Example Let us consider a VAR(1) for the one-month U.S. Treasury bill and the ten-year Treasury bond yields (the same series for the January 1990 - December 2016 sample that we have estimated in Example 3.2):

$$\begin{bmatrix} y_{1M,t} \\ y_{10Y,t} \end{bmatrix} = \begin{bmatrix} \underset{[-2.5382]}{-0.0490} \\ \underset{[0.8711]}{0.0080} \end{bmatrix} + \begin{bmatrix} \underset{[210.6540]}{0.9819} & \underset{[0.4077]}{0.0209} \\ \underset{[3.3784]}{0.0009} & \underset{[240.0320]}{0.9970} \end{bmatrix} \begin{bmatrix} y_{1M,t-1} \\ y_{10Y,t-1} \end{bmatrix} + \begin{bmatrix} u_{1M,t} \\ u_{10Y,t} \end{bmatrix},$$

with estimated covariance matrix of the reduced-form residuals:

$$\hat{\mathbf{\Sigma}}_u \begin{bmatrix} 0.0476 & 0.0013 \\ 0.0013 & 0.011 \end{bmatrix}.$$

As we shall recall from Section 2.1, applying a Choleski decomposition we get that $\mathrm{var}[u_{1M,t}] = \sigma_1^2$, $\mathrm{var}[u_{10Y,t}] = \sigma_2^2 - b_{2,1}\sigma_1^2$, $\mathrm{cov}[u_{1M,t}, u_{10Y,t}] = -b_{2,1}\sigma_1^2$. Therefore, $b_{2,1}$ is equal to

$$b_{2,1} = -\frac{\hat{\sigma}_{1,2}}{\hat{\sigma}_1^2} = -\frac{0.0013}{0.4762} = -0.0027,$$

and equations (3.29)-(3.30) become

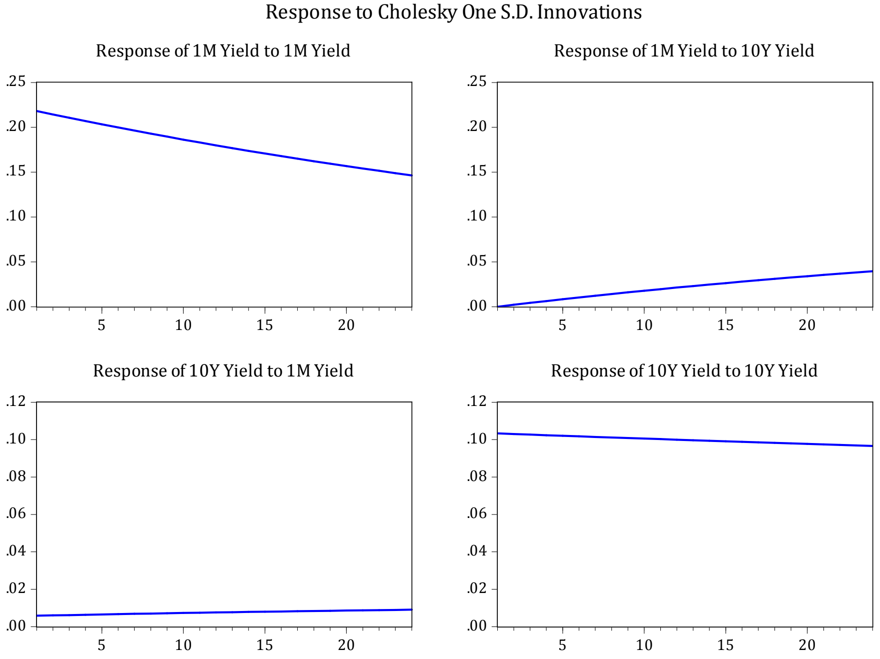
$$u_{1M,t} = \varepsilon_{1,t} ,$$
$$u_{10Y,t} = \varepsilon_{2,t} - b_{2,1}\varepsilon_{1,t} = \varepsilon_{2,t} - 0.0027\varepsilon_{1,t} .$$

This means that a shock to $\varepsilon_{1.t}$ equal to one-standard deviation (0.218, that is $\sqrt{0.0476}$) causes an immediate change by 0.218 in $u_{1M,t}$ (and thus in $y_{1M,t}$); in addition, it will also cause an immediate increase (albeit very small) of $0.218 \times 0.0027 = 0.0006$ in $u_{10Y,t}$ (and thus in the 10-year Treasury yield) because of the implicit correlation structure that is admissible under the selected Choleski scheme. At time $t+1$, the lagged value of the one-month yield enters the first equation with a coefficient 0.9819 and thus after one period the one-month yield will grow by $0.9819 \times 0.218 = 0.214$ (i.e., approximately 21 basis points, henceforth bps) above what it would have been without the shock. The ten-year yield would have been $0.9970 \times 0.0006 = 0.000598$ higher because of the effect of its own lag. In addition, the lagged value of the 1-month yield also enters the second equation with a coefficient 0.0009, and thus the 10-year Treasury yield will rise by an additional $0.0009 \times 0.218 = 0.000196$; in total, the 10-year Treasury yield would be approximately 0.00079 higher with respect to what it would have been without a shock to the 1-month yield. Therefore, one period after the one standard deviation shock to the one-month Treasury yield has occurred, the **cumulative response** of the one-month Treasury yield to its own shock would have been $0.218 + 0.214 = 0.432$, that is, 43 bps. In addition, the accumulated response of the ten-year Treasury yield to the one standard deviation shock to the one-month Treasury yield would have been $0.0006 + 0.00079 = 0.00139$. The process then progresses further over subsequent rounds of impulse and reaction.

Alternatively, it is easy to see what happens if we give a one standard deviation shock to $\varepsilon_{2,t}$ (equal to 0.105): $u_{10Y,t}$ immediately increases by 0.105 (and so does $y_{10Y,t}$), but nothing happens to $u_{1M,t}$. Therefore, at time $t+1$ the 10-year yield would be higher by $0.9970 \times 0.105 = 0.10469$ (i.e., approximately 10 bps) because of the effect of its own lag (for an accumulated response of

0.209). In addition, the lag of the ten-year yield now affects the 1-month yield with a coefficient of 0.0209 and therefore the one-month Treasury yield will be $0.0209 \times 0.10469 = 0.0021$ higher than it would have been without a shock happening to the 10-year Treasury yield.

Figure 3.3 depicts the impulse response functions to a one-standard deviation shock to the 1-month yield and to the 10-year yield on the basis of a Choleski triangular scheme that places the one-month yield at the top of the variable ordering.



<INSERT FIGURE 3.3 HERE>

Figure 3.3 – Impulse response functions to shocks to one-month and ten-year yields, ordered on the basis of a Choleski triangular scheme that places shocks to the one-month at the top of the ordering