Case Study
The Economic Value of Conditional Correlations

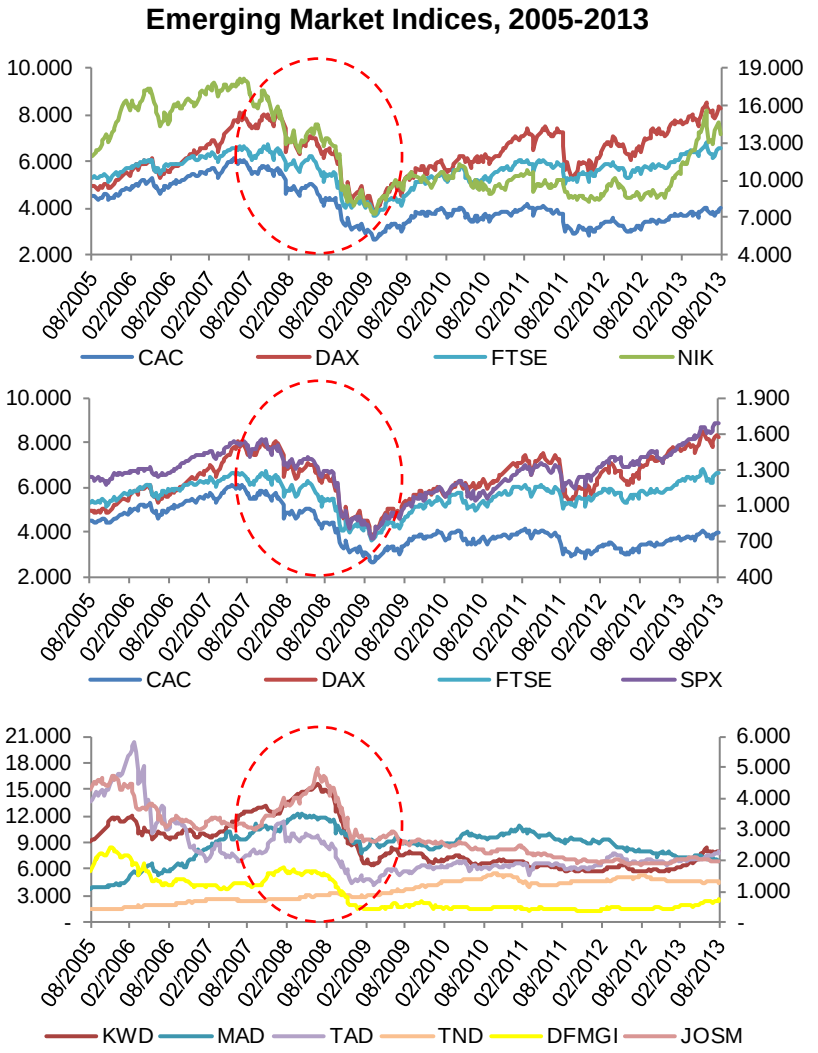
Review Session III

Outline

- Asset Span
- Preliminary Analysis
  - ✓ Unconditional Correlations
  - ✓ Time-Varying Correlations
- Modelling
  - ✓ *Dynamic Conditional Correlation* (DCC)
  - ✓ Statistical Tests
- Economic Value
  - ✓ Strategies and Performance Measures
  - ✓ Empirical Results
    - ✓ *Equally Weighted* (EW)
    - ✓ *Global Minimum Variance* (GMV)
    - ✓ *Mean – Variance* (MV)

Asset Span

- 11 Stock Market Indices:
  - ✓ **5 developed** markets: **France** (CAC 40), **Germany** (DAX 30), **Japan** (Nikkei 225), **USA** (S&P 500) e **UK** (FTSE 100)
  - ✓ **6 emerging** markets **MENA**: **Jordan** (JOSMGNFF), **Kuwait** (KWSEIDX), **Morocco** (MADEX),**Saudi Arabia** (TADAWUL), **Tunisia** (TUNINDEX) ed **UAE** (DFMGI)
- Weekly, August 2005 – August 2013
- 3 sub-samples:
  - ✓ Pre-Crisis: 08/2005-08/2007 (99 obs)
  - ✓ Crisis: 08/2007-08/2010 (149 obs)
  - ✓ Post-Crisis: 08/2010-08/2013 (152 obs)



Preliminary Analysis

Unconditional Correlations

- Developed Countries → High Correlations >70%
- Emerging Countries → Low Correlations < 50%
- Across the Crisis higher and stronger correlations

	CAC	DAX	NIK	SPX	FTSE	JOSM	KWD	MAD	TAD	TND	DFMGI
Intero campione											
CAC	1										
DAX	0,94 ***	1									
NIK	0,66 ***	0,65 ***	1								
SPX	0,84 ***	0,80 ***	0,63 ***	1							
FTSE	0,91 ***	0,86 ***	0,66 ***	0,86 ***	1						
JOSM	0,26 ***	0,30 ***	0,38 ***	0,19 ***	0,26 ***	1					
KWD	0,20 ***	0,21 ***	0,26 ***	0,16 ***	0,15 ***	0,44 ***	1				
MAD	0,08	0,09 *	0,10 **	0,08	0,07	0,23 ***	0,22 ***	1			
TAD	0,31 ***	0,32 ***	0,36 ***	0,26 ***	0,29 ***	0,40 ***	0,41 ***	0,08	1		
TND	0,09 *	0,10 **	0,12 **	0,07	0,08	0,20 ***	0,16 ***	0,14 ***	0,19 ***	1	
DFMGI	0,32 ***	0,32 ***	0,32 ***	0,29 ***	0,31 ***	0,45 ***	0,50 ***	0,13 ***	0,52 ***	0,15 ***	1
Pre-Crisi: 08/2005-08/2007											
CAC	1										
DAX	0,94 ***	1									
NIK	0,57 ***	0,54 ***	1								
SPX	0,71 ***	0,67 ***	0,36 ***	1							
FTSE	0,89 ***	0,82 ***	0,52 ***	0,68 ***	1						
JOSM	0,20 **	0,21 **	0,35 ***	0,08	0,21 **	1					
KWD	-0,08	-0,03	0,00	-0,11	-0,01	0,15	1				
MAD	0,20 **	0,22 **	0,08	0,12	0,12	0,20 **	0,06	1			
TAD	-0,08	-0,10	0,10	-0,13	-0,06	0,09	0,40 ***	0,07	1		
TND	0,08	0,08	-0,02	0,02	0,07	0,12	-0,09	0,22 **	-0,09	1	
DFMGI	-0,03	-0,02	0,08	-0,04	0,02	0,23 **	0,53 ***	0,13	0,45 ***	-0,09	1
Crisi: 08/2007-08/2010											
CAC	1										
DAX	0,95 ***	1									
NIK	0,75 ***	0,73 ***	1								
SPX	0,85 ***	0,81 ***	0,70 ***	1							
FTSE	0,92 ***	0,88 ***	0,71 ***	0,87 ***	1						
JOSM	0,36 ***	0,40 ***	0,47 ***	0,23 ***	0,30 ***	1					
KWD	0,28 ***	0,29 ***	0,34 ***	0,18 **	0,17 **	0,59 ***	1				
MAD	0,00	0,03	0,11	0,04	0,03	0,30 ***	0,41 ***	1			
TAD	0,52 ***	0,55 ***	0,55 ***	0,38 ***	0,43 ***	0,62 ***	0,48 ***	0,14 *	1		
TND	0,24 ***	0,24 ***	0,32 ***	0,17 **	0,19 **	0,37 ***	0,33 ***	0,15 *	0,34 ***	1	
DFMGI	0,42 ***	0,43 ***	0,40 ***	0,34 ***	0,36 ***	0,57 ***	0,51 ***	0,17 **	0,59 ***	0,30 ***	1
Post-Crisi: 08/2010-08/2013											
CAC	1										
DAX	0,92 ***	1									
NIK	0,53 ***	0,54 ***	1								
SPX	0,87 ***	0,85 ***	0,55 ***	1							
FTSE	0,90 ***	0,85 ***	0,59 ***	0,89 ***	1						
JOSM	0,09	0,14 *	0,18 **	0,17 **	0,19 **	1					
KWD	0,16 *	0,17 **	0,21 **	0,24 ***	0,20 **	0,21 **	1				
MAD	0,14 *	0,12	0,10	0,20 **	0,15 *	0,11	-0,07	1			
TAD	0,32 ***	0,32 ***	0,27 ***	0,38 ***	0,37 ***	0,24 ***	0,31 ***	0,05	1		
TND	-0,03	-0,01	-0,03	0,00	-0,01	0,06	0,17 **	0,04	0,37 ***	1	
DFMGI	0,33 ***	0,32 ***	0,27 ***	0,35 ***	0,38 ***	0,21 **	0,36 ***	0,09	0,51 ***	0,19 **	1

Preliminary Analysis

Time-Varying Correlations

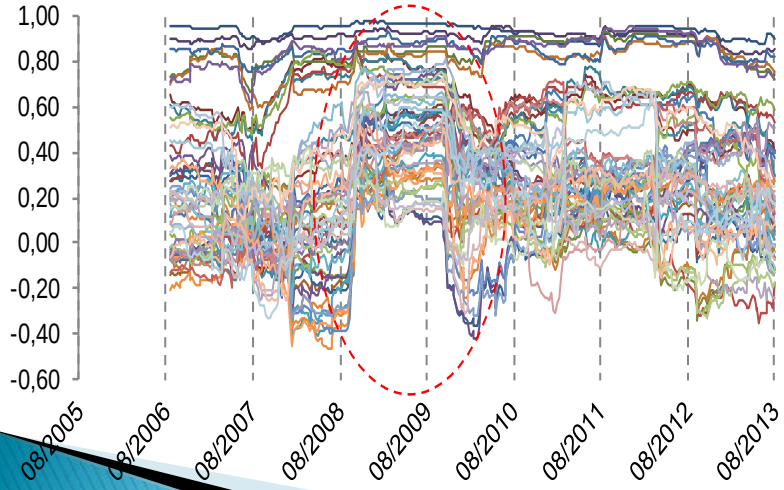
Rolling Window Correlation

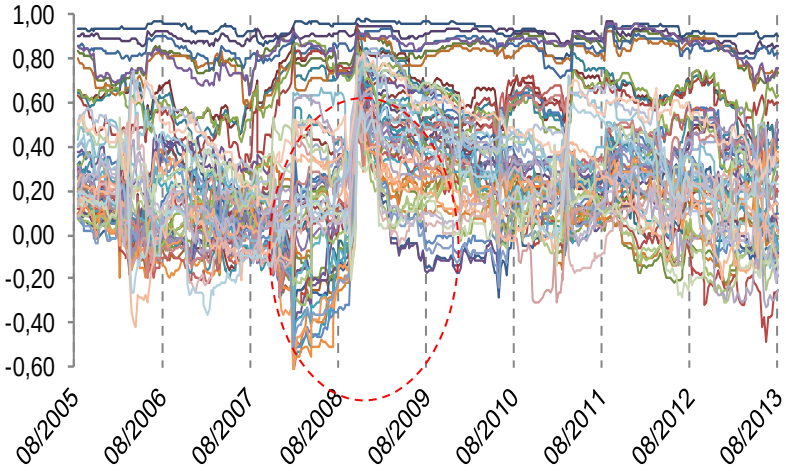
- Window *m=52* weeks
- Limits
  - ✓ Weight $1/m$ for each observation → Inefficient Dynamics
  - ✓ Strong dependence on the window size (the larger the size the smoother the dynamics)

Risk Metrics

$$\rho_{ij,t+1} = \frac{(1-\lambda) r_{i,t} r_{j,t} + \lambda \sigma_{ij,t}}{\sqrt{\left((1-\lambda) r_{i,t}^2 + \lambda \sigma_{i,t}^2\right)\left((1-\lambda) r_{j,t}^2 + \lambda \sigma_{j,t}^2\right)}}, \quad \forall i, j$$

- Advantages
  - ✓ Exponentially decreasing weights
  - ✓ One parameter λ (usually equal to 0.95 weekly)
- Limits
  - ✓ Infinite long-run volatilities
  - ✓ Permanent shocks in volatility





Dynamic Conditional Correlation

- Let us consider de-meaned asset returns $\boldsymbol{r}^*_{t+1} = \boldsymbol{r}_{t+1} - \boldsymbol{\mu}|\mathcal{F}_t \qquad \boldsymbol{r}^*_{t+1} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Omega}^{\mathrm{DCC}}_{t+1}\right)$
- The DCC model is based on a decomposition of the var-cov structure

$$\boldsymbol{\Omega}_{t+1} \equiv \mathbf{D}_{t+1}\boldsymbol{\Gamma}_{t+1}\mathbf{D}_{t+1}$$

Diagonal NxN of standard deviations $\sigma_{i,t+1}$

Correlation matrix $\rho_{ij,t+1}$

- Considering standardized residuals $z_{i,t+1} = r^*_{i,t+1}/\sigma_{i,t+1} \qquad z_{i,t+1} \sim \mathcal{N}(0,1)$ such that the conditional covariance is equivalent to the conditional correlation of the residuals
- Correlations are expressed in terms of auxiliary variables $q_{ij,t+1}$

$$\rho_{ij,t+1} = \frac{q_{ij,t+1}}{\sqrt{q_{ii,t+1}}\sqrt{q_{jj,t+1}}} \qquad q_{ij,t+1} = \rho_{ij} + \alpha\left(z_{i,t}z_{j,t} - \rho_{ij}\right) + \beta\left(q_{ij,t} - \rho_{ij}\right)$$

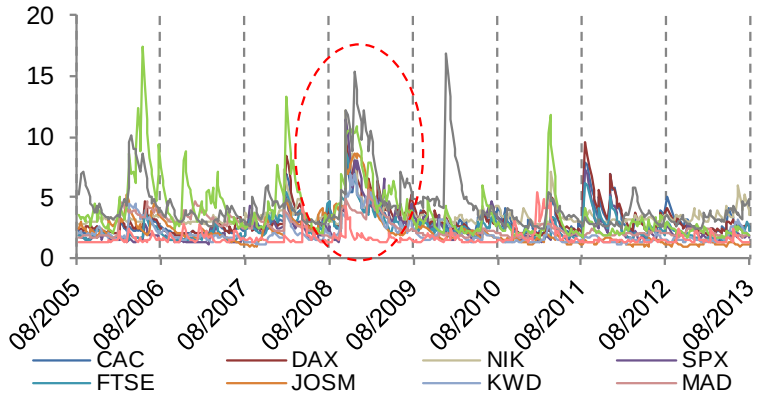
- Such DCC models are recursively estimated via *Quasi-Maximum Likelihood* in two steps:
  - ✓ Estimates of single-assets conditional volatility via a GARCH
  - ✓ Standardizing residuals by using volatilities estimates from the first step and estimate conditional correlations via auxiliary variables

Dynamic Conditional Correlations

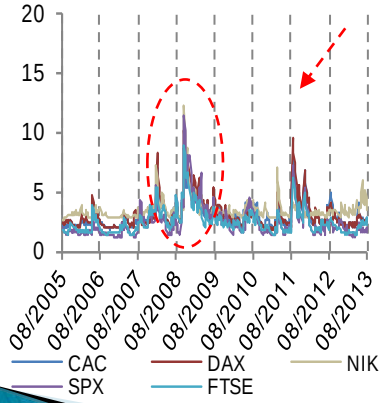
Results

- Higher volatilities after Lehman Brothers

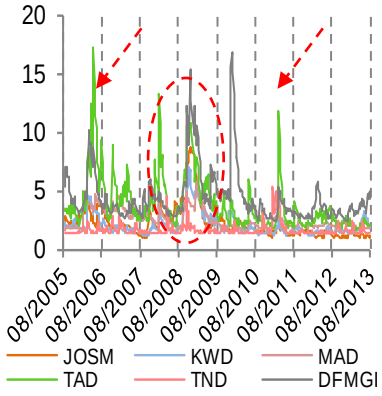
Conditional Volatility



Developing Countries



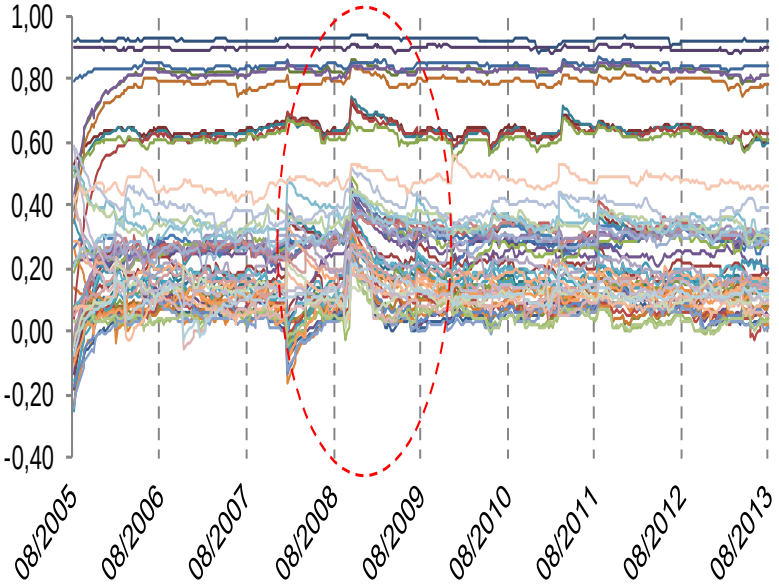
Emerging Markets



- Average Correlations

- ✓ France – Germany – UK – USA: 70%-95%
- ✓ Japan vs France, Germany, UK and USA: ~60%
- ✓ Saudi Arabia and UAE vs developed countries: ~30%
- ✓ Jordan and Kuwait vs developed countries: 15%-20%
- ✓ Morocco and Tunisia vs developed countries: 3%-6%

Dynamic Conditional Correlations



Economic Value

Strategies and performance measures

The Portfolio Strategies

- *Equally Weighted Portfolios* (EW)

- *Global Minimum Variance Porfolios* (GMV)
  - ✓ Minimize portfolio risk
  - ✓ No estimates of the conditional mean

$$\begin{aligned} \min_{w} \quad & w'\Sigma w \\ s.t. \quad & w'1 = 1 \\ & 0 \leq w \leq 1 \end{aligned}$$

- *Mean –Variance Porfolios* (MV)
  - ✓ Two estimation methods for the conditional mean
    - ✓ VAR (*Vector AutoRegressive Model*)

$$\begin{aligned} \max_{w} \quad & w'\mu - {}^{1}/_{2}\, w'\Sigma w \\ s.t. \quad & w'1 = 1 \\ & 0 \leq w \leq 1 \end{aligned}$$

Performance Measures

- *Sharpe Ratio* (SR)
  - ✓ Computed *out-of-sample* $SR_j = \frac{\bar{r}_{p,j}}{\sigma_{p,j}}$
  - ✓ Confidence Intervals $SR_j \pm 1{,}96 \times \sqrt{\frac{\left(1 + \frac{1}{2} SR_j^2\right)}{n}}$

- *Diversification Ratio* (DR)
  - ✓ Computed at time *t* as $DR_{P,t} = \frac{WAV_{P,t}}{\sigma_{P,t}}$

    Where the weighted average volatility is computed as

    $$\sum_{i=1}^{11} w_i \sigma_i$$

  - ✓ The DR of the *jth* strategy is equal to the average DRs across time
  - ✓ Greater than one if diversification benefit

Economic Value

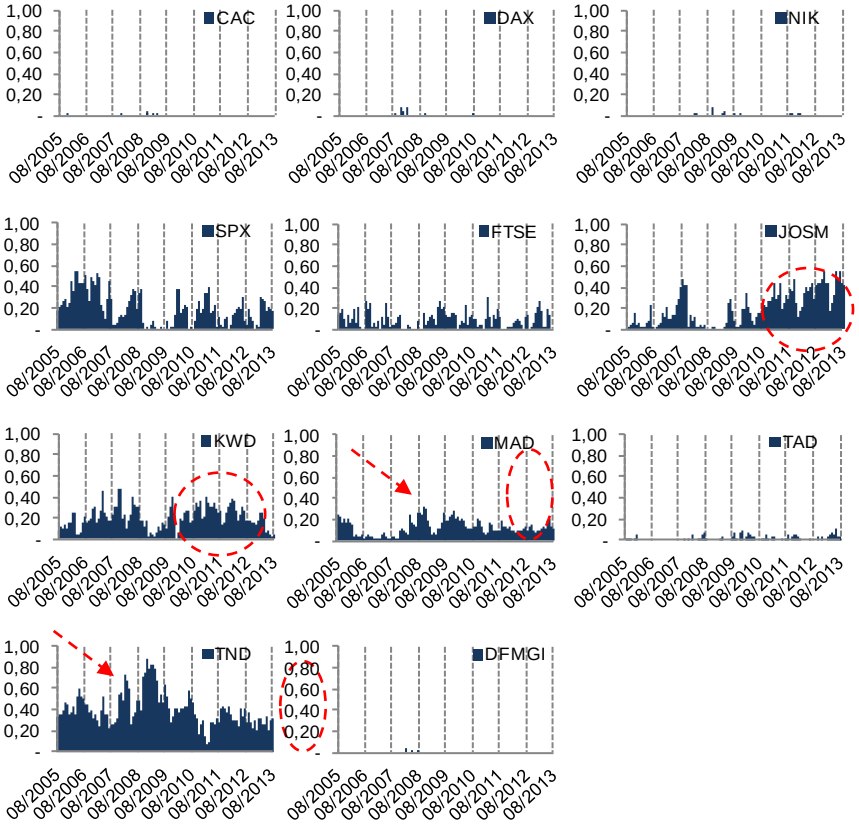
Equally Weighted Portfolios

	Mean	StDev	SR	IC(SR)	DR
Full Sample	0,097	14,547	0,007	[-0,098 , 0,111]	1,495
Pre-Crisis	12,968	6,817	1,902	[1,433 , 2,372]	2,264
Crisis	-9,375	19,087	-0,491	[-0,321 , -0,661]	1,404
Post-Crisis	5,182	10,699	0,484	[0,316 , 0,652]	1,709

- Unconditional Mean is not significant: 0,097%
  - ✓ Pre-Crisis: 13%
  - ✓ Crisis: -9%
  - ✓ Post-Crisis: 5%
- Opposite path for the standard deviation
  - ✓ Pre-Crisis: 7%
  - ✓ Crisis: 19%
  - ✓ Post-Crisis: 11%
- Negative Sharpe Ratio across the crisis
- DR: Diversification benefit
  - ✓ Portfolio volatilities are always lower than the weighted average volatility
  - ✓ Lower *Ratio* across the criis: Higher correlation among indices

Economic Value

Global Minimum Variance Portfolios



- The weights
  - ✓ CAC, DAX, NIK, TAD, DFMGI: No exposure
  - ✓ FTSE: Low exposure
  - ✓ JOSM, S&P, KWD: High exposure before and after the crisis
  - ✓ MAD,TND: Higher exposure across the financial crisis

	Mean	StDev	SR	IC(SR)	DR
Full Sample	4,640	8,628	0,538	[0,426 , 0,650]	1,523
Pre-Crisis	19,416	6,166	3,149	[2,465 , 3,832]	1,780
Crisis	4,935	11,137	0,443	[0,275 , 0,611]	1,454
Post-Crisis	-0,544	6,040	-0,090	[-0,249 , 0,069]	1,718

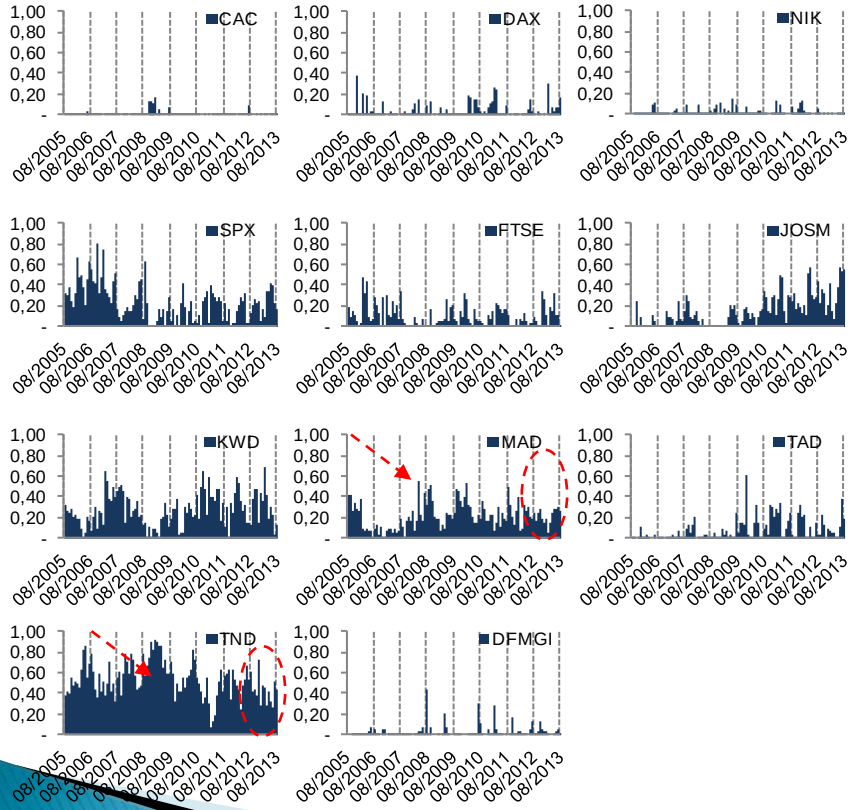
Economic Value

Mean –Variance Portfolios – VAR(1)

- DCC is applied on the residuals from VAR(1),

$$\boldsymbol{r}^*_{t+1} = \boldsymbol{r}_{t+1} - \boldsymbol{\mu}_{t+1} \qquad \boldsymbol{\mu}_{t+1|t} = \boldsymbol{A}_0 + \boldsymbol{A}_1 \boldsymbol{r}_t$$

$$\boldsymbol{r}^*_{t+1} \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Omega}^{DCC}_{t+1}\right)$$



- The weights
  - ✓ CAC, DAX, NIK, TAD, DFMGI: no exposure
  - ✓ SPX, FTSE: high exposure before the crisis
  - ✓ JOSM: high exposure post-crisis (50%-60%)
  - ✓ KWD: high exposure pre- and post-crisis
  - ✓ MAD,TND: high exposure throughout the sample (TND 80% across the crisis)

	Mean	StDev	SR	IC(SR)	DR
Full Sample	5,580	9,387	0,594	[0,481 , 0,708]	1,476
Pre-Crisis	27,223	5,708	4,769	[3,784 , 5,754]	1,671
Crisis	7,648	11,398	0,671	[0,493 , 0,849]	1,412
Post-Crisis	-3,342	7,851	-0,426	[-0,260 , -0,592]	1,583