Università Commerciale
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**MSc. Finance/CLEFIN**
**2014/2015 Edition**

# Advanced Tools for Risk Management and Asset Pricing

## May 2015 Exam for Non-Attending Students – Solutions

Time Allowed: 130 minutes

| Family Name (Surname) | First Name | Student Number (Matr.) |
|---|---|---|
| | | |

Please answer all questions by choosing the most appropriate alternative and/or by writing your answers in the spaces provided. You need to carefully justify and show your work in the case of "open" questions. There is only one correct answer(s) for each of the multiple choice questions: each selected alternative that is correct will be awarded one point. Only answers explicitly reported in the appropriate box will be considered. No other answers or indications pointing to potential answers will be taken into consideration. In the case of "open" questions, the maximum number of points is indicated.

**Question 1.** Which of the following statements about Correlation is FALSE?

- ☑ (A) Correlation is well defined for heavy-tailed distributions
- ☐ (B) Correlation measures the linear dependence and takes values in the interval [-1,1]
- ☐ (C) Correlation is invariant under strictly increasing linear transformations
- ☐ (D) Correlation is linked both to the copula function and to the marginal distributions

**Question 2.** Which of the following functions can NOT be a copula function?

- ☐ (A) $C(u_1, u_2) = u_1 u_2$
- ☐ (B) $C(u_1, u_2) = min(u_1, u_2)$
- ☑ (C) $C(u_1, u_2) = max(u_1 + u_2, 0)$
- ☐ (D) None of the above

**Question 3.** Which of the following statements about Rank Correlations is FALSE?

☑ (A) To compute rank correlations one needs to know both the numerical values of the variables and the ordering of the sample for each variable
☐ (B) Rank correlations are invariant under strictly increasing transformations
☐ (C) Rank correlations take a value of -1 when the variables are countermonotonic and the value 1 when the variables are comonotonic
☐ (D) The population version of Kendall's tau can be simply expressed as:

$$\rho_\tau(X_1, X_2) = E(sign((X_1 - \tilde{X}_1)(X_2 - \tilde{X}_2)))$$

**Question 4.** Which of the following statements about Copula Estimation and Calibration is FALSE?

☐ (A) A Gaussian copula is asymptotically independent in both tails
☐ (B) A Method-of-Moments approach involves calibration of a copula using empirical estimates for some rank correlation measures
☑ (C) Both Gumbel and Clayton copulas have upper tail dependence
☐ (D) Measures of extremal dependence between a pair of random variables $X_1$ and $X_2$ depend only on the copula of $X_1$ and $X_2$

**Question 5.** Which of the following statements about Stochastic Processes is TRUE?

☐ (A) Ito's Integral is a martingale
☐ (B) A Brownian motion is a stationary process with independent Gaussian increments.
☐ (C) The standard chain rule does NOT work for Ito's Integral.
☑ (D) All the above statements are TRUE

**Question 6.** In Vasicek's interest rate model, the dynamics of the short rate process $(r_t)$ are given by the SDE

$$dr_t = k(\vartheta - r_t)dt + \sigma dW_t,$$

where $k, \vartheta$ and $\sigma$ are strictly positive constants.
Show that the solution of SDE above is given by

$$r_t = \vartheta + (r^0 - \vartheta)e^{-kt} + \sigma e^{-kt}\int_0^t e^{ks}dW_s$$

*Hint*: Consider the Ito processes $(X_t)$ and $(Y_t)$ defined by

$$X_t = e^{kt} \text{ and } Y_t = r_t$$

and use integration by parts.

**Answer**.

a) We consider the Ito process $(X_t)$ and $(Y_t)$ defined by $X_t = e^{kt}$ and $Y_t = r_t$ and we note that

$$dX_t = ke^{kt}dt, X_0 = 1,$$
$$dY_t = k(\vartheta - r_t)dt + \sigma dW_t, Y_0 = r_0$$

Using the integration by parts formula, we calculate

$$e^{kt}r_t \equiv X_t Y_t$$
$$= X_0 Y_0 + \int_0^t Y_s dX_s + \int_0^t X_s dY_s + \int_0^t (dY_s)(dX_s)$$
$$= X_0 Y_0 + \int_0^t [ke^{ks}Y_s + k(\vartheta - r_t)X_s]ds + \int_0^t \sigma X_s dW_s$$
$$= r_0 + \int_0^t [ke^{ks}r_s + k(\vartheta - r_s)e^{ks}]ds + \int_0^t \sigma e^{ks}dW_s$$
$$= r_0 + \vartheta e^{kt} - \vartheta + \sigma \int_0^t e^{ks}dW_s .$$

This implies that

$$r_t = \vartheta + (r_0 - \vartheta)e^{-kt} + \sigma e^{kt} \int_0^t \sigma e^{ks}dW_s.$$

(b) Noting that the stochastic integral with respect to the Brownian motion has expectation 0, we can see that the expectation of the random variable r_{t} is given by

$$E[r_t] = \vartheta + (r^0 - \vartheta)e^{-kt}$$

Using Ito's isometry, we calculate

$$var(r_t) = E[(r_t - E[r_t])^2] = \sigma^2 e^{-2kt} E\left[\left(\int_0^t e^{ks}dW_s\right)^2\right]$$
$$= ((\sigma^2)/(2k))(1 - e^{-2kt})$$

**Question 7.** Assume a CDS quoted spread is 300 basis points and the recovery rate is estimated to be 40%. Under the assumptions that i) the premium leg of the CDS pays continuously and ii) the hazard rate is constant, what is the value of the hazard rate?

☑ (A) 5%
☐ (B) 7.5%
☐ (C) 500
☐ (D) 750

**Question 8.** The Gaussian Copula Approach allows to compute the joint probability of default of $n$ names. Such probability entails i) the calculation of a multi-dimensional integral and ii) the estimation of the correlation matrix among names. Given that $dim_I$= dimension of the integral and $dim_\rho$= number of free correlation parameters (entries of the correlation matrix), what are the values of these two parameters under the Single-Factor Gaussian Copula Approach?

☐ (A) $dim_I = n,\ dim_\rho = \frac{n(n-1)}{2}$
☐ (B) $dim_I = 1,\ dim_\rho = \frac{n(n-1)}{2}$
☐ (C) $dim_I = n,\ dim_\rho = n$
☑ (D) $dim_I = 1,\ dim_\rho = n$

**Question 9.** Which of the following statements about implied correlations is true?

☐ (A) The compound correlation is inconsistent at the level of a single tranche
☐ (B) The compound correlation for a given tranche is always unique
☑ (C) Base correlation can yield negative expected tranche losses
☐ (D)Base correlation depends on pairs of attachment points

**Question 10.** Which of the following statements about credit modeling in a multi-factor set up is false?

☐ (A) Sector concentration risk is due to the correlation structure of obligors inside the portfolio
☑ (B) Sector concentration risk affects only the conditional variance term
☐ (C) Name concentration risk is a second order effect
☐ (D) Name concentration risk can be diversified away

**Question 11.** Which of the following statements about the Limiting Portfolio Loss Distribution, introduced by Vasicek, is false?

☑ (A) By the law of large numbers, for an infinitely granular portfolio, the portfolio loss distribution converges to its expectation
☐ (B) The limiting loss distribution provides a good approximation to the portfolio loss, also for a portfolio consisting of uneven (but not dominating) weights
☐ (C) The probability density function of the portfolio loss can be U-shaped
☐ (D) The probability density function of the portfolio loss can be unimodal

**Question 12.** Which of the following statements about different approaches to price CDOs is true?

☐ (A) The Implied Copula Approach by Hull and White solves the problems of inconsistencies across the capital structure and maturities
☐ (B) The Gaussian Copula Approach is able to capture the phenomenon of clustered (sector) defaults associated to masses in the far right tail of the loss distribution
☑ (C) Dynamic(al) Loss Approaches model directly aggregated objects
☐ (D) Dynamic(al) Loss Approaches allow to calculate single name sensitivities

**Question 13.** Which of the following statements about Analytically Tractable First Passage (AT1P) models is false?

☐ (A) AT1P models assume the existence of a time-dependent deterministic barrier
☐ (B) AT1P models poses the problem of the correct determination of short term credit spreads
☑ (C) AT1P models implicitly assume that it is possible to reach a considerable probability of default in a very short time horizon smoothly
☐ (D)AT1P models implicitly assume that accounting data are fully reliable and transparent

**Question 14.** Which of the following statements about Reduced Form (Intensity) models is true?

☐ (A) Stochastic intensity models always allow to obtain large levels of option implied volatilities for CDS rates
☑ (B) In a multi-name setting, introducing dependence in stochastic intensities of different names can lead to unrealistically low levels of dependence across defaults
☐ (C) Negative values of the intensity can never be attained in CDS calibration
☐ (D) Calibration to CDS spreads is more stable with piece-wise linear intensities compared to piece-wise constant ones

**Question 15.** Which of the following statements about Reduced Form (Intensity) models is false?

☐ (A) Cox processes allow to take into consideration spread volatility
☑ (B) Default is described by an endogenous jump process
☐ (C) In deterministic intensity models, survival probabilities have the same structure as discount factors in short rate models
☐ (D) Survival probabilities obtained through calibration to CDS quotes can increase with time

**Question 16.** Consider a bespoke CDO that contains the same credits as the index portfolio. The recovery rate for the index is 60% and for the bespoke it is 0. Which of the following statements is false?

☑ (A) A 10% tranche on the bespoke will experience the same relative losses as a 6% tranche on the index
☐ (B) A 10% tranche on the bespoke will experience the same relative losses as a 4% tranche on the index
☐ (C) Losses on the bespoke are 2.5 times the losses on the index
☐ (D) The 10% strike on the bespoke should be priced with the same correlation as the 4% strike on the index

**Question 17.** Within the framework of structural models of default, the first model had been introduced by Merton in 1974. Briefly describe the assumptions underlying the model, its limits and extensions (write at most 20 lines of text).

**Answer**. See Lecture 9 "Structural Models":

- Assumptions: slide 8/65
- Limits: slide 21/65
- Extensions: slides 22-23/65

**Question 18.** In the framework of the Gaussian Copula approach, two types of implied correlation, i.e., compound correlation and base correlation have been introduced. Briefly describe them and their limitations (write at most 20 lines of text).

**Answer**. See Lecture 10 "CDO Basics"

- **Compound correlation**
  It is based on the assumption that each tranche [A,B] is characterized by a unique value of correlation $\rho_{AB}$. Typically, the compound correlation structure presents a smile.

  Limitations:
  1. It cannot be easily interpolated/extrapolated
  2. It is unable to price bespoke tranches
  3. It may not exist

- **Base correlation**
  It is based on the assumption that each equity tranche [0,X] is characterized by a unique value of correlation $\rho_{0X}$. It follows that a tranche [A,B] depends on two values of base correlation. Typically, the compound correlation structure presents a skew.

  Limitations:
  1. It is inconsistent at the level of single tranche
  2. It may yield negative expected tranche losses

**Question 19.** Considering the two termsheets below, which of the following statements is plausible:

| **Bonus Cap A** | |
|---|---|
| Underlying | Fiat |
| Maturity | 3 Years |
| Barrier | 70% |
| Barrier type | American |

| **Bonus Cap B** | |
|---|---|
| Underlying | Fiat |
| Maturity | 3 Years |
| Barrier | 70% |
| Barrier type | European |

☐ (A) Bonus Cap A has a Bonus equal to 115% and Bonus Cap B has a Bonus equal to 120%.
☑ (B) Bonus Cap A has a Bonus equal to 110% and Bonus Cap B has a Bonus equal to 108%.
☐ (C) Bonus Cap A has a Bonus equal to 108% and Bonus Cap B has a Bonus equal to 108%.
☐ (D) Bonus Cap A has a Bonus equal to 98% and Bonus Cap B has a Bonus equal to 95%.

**Question 20.** A Short Equity Protection which pays Eur 100 if at maturity the underlying (with price $S(t)$) is above the Strike and Eur $100 \times [1 + (1 - S(T)/S(0)\ )\ ]$ if the underlying is below the Strike is replicated by:

☐ (A) Buying a Zero Coupon Bond and selling an ATM put option
☐ (B) Buying a Zero Coupon Bond and buying an ATM call option
☐ (C) Buying a Zero Coupon Bond and selling an ATM call option
☑ (D) Buying a Zero Coupon Bond and buying an ATM put option

**Question 21.** You are structuring an Equity Protection certificate with 100% capital protection; the Zero Coupon Bond costs Eur 95. An ATM call option on the FTSE MIB index costs 7 Euro. Because you would like your product to offer 100% participation to any potential appreciation of the underlying, which strategy of selection of an underlying *different* from the FTSE MIB index would you consider?

☐ (A) I shall not need any alternative selection of the underlying asset because I can already offer 100% participation to any potential appreciation of the underlying
☑ (B) I will be looking for an underlying asset with lower volatility and a higher dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible
☐ (C) I will be looking for an underlying asset with lower volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible
☐ (D) I will be looking for an underlying asset with higher volatility and dividend yield than the FTSE MIB so that the option will be cheaper to try and aim at an option cost of Eur 5 to make a 100% protection possible

**Question 22.** An outperformance certificate is:

☐ (A) A certificate with capital protection and one-to-one participation to potential appreciation of the underlying
☐ (B) A certificate with capital protection and magnified participation to potential appreciation of the underlying
☐ (C) A certificate with conditional capital protection and one-to-one participation to potential appreciation of the underlying
☑ (D) None of the above

**Question 23.** Which of the options listed below will be the cheapest:

☑ (A) An ATM Asian option on the FTSE MIB with 3-year maturity
☐ (B) An ATM European option on the FTSE MIB with 3-year maturity
☐ (C) An ITM European option on the FTSE MIB with 3-year maturity
☐ (D) I cannot say

**Question 24.** Please read the description of the structured payoff below, draw the payoff graph (assume that strike is 100% of the value of the underlying at the issuance date) and explain which option strategy replicates such a payoff.

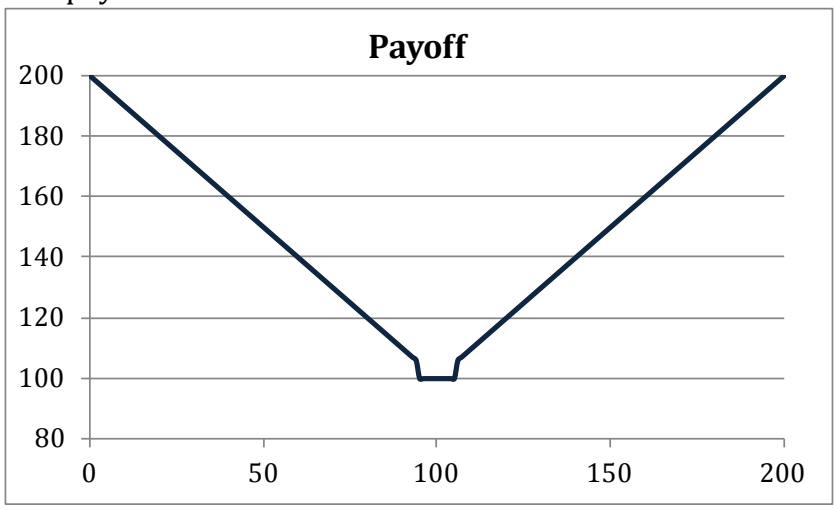
The certificate guarantees that at maturity 100% of the capital invested is paid back; in addition, if the value of the underlying at maturity is lower than 95% of the Strike, then the certificates pays

$$100\% \times (1 - \frac{S(t)}{Strike})$$

and if the value of the underlying at maturity is above 105% of the Strike then it pays

$$100\% \times (\frac{S(t)}{Strike} - 1)$$

**Answer**. This is special case of a double win certificate similar to the ones analyzed in our lectures in which the payoff function is as follows.



This structure may be replicated by buying a zero coupon bond with a notional amount of Eur 100, buying a European put with strike equal to 95% of the strike of the structure, and buying a European call with strike equal to 105% of the strike of the structure. The idea is that the capital saved with respect to the 100 investment in the ZCB should allow to purchase both the out-of-the money European puts and calls that the structure requires.