

Università Commerciale Luigi Bocconi

MSc. Finance/CLEFIN 2018/2019 Edition

THEORY OF FINANCE – PART 1

Mock Question 2 (total 5 points) Time Advised: 20-21 minutes (for this question) Difficulty Level: MEDIUM-HIGH

Question 2.A (3.75 points)

Define the absolute and relative risk aversion functions and explain why, in general, these are functions of an individual's wealth. What are the main economic interpretations/practical applications of ARA(W) and RRA(W)? Make sure to carefully justify your answer and, where necessary, provide examples. For the case of negative exponential utility with $\theta = 2$, use four plots to describe the behavior of ARA(W), RRA(W), the minimum odds $\pi(W; h)$, and the insurance risk premium $\Pi(W; h)$ for a fixed, small fair bet, i.e., h "close to zero".

Question 2.B (0.75 points)

Mary has an initial wealth of 100 and preferences for wealth described by a power utility function with $\gamma = 6$. Write her utility function and check whether it is monotonic increasing and strictly concave. Mary is now facing a small gamble *H* with stochastic returns \approx_{H} (-0.1 with prob. 2/3

$$\tilde{R}^{H} = \begin{cases} -0.1 & \text{with prob. } 2/3 \\ 0.6 & \text{with prob. } 1/3 \end{cases}$$

Compute the (financial) risk premium that Mary should demand in order to play this lottery.

Question 2.C (0.5 points)

For a small, fair gamble with size h, your artificial intelligence system has inferred that that John does not displays a non-monotone minimum odds function $\pi^{John}(W;h)$ (as a function of wealth). Bill is instead characterized by a monotone decreasing function $\pi^{Bill}(W;h)$; Rachel is characterized by a monotone increasing function $\pi^{Rachel}(W;h)$. We know that $\pi^{John}(W;h)$ always (for all wealth levels) exceeds $\pi^{Bill}(W;h)$ and $\pi^{Rachel}(W;h)$. Based on their checking account and credit card transactions as well as on a series of surveys they have filled out over time, the AI system has also established that the VNM utility function of Mary is related to John's by the following relationship:

$$U^{Mary}(W) = -600 + 2U^{John}(W)$$

John, Bill, and Rachel are all risk-averse investors and $\pi^{John}(W;h), \pi^{Bill}(W;h)$, and $\pi^{Rachel}(W;h)$ are everywhere continuous and differentiable. Based on the previous information and assuming the relationship between ARA and minimum odds functions $\pi^{John}(W;h)$ approximately holds, what will your AI system extrapolate from the previous information on the shape of the ARA function of Mary, $ARA^{Mary}(W)$? [*Hint*: probably it is a good idea to try and qualitatively plot the function $ARA^{Mary}(W)$ that the AI system may extrapolate]