On-Line Ex. 5B. Here we would like to compare the different degree of exposure to asymmetries of different asset classes. To this purpose, we use daily *excess* CRSP equity returns, weekly (negative) differences in 10-year US Treasury rates, and monthly US equity *total* returns from Bloomberg. It is interesting to compare the results from the first and the third series to investigate both the effects of the frequency at which a series is sampled and, at least possibly, the impact of subtracting short-term rates from equity returns. The following table conducts a step-by-step model specification search for each of the three series within the threshold-GARCH(p, d, q) class. In the light of earlier evidence, we start off with the case of both p and q being positive, i.e., we rule out simpler ARCH(p) models.

Table 5.B1 approximately here

Conditional Mean Model	Conditional Variance Model	p	d	q	Maximized Log-Lik	BIC	Hannan-Quinn	AIC
Daily 1963-2016 Excess Equity Returns from CRSP								
MA(1)	Homeskedastic	0	0	0	-19069.6	2.8070	2.8063	2.8059
MA(1)	GARCH(1,1)	1	0	1	-16255.1	2.3950	2.3932	2.3922
MA(1)	T-GARCH(1,1,1)	1	1	1	-16103.9	2.3735	2.3713	2.3702
MA(1)	T-GARCH(2,1,1)	2	1	1	-16102.3	2.3739	2.3714	2.3701
MA(1)	*T-GARCH(2,2,1)*	*2*	*2*	*1*	*-16091.2*	*2.3730*	*2.3700*	*2.3686*
MA(1)	T-GARCH(2,2,2)**	2	2	2	-16022.1	2.3635	2.3602	2.3586
MA(1)	T-GARCH(2,1,2)	2	1	2	-16099.2	2.3742	2.3712	2.3698
MA(1)	T-GARCH(3,2,2)**	3	2	2	-16035.8	2.3662	2.3626	2.3607
Weekly 1982-2016 10-year Treasury Yield Changes								
AR(1)	Homeskedastic	0	0	0	1257.2	-1.3744	-1.3763	-1.3774
AR(1)	*GARCH(1,1)*	*1*	*0*	*1*	*1424.7*	*-1.5457*	*-1.5536*	*-1.5578*
AR(1)	T-GARCH(1,1,1)	1	1	1	1425.1	-1.5420	-1.5515	-1.5571
AR(1)	T-GARCH(2,1,1)	2	1	1	1426.7	-1.5397	-1.5511	-1.5578
AR(1)	T-GARCH(2,2,1)**	2	2	1	1430.0	-1.5391	-1.5525	-1.5603
AR(1)	T-GARCH(2,2,2)**	2	2	2	1430.8	-1.5359	-1.5512	-1.5601
AR(1)	T-GARCH(2,1,2)	2	1	2	1427.0	-1.5358	-1.5492	-1.5570
AR(1)	T-GARCH(3,2,2)	3	2	2	1430.0	-1.5349	-1.5502	-1.5591
Monthly 1977-2016 Equity Returns								
CER	Homeskedastic	0	0	0	-1394.4	5.8228	5.8175	5.8141
CER	GARCH(1,1)	1	0	1	-1379.6	5.8000	5.7786	5.7649
CER	T-GARCH(1,1,1)	1	1	1	-1376.5	5.7998	5.7734	5.7563
CER	T-GARCH(2,1,1)**	2	1	1	-1375.0	5.8065	5.7748	5.7543
CER	T-GARCH(2,2,1)**	2	2	1	-1365.1	5.7782	5.7412	5.7173
CER	*T-GARCH(2,2,2)*	*2*	*2*	*2*	*-1365.2*	*5.7911*	*5.7488*	*5.7215*
CER	T-GARCH(2,1,2)	2	1	2	-1373.4	5.8125	5.7755	5.7516
CER	T-GARCH(3,2,2)**	3	2	2	-1364.7	5.8019	5.7544	5.7236

** = some of the ML estimates of GARCH coefficients turned out to be negative

Table 5.B1 – Information Criteria-Based Model Selection for Different Data Sets and Frequencies

The results are in some ways expected: stock return data contain strong evidence of a need to incorporate asymmetries in a GARCH model; bond returns data do not, and economically it would be more complex to find any justification for such asymmetries. On the oppo-

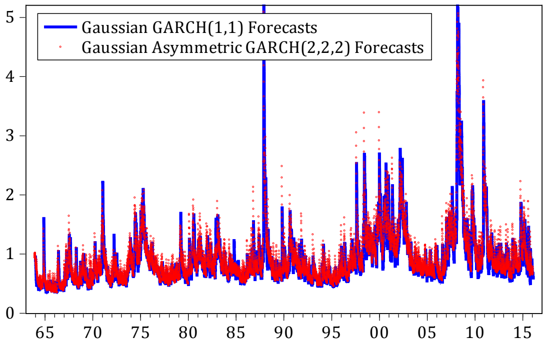
site, the frequency of the data does not seem to be crucial: stock index returns data contain leverage both at daily and monthly frequency and over two quite different time periods (the former is considerably longer, also including 1963-1976). Interestingly, for both equity data sets, the same, rich GARCH(2,2,2) model prevails. Technically, slightly more parsimonious GARCH(2,2,1) models prevail in terms of minimizing the information criteria, but these models are characterized by a few negative coefficients that make them usuitable to practical uses. This tendency of relatively large GARCH models to be selected by (all) standard information criteria is something relatively novel when compared to the empirical finance literature, possibly due to the fact that we are performing these estimations 30 years later the birth of the original GARCH model, so that much longer time series of data have become available.
To give an idea of typical estimates, in Figure 5.12, we report ML estimates and plot the forecasts of conditional volatility they imply. In the case of stock index returns, we also compare such forecasts with those that a symmetric GARCH(1,1) would obtain. P-values are in parentheses below ML estimates (obtained assuming normality); we also computed and report long-run ergodic volatilities obtained as the square root of long-run variances.

Figure 5.B1 approximately here

GARCH(2,2,2):

$$\sigma^2_{t+1|t} = \underset{(0.000)}{0.0003} + \underset{(0.340)}{0.003}\varepsilon_t^2 + \underset{(0.000)}{0.180} I_{\{\varepsilon_t<0\}}\varepsilon_t^2 + \underset{(0.909)}{0.000}\varepsilon_{t-1}^2 - \underset{(0.000)}{0.176} I_{\{\varepsilon_{t-1}<0\}}\varepsilon_{t-1}^2 + \underset{(0.000)}{0.794}\sigma^2_{t|t-1} + \underset{(0.000)}{0.202}\sigma^2_{t-1|t-2}$$

$$\bar{\sigma} = 0.958$$

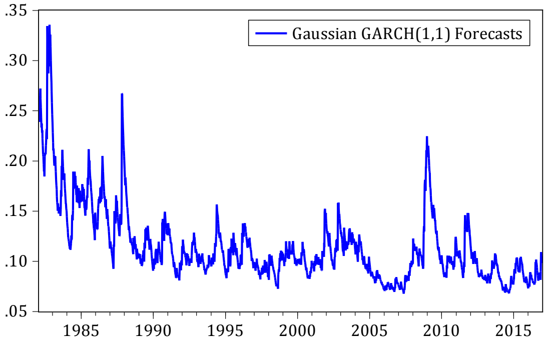


Panel (a) Daily US Equity Excess Returns

GARCH(1,0,1):

$$\sigma^2_{t+1|t} = \underset{(0.003)}{0.0002} + \underset{(0.000)}{0.073}\varepsilon_t^2 + \underset{(0.000)}{0.910}\sigma^2_{t|t-1}$$

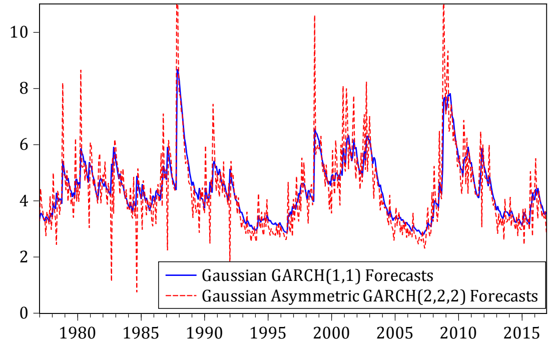
$$\bar{\sigma} = 0.117$$



Panel (b) Weekly 10-year Treasury Returns

GARCH(2,2,2)

$$\sigma^2_{t+1|t} = \underset{(0.130)}{0.749} + \underset{(0.000)}{0.103}\varepsilon_t^2 + \underset{(0.000)}{0.463} I_{\{\varepsilon_t<0\}}\varepsilon_t^2$$
$$+\underset{(0.000)}{0.235}\varepsilon_{t-1}^2 - \underset{(0.000)}{0.431} I_{\{\varepsilon_{t-1}<0\}}\varepsilon_{t-1}^2 +$$
$$+\underset{(0.000)}{0.778}\sigma^2_{t|t-1} + \underset{(0.797)}{0.049}\sigma^2_{t-1|t-2}$$
$$\bar{\sigma} = 5.353$$



Panel (c) Monthly US Equity Returns

Figure 5.B1 – Estimates and In-Sample Volatility Forecasts from Alternative GARCH Models

While in panel (a), the differences between plain vanilla GARCH(1,0,1) and a richer GARCH(2,2,2) with two lags of leverage effects are modest, in panel (c), at a monthly frequency, these become much more visible. Indeed, in panel (a) the correlation between the two sets of forecasts is 0.96 vs. 0.83 in panel (c). The threshold GARCH forecasts seem to jiggle around the symmetric GARCH one, and therefore appear "spikier" in both directions, in the sense there are a few months in the 1980s characterized by predicted volatility well below 2% per month that could not be forecast from the smoother GARCH(1,0,1). Such differences are presumably due to the fact that while in panel (a) both the standard ARCH coefficients are small and fail to be significant while the two asymmetric ARCH coefficients almost exactly cancel out, this does not occur in panel (c).