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Measurement theory and utility analysis in Suppes' early work, 1951–1958

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The paper reconstructs the connections between the evolution of Patrick Suppes' measurement theory from 1951 to 1958 and the research in utility analysis he conducted between 1953 and 1957 within the Stanford Value Theory Project. In particular, the paper shows that Suppes' superseding of the classical understanding of measurement, his endorsement of the representational view of measurement, and his conceiving of an axiomatic version of the latter were prompted by his research in utility analysis.

Keywords: Patrick Suppes; representational theory of measurement; axiomatization; utility theory; Stanford Value Theory Project

JEL Classification: B21; B31; B41; C91; D81

1. Introduction

In the historiography of measurement theory, it is customary to distinguish between the 'classical' and the 'representational' concepts of measurement. The former is labeled 'classical' because it dates back to Aristotle and dominated philosophy and science until the early decades of the twentieth century. The 'representational' view of measurement subsumes the classical view as a special case. It emerged in the first half of the twentieth century, primarily to account for the methods used in experimental psychology to quantify sensations, and was originally articulated by the Harvard psychologist Stanley Smith Stevens in 1946 (see e.g. Michell, 1999; Moscati, 2013a).

Both conceptions of measurement were, at some point, axiomatized. A first axiomatic version of the classical view was put forward by Otto Hölder (1901/1996); other versions were subsequently advanced by, among others, Suppes (1951) in his first published article. The axiomatic approach to the representational view of measurement was outlined by Suppes and Dana Scott (1958), developed by Suppes and Joseph Zinnes in a chapter of the *Handbook of Mathematical Psychology* (1963), and found its full-fledged expression in *Foundations of Measurement*, the book Suppes wrote in collaboration with David H. Krantz, R. Duncan Luce, and Amos Tversky and the first volume of which was published in 1971. In recent years, the axiomatic version of the representational view of measurement elaborated by Suppes and his co-authors has been criticized for paying inadequate attention to the empirical dimension of measurement (Boumans, 2015; Frigerio, Giordani, & Mari, 2010). However, it still represents the

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standard approach in measurement theory, and even scholars who are critical of it take the *Foundations of Measurement* as the starting point of their research.

In an autobiographical essay, Suppes (1979, pp. 13–16) recalled that the evolution of his view on measurement between 1951 and 1958 was driven by the work in utility analysis he conducted within the Stanford Value Theory Project between 1953 and 1957. So far as I am aware, however, there exists no systematic study of Suppes' early work in measurement theory and utility analysis that reconstructs in detail the connections between these two threads of his research. The present paper fills this lacuna, and shows how Suppes' superseding of the classical understanding of measurement, his endorsement of the representational view, and his conceiving of an axiomatic version of the latter were all prompted by his investigations in collaboration with Chen McKinsey, Donald Davidson, Muriel Winet, and Sidney Siegel at the Stanford Value Theory Project.

When Suppes entered utility analysis in the early 1950s, most research in the field was building on *Theory of Games and Economic Behavior*, the book mathematician John von Neumann and economist Oskar Morgenstern had published in 1944 (second edition 1947, third edition 1953). Besides originating modern game theory, the book made three further important contributions to economics. First, it introduced into the economic analysis of decision-making the set-theoretical axiomatic approach that was already mainstream in mathematics (Mongin, 2004). Second, it offered an axiomatic foundation to Expected Utility Theory (EUT), the theory of choices involving risk that had been adopted by early marginalists but had been subjected to sustained criticism in the 1930s and early 1940s. Between the late 1940s and early 1950s, von Neumann and Morgenstern's axiomatic version of EUT generated an intense debate in which many major economists of the period took part. By 1952–1953, however, a substantial consensus on EUT had formed, at least among American economists (Mongin, 2009; Moscati, 2016). Finally, since the expected utility formula features a cardinal utility function, i.e. a function unique up to linearly increasing transformations, the rise of EUT was associated with the rehabilitation of cardinal utility. This concept had been marginalized in the 1930s and early 1940 because it was at odds with the ordinal approach then dominant in utility analysis.[1]

Suppes' research on utility theory between 1953 and 1957 focused on the axiomatization of EUT and cardinal utility, and also on the experimental testing of EUT. In the course of this research, Suppes became aware that his axiomatizations of EUT and cardinal utility could be seen as particular instances of a more general exercise, namely that of specifying axiomatically the conditions that make a generic object – not only utility – measurable in one way rather than another. This paved the way for his axiomatization of the representational view of measurement, which he delineated in his 1958 article with Scott.

A final introductory remark is in order. Suppes wrote most of the works examined in the present paper in collaboration with other scholars (Davidson, McKinsey, Scott, Siegel, and Winet), and it is extremely difficult to disentangle his specific contributions from those of his co-authors. Therefore, in the paper I will treat each work as a joint product and assume that it fully reflected Suppes' views, as well as those of his co-authors. In fact, Suppes' co-authors dealt with measurement-theoretic issues only in those papers co-written with him, and therefore one could make the stronger case that those works reflected, first and foremost, Suppes' views on measurement. But such a strong claim is not needed for the purposes of this paper.

2. Axiomatizing classical measurement

Patrick Colonel Suppes (1922–2014) studied physics and meteorology at the University of Chicago (BS 1943) and, after serving in the Army Air Force during the war, in 1947 entered Columbia University as a graduate student in philosophy (Suppes, 1979). At Columbia, he came under the influence of the philosopher and measurement theorist Ernest Nagel, and also took courses in advanced mathematical topics. Around 1948, he was one of a group of Columbia PhD students who organized an informal seminar on von Neumann and Morgenstern's *Theory of Games*. He graduated in June 1950 under Nagel's supervision and, in September of the same year, joined the Department of Philosophy at Stanford University, where he remained for the rest of his working life.

In his first article, Suppes (1951) put forward a series of axioms that warrant the measurability of objects according to the classical conception of measurement. According to this conception, measuring the property of an object (e.g. the length of a table) consists of comparing it with some other object that displays the same property and is taken as a unit (e.g. a meter-long ruler) and then assessing the numerical ratio between the unit and the object to be measured (if the ratio is two to one, the table is two meters long). Over the course of the history of measurement theory, the key condition warranting the measurability of objects in the classical sense was identified in the possibility of adding objects in a sense analogous to that in which numbers are summed (two tables can be 'added' by placing them end to end, and the length of the resulting surface is equal to the sum of the lengths of the two single tables).

In 1901, the German mathematician Otto Hölder applied to measurement theory the axiomatic approach to geometry launched two years earlier by Hilbert (1899/1950). Hölder (1901/1996) laid down seven axioms on magnitudes and proved that, if magnitudes satisfy them, the ratio between any two magnitudes is well-defined, and one magnitude can be taken as a unit to measure the others. In other words, Hölder specified axiomatically the conditions warranting the measurability of objects in the classical sense.

Despite its importance for the theory of measurement, Hölder's article went unnoticed until the early 1930s when it was rediscovered by Nagel, who later became Suppes' mentor at Columbia. Nagel (1931) suggested a set of 12 axioms, slightly different from Hölder's, which should warrant the measurability of magnitudes in the classical sense. However, Nagel did not give any formal proof that his axioms actually deliver classical measurability.

Building on Hölder and Nagel, Suppes in his first article considered a set of objects, a binary relation between these objects interpretable as the inequality relation $\leq$, and a binary function interpretable as the operation of addition $+$. Suppes put forward seven axioms concerning the set of objects, the relation $\leq$, and the operation $+$, and proved that, although less restrictive than Hölder's, his axioms were nonetheless sufficient to warrant the measurability of the elements of the set in the classical sense.[2]

3. Stevens and the representational view of measurement

In 1951, Suppes did not refer to Stanley Smith Stevens and the representational conception of measurement the latter had expounded in 1946. Nevertheless, a brief aside on Stevens' measurement theory is in order at this point in the narrative.

For Stevens (1946, p. 677), measurement consists of 'the assignment of numerals to objects or events according to rules.' Since there are various rules for assigning numbers to objects, there are various forms, or ‘scales’, of measurement. Generally speaking, measurement is possible in the first place because there is ‘a certain isomorphism’ (p. 677), i.e. a certain similarity, between some empirical relations between the objects to be measured and some formal relations between the numbers assigned to those objects. Thus, for example, there exists an isomorphism between the empirical relation ‘longer than’ between tables, and the formal relation ‘greater than’ between numbers.

Stevens (1946, pp. 678–680) identified four basic measurement scales and characterized each one by a set of empirical operations and by the class of mathematical transformations the numbers in the scale can be subjected to without altering the scale’s capacity of representing the empirical relations among objects. For instance, he characterized the interval scale by three operations: (i) the determination of whether two objects are equal with respect to some trait; (ii) the ranking of objects; (iii) the determination of equality of differences between objects. Mathematically, the interval scale is identified by linearly increasing transformations, i.e. those of the form $f(x) = \alpha x + \beta$, where α is a positive constant, and β is a constant that can also be negative.[3] As an example of interval scales in physics, Stevens considered the Centigrade and Fahrenheit scales of temperature. In economics, we may add, cardinal utility is measurable according to an interval-scale.

For Stevens, the classical conception of measurement is a narrow one which identifies measurement with a specific, and quite demanding, form of measurement, namely ratio-scale measurement. Most of the entities studied by physics and other natural disciplines can be added, and are therefore measurable according to a ratio-scale, i.e. in the classical sense. By contrast, sensations, intellectual abilities, utility, and other entities studied by the behavioral and social sciences cannot be added, and thus cannot be measured according to a ratio-scale. However, for Stevens, these entities might be subject to other empirical operations and hence measured according to other scales.

Two final comments on Stevens’ representational theory of measurement are in order. First, Stevens’ theory was not an axiomatic one. He associated each scale of measurement with a set of empirical operations and a class of mathematical transformations, but did not attempt to specify under what exact conditions, i.e. axioms, certain operations are feasible while others are not. Steven did not even attempt to prove mathematically how a specific class of admissible transformations draws from a given set of operations and, accordingly, there are no ‘representation theorems’ in his work.[4]

Second, Stevens (1946) theory of measurement displays significant similarities, but also differences, with the theory of measurement outlined by von Neumann and Morgenstern in the first chapter of *Theory of Games* (von Neumann & Morgenstern, 1944). Like Stevens, von Neumann and Morgenstern related measurement to the assignment of numbers to objects, and connected different types of measurement with different classes of mathematical transformations. However, they associated the possibility of proper measurement with the possibility of adding the objects to be measured, which confers a classical flavor to their theory of measurement.[5]

Returning to Suppes, we should note that neither his exclusive focus on the classical conception of measurement in his 1951 article nor his apparent ignorance of Stevens’ measurement theory are surprising. This is because Suppes’ background was not in psychology or economics, but in disciplines such as philosophy, mathematics, and physics, in which the classical conception of measurement remained unrivaled. The articles Suppes published between 1951 and 1953 also belonged to these latter disciplines. In these works, Suppes and his co-authors advanced a set-theoretical axiomatic foundation of particle mechanics (McKinsey, Sugar, & Suppes, 1953; McKinsey & Suppes, 1953a, 1953b). Regarding measurement issues, these articles remained within the orbit of the classical conception. In 1953, however, things changed rapidly.

4. The Stanford Value Theory Project

In the early 1950s, two main factors contributed to a shift in Suppes' research interests toward economics, psychology, and the behavioral sciences in general. The first factor was the influence of J.C.C. 'Chen' McKinsey, Suppes' postdoctoral tutor at Stanford. McKinsey (1908–1953) was a logician who had worked intensively on game theory at the RAND Corporation, a think tank created by the US Air Force in 1946 and located in Santa Monica, California. In 1951, McKinsey, after being forced to leave RAND because his homosexuality was considered a security risk (Nasar, 1998), joined Stanford's Philosophy Department. At the time, he was completing his *Introduction to the Theory of Games* (McKinsey, 1952), which would become the first textbook in game theory. Suppes' familiarity with game theory and decision analysis was further enhanced by his summer research position in the early 1950s, working with David Blackwell and Meyer A. Girshick while they were writing their book *Theory of Games and Statistical Decisions* (Blackwell & Girshick, 1954), in which the tools of decision theory and game theory were employed to evaluate statistical procedures.

From McKinsey, Suppes learned not only game theory, but also the set-theoretical methods that would play a crucial role in his subsequent work. McKinsey also encouraged Suppes to attend the seminar conducted by the eminent logician Alfred Tarski at the University of California at Berkeley. According to Suppes (1979, p. 8), 'it was from McKinsey and Tarski that I learned about the axiomatic method and what it means to give a set-theoretical analysis of a subject.'[6]

The second, and possibly more powerful, factor that contributed to shift Suppes' research interests toward the behavioral sciences was funding. In 1953, John Goheen, the chair of Stanford's Department of Philosophy, obtained a grant from the Ford Foundation for a study on 'Value, Decision and Rationality.' Around the same time, Goheen negotiated contracts with two military agencies, namely the Office of Naval Research and the Office of Ordnance Research of the US army, for work on the theory of decisions involving risk. Goheen entrusted McKinsey and Suppes with the project, which was renamed the 'Stanford Value Theory Project' (Isaac, 2013; Suppes, 1979).[7]

McKinsey and Suppes co-opted into the enterprise Donald Davidson, another philosopher who had joined the Philosophy Department in January 1951. Davidson (1917–2003) had studied at Harvard University (BA 1939, PhD 1949), where he was influenced by logician and analytical philosopher W.V.O. Quine. Today, Davidson is best known for his influential work on the philosophy of mind and action, the philosophy of language, and epistemology. However, he published these works only from the early 1960s on. In the 1950s, he was still very busy with teaching and did not as yet have a clear philosophical project. As he explained in a later interview: 'Suppes and McKinsey took me under their wing […] because they thought this guy [Davidson] really ought to get some stuff out' (Lepore, 2004, p. 252).

Most of the research connected with the Stanford Value Theory Project was conducted between 1953 and 1955, and appeared in print between 1955 and 1957. The final output consisted of three articles, each of a theoretical character (Davidson, McKinsey, & Suppes, 1955; Davidson & Suppes, 1956; Suppes & Winet, 1955), and a book presenting the results of an experiment to measure the utility of money (Davidson, Suppes,

& Siegel, 1957). However, McKinsey contributed only to the setting up and very early work of the Project because in October 1953 he committed suicide.

5. Axiomatizing cardinal utility by utility differences

October 1953 is relevant for our narrative also because in that month Suppes and his doctoral student Muriel Winet completed a first version of one of the Stanford Value Theory Project papers, namely, 'An Axiomatization of Utility Based on the Notion of Utility Differences' (Suppes & Winet 1953/1954). The paper was presented in November 1953 at a meeting of the American Mathematical Society held at the California Institute of Technology, and was later published in the April–July 1955 issue of the first volume of *Management Science*, a newly founded journal that was open to studies in decision theory from different disciplines.[8] In their work, Suppes and Winet (1955) advanced an axiomatization of cardinal utility based on the assumption that individuals are not only able to rank the utility of different commodities, as is assumed in the ordinal approach to utility, but are also capable of ranking the differences between the utilities of commodities.

There are different methods for arriving at a cardinal utility function. The method popular in the early 1950s was based on the EUT axioms. As already mentioned, it was in fact through the association with EUT that cardinal utility was rehabilitated in economic theory in this period. An older way of attaining cardinal utility was based on the assumption that the utilities of different commodities are independent. However, from the 1910s the majority of utility theorists had rejected this assumption because it rules out conspicuous economic phenomena such as the complementarity and substitutability of goods. A third route, based on the ranking of utility differences, had been extensively discussed in the 1930s by Lange (1934), Phelps Brown (1934), Alt (1936/1971), Samuelson (1938) and other economists. At that time, however, most utility theorists had remained skeptical about the ranking of utility differences because it has no clear observable counterpart in terms of acts of choice, and therefore relies only upon introspection, which was not considered a reliable source of evidence (see Baccelli & Mongin, 2016; Moscati, 2013b).

Suppes and Winet (1955, p. 259) mentioned that the notion of utility differences had been already discussed in economics, and cited Lange's (1934) article on the topic. They also took a stance against the economists' opposition to introspection that, since the mid-1930s, had played a crucial role in the marginalization of utility differences in economic analysis. They claimed that in many areas of economic theory 'there is little reason to be ashamed of direct appeals to introspection', and that there are sound arguments for justifying 'the determination of utility differences by introspective methods' (p. 261). They then affirmed that, despite the importance and legitimacy of utility differences, to the best of their knowledge 'no adequate axiomatization for this difference notion has yet been given' (p. 259). Evidently, they were ignorant of the fact that almost 20 years earlier, in the debate initiated by Lange, the Viennese mathematician Alt (1936/1971) had provided a rigorous axiomatization of cardinal utility anticipating in significant respects that of Suppes and Winet.[9]

Like Alt, Suppes and Winet considered two order relations – Q and R – over the elements of an abstract set K. Q is a standard, binary preference relation: xQy means that x is not preferred to y. R is a quaternary relation concerning 'differences' or 'intervals' between alternatives: $(x,y)R(z,w)$ means that the interval between x and y is not greater than the interval between z and w. Suppes and Winet imposed 11 axioms on the set K, the relations Q and R. The conditions these axioms impose are analogous to the conditions defined by Alt: completeness, transitivity, continuity, and some form of additivity for the two order relations, and an Archimedean property on R.[10]

Based on their 11 axioms, Suppes and Winet proved what they explicitly called a 'representation theorem' (p. 265): the axioms imply the existence of a function u that can be interpreted as a utility function, is unique up to linearly increasing transformations, and is such that the interval between x and y is smaller than the interval between z and w if and only if the absolute utility difference between x and y is smaller than the absolute utility difference between z and w (pp. 265–270). More formally, xQy if and only if $u(x) \leq u(y)$, and $(x,y)R(z,w)$, if and only if $|u(x)–u(y)| \leq |u(z)–u(w)|$.[11]

In the terminology of Stevens' representational theory of measurement, Suppes and Winet's axioms identify a set of operations that allow one to assign (utility) numbers to objects according to an interval rather than a ratio-scale. In their article, Suppes and Winet referred to two papers the Harvard psychologist had published before 1946 (Stevens, 1936; Stevens & Volkmann, 1940), but did not mention Stevens' measurement theory, did not use his scale-of-measurement terminology, and, accordingly, did not label cardinal utility as utility measurable on an interval scale.

Nevertheless, by identifying conditions that warrant the interval-scale measurement of an object, Suppes overcame the classical conception that confines measurement to ratio measurement. Suppes' superseding of the classical view of measurement and his endorsement of the representational view became more explicit in a paper he read at the annual meeting of the Pacific Division of the American Philosophical Association that was held at Stanford University in December 1953.

6. Endorsing the representational view

This paper, which does not belong to those funded by the Stanford Value Theory Project, bore the title 'Some Remarks on Problems and Methods in the Philosophy of Science,' and was published the following year in *Philosophy of Science*. In this, Suppes (1954) made various programmatic proposals for the advancement of the philosophy of science, some of which concerned the theory of measurement.

Suppes argued that the most urgent task in the philosophy of science was that of 'axiomatizing the theory of all developed braches of empirical science' (p. 244). In particular, for Suppes this axiomatization program should not be based on logic or other metamathematical languages but on set-theoretical methods:

> We can pursue a program of axiomatization without constructing any formal languages. [...] The basic methods appropriate for axiomatic studies in empirical sciences are not metamathematical (and thus syntactical and semantical), but set-theoretical (p. 244).In Suppes' view, the axiomatization task could be divided into four steps: (i) listing the primitive notions of a given theory and characterizing them in set-theoretical terms; (ii) indicating the axioms that the notions must satisfy; (iii) investigating the deductive consequences of the axioms; and (iv) providing an empirical interpretation of the axiomatized theory. Suppes claimed that measurement theory relates to the fourth step and should show us how 'we may legitimately pass from the rough and ready region of qualitative, commonsense observations to the precise and metrical realm of systematic science' (p. 246).

At this point, Suppes explicitly asserted that there are 'various types of measurement', and noticed that in the literature there are 'several sets of axioms for measurement of different sorts' (p. 246). As examples, he listed the measurement axioms put forward by Hölder (1901/1996), Nagel (1931), von Neumann and Morgenstern (1944), Suppes

(1951), and Suppes and Winet (1953/1954). In a footnote, Suppes further noted that, from a mathematical viewpoint, the task of axiomatizing a given type of measurement is equivalent to 'the search for a representation theorem in the domain of real numbers, with the representation unique up to the appropriate transformations' (p. 246, footnote 9).

The above-quoted passages show that, by the end of 1953, Suppes had abandoned the classical understanding of measurement that had informed his 1951 article and endorsed the representational conception. Unlike Stevens, however, Suppes advocated an axiomatic approach, and more specifically a set-theoretical axiomatic approach, to the representational theory.

7. A coherent theory of measurement

A further step toward the elaboration of a systematic axiomatic theory of representational measurement can be found in the second installment of the Stanford Value Theory Project. This is the paper entitled 'Outlines of a Formal Theory of Value. I', which Suppes co-authored with Davidson and McKinsey. McKinsey had read an earlier and much shorter version of the paper in May 1953 at a seminar at the University of California at Los Angeles. After his death, Davidson and Suppes expanded and revised the work, which was eventually published in the April 1955 issue of *Philosophy of Science*.

In this article, Davidson et al. (1955, p. 141) put forward different sets of axioms identifying normative conditions for a 'rational preference pattern' in choice behavior. In particular, they claimed that a rational preference pattern must satisfy transitivity and, in support of this claim, they put forward the argument for which the article is usually cited in the economics literature, namely the so-called 'money pump argument.' Basically, Davidson, McKinsey, and Suppes showed that an individual with intransitive preferences – a hypothetical university professor they called Mr. S. – can be induced to pay money for nothing (pp. 145, 146).[12] For our purposes, however, the most important part of their article is the aside on measurement theory contained in Section 3.

In this section Davidson, McKinsey, and Suppes explicitly adopted the scale-of-measurement terminology and, without citing Stevens, they introduced a classification of measurement scales that closely resembles the one introduced by the Harvard psychologist in 1946. They identified, in order of increasing strength, the 'ordinal,' 'interval,' and 'ratio' scales, already discussed by Stevens, as well as the 'absolute scale,' which does not admit any arbitrary element and can be transformed only by the identity function $f(x) = x$. A magnitude measurable according to the absolute scale is probability.

Furthermore, just like Stevens, Davidson, McKinsey and Suppes criticized the traditional identification of measurement with classical, i.e. ratio, measurement, and noticed that such identification was still widespread in a number of philosophical treatises of their day. They argued that 'this ratio requirement is too rigid,' and that the erroneous identification of measurement with ratio measurement 'lies in the assumption that the only things which are measurable in a strong sense are […] magnitudes for which there exists a natural operation corresponding closely to the addition of numbers' (p. 151). This error also 'led to the erroneous view that no kind of measurement appropriate to physics is applicable to psychological phenomena' (p. 151, footnote 8).

Moving to the measurement of preferences, Davidson, McKinsey, and Suppes agreed that preferences 'cannot be measured in the sense of a ratio scale,' but added that this circumstance does not exclude 'the possibility […] that preferences can be measured in the sense of an interval scale' (p. 151), i.e. using a cardinal utility function, which latter was the view they supported.

The aside on measurement concludes with a passage in which Davidson, McKinsey and Suppes defined the concept of 'coherent theory of measurement.' This concept expresses, succinctly but with great precision, the conception of measurement Suppes was going to articulate in his subsequent works:

> A coherent theory of measurement is given by specifying axiomatically conditions imposed on a structure of empirically realizable operations and relations. The theory is formally complete if it can be proved that any structure satisfying the axioms is isomorphic to a numerical structure of a given kind (p. 151).

8. Experimental utility measurement

Part of the Stanford Value Theory Project centered on an experiment aimed at measuring the utility of money of a number of individuals on the basis of their preferences between gambles where small amounts of real money were at stake. The experiment relied on an axiomatization of EUT elaborated for the occasion by Davidson and Suppes (1956) – in the third Stanford Value Theory Project paper – and generated cardinal measures of utility.[13] In a second phase of the experiment, these cardinal measures of utility were used to test the validity of EUT.

An earlier experiment to measure the utility of money within the EUT framework had been conducted between 1948 and 1949 by Frederick Mosteller, a Harvard statistician who, in the late 1940s, had become interested in experimental psychology, and Philip Nogee, then a Harvard PhD student in psychology (Mosteller & Nogee, 1951). Beginning in November 1953, Davidson and Suppes began thinking about an experiment that could overcome a number of limitations they saw in the Mosteller-Nogee design. However, neither Davidson nor Suppes had any previous experience in experimental investigation, and they therefore brought Sidney Siegel into the project. Siegel (1916–1961), then completing his PhD in psychology at Stanford, had begun his doctoral studies in 1951 at the age of 35, having previously taken a rather singular biographical and educational path (Engvall Siegel, 1964). In the doctoral dissertation he completed in fall 1953, Siegel presented a possible measure of authoritarianism based on experimental techniques (Siegel, 1954).

Davidson, Suppes, and Siegel conducted their experiment in spring 1954. They presented their experimental results in a Stanford Value Theory Project report published in August 1955 and then, two years later, in the book *Decision Making: An Experimental Approach* (1957). Like Mosteller and Nogee, Davidson, Suppes, and Siegel concluded that it is feasible to measure experimentally the utility of money in a cardinal way, and also that their experimental findings supported the validity of EUT. I have discussed in detail the design and findings of their experiment, as well as its relationship with the Mosteller-Nogee study, in another paper (Moscati, in press). Here, I would like to call attention to the measurement-theoretic view underlying the Davidson-Suppes-Siegel study.

At the beginning of the book, the role of the Davidson-Suppes axiomatization of EUT used in the experiment is presented in unmistakably representational terms:

> We require that the axioms […] permit us to prove (a) that is possible to assign numbers to the elements of any set […] in such a way as to preserve the structure imposed on such

> sets by the axioms, and (b) any two assignments of numbers […] are related by some specified group of transformations (Davidson et al., 1957, p. 6).

In particular, the authors specify that the axioms 'yield interval measurement of utility.' (p. 6). They also present the findings of the experiment using the scale-of-measurement terminology of the representational view:

> The chief experimental result may be interpreted as showing that for some individuals and under appropriate circumstances it is possible to measure utility in an interval scale. (p. 19)

The book was the last research item of the Stanford Value Theory Project to be published. As we have seen, between the beginning of the project in 1953 and the publication of its last item in 1957, Suppes' views on measurement changed significantly. He moved beyond the classical understanding of measurement that he initially reckoned on, definitely embraced the representational view, and envisaged, under the label of 'a coherent theory of measurement,' the project of a thoroughgoing axiomatization of the representational approach to measurement. His first decisive step toward the realization of this project is represented by a paper he co-authored with Dana Scott.

9. A conceptual framework for axiomatic measurement theory

Dana S. Scott (born 1932) studied mathematics and logic at the University of California at Berkeley, where he became a pupil of Alfred Tarski (Burdman Feferman and Ferferman 2004). Suppes had met Scott in 1952, when the latter was an undergraduate student in a course on the philosophy of science that Suppes taught at Berkeley (Suppes, 1979). After completing his BA in 1954, Scott moved to Princeton University for doctoral studies under Alonzo Church, another prominent logician, and received his PhD in 1958.

Scott and Suppes completed a first version of their joint paper, which is entitled 'Foundational Aspects of Theories of Measurement', in April 1957; the paper was later published in the June 1958 issue of the *Journal of Symbolic Logic*.[14] In the article, Scott and Suppes did not work out a systematic treatment of the axiomatic theory of measurement but rather delineated the conceptual framework in which such a treatment could be developed. Although this conceptual framework remained fundamentally set-theoretical in nature, it also incorporated some elements of the logic-based approach to axiomatization about which, as mentioned above, Suppes (1954) had expressed some skepticism. These logic-based elements can be explained by the fact that Suppes' co-author was a logician and that the article was published in a symbolic-logic journal.

Scott and Suppes (1958) grounded their conceptual framework in the notion of a *relational system* that Tarski (1954/1955) had recently introduced. A relational system $\mathfrak{A} = \langle A, R_1, \ldots, R_n \rangle$ is a set-theoretical structure in which A is a non-empty set of elements called the *domain* of $\mathfrak{A}$, and R_1, …, R_n are relations between one, two, or more elements of A (functions connecting elements of A can be conceived as relations). For instance, A_1 can be a set of sounds and R_1 the binary relation 'louder than' expressing the acoustic judgment of a given subject; then $\mathfrak{A}_1 = \langle A_1, R_1 \rangle$ is a relational system. Using the notion of relational system, the intuitive idea of isomorphism can be made precise. Two relational system $\mathfrak{A} = \langle A, R_1, \ldots, R_n \rangle$ and $\mathfrak{B} = \langle B, S_1, \ldots, S_n \rangle$ are said to be isomorphic if there exists a one-to-one function f from A onto B such that, for each relation R_i and for each sequence $\langle a_1, \ldots, a_m \rangle$ of elements of A, the relation R_i holds on these elements if and only if a relation S_i holds for the image of $\langle a_1, \ldots, a_m \rangle$ in B through f, that is, $R_i(a_1, \ldots, a_m)$if and only if $S_i(f(a_1), \ldots, f(a_m))$.

A *numerical relational system* $\mathfrak{N} = \langle \mathbb{R}, N_1, \ldots, N_n \rangle$ is a system whose domain of elements is the set $\mathbb{R}$ of real numbers. A *numerical assignment* for a relation system $\mathfrak{A}$ with respect to a numerical relational system $\mathfrak{N}$ is a function f from A onto $\mathbb{R}$ such that $R_i(a_1, \ldots, a_m)$ if and only if $N_i(f(a_1), \ldots, f(a_m))$. The function f does not need to be one-to-one, for in many cases we may want to assign the same number to two distinct objects. If such a numerical function f exists, Scott and Suppes call the relation system $\mathfrak{A}$ *imbeddable* in $\mathfrak{N}$. Using a less esoteric terminology, we could say that the relation system $\mathfrak{A}$ is represented by the numerical relational system $\mathfrak{N}$.

After all these preliminaries, by the end of the first section of their paper Scott and Suppes (1958, p. 115) could finally define 'a theory of measurement' as a class K of isomorphic relational systems for which there exists 'a numerical relational system $\mathfrak{N}$ […] such that all relational systems in K are […] imbeddable in $\mathfrak{N}$.'

In the second section, Scott and Suppes investigated the conditions under which a theory of measurement for a given class of relational systems exists. In the 3 Section, they moved to the issue of axiomatizability. They called a theory of measurement *axiomatizable* if there exists 'a set of sentences of first-order logic (the axioms of the theory)', such that 'a relational system is in the theory if and only if the system satisfies all the sentences in the set' (p. 123).[15] Without entering here into technical details, we can say that Scott and Suppes showed that the existence and axiomatizability of a theory of measurement depend, among other things, on whether the domain of the relational system is finite or infinite, and on which type of numerical relations N_i are considered.

For our purposes, what is particularly important is that Scott and Suppes illustrated their new conceptual framework for measurement theory with examples taken from utility analysis. In these examples, the domain A of a relational system is constituted by a set of choice objects, and the relations $\langle R_1, \ldots, R_n \rangle$ on A are preference relations over these objects satisfying certain axioms. In particular, Scott and Suppes (pp. 118–122) considered preference axioms analogous to those used by Suppes and Winet (1955) in their utility-difference model, the axioms introduced by Luce (1956) in his model of semi-ordered preferences, as well as the axioms used in the probabilistic theory of choice, which was an approach to utility analysis quite fashionable in the second half of the 1950s.[16]

Given that Scott's background was in mathematics and logic, it seems fair to assume that these examples taken from utility analysis are due to Suppes. This circumstance reinforces the main point made in the present paper, namely that Suppes' early work in measurement theory was crucially influenced by his contemporaneous engagement with utility analysis.

10. Epilogue and summary

In their 1958 article, Scott and Suppes explained how certain traditional problems of measurement theory, namely those related to the assignment of numbers to objects, could be restated and clarified within the new conceptual framework they put forward. However, they barely discussed the problems related to the class of transformations the assigned numbers can be legitimately subjected to, and did not explain how these problems could be expressed within their new framework. Moreover, although Scott and Suppes illustrated their framework by examples taken from utility analysis, these illustrations were far from systematic. In particular, they did not explain how the theory for each of the four scales typically discussed in measurement analysis would look like in the new framework. In the following years, this work of clarification and systematization was undertaken by Suppes and Zinnes in the first chapter of the *Handbook of Mathematical Psychology* (1963), and finalized by Krantz, Luce, Suppes, and Tversky in the *Foundations of Measurement* (1971, 1989, 1990). A discussion of these developments is beyond the scope of the present paper. However, it is worth noting that both the Suppes-Zinnes chapter and the *Foundations of Measurement* adopt a set-theoretical rather than logical approach.

This paper has reconstructed the connections between the work Suppes did in utility theory between 1953 and 1957 within the Stanford Value project, and the evolution of his measurement theory from 1951 to 1958. The constant element in Suppes' approach to measurement theory was his steadfast reliance on the set-theoretical axiomatic method. What changed was his conception of what it means to measure a thing. Because his background was in philosophy, mathematics, and physics, Suppes in 1951 conformed to the classical conception of measurement dominant in these disciplines. But his subsequent involvement with psychology, economics, and the behavioral sciences in general helped him understand that the classical view was too narrow to account for the quantification practices used in these disciplines. Thus, he embraced the broader representational concept of measurement that had been expounded by Stevens in the 1940s. But while Stevens was not interested in giving the representational theory an axiomatic treatment, Suppes was. The first, decisive step in the realization of this ambitious project was the article Suppes co-authored with Scott in 1958.

In an autobiographical article, Suppes (1979) recalled that his views on measurement had its origins in the work he conducted within the Stanford Value Theory Project. Nevertheless, hitherto there has existed no systematic study integrating Suppes' memories with a detailed analysis of his early research in measurement theory and utility analysis. The present essay has filled this lacuna, and in doing so has showed that Suppes' superseding of the classical understanding of measurement, his endorsement of the representational view, and his conceiving of an axiomatic version of the latter were all driven by the research in utility analysis that he carried out within the Project.

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Notes

1. Since the early 1950s Milton Friedman and Savage (1952), and also other economists, had pointed out that from a strictly theoretical viewpoint EUT does not imply a return to cardinal utility (Moscati, 2016). Historically, however, this ordinal interpretation of EUT was not shared by all users of EUT, who included, besides economists, operation research scholars, philosophers, psychologists, and mathematicians (Erikson & others, 2013; Heukelom, 2014). In particular, in the works of Suppes, and notably in those he published between

1951 and 1958, I was not able to find any trace of the ordinal interpretation of EUT. For Suppes, the mathematical function featuring in the expected utility formula *was* a cardinal utility function. For a more extended discussion of Suppes' approach to cardinal utility, see Baccelli and Mongin (2016).

2. Suppes' seven axioms (1951, pp. 164, 165) are as follows: (1) $\leq$ is transitive; (2) + is closed in the set of objects K; (3) + satisfies the associative law; (4) if x, y and z are in K, and $x \leq y$, then $(x + z) \leq (y + z)$, i.e., 'adding' the same element does not alter order; (5) if x and y are in K, and not $x \leq y$, then there is a z in K such that $x \leq y + z$ and $y + z \leq x$, that is, any element x may be obtained by 'summing' two other elements y and z; 6: if x and y are in K, then not $x + y \leq x$, that is, 'sum' is greater than the 'summands'; 7: if x and y are in K and $x \leq y$, then there is a number n such that $y \leq nx$ (Archimedean property).
3. The other three scales are identified by the following classes of transformations. Nominal scale: transformations by any one-to-one function $f(x)$. Ordinal scale: transformations by monotonically increasing functions, i.e. by any $f(x)$ with positive first derivative f'. Ratio scale: proportional transformations by functions of the form $f(x) = \alpha x$, with $\alpha > 0$.
4. Stevens' non-axiomatic stance is related to his endorsement of the operational epistemology put forward by Bridgman (1927). On Stevens' operationalism and its relationship with his measurement theory, see Hardcastle (1995) and Michell (1999).
5. In his 1946 article, Stevens did not cite *Theory of Games*. However, in a footnote in a later paper, Stevens (1951, p. 23) acknowledged the similarity between his measurement theory and that of von Neumann and Morgenstern. In order to anticipate possible priority issues, Stevens claimed that he had presented his theory 'before the International Congress for the Unity of Science, September 1941.' I have found no evidence refuting this claim. Apparently, Stevens and von Neumann-Morgenstern arrived at similar theories of measurement in an independent way around the same time.
6. More on McKinsey's scientific contributions and personality in Davidson, Goheen, and Suppes (1954), Lepore (2004), and Burdman Feferman, and Feferman (2004). This last book is also a good reference for Tarski's work in logic.
7. On the reasons why in the 1950s the Ford Foundation and military agencies such as the Office of Naval Research were interested in funding research on decision-making, see Pooley and Solovey (2010), and Erikson and others (2013).
8. In 1957, Winet completed her PhD at Stanford under Suppes with a dissertation on *Interval Measurement of Subjective Magnitudes with Subliminal Differences* (Wood Winet Gerlach, 1957), a topic strictly related to that of the article co-authored with her advisor.
9. Alt's article had been published in German in the Austrian journal *Zeitschrift für Nationalökonomie*, which in 1936 was directed by Morgenstern. Although Morgenstern, Lange, and Samuelson were aware of its existence, Alt's article became known to a larger English-speaking public only after it was mentioned by Joseph Schumpeter in his *History of Economic Analysis* (Schumpeter, 1954, p. 1063). More on Alt, his axiomatization of cardinal utility, and the fortunes of his article in Moscati (2015).
10. Suppes and Winet's axioms are as follows. Axioms 1–4 require that both Q and R are complete and transitive. Axiom 5 imposes that only the 'extension' of the interval between two elements x and y matter, and not the relative order of x and y; thus interval (x,y) is equivalent to interval (y,x). Axiom 6 means that any interval (x,y) can be bisected, i.e. that for any two elements x and y there exists a midpoint element t such that interval (x,t) is equivalent to interval (t,y). Axiom 7 states that, if two elements x and y are indifferent, then one can be substituted for the other without modifying the order relationships among intervals: if xQy, yQx, and $(x,z)R(u,v)$, then $(y,z)R(u,v)$. Axiom 8 requires that, if y is between x and z, then the interval between x and y is smaller than the interval between x and z. Axiom 9 is an additivity assumption on R: if interval (x,y) is smaller than interval (u,w), and interval (y,z) is smaller than interval (w,v), then the 'sum' (x,z) of the two smaller intervals is smaller than the 'sum' (u,v) of the two larger intervals. Axiom 10 imposes a continuity property: if interval (x,y) is strictly smaller than interval (u,v), then there is an element t between u and v such that interval (x,y) is still not greater than interval (u,t). Axiom 11 is an Archimedean assumption; it fundamentally states that each interval can be expressed as the sum of a finite sequence of smaller, equivalent intervals.
11. Baccelli and Mongin (2016) discuss the problems associated with Suppes and Winet's focus on absolute rather than simple (i.e. algebraic) utility differences.

12. For an introduction to the money pump argument, see Anand (1993).
13. The Davidson-Suppes axiomatization has two main distinguishing features: (i) it is based on the subjective approach to probability and EUT pioneered by Frank Ramsey (1926/1950) and developed by Savage (1954), and (ii) it is 'finitistic,' in the sense that the set of alternatives over which the individual's preferences are defined is not infinite, as it is in the Ramsey-Savage framework, but finite. Suppes (1956) presented an alternative version of the Davidson-Suppes axiomatization of subjective EUT. Neither Davidson and Suppes (1956) nor Suppes (1956) addressed measurement issues in detail, and therefore these articles are of only modest relevance for the present paper. For a more extended discussion of Suppes' subjective version of EUT, see Baccelli and Mongin (2016).
14. In April 1957, Suppes also completed an introductory textbook on logic, which he dedicated to the memory of McKinsey (Suppes, 1957, p. v). The last chapter of the book is devoted to the set-theoretical foundations of the axiomatic method, while elsewhere in it Suppes briefly discusses measurement theory (pp. 265, 266). This discussion, however, does not add any new elements to the picture drawn so far.
15. First-order logic allows for quantification over objects. In particular, it admits the universal quantifier ∀ ('for every object') and the existential quantifier ∃ ('there exists an object'). However, first-order logic does not permit quantification over sets of objects ('for every set of objects'), for which second-order logic is required.
16. In order to account for situations in which an individual cannot discriminate between similar objects, Luce (1956) considered a generalization of standard utility theory in which the relation of strict preference between objects is transitive while the indifference relation is not. Such a preference structure is called a 'semiorder,' and Luce showed how to construct a utility function that assigns numbers to semi-ordered objects. In the probabilistic approach to choice, put forward by Marschak (1955), Quandt (1956) and other economists, it is assumed that when a subject chooses between the same pair of objects x and y more than once, because of judgment errors or other factors, sometimes he ranks x over y, and sometimes y over x. The subject is said to prefer x to y if the probability p_{xy} that he chooses x over y is at least 0.5, i.e., if $p_{xy} \geq 0.5$. If the probability p_{xy} is at least as large as the probability p_{zw} that he chooses z over w, i.e. if $p_{xy} \geq p_{zw}$, then the utility difference between x and y is considered to be at least as large as the utility difference between z and w. Under certain assumptions, probabilistic preferences can be represented by a cardinal utility function.

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