

Università Commerciale Luigi Bocconi

MSc. Finance/CLEFIN 2018/2019 Edition

# **THEORY OF FINANCE – PART 1**

## Mock Question 2 (total 5 points) Time Advised: 20-21 minutes (for this question) Difficulty Level: MEDIUM-HIGH

## Question 2.A (3.75 points)

Define the absolute and relative risk aversion functions and explain why, in general, these are functions of an individual's wealth. What are the main economic interpretations/practical applications of ARA(W) and RRA(W)? Make sure to carefully justify your answer and, where necessary, provide examples. For the case of negative exponential utility with  $\theta = 2$ , use four plots to describe the behavior of ARA(W), RRA(W), the minimum odds  $\pi(W; h)$ , and the insurance risk premium  $\Pi(W; h)$  for a fixed, small fair bet, i.e., h "close to zero".

### **Question 2.B (0.75 points)**

Mary has an initial wealth of 100 and preferences for wealth described by a power utility function with  $\gamma = 6$ . Write her utility function and check whether it is monotonic increasing and strictly concave. Mary is now facing a small gamble *H* with stochastic returns  $\approx_{H}$  (-0.1 with prob. 2/3

$$\tilde{R}^{H} = \begin{cases} -0.1 & \text{with prob. } 2/3 \\ 0.6 & \text{with prob. } 1/3 \end{cases}$$

Compute the (financial) risk premium that Mary should demand in order to play this lottery.

#### **Question 2.C (0.5 points)**

For a small, fair gamble with size h, your artificial intelligence system has inferred that that John does not displays a non-monotone minimum odds function  $\pi^{John}(W;h)$  (as a function of wealth). Bill is instead characterized by a monotone decreasing function  $\pi^{Bill}(W;h)$ ; Rachel is characterized by a monotone increasing function  $\pi^{Rachel}(W;h)$ . We know that  $\pi^{John}(W;h)$  always (for all wealth levels) exceeds  $\pi^{Bill}(W;h)$  and  $\pi^{Rachel}(W;h)$ . Based on their checking account and credit card transactions as well as on a series of surveys they have filled out over time, the AI system has also established that the VNM utility function of Mary is related to John's by the following relationship:

$$U^{Mary}(W) = -600 + 2U^{John}(W)$$

John, Bill, and Rachel are all risk-averse investors and  $\pi^{John}(W;h), \pi^{Bill}(W;h)$ , and  $\pi^{Rachel}(W;h)$  are everywhere continuous and differentiable. Based on the previous information and assuming the relationship between ARA and minimum odds functions  $\pi^{John}(W;h)$  approximately holds, what will your AI system extrapolate from the previous information on the shape of the ARA function of Mary,  $ARA^{Mary}(W)$ ? [*Hint*: probably it is a good idea to try and qualitatively plot the function  $ARA^{Mary}(W)$  that the AI system may extrapolate]