



Advanced Tools for Risk Management and Asset Pricing

May 2015 Exam for Attending Students

Time Allowed: 60 minutes

Family Name (Surname)	First Name	Student Number (Matr.)

Please answer all questions by choosing the most appropriate alternative(s) and/or by writing your answers in the spaces provided. You need to carefully justify and show your work in the case of “open” questions. There is only one correct answer(s) for each of the multiple choice questions: each selected alternative that is correct will be awarded one point; wrong answers will be penalized with minus 0.5 point. Correct answers not selected and questions that have been left blank will receive zero points. Only answers explicitly reported in the appropriate box will be considered. No other answers or indications pointing to potential answers will be taken into consideration. In the case of “open” questions, the maximum number of points is indicated.

Question 1. Which of the following functions is NOT a copula function?

- ☐ (A) $C(u_1, u_2) = u_1 u_2$
- ☐ (B) $C(u_1, u_2) = \min(u_1, u_2)$
- ☒ (C) $C(u_1, u_2) = \max(u_1 + u_2, 0)$
- ☐ (D) None of the above

Question 2. Which of the following statements about Rank Correlations is FALSE?

- ☒ (A) To compute rank correlations one needs to know both the numerical values of the variables and the ordering of the sample for each variable
- ☐ (B) Rank correlations are invariant under strictly increasing transformations
- ☐ (C) Rank correlations take value of -1 when the variables are countermonotonic and the value 1 when the variables are comonotonic
- ☐ (D) A population version of Kendall's tau can be simply expressed as:

$$\rho_\tau(X_1, X_2) = E(\text{sign}((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2)))$$

Question 3. Which of the following statements about Copulas Estimation and Calibration is FALSE?

- ☐ (A) A Gaussian copula is asymptotically independent in both tails
- ☐ (B) A Method-of-Moment approach involves calibration of copula using an empirical estimate for some rank correlation measures
- ☒ (C) Both the Gumbel and Clayton copulas display upper tail dependence
- ☐ (D) Measures of extremal dependence between a pair of random variables X_1 and X_2 depend only on the copula of X_1 and X_2

Question 4. Consider a sample of n observations from a vector (X_1, X_2) of continuous random variables.

1. Define sample versions of Kendall's Tau and Spearman's Rho
2. Let's assume that we observe the following realizations of X_1 and X_2 :

obs	X_1	X_2
1	1	2
2	1	3
3	1	4
4	1	5
5	2	3
6	2	4
7	2	5
8	3	4
9	3	5
10	4	5

Calculate the value of Kendall's Tau and Spearman's Rho.

Answer.

1. Sample version of Kendall's Tau:

Consider a sample of n observations from a vector (X_1, X_2) of continuous random variables. There are $\frac{n(n-1)}{2}$ distinct pairs (x_{1i}, x_{2i}) and (x_{1j}, x_{2j}) and each pair is either concordant or

discordant. Let c denote the number of concordant pairs and d the number of discordant pairs. The Kendall's tau for the sample is defined as:

$$\hat{\rho}_\tau = \frac{c - d}{n(n-1)/2}$$

Sample version of Spearman's Rho:

- Consider a sample of n observations from a vector (X_1, X_2) of continuous random variables
- Rank the n observations of X_1 and create the rank variable R_1
- Rank the n observations of X_2 and create the rank variable R_2
- Compute the sample correlation between the rank variables:

$$\hat{\rho}_s(X_1, X_2) = \rho(R_1, R_2)$$

2. Kendall's Tau

Any pair of observations (x_{1i}, x_{2i}) and (x_{1j}, x_{2j}) are said to be concordant if the ranks for both elements agree: that is, if both $x_{1i} > x_{1j}$ and $x_{2i} > x_{2j}$, or if both $x_{1i} < x_{1j}$ and $x_{2i} < x_{2j}$. They are said to be discordant, if $x_{1i} > x_{1j}$ and $x_{2i} < x_{2j}$, or if $x_{1i} < x_{1j}$ and $x_{2i} > x_{2j}$. If $x_{1i} = x_{1j}$ and $x_{2i} = x_{2j}$ the pairs are neither concordant nor discordant.

We have to consider all $\frac{n(n-1)}{2}$ distinct pairs. After that we determine the type of the pair: concordant, discordant or neither. The last step is to calculate the Kendall's Tau using the formula described in point 1.

Spearman's Rho:

- Rank n observations of X_1 and create the rank variable R_1 and rank the n observations of X_2 and create the rank variable R_2 :

obs	R_1	R_2
1	2.5	1
2	2.5	2.5
3	2.5	5
4	2.5	8.5
5	6	2.5
6	6	5
7	6	8.5
8	8.5	5
9	8.5	8.5
10	10	8.5

- Compute the sample correlation between the rank variables:

$$\hat{\rho}_S(X_1, X_2) = \rho(R_1, R_2) = 0.477$$

Question 5. Assume a CDS quoted spread is 300 basis points and the recovery is estimated to be 40%. Under the assumptions that i) the premium leg of the CDS pays continuously and ii) the hazard rate is constant, what is the value of the hazard rate?

- ☒ (A) 5%
- ☐ (B) 7.5%
- ☐ (C) 500
- ☐ (D) 750

Question 6. The Gaussian Copula Approach allows to compute the joint probability of default of n names. Such probability entails i) the calculation of a multi-dimensional integral and ii) the estimation of the correlation matrix among names. Given that dim_I = dimension of integral and dim_ρ = number of free correlation parameters (entries of the correlation matrix), what are the values of these two parameters under the Single-Factor Gaussian Copula Approach?

- ☐ (A) $dim_I = n, dim_\rho = \frac{n(n-1)}{2}$
☐ (B) $dim_I = 1, dim_\rho = \frac{n(n-1)}{2}$
☐ (C) $dim_I = n, dim_\rho = n$
☒ (D) $dim_I = 1, dim_\rho = n$

Question 7. Which of the following statements about implied correlations is true?

- ☐ (A) Compound correlations are inconsistent at the level of single tranche
☐ (B) Compound correlation for a given tranche is always unique
☒ (C) Base correlation can yield negative expected tranche losses
☐ (D) Base correlation depends on pairs of attachment points

Question 8. Which of the following statements about credit modeling in a multi-factor set up is false?

- ☐ (A) Sector concentration risk is due to the correlation structure of obligors inside the portfolio
☒ (B) Sector concentration risk affects only the conditional variance term
☐ (C) Name concentration risk is a second order effect
☐ (D) Name concentration risk can be diversified away

Question 9. Considering the two termsheets below, which of the following statements is plausible:

Bonus Cap A	
Underlying	Fiat
Maturity	3 Years
Barrier	70%
Barrier type	American

Bonus Cap B	
Underlying	Fiat
Maturity	3 Years
Barrier	70%
Barrier type	European

- ☐ (A) Bonus Cap A has a Bonus equal to 115% and Bonus Cap B has a Bonus equal to 120%.
☒ (B) Bonus Cap A has a Bonus equal to 110% and Bonus Cap B has a Bonus equal to 108%.
☐ (C) Bonus Cap A has a Bonus equal to 108% and Bonus Cap B has a Bonus equal to 108%.
☐ (D) Bonus Cap A has a Bonus equal to 98% and Bonus Cap B has a Bonus equal to 95%.

Question 10. You are structuring an Equity Protection certificate with 100% capital protection; the Zero Coupon Bond costs Eur 95. An ATM call option on the FTSE MIB index costs 7 Euro. Because you would like your product to offer 100% participation to any potential appreciation of the underlying, which strategy of selection of an underlying *different* from the FTSE MIB index would you consider?

- ☐ (A) I shall not need any alternative selection of the underlying asset because I can already offer 100% participation to any potential appreciation of the underlying
- ☒ (B) I will be looking for an underlying asset with lower volatility and a higher dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible
- ☐ (C) I will be looking for an underlying asset with lower volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible
- ☐ (D) I will be looking for an underlying asset with higher volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible

Question 11. An outperformance certificate is:

- ☐ (A) A certificate with capital protection and one-to-one participation to potential appreciation of the underlying
- ☐ (B) A certificate with capital protection and magnified participation to potential appreciation of the underlying
- ☐ (C) A certificate with conditional capital protection and one-to-one participation to potential appreciation of the underlying
- ☒ (D) None of the above

Question 12. Please read the description of the structured payoff below, draw the payoff graph (assume that strike is 100% of the value of the underlying at the issuance date) and explain which option strategy replicates such a payoff.

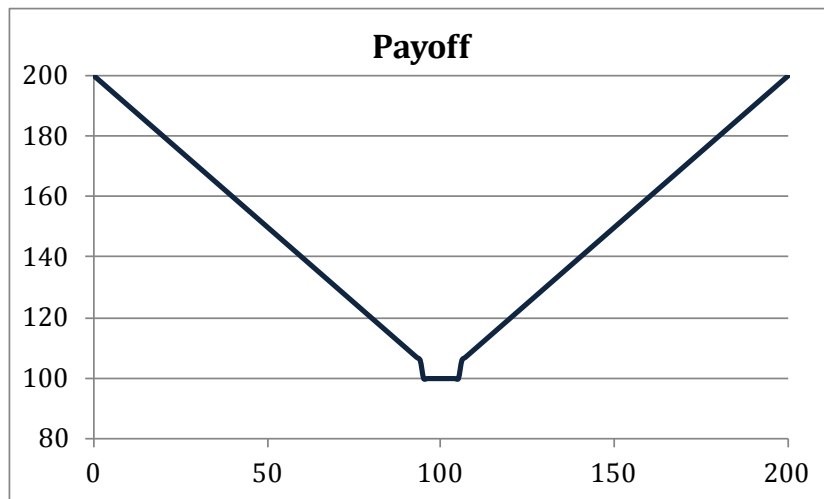
The certificate guarantees that at maturity 100% of the capital invested is paid back; in addition, if the value of the underlying at maturity is lower than 95% of the Strike, then the certificates pays

$$100\% \times \left(1 - \frac{S(t)}{\text{Strike}}\right)$$

and if the value of the underlying at maturity is above 105% of the Strike then it pays

$$100\% \times \left(\frac{S(t)}{\text{Strike}} - 1\right)$$

Answer. This is special case of a double win certificate similar to the ones analyzed in our lectures in which the payoff function is as follows.



This structure may be replicated by buying a zero coupon bond with a notional amount of Eur 100, buying a European put with strike equal to 95% of the strike of the structure, and buying a European call with strike equal to 105% of the strike of the structure. The idea is that the capital saved with respect to the 100 investment in the ZCB should allow to purchase both the out-of-the money European puts and calls that the structure requires.