a **long** position in a **call ATM**
- a **short** position in a **call OTM**

Example

- invest 96.80% of the capital today to get 100% back in 5 years
- buy a **call ATM** on Euro STOXX 50 (Strike = 3240 index points); assume that it costs 6.40%
- I need to find what is the strike of an OTM call that is worth approximately 3.20%

Equity Protection Caps (Collar ICs)

For example, assume the following prices of different call options on the Euro STOXX 50 (SX5E)

- Strike 3300: 6.00%
- Strike 3350: 5.50%
- Strike 3400: 4.5%
- Strike 3500: 4.00%
- Strike 3550: 3.50%
- Strike 3600: 3.20%
- Strike 3650: 3.00%

CAP = 11%

Payoff (if SX5E > Strike):

**100% +
Min[SX5E_Final/SX5E_Strike - 1;
11%]**

TRADE OFF:

- The more we go OTM, the higher the cap, and thus the more the investor will participate to the upside
- The more we go OTM the less the price that we get from the sale of the option

Equity Protection: The Greeks

The price of an equity protection depends (it is sensible to) a number of factors

Sensitivity of the price of an equity protection certificate:

- PRICE OF THE UNDERLYING: **POSITIVE** (however, in case of capped version, the max price of the certificate is equal to the cap)
- VOLATILITY: **POSITIVE** (in case of capped version, note that the ATM call has a higher vega than the OTM one)
- INTEREST RATES: **NEGATIVE** (“more discounting” if interest rates increase)
- DIVIDEND YIELD: **NEGATIVE** (you are buying the stock but are not entitled to dividend distribution)
- TIME: **NEGATIVE** (time decay, but effect not clear in case of cap)

Equity Protection: The Greeks

Let's put ourselves in the shoes of a structurer...what do these sensitivities imply from her point of view?

Exercise (1)

- Suppose that you want to structure an equity protection with 5Y tenor. Given an initial offer price of € 100, and a price of the ZCB of € 95 (we have 5 € to spend in the option), which of these stocks is likely to yield the highest participation rate (ceteris paribus)?
 - **Eni**: implied volatility of a 5Y ATM option is 20%
 - **Intesa San Paolo**: implied volatility of a 5Y ATM option is 27%
 - **Telecom**: implied volatility of a 5Y ATM option is 18%

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THE LOWER THE VOLATILITY, THE CHEAPER THE OPTION
THAT YOU NEED TO BUY

Equity Protection: The Greeks

Let's put ourselves in the shoes of a structurer...what do these sensitivities imply from her point of view?

Exercise (2)

- Suppose that you want to structure an equity protection with 5Y tenor. Given an initial offer price of € 100, and a price of the ZCB of € 95 (we have 5 € to spend in the option), which of these stocks is likely to yield the highest participation rate (ceteris paribus)?
 - **Eni**: dividend yield 2%
 - **Intesa San Paolo**: dividend yield 1%
 - **Telecom**: dividend yield 3%

Equity Protection: The Greeks

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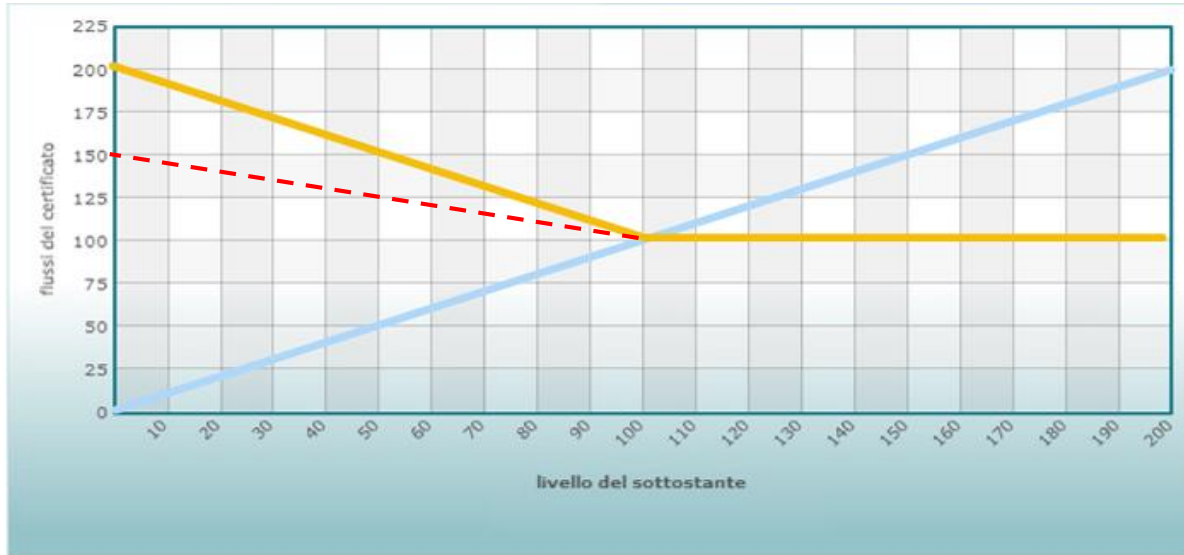
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 - **Eni**: dividend yield 2%
 - **Intesa San Paolo**: dividend yield 1%
 - **Telecom**: dividend yield 3%

THE HIGHER THE DIVIDEND YIELD, THE CHEAPER THE OPTION THAT YOU NEED TO BUY

Equity Protection Short

If the expectation is that the price of the underlying is going to decline, an investor may be willing to buy a “short” version of the equity protection



Equal to:

- a long position in a ZCB
- a **long** position in a **put ATM**

EXAMPLE (Euro STOXX 50, see before)

- Strike=3240. Below the strike the put is ITM and the investor receives $(3240 - \text{Eurostoxx_Final}) / 3240$ (from the option) + € 100 (from ZCB)
- Above the strike the put is OTM, the investor receives € 100

Equity Protection Short: The Greeks

The price of an equity protection short depends (it is sensible to) a number of factors

Sensitivity of the price of an equity protection short certificate:

- PRICE OF THE UNDERLYING: **NEGATIVE**
- VOLATILITY: **POSITIVE** (we are still buying an option)
- INTEREST RATES: **NEGATIVE** (“more discounting” if interest rates increase)
- DIVIDEND YIELD: **POSITIVE** (you are selling a stock but you do not have to pay the dividends)
- TIME: unclear (depend if the winner is the decay on the option or the effect of the increase in the price of the ZCB)

WHAT ARE THE CONSEQUENCES FOR A STRUCTURER?

Structuring: from Linear to Asymmetric (Nonlinear) Payoffs

ZCB can be combined not only with vanilla (European and/or American) call and put options but also with **exotics**, to obtain, peculiar (**often asymmetric/non-linear**) risk-return profiles

- Examples of exotic options are:
 - **Digital Options** (which pay a fixed amount if the underlying is above a certain level);
 - **Asian Options** (which pay the average performance of the underlying);
 - **Barrier Options** (which come into life / expiry if the knock – in / knock – out event happens, i.e. if the barrier is touched/crossed)

Exotic Options: Digital

Under Black-Scholes, the pricing of **digital options only depends on the (risk-neutral) probability that the underlying will be above the strike at maturity**

- **Under BS**, the pricing of the digital options is straightforward, because they only depend on the (risk-neutral) probability that the underlying will be above the strike at maturity:

$$P = \text{Fixed Amount} \times N(d_2) \times DF$$

where $N(d_2)$ is the probability that the Euro STOXX 50 will exceed the strike at the expiry of the option and DF is the discount factor

- From BS formula you know that:

$$d_2 = \frac{\ln\left(\frac{St}{K}\right) + r\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} - \sigma\sqrt{T-t}$$

Exotic Options: Digital

Suppose that, instead of paying out the performance of the SX5E at maturity, our Equity Protection pays a digital coupon every year, if the value of SX5E is above the strike

- Suppose that at issuance of , the value of Euro STOXX 50 is 3,240 points
- We structure the 5Y Equity Protection so that at the end of each year the bond pays a fixed amount if the value of the Euro STOXX 50 is above 3,240 points
- Suppose that the ZCB costs 92 Euro so that you have 8 Euros to spend
- Instead of buying a 5-year ATM call, we invest the 8 Eur to buy 5 European digitals (a 1-year digital, a 2-year digital option, etc.)

Exotic Options: Digital

How do we calculate the fixed amount that we can afford to pay?

- I calculate the value of the 5 digital options (a 1-year digital option, a 2-year digital option etc.) paying a fixed amount equal to 1 Euro;
- I then sum them and divide 8 Euros (money to be spent) by that number (cost of a strip of 5 digitals paying 1 Euro at maturity)
- E.g. if the 5 digitals paying 1 Euro cost 2.40 Euros I can afford to pay a fixed amount of 3.33 Euros

<i>Maturity</i>	<i>DF</i>	<i>N(d2)</i>	<i>Premium</i>
<i>1</i>	<i>0.9949</i>	<i>0.52</i>	<i>0.5174</i>
<i>2</i>	<i>0.9816</i>	<i>0.51</i>	<i>0.5006</i>
<i>3</i>	<i>0.9623</i>	<i>0.50</i>	<i>0.4811</i>
<i>4</i>	<i>0.9403</i>	<i>0.49</i>	<i>0.4608</i>
<i>5</i>	<i>0.9178</i>	<i>0.48</i>	<i>0.4406</i>
<i>Sum</i>			<i>2.4000</i>

Exotic Options: Asians

An Asian option has a payoff that depends **on the average value of the underlying** at some predetermined dates (or during the whole life of the option)

- An **Asian option** has a payoff that depends on the average value of the underlying at some predetermined dates (or during the whole life of the option)
 - E.g., the payoff of an Asian call option with n observation dates is:

$$\text{Payoff } f_n = NA \times \frac{1}{K} \times \text{Max} \left\{ \frac{\sum_{t=1}^n S_t}{t} - K, 0 \right\} = NA \times \text{Max} \left\{ \frac{S_{\text{Average}}}{K} - 1, 0 \right\}$$

Strike price

Average of the underlying price at the n observation dates

Exotic Options: Asians

Suppose that your structured bond will pay, instead of the performance of the underlying, the average performance computed at the end of each year

- Consider an Asian option on the Euro STOXX 50 with strike equal to 3,656 and 5 yearly observation dates
- The payoff of the Asian option at maturity will be:

t	S_t
1	3,700
2	3,860
3	3,850
4	3800
5	3,750
Average	3,792

$$NA \times \text{Max} \left\{ \frac{S_{\text{Average}}}{K} - 1, 0 \right\} = NA \times \text{Max} \left\{ \frac{3,792}{3,656} - 1, 0 \right\} = NA \times 3.72\%$$

- If the premium of the Asian option is equal to 10% and we have 8% to spend, we can afford 80% participation to any positive returns on the underlying so that at maturity, the Equity Linked Bond / Certificate (ELB) will pay:

$$ELB = 100 \times [1 + 0.8 \times 3.72\%] = 103$$

- Obviously, being path-dependent, an Asian call will always be **cheaper** than the equivalent European call option

Exotic Options: Barriers

A barrier option comes into life/ disappears if a certain barrier is touched

- A barrier option may be knock-in (if it comes into life when the barrier is touched) or knock-out if it vanishes when the barrier is touched)
 - According to level of barrier vs. initial underlying price, we distinguish up-and-out, down-and-out, up-and-in, and up-and-out options

Type	Barrier Location	Description
Down&In	Below	Price has to decrease below the barrier for the option to come into life
Up&In	Above	Price has to increase above the barrier for the option to come into life
Down&Out	Below	Price has not to decrease below the barrier for the option to stay alive
Up&Out	Above	Price has not to increase above the barrier for the option to stay alive

- The barrier can be observed only at maturity (European barrier) or during the whole life of the option (American barrier)

Exotic Options: Barriers

- Let's analyze an example of how a barrier option works:
 - Consider an ATM call option on Fiat with spot price = Strike = Eur 15
 - A European Up&Out barrier at Eur 19 is written: if at maturity the price of Fiat is equal to Eur 19 the option will expire and nothing will be paid
 - Instead, if the price at maturity is equal to Eur 17, the pay-out of the option will be equal to 2
 - From the payoff table below, a barrier option with a barrier $K = 19$, should be cheaper than a standard, plain vanilla option (even if $K = 15$ for both)

	Payoff	
S_t	Knock-Out _{B=19} Call	Standard Call
14	0	0
15	0	0
16	1	1
17	2	2
18	3	3
19	0	4
20	0	5

Exotic Options: Barriers

- This difference in the prices of a standard option vs. a barrier option with the same strike is exactly the reason why someone may wish to buy a barrier
 - Suppose, as an example, that you believe that Fiat will slightly increase in the next three months to a target price of 17 Eur
 - You have three options to bet on this increase of Fiat
 - (a) buy Fiat at 15 Eur and sell it in three-month's time; if you have 1,500 Eur to invest you will buy 100 shares and then sell them in three months
 - (b) buy an ATM option with three-month maturity; if the premium of the option is 1.5 Eur and you have 1,500 Eur to invest then he can underwrite an option on a notional of 15,000 Eur (1,000 shares)
 - (c) buy an ATM option with Up&Out barrier (barrier equal to 18 Eur) with a three-month maturity; if the premium of the option is 1 Eur and you have 1,500 Eur to invest you can underwrite an option on a notional of 22,500 Eur (1,500 shares)

Exotic Options: Barriers

Barrier options may (ex-post) maximize the profits from strategies based on (ex-post accurate) range-level forecasts of the underlying

- The table below considers the possible scenarios at maturity:

S_t	P&L(a)	P&L(b)	P&L(c)
13	-200	-1500	-1500
14	-100	-1500	-1500
15	0	-1500	-1500
16	100	-500	0
17	200	500	1500
18	300	1500	0
19	400	2500	0

- In essence, if you are not interested in the upside of Fiat above 18 Eur, the Up&Out option allows you not to pay for it
- Conversely, an Up&In option will be bought by a client that believes that the price increase will be higher than a certain level
- If you believe that Fiat will quote higher than 18 Eur in three months, buying an ATM Up&In call option with barrier equal to 18 Eur will be cheaper than buying a plain vanilla ATM option

Reverse Convertible Products

A reverse convertible is a structured product that pays a fixed coupon but **refunds a portion of the notional principal that depends on the behavior of the price of one underlying** security

- A **standard Reverse Convertible** is a note that pays an unconditional coupon at maturity (e.g., 10%) regardless of the behavior of the underlying
- In addition, if the price of the underlying has not declined, the note also pays back its notional
- On the contrary, if the underlying has depreciated, the investor obtains a number of shares equal to the notional divided by the Strike Price (also known as Conversion Price)

- As an example, consider a 1-year Reverse Convertible note on Fiat:

Reverse Convertible Termsheet/Features	
Tenor	1 year
Underlying	Fiat
Strike Price (conversion price)	16 Euro
Coupon	10%
Notional	100 Euro (equal to 6.25 Fiat shares)
Payoff in case Fiat \geq 16 Euro	110 Euro
Payoff in case Fiat $<$ 16 Euro	6.25 shares plus 10 Euro

Reverse Convertible Products

In a reverse convertible, **an investor absorbs downside market risk in exchange of a coupon rate higher than the risk-free**

- This payoff is replicated with a purchase of a ZCB plus the sale of a put option with 100% strike
 - Indeed, the payoff of the ZCB at maturity is equal to 100 Euro
 - If the value of Fiat is below 16 Euro the put option will be exercised and the payoff is: $-(K - S)$
 - Consequently the total payoff will be: $100 - 100 + S = S$
 - Instead, if Fiat is above 16 Euro, the put option will be OTM, so it will not be exercised
- The proceeds from the sale of the put are invested at the risk free rate to delivery a fixed coupon at maturity which is higher than the one that could be simply obtained from investing in a ZCB
- The investor is buying downside risk to obtain a higher coupon
- Note that instead of receiving a coupon at maturity, the investor may prefer to buy the note at a discounted price, the case of **Discount Certificates**

Reverse Convertible: The Greeks

The price of a Reverse Convertible depends (it is sensible to) a number of factors

Sensitivity of the price of an equity protection certificate:

- PRICE OF THE UNDERLYING: **POSITIVE** (the investor is SHORT of a put option)
- VOLATILITY: **NEGATIVE** (the vega of short options is negative)
- INTEREST RATES: **NEGATIVE** (“more discounting” if interest rates increase)
- DIVIDEND YIELD: **NEGATIVE** (a put costs more if the dividend is high, but you are selling it)
- TIME: generally **POSITIVE** (but depends...)