Università Commerciale
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MSc. Finance/CLEFIN
2018/2019 Edition

THEORY OF FINANCE – PART 1

Mock Question 1 (total 5 points)
Time Advised: 20-21 minutes (for this question)
Difficulty Level: EASY

Question 1.A (3.75 points)
Describe in precise mathematical terms how would you go about setting up a typical portfolio selection problem, when your goal is to involve adequate computing power. Make sure to carefully define all symbols and quantities invoked in your answer.

Debriefing:

Dynamic asset allocation: what is it?

An asset allocation problem maps an asset menu, a set of preferences, and a sequence of subjective discount factors, into a sequence of optimal portfolio weights, possibly subject to constraints

- Let's examine a first, sample portfolio choice problem written as:

$$\max_{\{\omega_t\}_{t=0}^{T-1}} \sum_{t=0}^{T} \beta^t U(P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t))$$
$$\text{s.t.} \quad \boldsymbol{\omega}_t \in \mathbb{C}$$

- It is a maximization problem
- W.r.t. a N x 1 vector of portfolio weights ω, where N is the number of alternative assets (possibly, securities, we may choose from)
  - ✓ N and which securities we can choose from define the asset menu
- We maximize the present discounted sum of the realized values of some function U(•) that measures our level of happiness and therefore reflects preferences
  - ✓ We shall often write of a (Von Neumann-Morgenstern) utility function

Intro Lecture 9

Dynamic asset allocation: an overview

An asset allocation problem maps an asset menu, a set of preferences, and a sequence of subjective discount factors, into a sequence of optimal portfolio weights, possibly subject to constraints

$$\max_{\{\omega_t\}_{t=0}^{T-1}} \sum_{t=0}^{T} \beta^t U(P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t))$$
$$\text{s.t.} \quad \boldsymbol{\omega}_t \in \mathbb{C}$$

- The sum of "felicity levels" is discounted at a subjective rate β
  - ✓ Time "0" means today, now
  - ✓ Subjective because β does not have to be an interest rate
  - ✓ Because β performs a discounting operation, we normally assume that $\beta \in (0,1]$
- $P_t(\omega_0, \omega_1, \ldots, \omega_t)$ is a performance criterion, for instance, realized portfolio return:

$$P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t) = R_t^p = \sum_{n=1}^{N} \sum_{\tau=1}^{t} \omega_{n,\tau-1} r_{n,\tau}$$

Intro Lecture 11

Dynamic asset allocation: an overview

An asset allocation problem maps an asset menu, a set of preferences, a sequence of subjective discount factors into a sequence of optimal portfolio weights, possibly subject to constraints

$$\max_{\{\omega_t\}_{t=0}^{T-1}} \sum_{t=0}^{T} \beta^t U(P_t(\boldsymbol{\omega}_0, \boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t))$$
$$\text{s.t.} \quad \boldsymbol{\omega}_t \in \mathbb{C}$$

- The sequence of portfolio weights may be restricted to belong to some set $\mathbb{C}$
  - ✓ E.g., short sales may be restricted or impossible outright
  - ✓ Almost all institutional investors must fulfill maximum investment rules/criteria (positive diversification constraints)
  - ✓ Occasionally, more technical constraints may be imposed: e.g., a portfolio that maximizes utility subject to a value-at-risk constraint
- Even though the entire sequence $\{\omega_0, \omega_1, \ldots, \omega_t\}$ gets selected, many problems do allow that future weight revisions be taken into account (rebalancing case)

Intro Lecture 12

Question 1.B (0.75 points)

John and Mary use the mathematical set up that you have defined in question 1.A to get support from a computer network system in solving the task of computing their optimal asset allocation. In what way do you think that they may use the mathematical set up in question 1.A to capture the fact that they are different individuals with different personal attitudes?

Debriefing:

As mentioned in the lectures, John and Mary may represent to a computer their differences by communicating:

- Different utility (of monetary wealth) functions, $U(\cdot)$;
- Different subjective discount factors, β;
- Different performance functions, $P_t(\cdot)$, for instance John may care for the utility of portfolio returns while Mary may care for the ratio of portfolio returns vs. some benchmark that she would like to outperform;
- Different asset menus, i.e., collection of securities or more generally investable assets to invest in or to short, if required (e.g., John may entertain investing in derivatives and hedge funds, while Mary may want to rule that out);
- Different sets of constraints to be imposed on the weights, for instance, Mary may want to prevent short sales, while John allow them but impose that each asset class may receive at most a weight of $\kappa \leq 1/N$).

You can even find more ways for John and May to communicate how different they are and you were not expected to list all these items, but at least 3-4 of them.

Question 1.C (0.5 points)

Suppose that $P_t(\omega_0, \omega_1, \ldots, \omega_t)$ = *monetary wealth* at time t and that you are told that John is a very patient investor who is indifferent about risk, while Mary is an impatient investor very much risk averse. Both John and Mary are expected utility maximizers. With reference to the mathematical set up that you have defined in question 1.A, how will you communicate such differences to a computer network system that solves the task of computing their optimal asset allocation?

Debriefing:

As examined later in lecturer 1, given that $P_t(\omega_0, \omega_1, \ldots, \omega_t) = W_t$, then:

- $U^{John}(W_t)$ = is monotone increasing but linear and β^{John} will be very close to 1 (or just 1);
- $U^{Mary}(W_t)$ = is monotone increasing and strictly concave and β^{Mary} will be well below 1 (or even close to zero).