Università Commerciale Luigi Bocconi

MSc. Finance/CLEFIN
2017/2018 Edition

FINANCIAL ECONOMETRICS AND EMPIRICAL FINANCE - MODULE 2

Mock Exam
Time Allowed: 1 hour and 10 minutes

Family Name (Surname)	First Name	Student Number (Matr.)

Please answer all questions by writing your answers in the spaces provided. No additional papers will be collected and therefore they will not marked or taken into consideration. You always need to carefully justify and show your work. The total score is 50.

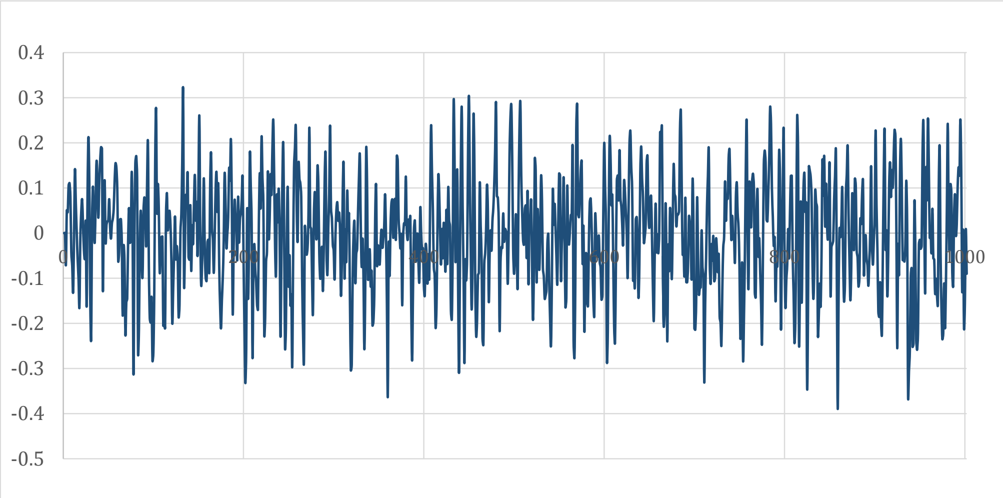
Question 1.A (13 points)
Write in formal terms an AR(p) model with $p \geq 1$, making sure to explain what each term represents and whether each term is an observable random variable, a latent shock, or a parameter; also explain the economic intuition for the model, if any. What does it mean, both in logical and in statistical terms, that an AR(p) time series process is *stationary*? Assuming stationarity, make sure to discuss what the relevant *population* moments of the process are, also providing a few examples of the corresponding closed-form formulas.

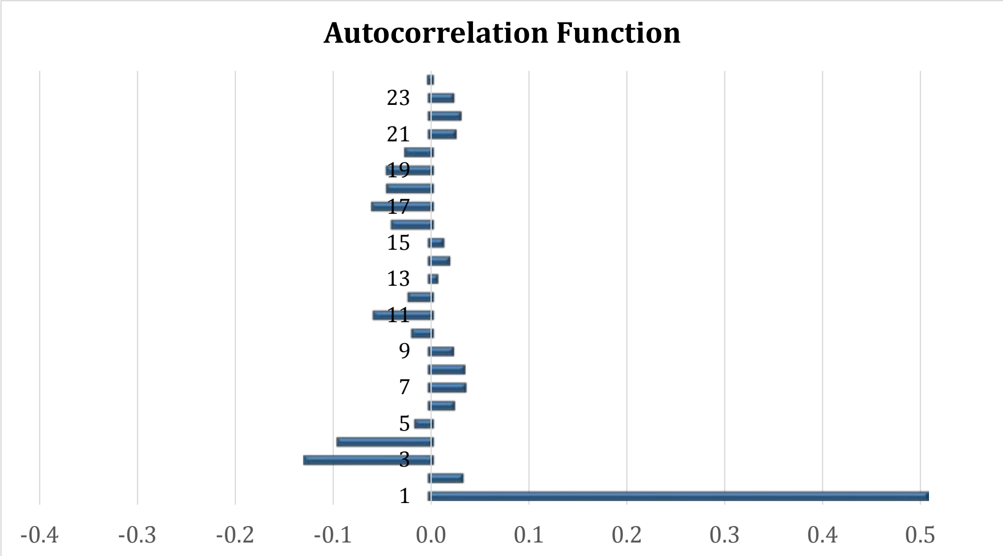
Question 1.B (2 points)

Using the lag operator L, write an AR(2) process in "lag operator-polynomial" form and discuss how would you go about testing whether the process is stable and hence stationary. Will the resulting stationarity, if verified, be strong or weak? Make sure to explain your reasoning.

Question 1.C (1 point)

Consider the following data and the corresponding sample ACF:





What is the most likely type of ARMA(p, q) process that may have originated this SACF? What other type of information would you be needing in order to make sure of your answer? Make sure to carefully justify your arguments.

Question 2.A (14 points)

Consider a bivariate VAR(2) model for S&P 500 returns and the log changes in the VIX volatility index ($R_t^{S\&P}$ and $\Delta lnVIX_t$). Write:

- The structural, unconstrained VAR(2) that includes contemporaneous effects across the two markets.
- The associated unconstrained reduced-form VAR(2).

Explain through which steps it is possible to transform the structural VAR model into the reduced-form one (algebra is not required, unless it helps you provide an efficient answer). How would/could you estimate the structural VAR? How would/could you estimate the reduced-form model? Explain what are the issues/limitations caused by the transformation of a structural VAR into a reduced-form model.

Question 2.B (2 points)

Suppose that the bivariate structural VAR(2) is to be exactly identified by imposing either of the two possible Choleski triangularization schemes:

$$\boldsymbol{B}' = \begin{bmatrix} 1 & 0 \\ b_{21} & 1 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{B}'' = \begin{bmatrix} 1 & b_{12} \\ 0 & 1 \end{bmatrix}$$

Carefully explain the implications and differences in economic interpretations of the estimated, corresponding reduced-form model deriving from imposing the restriction in $\boldsymbol{B}'$ instead of $\boldsymbol{B}''$. How does your answer change when the restriction

$$\boldsymbol{B}''' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is imposed instead?

Question 2.C (1 point)

Suppose that the estimation of a constrained, reduced-form VAR(2) has provided the following ML estimates of the conditional mean function and of the covariance matrix of the reduced-form shocks (p-values are in parentheses):

$$\begin{cases} R_t^{S\&P} = \underset{(0.044)}{0.006} + \underset{(0.093)}{0.053} R_{t-1}^{S\&P} - \underset{(0.003)}{0.473} \Delta lnVIX_{t-1} + \underset{(0.045)}{0.113} \Delta lnVIX_{t-2} + u_t^{S\&P} \\ \Delta lnVIX_t \; = -\underset{(0.149)}{0.194} - \underset{(0.024)}{0.375} R_{t-1}^{S\&P} + \underset{(0.050)}{0.094} R_{t-2}^{S\&P} + \underset{(0.000)}{0.804} \Delta lnVIX_{t-1} + u_t^{VIX} \end{cases}$$

$$Var\left(\begin{bmatrix} u_t^{S\&P} \\ u_t^{VIX} \end{bmatrix}\right) = \begin{bmatrix} \underset{(0.000)}{0.008} & \underset{(0.007)}{-0.016} \\ \underset{(0.007)}{-0.016} & \underset{(0.000)}{0.014} \end{bmatrix}$$

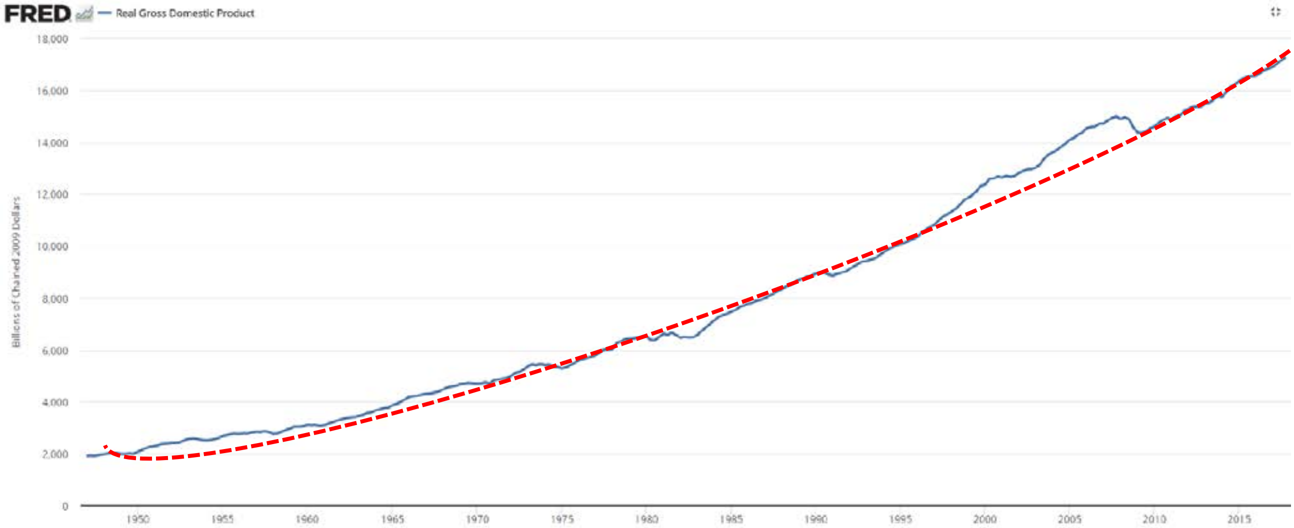
You would like to recover the original structural parameters, including the contemporaneous, average impact of both VIX changes on S&P 500 returns and vice-versa. Is there a chance that this may be possible even though you are *not* ready to impose a Choleski ordering on the two variables?

Question 3.A (13 points)

Define a stochastic trend and indicate what is the relationship between a stochastic trend and a random walk, with and without drift, for the special case of a I(1) process. For this case, comment on (or show, as you deem most appropriate) the stationarity or lack thereof of a random walk and explain why this may represent a problem in empirical work. Indicate how would you proceed to make a I(d) time series, $\{y_t\}$, with $d \geq 2$, stationary. Would the choice of considering $\{y_t - y_{t-d}\}$ instead of $\{y_t\}$ be an appropriate one? Make sure to carefully explain your answers.

Question 3.B (2.5 points)

An analyst at Charles Thomas and Associates has just downloaded the following series of data on the quarterly US real GDP (in constant dollars, expressed as 2009 billions).



He has proposed to make this series stationary by first fitting (by simple OLS) a quadratic function of time (shown as a dashed red line in the picture) and then replace the time series of real GDP with the OLS residuals from such a quadratic trend regression. What are the risks that the analysts is exposing himself and his firm to by adopting this simple procedure?

Question 3.C (1.5 points)

You know that a time series $\{y_t\}$ was originally suspected to be I(d) with $d \geq 1$. A fellow quant analyst, Ms. Maria Delas, has then transformed it by differentiating it three times, in the attempt to make it stationary and delivered the series to you. Upon your own analysis, you determine that the series contains now 2 unit roots in its MA component (i.e., the residuals need to be differentiated twice for them to be "well-behaved", that we may have called invertible). What do you know about the d characterizing the original series?