We use daily data on US aggregate excess market returns (x_{t+1}) obtained as the difference between CRSP value-weighted stock returns (concerning all NYSE, NASDAQ, and AMEX listed stocks, over the relevant periods) for the long sample period Jan. 2, 1963 – Dec. 31, 2016, for a total of 13,594 observations. In particular, we specify a simple Gaussian AR(1) model for the conditional mean function and a Riskmetrics model for the conditional variance function:

$$x_{t+1} = \phi_0 + \phi_1 x_t + \varepsilon_{t+1} \qquad \varepsilon_{t+1} \ \text{IID} \ \text{N}(0, \sigma^2_{t+1|t}(\lambda))$$
$$\sigma^2_{t+1|t} = (1-\lambda)\varepsilon_t^2 + \lambda \sigma^2_{t|t-1},$$

The conditional mean function is $\mu_t(\phi_0, \phi_1) = \phi_0 + \phi_1 x_t$. Using E-Views, we have estimated by ML the model obtaining the following estimates (p-values are in parentheses underneath the corresponding coefficient):

$$x_{t+1} = \underset{(0.000)}{0.035} + \underset{(0.000)}{0.123} x_t + \varepsilon_{t+1} \qquad \varepsilon_{t+1} \ \text{IID} \ \text{N}(0, \sigma^2_{t+1|t})$$
$$\sigma^2_{t+1|t} = \underset{(0.000)}{0.064} \varepsilon_t^2 + \underset{(0.000)}{0.936} \ \sigma^2_{t|t-1},$$

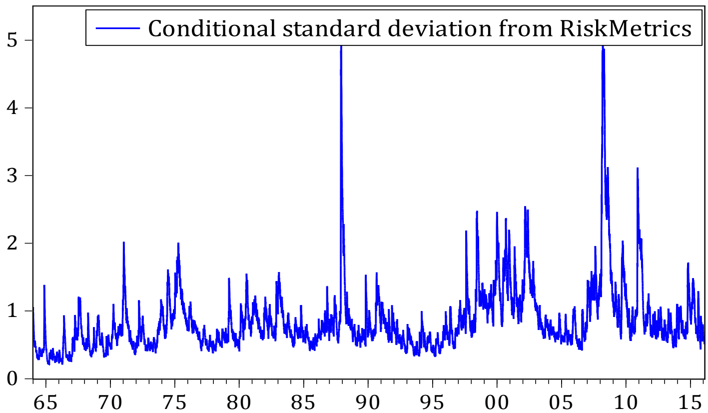

Figure 5A.1 – Plot of One-Day RiskMetrics Volatility Forecasts for US Excess Aggregate Stock Returns

We use the estimated model to forecast the conditional standard deviation of the excess stock return process, since

$$Var_t[x_{t+1}] = \phi_0 + \phi_1 Var_t[x_t] + Var_t[\varepsilon_{t+1}] = \sigma^2_{t+1|t},$$

so that it is natural to use $\sigma_{t+1|t} \equiv \sqrt{\sigma^2_{t+1|t}}$ as a forecast of volatility. Figure 5A.1 shows such forecasts.

Once more, the "law of the 0.94" estimate strikes: almost 30 years later, we find that $\hat{\lambda}=0.936$, which is close to 0.94 indeed. We move one step further and test this law on a different series of equity-related returns, those on the SMB ("Small-minus-Big") portfolio that goes long in the lowest quintile of the CRSP universe stocks in terms of market value and finances that position by shorting the highest quintile of CRSP stocks when sorted by their total market value. ML estimates are (p-values are in parentheses):

$$x_{t+1}=\underset{(0.000)}{0.010}+\underset{(0.000)}{0.111}x_t+\varepsilon_{t+1} \qquad \varepsilon_{t+1}\ \text{IID}\ \text{N}(0,\sigma^2_{t+1|t})$$

$$\sigma^2_{t+1|t}=\underset{(0.000)}{0.065}\varepsilon_t^2+\underset{(0.000)}{0.935}\ \sigma^2_{t|t-1},$$

Strikingly, even though the portfolio is very different (just think this is a long-short portfolio that in principle has no or small net beta exposure on the aggregate market portfolio), we obtain similar parameter estimates and also in this case $\hat{\lambda}=0.935$ falls very close to the 0.94 often recommended by the RiskMetrics experts. We use the estimated model to forecast the conditional standard deviation of the excess stock return process, as shown below.

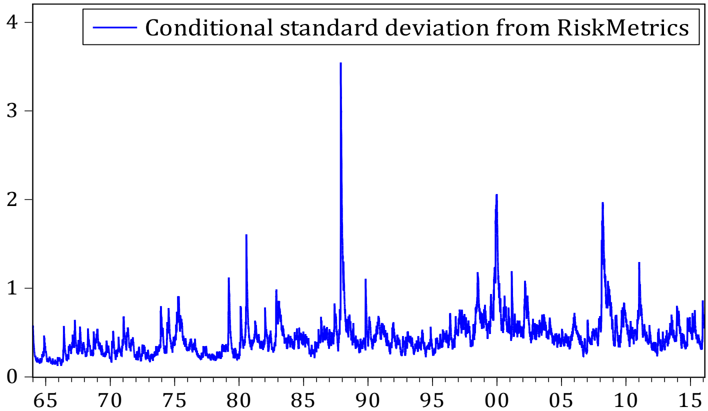

Figure 5A.2 – One-Day RiskMetrics Volatility Forecasts for SMB Returns