Università Commerciale Luigi Bocconi

Correlations and Structured Products: Basket Derivatives and Certificates

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20541– Advanced Tools for Risk Management and Pricing

Spring 2019

Multi-Underlyings Structured Products

Until now we have only considered structured products composed by derivatives written on a single underlying; yet structured products on more than one underlying are common

- Many types of multi-underlying structured products:
  - Products (notes/certificates) on a basket of underlyings (e.g.., an equally weighted basket of shares)
  - Products whose payoff depends on the performance of the worst performing underlying
    - An option on the worst performing of a basket of underlying will always cost less than the option on the linear basket of the same underlyings
  - Products whose payoff depends on the performance of the best performing underlying
    - An option on the best performing of a basket of underlying will always cost more than the option on the linear basket of the very same underlyings

Pricing of Multi-Underlyings Structured Products

Pricing Multi-Underlyings structured products is similar to pricing products on a single underlying, but we need an additional input: the correlation among the underlying assets – i.e. their correlation matrix

- Consider, for example, the correlation matrix of IBM, EADS and Rio Tinto:

	IBM	EADS	Rio Tinto
IBM	1.00	0.36	0.22
EADS	0.36	1.00	0.08
Rio Tinto	0.22	0.08	1.00

Correlation Coefficients

Cholesky decomposition

1.0000	0.0000	0.0000
0.3600	0.9329	0.0000
0.2227	0.0020	0.9749

Pricing of Multi-Underlyings Structured Products

The Choleski decomposition of the correlation matrix is then used to obtain/generate correlated random variables ϵ

- Pricing simulated paths:
  1. Generate a set of three correlated random numbers
  2. Multiply this set of numbers by the Cholesky decomposition of the correlation matrix

$$\underset{(1)}{\begin{bmatrix} -0.063 \\ 0.836 \\ -0.558 \end{bmatrix}} \underset{(2)}{\begin{bmatrix} 1.000 & 0.000 & 0.000 \\ 0.360 & 0.933 & 0.000 \\ 0.223 & 0.002 & 0.975 \end{bmatrix}} = \begin{bmatrix} -0.063 \\ 0.757 \\ -0.556 \end{bmatrix}$$

Correlated random variables ϵ

Pricing of Multi-Underlyings Structured Products

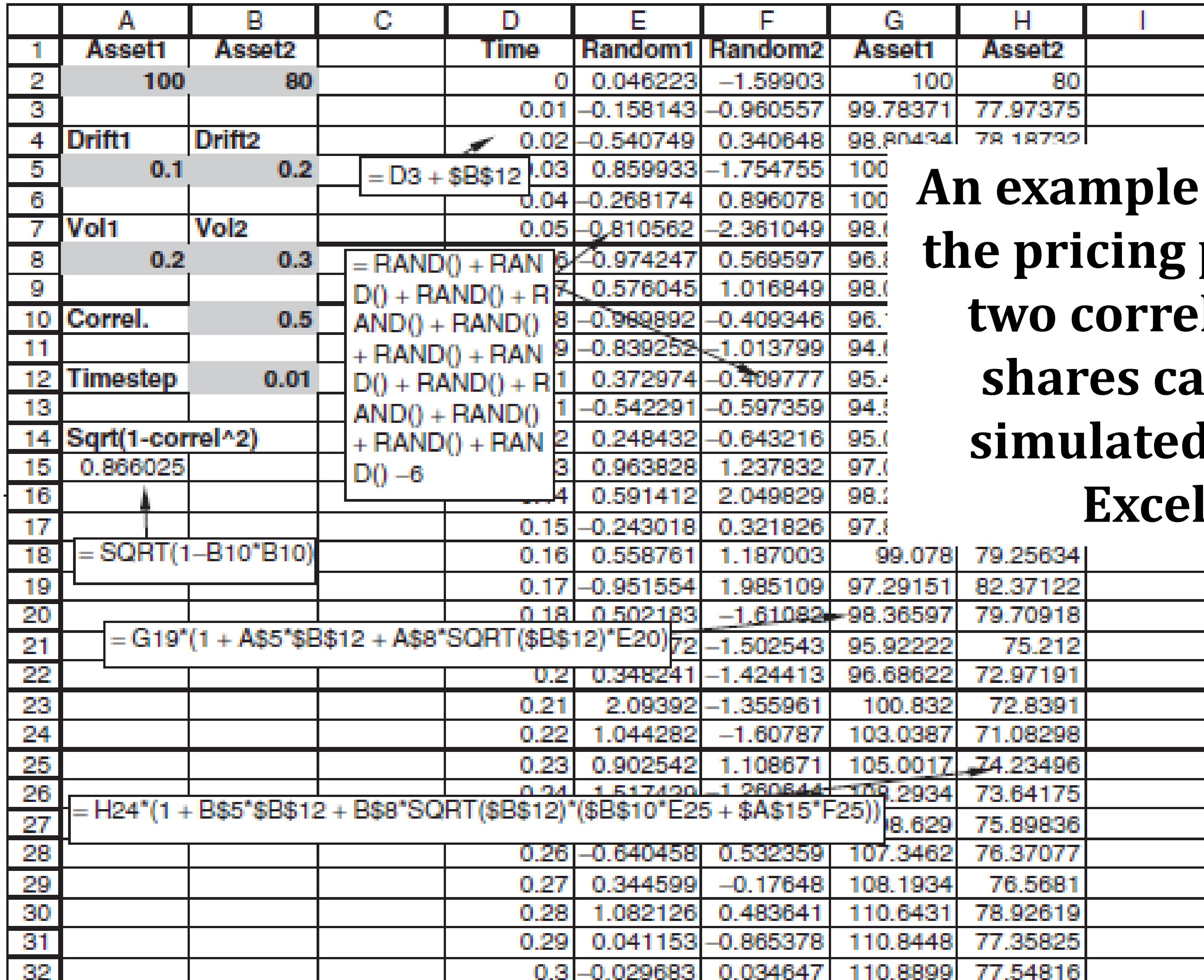
The Choleski decomposition of the correlation matrix is then used to obtain correlated random variables ϕ

- Pricing paths:
  1. Generate a set of three correlated random numbers
  2. Multiply this set of numbers by the Cholesky decomposition of the correlation matrix
  3. Integrate the correlated random numbers into the pricing path of the shares:

$$S_i(t + \delta t) = S_i(t) \exp\left(\left(r - \frac{1}{2}\sigma_i^2\right)\delta t + \sigma_i\sqrt{\delta t}\,\phi_i\right)$$

Where ϕ_i is the random shock originated as discussed in the previous slide

Pricing of Multi-Underlyings Structured Products



	A	B	C	D	E	F	G	H	I
1	Asset1	Asset2		Time	Random1	Random2	Asset1	Asset2	
2	100	80		0	0.046223	−1.59903	100	80	
3				0.01	−0.158143	−0.960557	99.78371	77.97375	
4	Drift1	Drift2		0.02	−0.540749	0.340648	98.80434	78.18732	
5	0.1	0.2		.03	0.859933	−1.754755	100		
6				0.04	−0.268174	0.896078	100		
7	Vol1	Vol2		0.05	−0.810562	−2.361049	98.		
8	0.2	0.3		6	−0.974247	0.569597	96.8		
9				7	0.576045	1.016849	98.		
10	Correl.	0.5		8	−0.969892	−0.409346	96.		
11				9	−0.839252	−1.013799	94.		
12	Timestep	0.01		1	0.372974	−0.409777	95.		
13				1	−0.542291	−0.597359	94.		
14	Sqrt(1-correl^2)			2	0.248432	−0.643216	95.		
15	0.866025			3	0.963828	1.237832	97.		
16				.4	0.591412	2.049829	98.		
17				0.15	−0.243018	0.321826	97.		
18				0.16	0.558761	1.187003	99.078	79.25634	
19				0.17	−0.951554	1.985109	97.29151	82.37122	
20				0.18	0.502183	−1.61082	98.36597	79.70918	
21					72	−1.502543	95.92222	75.212	
22				0.2	0.348241	−1.424413	96.68622	72.97191	
23				0.21	2.09392	−1.355961	100.832	72.8391	
24				0.22	1.044282	−1.60787	103.0387	71.08298	
25				0.23	0.902542	1.108671	105.0017	74.23496	
26				0.24	[illegible]	[illegible]	.2934	73.64175	
27							8.629	75.89836	
28				0.26	−0.640458	0.532359	107.3462	76.37077	
29				0.27	0.344599	−0.17648	108.1934	76.5681	
30				0.28	1.082126	0.483641	110.6431	78.92619	
31				0.29	0.041153	−0.865378	110.8448	77.35825	
32				0.3	−0.029683	0.034647	110.8899	77.54816	

An example of how the pricing path of two correlated shares can be simulated into Excel

Pricing of Multi-Underlyings Structured Products

Virtually any of the payoffs studied in the previous lectures can have multi-underlyings; let's consider for example a Bonus Cap

- At maturity a Bonus Cap pays an amount higher than Eur 100 if the underlying has never touched the barrier during the life of the product, otherwise it pays Min [Cap, Underlying(T)/Strike]
  - The underlying may be a single shares or a single index

  => Already discussed
  - The underlying can be an equally weighted basket of shares/indices

  => Underlying value at any t is equal to the average of the values of the shares / indices at time t
  - The underlying can have a "worst of" feature on a basket of shares/indices

  => Only the worst performing share/index is observed

Pricing of Multi-Underlyings Structured Products

All else being equal, which certificate will have the higher Bonus amount? The one on linear basket or the one with Worst Of feature?

- It is evident that a certificate with the Worst Of feature will deliver a higher bonus amount than a certificate on a linear basket, as it implies higher risk, as it is showed in the next slides

Payoff 1

Bonus Cap on linear Basket of Fiat and Eni

Strike = 15 Eur (average 13 and 17)

Barrier = 80% (12 Eur), only observed at maturity

Maturity = 3 months

Payoff 2

Bonus Cap Worst of on Fiat and Eni

Strike (Fiat 13 Eur; Eni =17 Eur)

Barrier = 80% (Fiat =10.4; Eni=13.6) only observed at maturity

Maturity = 3 months

Pricing of Multi-Underlyings Structured Products

All else being equal, which certificate will have the higher Bonus amount? The one on linear basket or the one with Worst Of feature?

- Scenario 1 = Both Eni and Fiat at maturity are above 80% of their initial value

Payoff 1 ➔ Bonus Amount

Payoff 2 ➔ Bonus Amount

- Scenario 2 = One of the shares (suppose Eni) is below 80% of its initial value (suppose that Eni = 12 Eur , Fiat = 15 Eur, average =13.5 Eur)

Payoff 1 ➔ Bonus Amount (13.5 is higher than 12)

Payoff 2 ➔ Eur 100 x WorstOf (T)/WorstOf (0) = Eur 100 x 12/17 = 70.5 Euro

Higher risk, higher return (higher Bonus amount)

Bonus Cap Example

Choosing shares with higher correlation increases the Bonus amount of a Bonus Cap on a linear portfolio; shares with higher correlation decreases the Bonus of a Bonus Cap under Worst Of

- You recall that the Bonus Cap is replicated with ZCB plus a short Down & In put with barrier equal to the barrier of the product
  - The Down & In put is more expensive when the correlation is higher
  - You have more money to spend for the Bonus which will increase as correlations increase
  - When the Bonus Cap includes the “Wost Of” features, as correlations increase, the value of the put declines because it becomes unlikely that one underlying may decouple from others
  - You have less money to spend for the Bonus which will decrease as correlations increase