



FINANCIAL ECONOMETRICS AND EMPIRICAL FINANCE - MODULE 2

Mock Question 2 (total 17 points, out of 50 from 3 questions) Time Advised: 24 minutes (for this question)

Question 2.A (14 points)

Consider a bivariate VAR(2) model for S&P 500 returns and the log changes in the VIX volatility index ($R_t^{S\&P}$ and $\Delta \ln VIX_t$). Write:

- The structural, unconstrained VAR(2) that includes contemporaneous effects across the two markets.
- The associated unconstrained reduced-form VAR(2).

Explain through which steps it is possible to transform the structural VAR model into the reduced-form one (algebra is not required, unless it helps you provide an efficient answer). How would/could you estimate the structural VAR? How would/could you estimate the reduced-form model? Explain what are the issues/limitations caused by the transformation of a structural VAR into a reduced-form model.

Question 2.B (2 points)

Suppose that the bivariate structural VAR(2) is to be exactly identified by imposing either of the two possible Choleski triangularization schemes:

$$\mathbf{B}' = \begin{bmatrix} 1 & 0 \\ b_{21} & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B}'' = \begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix}$$

Carefully explain the implications and differences in economic interpretations of the estimated, corresponding reduced-form model deriving from imposing the restriction in \mathbf{B}' instead of \mathbf{B}'' .

How does your answer change when the restriction

$$\mathbf{B}''' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is imposed instead?

Question 2.C (1 point)

Suppose that the estimation of a constrained, reduced-form VAR(2) has provided the following ML estimates of the conditional mean function and of the covariance matrix of the reduced-form shocks (p-values are in parentheses):

$$\begin{cases} R_t^{S\&P} = \frac{0.006}{(0.044)} + \frac{0.053}{(0.093)} R_{t-1}^{S\&P} - \frac{0.473}{(0.003)} \Delta \ln VIX_{t-1} + \frac{0.113}{(0.045)} \Delta \ln VIX_{t-2} + u_t^{S\&P} \\ \Delta \ln VIX_t = -\frac{0.194}{(0.149)} - \frac{0.375}{(0.024)} R_{t-1}^{S\&P} + \frac{0.094}{(0.050)} R_{t-2}^{S\&P} + \frac{0.804}{(0.000)} \Delta \ln VIX_{t-1} + u_t^{VIX} \end{cases}$$
$$Var \left(\begin{bmatrix} u_t^{S\&P} \\ u_t^{VIX} \end{bmatrix} \right) = \begin{bmatrix} 0.008 & -0.016 \\ (0.000) & (0.007) \\ -0.016 & 0.014 \\ (0.007) & (0.000) \end{bmatrix}$$

You would like to recover the original structural parameters, including the contemporaneous, average impact of both VIX changes on S&P 500 returns and vice-versa. Is there a chance that this may be possible even though you are *not* ready to impose a Choleski ordering on the two variables?