Social Decision Theory: Choosing within and between Groups^{*}

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Abstract

We introduce a theoretical framework to study interdependent preferences, where the outcome of others affects the welfare of the decision maker. The dependence may take place in two conceptually different ways, which depend on how the decision maker evaluates his and others' outcomes. In the first he values his outcomes and those of others on the basis of his own utility. In the second, he ranks outcomes according to a social value function. These two representations express two different views of the nature and functional role of interdependent preferences. The first is Festinger's view that the evaluation of peers' outcomes is useful to improve individual choices by learning from the comparison. The second is Veblen's view that interdependent preferences keep track of social status derived from a social value attributed to the goods one consumes.

We give different axiomatic foundations to these two different, but complementary, views of the nature of the interdependence. On the basis of this axiomatic foundation we build a behavioral theory of comparative statics within subjects and across subjects. We characterize preferences according to the relative importance assigned to social gains and losses, that is, pride and envy. This parallels the standard analysis of private gains and losses (as well as that of regret and relief). We give an axiomatic foundation of inter personal comparison of preferences, ordering individuals according to their sensitivity to social ranking. These characterizations provide the behavioral foundation for applied analysis of market and game equilibria with interdependent preferences.

1 Introduction

Decision Theory has been mainly concerned with the private side of economic choices: standard preference functionals give no importance to the comparison of the decision makers' outcomes with those of their peers. This is in stark contrast with the large empirical literature that emphasizes the importance of relative outcomes in economic choice: the empirical significance of relative income and consumption has been widely studied, from Dusenberry's early contribution to the many recent works on external habits, the *keeping up the with the Joneses* phenomenon.

Our first purpose is to fill this important gap between theory and empirical evidence by providing a general choice model that takes into account the concern for relative outcomes. We generalize

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the classic subjective expected utility model by allowing decision makers' preferences to depend on their peers' outcomes. In order to do so we consider preferences defined over "act profiles," that is, vectors of acts whose first component is agent's own act and the other ones are his peers' acts. The axiomatic system and the representation are simple, and reduce naturally to the standard theory when the decision maker is indifferent to the outcome of others. In particular preferences in our theory are transitive, a fundamental property for economic applications. The representation is described in Section 1.1.

Our second purpose is to provide a sound basis for comparative statics analysis for interdependent preferences. How the relative standing of peers' outcomes affects preferences depends on the decision makers' attitudes toward social gains and losses, that is, on their feelings of envy and pride. We call envy the negative emotion that agents experience when their outcomes fall below those of their peers, and we call pride the positive emotion that agents experience when they have better outcomes than their peers. Attitudes toward social gains and losses describe the way concerns for relative outcomes affect individual preferences. These attitudes differ across individuals. Our second purpose is thus to provide the conceptual tools to make meaningful intra-personal comparisons ("a person is more proud than envious") and inter-personal comparisons ("a person is more envious than another one"). The psychological motivations for the concern for relative outcomes and their main characteristics are discussed in Section 1.2.

1.1 The Representation

We consider preferences of an agent o. Let $(f_o, (f_i)_{i \in I})$ represent the situation in which agent o takes act f_o , while each member i of the agent's reference group I takes act f_i . According to our main representation result, Theorem 3, agent o evaluates this situation according to:

$$V\left(f_{o},\left(f_{i}\right)_{i\in I}\right) = \int_{S} u\left(f_{o}\left(s\right)\right) dP\left(s\right) + \int_{S} \varrho\left(v\left(f_{o}\left(s\right)\right), \sum_{i\in I} \delta_{v\left(f_{i}\left(s\right)\right)}\right) dP\left(s\right).$$
(1)

The first term of this representation is familiar. The index $u(f_o(s))$ represents the agent's intrinsic utility of the realized outcome $f_o(s)$, while P represents his subjective probability over the state space S. The first term thus represents the agent's subjective expected utility from act f_o . The effect on o's welfare of the outcome of the other individuals is reported in the second term. The index $v(f_o(s))$ represents the social value, as subjectively perceived by o, of the outcome $f_o(s)$.

Given a profile of acts, agent's peers will get outcomes $(f_i(s))_{i \in I}$ once state s obtains. If o does not care about the identity of who gets the value $v(f_i(s))$, then he will only be interested in the distribution of these values. This distribution is represented by the term $\sum_{i \in I} \delta_{v(f_i(s))}$ in (1) above, where δ_x is the measure giving mass one to x. Finally, the function ρ is increasing in the first component and stochastically decreasing in the second. This term, which we call the positional index, represents o's satisfaction that derives from the comparison of his outcome with the distribution of outcomes in his reference group.

The choice criterion (1) is an ex ante evaluation, combining standard subjective expected utility and the ex post envy/pride feeling that decision makers anticipate. In choosing among acts decision makers consider both the private benefit of their choices and the externality derived from social comparison. Standard theory is the special case where the function ρ is identically zero. We consider this ex ante compromise as the fundamental trade-off that social decision makers face. This compromise takes a simple additive form in (1), which is a parsimonious extension of standard theory able to deal with concerns for relative outcomes. Behavioral foundation and parsimony are thus two major features of our criterion (1). In contrast, the ad hoc specifications used in empirical work often overlook this key trade-off and focus only on relative outcome effects, that is, on the ρ component of (1).

Finally, observe that for fixed $(f_i)_{i \in I}$ the preference functional (1) represents agent's within group preferences over acts, which are conditional on a group having $(f_i)_{i \in I}$. For fixed f_o , the preference functional (1) instead represents between groups preferences, which are conditional on the agent's act. Depending on which argument in V is fixed, either f_o or $(f_i)_{i \in I}$, the functional V thus represents preferences within or between groups.

1.2 Private and Social Emotions

The index ρ in representation (1) describes the effect on the decision maker's well being of the social profile of outcomes. The social value of these outcomes, as perceived by the decision maker, is recorded by the index v. If v is equal to the index u, the representation is derived in Theorem 2; if it is different, the representation is derived in Theorem 3. These two different representations correspond to two different views and explanations of the effect of the fortune of others on our preferences. To focus our analysis, we concentrate on envy. We propose two, complementary, explanations of this key social emotion, based on learning, dominance, and competition.

A Private Emotion Introspective analysis suggests that when we are envious we consider the outcomes of others, like goods and wealth, thinking how we would enjoy them, evaluating those goods from the point of view of our own utility, and comparing it to the utility that we derive from our own goods and wealth. This interpretation requires u = v in the representation (1).

This view of envy points to a possible functional explanation: the painful awareness that others are achieving something we consider enjoyable reminds us that perhaps we are not doing the best possible use of our abilities. Envy is a powerful tool of learning how to deal with uncertainty, by forcing us to evaluate what we have compared to what we could have. Envy is, from this point of view, the social correspondent of regret. These two emotions are both based on a counterfactual thought. Regret reminds us that we could have done better, had we chosen a course of action that was available to us, but we did not take. Envy reminds us that we could have done better, had we chosen a course of action that was available to us, but someone else actually chose, unlike us. In both cases, we are evaluating the outcome of chosen and discarded alternatives from the standpoint of our own utility function; that is, u = v in (1). We regret we did not buy a house that was cheap because we like the house and we envy the neighbor who bought it for the same reason. In both cases, we learn that next time we should be more careful in our choices.

A related view of envy motivates the classic theory of social comparison developed in the field of social psychology by Festinger (1954a, 1954b): People have a drive for a precise evaluation of their own abilities, and an important source of information for such an evaluation is provided by the outcome of others. A corollary of this premise is the similarity hypothesis: individuals will typically be more interested in the outcome of others who are similar, the peers, rather than dissimilar.¹

Envy is, however, an essentially social emotion. We do care whether the successful outcome is simply a counterfactual thought (as in regret) or the concrete good fortune of someone else. We may feel envy even if we do not like at all the good that the other person has. There must be an additional reason for envy, a purely social one.

¹This theory has been tested and further developed in the last fifty years: see, e.g., Suls, Martin, and Wheeler (2002) for an introduction to recent advances in Social Comparison Theory.

A Social Emotion The search for dominance through competition is a most important force among animals because of the privileged access to resources, most notably food and mates, that status secures to dominant individuals. The organization of societies according to a competition and dominance ranking is thus ubiquitous, extending from plants to ants to primates. Quite naturally, competition and dominance feelings play a fundamental role in human societies too, whose members have a very strong preference for higher positions in the social ranking: the proposition has been developed in social psychology, from Maslow (1937) to Hawley (1999) and Sidanius and Pratto (1999). Envy induced by the success of others is the painful awareness that we have lost relative positions in the social ranking. In this view the good that the other is enjoying is not only important for the utility it can provide and we do not enjoy, but also for its cultural and symbolic meaning, that is, for the signal it sends.

Since it is perceived in a social environment, the way in which it is evaluated has to be social and different from the way in which we privately evaluate it. We may secretly dislike, or fail to appreciate, an abstract painting. But, we may proudly display it in our living room if we think that the signal it sends about us, our taste, our wealth and our social network, is valuable. Therefore, we may envy someone who has it, even if we would never hang it in our bedroom if we had it. When the effect of the outcome of others is interpreted in this way, the evaluation index for outcomes is a function v, possibly different from the private utility function u.

The social index v is as subjective as u is: even if they evaluate the outcome of others according to v, individuals have ultimately personal views on what society considers important. An individual may have a completely wrong view of what peers deem socially important; but it is this perception, as opposed to what the decision maker privately values and likes, that is taken into account when evaluating peers' outcomes. This is what *subjectively* (as everything else in decision theory) the individual regards as *socially* valuable.

Though conceptually different, the private and social views of envy (and pride) are complementary: we can envy our neighbor's Ferrari both because we would like to drive it (user value) and because of its symbolic value. The index v reflects the overall, cumulative, "outcome externality" that the decision maker perceives, that is, his overall relative outcome concerns. The scope of the social emotion, however, can be behaviorally revealed when the indexes u and v differ. For example, if two outcomes are equally ranked by u, but differently by v, then the different v ranking can be properly attributed to the outcomes' symbolic value. The classic silver spoon example of Veblen well illustrates this case: though the aluminium and silver spoons likely share the same u value, they might well have different v values, entirely due to their different symbolic value. In Section 5 we discuss in some more detail these issues.

1.3 Related Literature

The modern economic formulation of the idea that the welfare of an agent depends on both the relative and the absolute consumption is usually attributed to Veblen (1899). His underlying assumption is that agents have a direct preference for ranking.² Fifty years after *The Theory of the Leisure Class*, social psychology dealt with the issue of social comparison with the works of Festinger (1954a, 1954b). The focus of Festinger's theory is orthogonal to Veblen's: the comparison with others is motivated by learning, and the outcome of the others is relevant to us only because it provides information that may be useful in improving our performance. Veblen and Festinger provide the two main directions in research on social emotions. Our work is an attempt to provide a structure in which these two views can be compared and experimentally tested. Veblen's view has been dominant or even exclusive in

 $^{^{2}}$ For example, he writes that "In any community ... it is necessary ... that an individual should possess as large a portion of goods as others with whom he is accustomed to class himself; and it is extremely gratifying to possess something more than others ..."

inspiring research in Economics (e.g., Duesenberry 1949, Easterlin 1974, Frank 1985). We hope that our paper may help in restoring a more balanced view.

The significance of one's relative outcome standing has been widely studied in the economics and psychology of subjective well-being (e.g., Easterlin 1995 and Frey and Stutzer 2002, and the references therein) and there is a large body of direct and indirect empirical evidence in support of this fundamental hypothesis (e.g., Easterlin 1974, van de Stadt, Kapteyn, and van de Geer 1985, Tomes 1986, Clark and Oswald 1996, McBride 2001, Zizzo and Oswald 2001, Luttmer 2005).

A first theoretical approach had been proposed in Michael and Becker (1973), Becker (1974), and Stigler and Becker (1977) in which utility analysis is reformulated by considering basic needs as inputs of agents' objective functions, in place of market consumption goods. For example, Becker (1974) considers an objective function with the "need of distinction" as a primitive argument. The latter are viewed as inputs in household production functions.

More recent work has provided an endogenous explanation for such need for distinction, either for evolutionary reasons or as behavior resulting at equilibrium in a market or a game. For example, the theoretical studies of Samuelson (2004) and Rayo and Becker (2007) investigate the emergence of relative outcome concerns from an evolutionary point of view. They show how it can be evolutionary optimal to build relative outcome effects directly into the utility functions. Bagwell and Bernheim (1996) develop a similar intuition in a market economy with conspicuous goods (their consumption is observed) and un-conspicuous goods: agents are willing to bear a cost to signal wealth through conspicuous consumption (see also Ireland, 1994, and Hopkins and Kornienko, 2004, for a related game-theoretic perspective on relative outcome concerns).

A possible explanation of social preferences within a neoclassical setting has been pursued by Cole, Mailath, and Postlewaite in a series of influential papers starting with their 1992 article.³ In their analysis, the wealth of an agent allows acquisition of goods that are assigned with a matching mechanism and are not available in the market: instead, they are assigned according to the position of the individual in some ranking (for example, in the ranking induced by wealth). Thus, a higher status allows better consumption because it gives access to goods that are not otherwise available. The non market sector generates endogenously a concern for relative position.

Our aim here is different: we want to provide behavioral foundation for preference functionals that incorporate concern for status, providing a link between choices of agents and their preferences. On this basis we then show how to recover the social values that agents attribute to goods, and other fundamental features of the preferences, in particular the comparative statics analysis. Through these results one can then analyze features of preferences as derived, for instance, in model providing endogenously generated concerns for social position.

Closer to our approach are the papers of Ok and Koçkesen (2000), Gilboa and Schmeidler (2001), Gul and Pesendorfer (2005), Karni and Safra (2002). Ok and Koçkesen consider negative interdependent preferences over income distributions x and provide an elegant axiomatization of the relative income criterion $x_o f(x_o/\overline{x})$, where \overline{x} is the society average income and f is a strictly increasing function. In deriving their criterion, Ok and Koçkesen emphasize the distinction in agents' preferences over income distributions between relative and individual income effects, modeled by $f(x_o/\overline{x})$ and x_o , respectively. This distinction is a special instance of the general trade-off between private benefits and social externalities we discussed before.⁴

³See Postlewaite (1998) for a review.

⁴In particular, the ordinal logarithmic transformation of the criterion $x_o f(x_o/\overline{x})$ is a special case of Theorem 4 below, in which we axiomatize a version of the general criterion where decision makers only care about average outcomes.

1.4 Organization of the Paper

The rest of the paper is organized as follows. Section 2 presents some preliminary notions, used in Section 3 to state our basic axioms. Sections 4 and 5 contain our main results; in particular, in Section 4 we prove the private utility representation and in Section 5 we derive the social value one. The relation between the two orders is analyzed in Section 6. Section 7 considers a very important special case, where the decision maker is only sensitive to the average of others' payoffs (the most common synthetic representation of society's consumption). Sections 8 and 9 provide behaviorally based conditions on the shapes of the elements of the representations. Inequity Aversion is shown to be a special case of our analysis in Section 10. Finally, Section 11 contains some concluding remarks. All proofs are collected in the Appendix.

2 Preliminaries

We consider a standard Anscombe and Aumann (1963) style setting. Its basic elements are a set S of states of nature, an algebra Σ of subsets of S called *events*, and a convex set C of consequences. We denote by o a given *agent* and by N the non-empty, possibly infinite, set of all agents in o's world that are different from o himself, that is, the set of all his possible peers.

We denote by $\wp(N)$ the set of all finite subsets of N, with $\emptyset \in \wp(N)$. Throughout the paper, I denotes a generic element of $\wp(N)$. For every I, we denote by I_o the set $I \cup \{o\}$; similarly, if j does not belong to I, we denote by I_j the set $I \cup \{j\}$.

An act is a finite-valued, Σ -measurable function from S to C. We denote by \mathcal{A} the set of all acts and by \mathcal{A}_i the set of all acts available to agent $i \in N_o$; finally

$$\mathcal{F} = \left\{ \left(f_o, (f_i)_{i \in I} \right) : I \in \wp(N), \ f_o \in \mathcal{A}_o, \text{ and } f_i \in \mathcal{A}_i \text{ for each } i \in I \right\}$$

is the set of all *act profiles*. Each act profile $f = (f_o, (f_i)_{i \in I})$ describes the situation in which o selects act f_o and his peers in I select the acts f_i . When I is the empty set (i.e., o has no reference group of peers), we have $f = (f_o)$ and we often will just write f_o to denote such profile.

The constant act taking value c in all states is still denoted by c. With the usual slight abuse of notation, we thus identify C with the subset of the constant acts. The set of acts profiles consisting of constant acts is denoted by \mathcal{X} , that is,

$$\mathcal{X} = \left\{ \left(x_o, (x_i)_{i \in I} \right) : I \in \wp(N), \ x_o \in C \cap \mathcal{A}_o, \text{ and } x_i \in C \cap \mathcal{A}_i \text{ for each } i \in I \right\}.$$

Clearly, $\mathcal{X} \subseteq \mathcal{F}$ and we denote by c_{I_o} an element $x = (x_o, (x_i)_{i \in I}) \in \mathcal{X}$ such that $x_i = c$ for all $i \in I_o$.⁵

Throughout the paper we make the following structural assumption.

Assumption. $\mathcal{A}_o = \mathcal{A}$ and each \mathcal{A}_i contains all constant acts.

In other words, we assume that o can select any act and that his peers can, at least, select any constant act. This latter condition on peers implies that the consequences profile at s, $f(s) = (f_o(s), (f_i(s))_{i \in I})$, belong to \mathcal{X} for all $f = (f_o, (f_i)_{i \in I}) \in \mathcal{F}$ and all $s \in S$.

We now introduce distributions, which play a key role in the paper. Let A be any set, for example a set of outcomes or payoffs. If $I \in \wp(N)$ is not empty, set $A^I = \times_{i \in I} A$. Given a vector $\mathbf{a} = (a_i)_{i \in I} \in A^I$, we denote by $\mu_{\mathbf{a}} = \sum_{i \in I} \delta_{a_i}$ the distribution of $\mathbf{a}^{.6}$ That is, for all $b \in A$,

$$\mu_{\mathbf{a}}(b) = \sum_{i \in I} \delta_{a_i}(b) = |\{i \in I : a_i = b\}|.$$

⁵Similarly, c_I denotes a constant $(x_i)_{i \in I}$.

⁶Remember that δ_a is the measure on the set of all subsets of A assigning weight 1 to sets containing a and 0 otherwise.

In other words, $\mu_{\mathbf{a}}(b)$ is the number of indices *i*, that is, of agents, that get the same element *b* of *A* under the allocation **a**.

Let $\mathcal{M}(A)$ be the collection of all integer valued measures μ on the set of all subsets of A, with finite support, and such that $\mu(A) \leq |N|$. That is,⁷

$$\mathcal{M}(A) = \left\{ \sum_{i \in I} \delta_{a_i} : I \in \wp(N) \text{ and } a_i \in A \text{ for all } i \in I \right\}.$$

In other words, $\mathcal{M}(A)$ is the set of all possible distributions of vectors $\mathbf{a} = (a_i)_{i \in I}$ in A^I , while I ranges in $\wp(N)$.

Set

$$pim(A) = A \times \mathcal{M}(A).$$
⁽²⁾

For example, when A is a set of payoffs, pairs $(z, \mu) \in pim(A)$ are understood to be of the form

(payoff of *o*, distribution of peers' payoffs).

A function ρ : pim $(A) \to \mathbb{R}$ is *diago-null* if

$$\varrho(z, n\delta_z) = 0, \qquad \forall z \in A, 0 \le n \le |N|.$$
(3)

For example, when A is a set of outcomes, a diago-null function ρ is zero whenever o and all his peers are getting the same outcome.

When $A \subseteq \mathbb{R}$, a companion set to pim (A) is pid (A), the set of triplets (z, μ, μ') such that $z \in A$, μ and μ' are positive integer measures finitely supported in $\{a \in A : a < z\}$ and $\{a \in A : a \ge z\}$, with $(\mu + \mu')(A) \le |N|$. If A is a set of payoffs, the elements of pid (A) distinguish peers that are worse off and peers that are better off than o. The natural variation in the definition of diago-nullity for a function ρ defined on pid (A) requires that $\rho(z, 0, n\delta_z) = 0$ for all $z \in A$ and $0 \le n \le |N|$.

In the case $A \subseteq \mathbb{R}$ the order structure of \mathbb{R} makes it possible to introduce monotone distribution functions. Specifically, given $\mathbf{a} \in \mathbb{R}^{I}$,

$$F_{\mathbf{a}}(t) = \mu_{\mathbf{a}}((-\infty, t]) = |\{i \in I : a_i \le t\}|$$

and

$$G_{\mathbf{a}}(t) = \mu_{\mathbf{a}}((t,\infty)) = |\{i \in I : a_i > t\}| = |I| - F_{\mathbf{a}}(t)$$

are the increasing and decreasing distribution functions of \mathbf{a} , respectively.⁸

Given two vectors $\mathbf{a} = (a_i)_{i \in I} \in \mathbb{R}^I$ and $\mathbf{b} = (b_j)_{j \in J} \in \mathbb{R}^J$, we say that:

- (i) $\mu_{\mathbf{a}}$ upper dominates $\mu_{\mathbf{b}}$ if $G_{\mathbf{a}}(t) \geq G_{\mathbf{b}}(t)$ for all $t \in \mathbb{R}$,
- (ii) $\mu_{\mathbf{a}}$ lower dominates $\mu_{\mathbf{b}}$ if $F_{\mathbf{a}}(t) \leq F_{\mathbf{b}}(t)$ for all $t \in \mathbb{R}$,
- (iii) $\mu_{\mathbf{a}}$ stochastically dominates $\mu_{\mathbf{b}}$ if $\mu_{\mathbf{a}}$ both upper and lower dominates $\mu_{\mathbf{b}}$.

Notice that (i) and (ii) are equivalent when |I| = |J|. In this case it is enough, for example, to say that $\mu_{\mathbf{a}}$ stochastically dominates $\mu_{\mathbf{b}}$ if $F_{\mathbf{a}}(t) \leq F_{\mathbf{b}}(t)$ for all $t \in \mathbb{R}$.

⁷We adopt the convention that any sum of no summands (i.e., over the empty set) is zero.

⁸When $I = \emptyset$, then $\mu_{\mathbf{a}} = 0$ and so $F_{\mathbf{a}} = G_{\mathbf{a}} = 0$.

3 Basic Axioms and Representation

Our main primitive notion is a binary relation \succeq on the set \mathcal{F} that describes o's preferences. The ranking

$$(f_o, (f_i)_{i \in I}) \succeq (g_o, (g_j)_{j \in J})$$

indicates that agent o weakly prefers society $f = (f_o, (f_i)_{i \in I})$ where o takes act f_o and each $i \in I$ takes act f_i , to society $g = (g_o, (g_j)_{j \in J})$. Note that the peer groups I and J in the two act profiles may be different.

Axiom A. 1 (Nontrivial Weak Order) \succeq is nontrivial, complete, and transitive.

Axiom A. 2 (Monotonicity) Let $f, g \in \mathcal{F}$. If $f(s) \succeq g(s)$ for all s in S, then $f \succeq g$.

Axiom A. 3 (Archimedean) For all $(f_o, (f_i)_{i \in I})$ in \mathcal{F} , there exist \underline{c} and \overline{c} in C such that

$$\underline{c}_{I_o} \precsim (f_o, (f_i)_{i \in I}) \text{ and } (f_o, (f_i)_{i \in I}) \precsim \overline{c}_{I_o}.$$

Moreover, if the above relations are both strict, there exist $\alpha, \beta \in (0,1)$ such that

$$\left(\alpha\underline{c} + (1-\alpha)\overline{c}\right)_{I_o} \prec \left(f_o, (f_i)_{i\in I}\right) \text{ and } \left(f_o, (f_i)_{i\in I}\right) \prec \left(\beta\underline{c} + (1-\beta)\overline{c}\right)_{I_o}$$

These first three axioms are social versions of standard axioms. By Axiom A.1, we consider a complete and transitive preference \succeq . For transitivity, which is key in economic applications,⁹ it is important that the domain of preferences includes peers' acts. For, suppose there are two agents, o and -o. The following choice pattern

$$(f_o, g_{-o}) \succ (g_o, f_{-o})$$
 and $(g_o, h_{-o}) \succ (h_o, g_{-o})$ and $(h_o, f_{-o}) \succ (f_o, h_{-o})$

does not violate transitivity. If, however, one observed only the projection on the first component $f_o \succ g_o \succ h_o \succ f_o$, one might wrongly conclude that a preference cycle is exhibited by these preferences. But, this would be due to the incompleteness of the observation, which ignores the presence of a society, and not to actual intransitive behavior of the agent.

The monotonicity Axiom A.2 requires that if an act profile f is, state by state, better than another act profile g, then $f \succeq g$. Note that in each state the comparison is between social allocations, that is, between elements of \mathcal{X} . In each state, o is thus comparing outcome profiles, not just his own outcomes.

Finally, Axiom A.3 is an Archimedean axiom that reflects the importance of the decision makers' private benefit that they derive from their own outcomes, besides any possible relative outcome concern. In fact, according to this axiom, given any profile $(f_o, (f_i)_{i \in I})$ it is always possible to find an egalitarian profile \overline{c}_{I_o} that o prefers. In this egalitarian profile there are no relative concerns and so only the private benefit of the outcome is relevant. When large enough, such benefit is able to offset any possible relative effect that arises in the given profile $(f_o, (f_i)_{i \in I})$. In a similar vein, also a dispreferred egalitarian profile \underline{c}_{I_o} can be always found.

Axiom A. 4 (Independence) Let α in (0,1) and f_o, g_o, h_o in \mathcal{A}_o . If $(f_o) \succ (g_o)$, then

$$\left(\alpha f_{o} + (1 - \alpha) h_{o}\right) \succ \left(\alpha g_{o} + (1 - \alpha) h_{o}\right).$$

⁹In fact, these applications are typically based on some optimization problem that, without transitivity, in general does not have solutions.

This is a classic *independence axiom*, which we only require on preferences for a single individual, with no peers.

We have so far introduced axioms which are adaptations of standard assumptions to our setting. The next axioms, instead, are peculiar to our analysis.

Axiom A. 5 (Conformistic Indifference) For all c in C, I in $\wp(N)$, and j not in I, $c_{I_o} \sim c_{I_o \cup \{i\}}$.

According to this axiom, for agent o it does not matter if to an "egalitarian" group, where everybody has the same outcome c, is added a further peer with that outcome c. Axiom A.5 thus describes a very simple form of the trade-off, from the standpoint of the preferences of o, between an increase in the size of the society and the change in the outcome necessary to keep him indifferent. In the representation this axiom translates into the condition that the externality function ρ is zero when all members of the group have the same outcome.

Different trade-offs have a similar axiomatization. For example, if o prefers, for the same outcome c, a smaller society, then a similar axiom would require that, for some improvement over c, he would feel indifferent between the smaller society with a less preferred outcome and a larger one with better common outcome. With this more general axiom, the externality function would also depend on the size of the group.¹⁰

Axiom A.5 per se is especially appealing for large groups; in any case, we regard it as a transparent and reasonable simplifying assumption, whose weakening would complicate the derivation without a comparable benefit for the interpretation.

The next final basic axiom is an anonymity condition, which assumes that decision makers do not care about the identity of who, among their peers, gets a given outcome. This condition requires that only the distribution of outcomes matters, without any role for possible special ties that decision makers may have with some of their peers. This allows to study relative outcomes effects in "purity," without other concerns intruding into the analysis.

Axiom A. 6 (Anonymity) Let $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J})$ in \mathcal{X} . If there is a bijection $\pi : J \to I$ such that $y_j = x_{\pi(j)}$ for all $j \in J$, then $(x_o, (x_i)_{i \in I}) \sim (x_o, (y_j)_{j \in J})$.

Axioms A.1-A.5 guarantee that for each $(f_o, (f_i)_{i \in I}) \in \mathcal{F}$ there exists a $c_o \in C$ such that $(f_o, (f_i)_{i \in I}) \sim (c_o)$. Such element c_o will be denoted by $c(f_o, (f_i)_{i \in I})$.¹¹

Preferences that satisfy our basic axioms have a basic representation, which separates in an additive way the direct utility of the decision maker on own outcomes (the function u) and an externality term (the function ρ) on own and others' outcomes. The comparative statics results hold for this general representation, providing a behavioral characterization of general properties of this externality function.

Theorem 1 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.6 if and only if there exist a nonconstant affine function $u: C \to \mathbb{R}$, a diago-null function $\varrho: \text{pim}(C) \to \mathbb{R}$, and a probability P on Σ such that

$$V(f) = \int_{S} u(f_{o}(s)) dP(s) + \int_{S} \rho\left(f_{o}(s), \sum_{i \in I} \delta_{f_{i}(s)}\right) dP(s)$$

$$\tag{4}$$

represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

¹⁰In this case, $\rho(z, n\delta_z)$ would be a suitable decreasing function of n.

¹¹The existence of c_o is proved in Lemma 4 of the Appendix, which also shows that Axioms A.1-A.5 deliver a first simple representation.

In this basic representation relative outcome concerns are captured by the externality function ϱ : pim $(C) \to \mathbb{R}$, which depends on both agent's o own outcome $f_o(s)$ and on the distribution $\sum_{i \in I} \delta_{f_i(s)}$ of peers' outcomes. In fact, a pair $(z, \mu) \in \text{pim}(C)$ reads as

(outcome of *o*, distribution of peers' outcomes).

Such representation is essentially unique:

Proposition 1 Two triples (u, ϱ, P) and $(\hat{u}, \hat{\varrho}, \hat{P})$ represent the same relation \succeq as in Theorem 1 if and only if $\hat{P} = P$ and there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$ and $\hat{\varrho} = \alpha \varrho$.

4 Private Utility Representation

In this section we present our first representation, which models the private emotion discussed in the Introduction.

The basic Axioms A.1-A.6 are common to our two main representations, the private and the more general social one. The next two axioms are, instead, peculiar to the private representation. They only involve deterministic act profiles, that is, elements of \mathcal{X} .

Axiom B. 1 (Negative Dependence) If $\overline{c} \succeq \underline{c}$ then

$$\left(x_o, (x_i)_{i \in I}, \underline{c}_{\{j\}}\right) \succeq \left(x_o, (x_i)_{i \in I}, \overline{c}_{\{j\}}\right)$$

$$\tag{5}$$

for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$.

Axiom B.1 is a key behavioral condition because it captures the negative dependence of agent o welfare on his peers' outcomes. In fact, according to Axiom B.1 the decision maker o prefers, *ceteris paribus*, that a given peer j gets an outcome that he regards less valuable. In this way, he behaviorally reveals his envious/proud nature.

Axiom B. 2 (Comparative Preference) Let $(x_o, (x_i)_{i \in I}), (y_o, (x_i)_{i \in I}) \in \mathcal{X}$. If $x_o \succeq y_o$, then

$$\frac{1}{2}c\left(x_{o}, (x_{i})_{i \in I}\right) + \frac{1}{2}y_{o} \succeq \frac{1}{2}x_{o} + \frac{1}{2}c\left(y_{o}, (x_{i})_{i \in I}\right).$$

Axiom B.2 is based on the idea that the presence of a society stresses the perceived differences in consumption. For example, interpreting x_o as a gain and y_o as a loss, the idea is that winning in front of a society is better than winning alone, losing alone is better than loosing in front of a society, and, "hence," a fifty-fifty randomization of the better alternatives is preferred to a fifty-fifty randomization of the worse ones.

We can now state the private utility representation, where we use the notation introduced in Section 2.2. In particular, recall from (2) that $pim(u(C)) = u(C) \times \mathcal{M}(u(C))$.

Theorem 2 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.6 and B.1-B.2 if and only if there exist a non-constant affine function $u: C \to \mathbb{R}$, a diago-null function $\varrho: \text{pim}(u(C)) \to \mathbb{R}$ increasing in the first component and decreasing (w.r.t. stochastic dominance) in the second one, and a probability Pon Σ such that

$$V\left(f_{o},(f_{i})_{i\in I}\right) = \int_{S} u\left(f_{o}\left(s\right)\right) dP\left(s\right) + \int_{S} \varrho\left(u\left(f_{o}\left(s\right)\right),\sum_{i\in I}\delta_{u\left(f_{i}\left(s\right)\right)}\right) dP\left(s\right)$$

$$\tag{6}$$

represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

In this representation relative outcome concerns are captured by the positional index ρ : pim $(u(C)) \rightarrow \mathbb{R}$, which depends on agent's o evaluation via his utility function u of both his own outcome $f_o(s)$ and of the distribution $\sum_{i \in I} \delta_{u(f_i(s))}$ of peers' outcomes. This dependence is increasing in o's payoff and decreasing (w.r.t. stochastic dominance) in the peers' outcome distribution. This reflects the negative dependence behaviorally modelled by Axiom B.1.

The preferences described by Theorem 2 can be represented by a triplet (u, ρ, P) . Next we give the uniqueness properties of this representation.

Proposition 2 Two triplets (u, ϱ, P) and $(\hat{u}, \hat{\varrho}, \hat{P})$ represent the same relation \succeq as in Theorem 2 if and only if $\hat{P} = P$ and there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$, and

$$\hat{\varrho}\left(z,\sum_{i\in I}\delta_{z_i}\right) = \alpha \varrho\left(\alpha^{-1}\left(z-\beta\right),\sum_{i\in I}\delta_{\alpha^{-1}\left(z_i-\beta\right)}\right),$$

for all $(z, \sum_{i \in I} \delta_{z_i}) \in \operatorname{pim}(\hat{u}(C)).$

5 Social Value Representation

We turn now to the possibility that agents might experience feelings of envy or pride also because of the outcomes' symbolic value. An object may be valuable for the utility it provides to the user, abstracting from the social signal it sends: obviously this is the case for example if the object is used in private. An additional value may derive from the social signal. To illustrate, consider the famous "silver spoon" example of Veblen (1899), which clearly brings out the contrast between user and symbolic values of objects:

A hand-wrought silver spoon, of a commercial value of some ten to twenty dollars, is not ordinarily more serviceable – in the first sense of the word – than a machine-made spoon of the same material. It may not even be more serviceable than a machine-made spoon of some "base" metal, such as aluminum, the value of which may be no more than some ten to twenty cents.

The conceptual structure that we have developed so far allows us to make more precise and behaviorally founded the classic Veblenian distinction between the two values of an object. We formalize this idea by introducing an induced preference \succeq on C, which will be represented by a social value function v.

Definition 1 Given any \overline{c} and \underline{c} in C, set

$$\bar{c} \stackrel{\cdot}{\succeq} \underline{c}$$
 (7)

if

$$\left(x_o, (x_i)_{i \in I}, \underline{c}_{\{j\}}\right) \succeq \left(x_o, (x_i)_{i \in I}, \overline{c}_{\{j\}}\right) \tag{8}$$

for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$.

In other words, we have $\overline{c} \succeq \underline{c}$ when in all possible societies to which the decision maker can belong, he always prefers that, *ceteris paribus*, a given peer has \underline{c} rather than \overline{c} : the externality is thus negative in every case. In particular, only peer j's outcome changes in the comparison (8), while both the decision maker's own outcome x_o and all other peer's outcomes $(x_i)_{i \in I}$ remain the same. The ranking (8) thus reveals through choice behavior a negative outcome externality of j on o. This negative externality can be due to the private emotion we discussed before; in this case Axiom B.1 holds and, under mild additional assumptions,¹² the rankings \succeq and \succeq are then easily seen to agree on C (i.e., u = v in the representation). More generally, however, this externality can be also due to a cultural/symbolic aspect of j's outcome. For instance, the Veblen silver and aluminum spoons are presumably ranked indifferent by \succeq when this order does not involve social comparisons (that is, when $I = \emptyset$), but not by \succeq . That is, they have similar u values, but different v values. On the other hand, the two evaluations do not necessarily contradict each other, or even differ, as the case u = vindicates. Social and private value are different conceptually, not necessarily behaviorally. Summing up, we interpret $\overline{c} \succeq \underline{c}$ as revealing, via choice behavior, that our envious/proud decision maker regards outcome \overline{c} to be more socially valuable than \underline{c} . If \succeq and \succeq do not agree on C, this can be properly attributed to the outcomes' symbolic value.

A simple, but important, economic consequence of the disagreement between \succeq and \succeq (and so between u and v) are the classic Veblen effects, which occur when decision makers are willing to pay different prices for functionally equivalent goods (see Fershtman, 2008). Our approach actually suggests a more subjective view of Veblen effects, in which they arise when the goods share a similar u value, possibly because they are functionally equivalent. A caveat is, however, in order. In our envy/pride interpretation the decision maker considers \bar{c} more socially valuable than \underline{c} and prefers others to have less socially valuable goods. An alternative interpretation is that the decision maker, instead, considers \underline{c} more socially valuable and prefers others to have more. Choice behavior per se is not able to distinguish between these two, equally legitimate, interpretations.

The relation \succeq is trivial for conventional decision makers who ignore the outcome of others, because for them it is always true that

$$\left(x_o, (x_i)_{i \in I}, \underline{c}_{\{j\}}\right) \sim \left(x_o, (x_i)_{i \in I}, \overline{c}_{\{j\}}\right) \sim (x_o).$$

That is, associal decision makers are characterized by the general social indifference $\overline{c} \sim \underline{c}$ for all $\overline{c}, \underline{c} \in C$.

We now present the counterparts of Axioms B.1 and B.2 on \succeq . For simplicity, we present the axiom directly on the order \succeq rather than on the primitive order \succeq .

Axiom A. 7 (Social Order) \succeq is a nontrivial, Archimedean, and independent weak order.

Note that the order \succeq on C has the properties stated in Axiom A.7, and this guarantees that it has a representation by a utility function u. Here, Axiom A.7 guarantees that the order \succeq has a representation by a real-valued affine function v, the social value function. Since \succeq is defined in terms of the primitive order \succeq , Axiom A.7 can be formulated directly in terms of the properties of \succeq . This formulation, which makes the axioms testable, is presented in the Appendix (Section A.10).

The final axiom we need for the social representation is simply the social version of Axiom B.2.

Axiom A. 8 (Social Comparative Preference) Let $(x_o, (x_i)_{i \in I}), (y_o, (x_i)_{i \in I})$ in \mathcal{X} . If $x_o \succeq y_o$, then

$$\frac{1}{2}c\left(x_{o}, (x_{i})_{i \in I}\right) + \frac{1}{2}y_{o} \gtrsim \frac{1}{2}x_{o} + \frac{1}{2}c\left(y_{o}, (x_{i})_{i \in I}\right)$$

We can now state our more general representation result.

Theorem 3 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.8 if and only if there exist two nonconstant affine functions $u, v : C \to \mathbb{R}$, a diago-null function $\varrho : \operatorname{pim}(v(C)) \to \mathbb{R}$ increasing in the

 $^{^{12}}$ See Section 6 for details.

first component and decreasing (w.r.t. stochastic dominance) in the second one, and a probability P on Σ , such that v represents \succeq and

$$V\left(f_{o},(f_{i})_{i\in I}\right) = \int_{S} u\left(f_{o}\left(s\right)\right) dP\left(s\right) + \int_{S} \varrho\left(v\left(f_{o}\left(s\right)\right), \sum_{i\in I} \delta_{v\left(f_{i}\left(s\right)\right)}\right) dP\left(s\right)$$

$$\tag{9}$$

represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

Relative to the private representation (6), there is now a non-constant affine function $v: C \to \mathbb{R}$ that represents \succeq and so quantifies the social valuation of outcomes. The function v replaces u in the positional index ϱ , and so here agent o evaluates with v both his own payoff and the peers' outcome. Like u, also v is a purely subjective construct because \succeq is derived from the subjective preference \succeq . As such, it may depend solely on subjective considerations.

The preferences described by Theorem 3 are thus represented by a quadruple (u, v, ρ, P) . Next we give the uniqueness properties of this representation.

Proposition 3 Two quadruples (u, v, ρ, P) and $(\hat{u}, \hat{v}, \hat{\rho}, \hat{P})$ represent the same relations \succeq and \succeq as in Theorem 3 if and only if $\hat{P} = P$ and there exist $\alpha, \beta, \dot{\alpha}, \dot{\beta} \in \mathbb{R}$ with $\alpha, \dot{\alpha} > 0$ such that $\hat{u} = \alpha u + \beta$, $\hat{v} = \dot{\alpha}v + \dot{\beta}$, and

$$\hat{\varrho}\left(z,\sum_{i\in I}\delta_{z_i}\right) = \alpha \varrho\left(\dot{\alpha}^{-1}\left(z-\dot{\beta}\right),\sum_{i\in I}\delta_{\dot{\alpha}^{-1}\left(z_i-\dot{\beta}\right)}\right)$$

for all $(z, \sum_{i \in I} \delta_{z_i}) \in \text{pim}(\hat{v}(C)).$

6 Private versus Social

The fact that the preference functional (6) in Theorem 2 is a special case of (9) in Theorem 3 might suggest that Theorem 2 is a special case of Theorem 3. Because of the requirement in Theorem 3 that v represents \succeq , this is true provided u also represents \succeq , that is, provided \succeq and \succeq agree on C. Notice that Axiom B.1 guarantees that \succeq implies \succeq . The converse implication is obtained by strengthening Axiom B.1 as follows.

Axiom B. 3 (Strong Negative Dependence) \succeq satisfies Axiom B.1 and, if the first relation in (5) is strict, the second relation too is strict for some $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$.

This axiom thus requires that the agent be "sufficiently sensitive to externalities."

Proposition 4 Let \succeq on \mathcal{F} be a binary relation that satisfies Axioms A.1-A.6. The following statements are equivalent:

- (i) \succeq satisfies Axioms A.7 and B.1;
- (ii) \succeq satisfies Axiom B.3;
- (iii) \succeq coincides with $\stackrel{\cdot}{\succeq}$ on C.

Remark 1 If \succeq is represented as in Theorem 2, then Axiom B.3 is clearly satisfied whenever ρ is strictly increasing in the second component (w.r.t. stochastic dominance). On the contrary, if $\rho \equiv 0$ we are in the standard expected utility case: Axiom B.1 is satisfied, while Axiom B.3 is violated.

As already observed, Axiom B.1 guarantees that \succeq is coarser than \succeq . The next example shows that this can happen in nontrivial ways.

Example 1 Assume |S| = |N| = 1 and $C = \mathbb{R}$, and consider the preferences on \mathcal{F} represented by

$$V(x_o) = x_o,$$

 $V(x_o, x_{-o}) = x_o + ((x_o)^+ - (x_{-o})^+)^{1/3},$

for all $x_o, x_{-o} \in \mathbb{R}$. They have a natural interpretation: there is a "poverty line" at 0 and agents do not care about peers below that line. Using Theorem 2, it is easy to check that these preferences satisfy Axioms A.1-A.6 and B.1-B.2. Moreover, it is easy to check that Axiom B.3 is violated. In fact, \succeq coincides on \mathbb{R} with the usual order, while \succeq is trivial on \mathbb{R}_- and it is the usual order on \mathbb{R}_+ (Proposition 4 implies that Axiom B.3 and Axiom A.7 are violated).

7 Average Payoff

In view of applications, in this section we study the special case of Theorem 3 in which the positional index ρ only depends on peers' average social payoff. This case is especially tractable from an analytic standpoint and, for this reason, it is often considered in empirical work.

This form of ρ reduces social comparisons to a simple comparison between the decision maker and a single other individual, a *representative other*, holding this average. Other specifications are possible in our setup, for example the one in which only the best and worst outcomes matter. For a detailed treatment see the working paper version Maccheroni, Marinacci, and Rustichini (2009a) of this paper.

Let n be a positive integer and $(x_o, (x_i)_{i \in I})$ an element of \mathcal{X} . Intuitively, we define an *n*-replica of $(x_o, (x_i)_{i \in I})$ as a society in which each agent i in I has spawned n-1 clones of himself, each with the same endowment x_i . Formally, we define an *n*-replica any element

$$\left(x_o, \left(x_{i_{J_i}}\right)_{i\in I}\right) \in \mathcal{X},$$

where $\{J_i\}_{i \in I}$ is a class of disjoint subsets of N with $|J_i| = n$ for all $i \in I$. We denote the *n*-replica by $(x_o, n(x_i)_{i \in I})^{13}$

Axiom A. 9 (Replica Independence) Let $(x_o, (x_i)_{i \in I}), (y_o, (y_i)_{i \in I}) \in \mathcal{X}$. Then

$$(x_o, (x_i)_{i \in I}) \succeq (y_o, (y_i)_{i \in I}) \Longrightarrow (x_o, n(x_i)_{i \in I}) \succeq (y_o, n(y_i)_{i \in I}), \quad \forall n \in \mathbb{N}.$$

When N is infinite, the addition of this axiom allows to replace in (9) the distribution $\sum_{i \in I} \delta_{v(f_i(s))}$ with its (normalized) frequency $|I|^{-1} \sum_{i \in I} \delta_{v(f_i(s))}$ (if $I \neq \emptyset$, otherwise the second term vanishes).

Axiom A. 10 (Randomization Independence) Let $(x_o, (x_i)_{i \in I}), (x_o, (y_i)_{i \in I}) \in \mathcal{X}$. If

$$\left(x_o, \left(\alpha x_i + (1-\alpha) w_i\right)_{i \in I}\right) \succ \left(x_o, \left(\alpha y_i + (1-\alpha) w_i\right)_{i \in I}\right)$$

for some α in (0,1] and $(x_o, (w_i)_{i \in I}) \in \mathcal{X}$, then

$$\left(x_o, \left(\beta x_i + (1-\beta) z_i\right)_{i \in I}\right) \succeq \left(x_o, \left(\beta y_i + (1-\beta) z_i\right)_{i \in I}\right)$$

for all β in (0,1] and $(x_o, (z_i)_{i \in I}) \in \mathcal{X}$.

Axioms A.9 and A.10 say, respectively, that the agent's preferences are not reversed either by an *n*-replica of the societies $(x_i)_{i \in I}$ and $(y_i)_{i \in I}$ or by a randomization with a common society $(w_i)_{i \in I}$.

Next we have a standard continuity axiom.

¹³Remember that $x_{i_{J_i}}$ is the constant vector taking value x_i on each element of J_i . Notice also that, if $I = \emptyset$, then $(x_o, n(x_i)_{i \in I}) = (x_o, (x_i)_{i \in I})$, whereas if n |I| > |N|, then $(x_o, (x_i)_{i \in I})$ admits no *n*-replicas.

Axiom A. 11 (Continuity) For all $(x_o, (x_i)_{i \in I}), (x_o, (y_i)_{i \in I}), (x_o, (w_i)_{i \in I}) \in \mathcal{X}$, the sets

$$\left\{ \alpha \in [0,1] : \left(x_o, (\alpha x_i + (1-\alpha) w_i)_{i \in I} \right) \succsim \left(x_o, (y_i)_{i \in I} \right) \right\}, \text{ and} \\ \left\{ \alpha \in [0,1] : \left(x_o, (\alpha x_i + (1-\alpha) w_i)_{i \in I} \right) \precsim \left(x_o, (y_i)_{i \in I} \right) \right\},$$

are closed.

To state our result we need some notation. The natural version of diago-nullity for a function ρ on $K \times (K \cup \{\infty\})$ requires that $\rho(z, z) = 0 = \rho(z, \infty)$ for all $z \in K$.¹⁴ Moreover, a function $\varphi : K \to \mathbb{R}$ is *continuously decreasing* if it is a strictly increasing transformation of a continuous and decreasing function $\psi : K \to \mathbb{R}$.¹⁵

If we add Axioms A.9-A.11 to those in Theorem 3, then we obtain the following representation:

Theorem 4 Let N be infinite. A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.11 if and only if there exist two non-constant affine functions $u, v : C \to \mathbb{R}$, a diago-null function $\varrho : v(C) \times (v(C) \cup \{\infty\}) \to \mathbb{R}$ increasing in the first component and continuously decreasing in the second one on v(C), and a probability P on Σ such that v represents \succeq and

$$V(f_{o},(f_{i})_{i\in I}) = \int_{S} u(f_{o}(s)) dP(s) + \int_{S} \varrho\left(v(f_{o}(s)), \frac{1}{|I|} \sum_{i\in I} v(f_{i}(s))\right) dP(s)$$
(10)

represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

In the representation (10) decision makers only care about the average social value. For example, if C is a set of monetary lotteries and for monetary outcomes v(x) = x, then – according to (10) – monetary acts are evaluated through

$$V\left(f_{o},\left(f_{i}\right)_{i\in I}\right) = \int_{S} u\left(f_{o}\left(s\right)\right) dP\left(s\right) + \int_{S} \varrho\left(f_{o}\left(s\right), \frac{1}{\left|I\right|} \sum_{i\in I} f_{i}\left(s\right)\right) dP\left(s\right),$$

where only the average outcome appears, that is, decision makers only react to the peers' average outcome. This is assumed in many specifications used in empirical work. In this vein, it is also possible to give conditions such that $\rho(z,t) = \gamma(z-t)$ on $v(C) \times v(C)$ for some increasing $\gamma : \mathbb{R} \to \mathbb{R}$ with $\gamma(0) = 0$. For example, this specification is considered by Clark and Oswald (1998) in their analysis of relative concerns: specifically, in their Eq. 1 p. 137, γ corresponds to sv while u corresponds to (1-s)u-c.

Finally, the uniqueness properties of representation (10) are, by now, standard and given in Proposition 10 of the Appendix.

8 Attitudes to Social Gains and Losses

The axiomatization of preferences given in the first two basic theorems opens now the way to a behavioral foundation of the analysis of preferences. In this section we assume that \succeq satisfies Axioms A.1-A.11, so that the representation (10) holds. c

Throughout this section we denote by D a convex subset of C. An event $E \in \Sigma$ is *ethically neutral* if $\overline{c}E\underline{c} \sim \underline{c}E\overline{c}$ for some $\overline{c} \nsim \underline{c}$ in C. Representation (10) guarantees that this amounts to say that the agent assigns probability 1/2 to event E.¹⁶

¹⁴Here K is a nontrivial interval and we adopt the convention $0/0 = \infty$.

¹⁵For example, strictly decreasing functions $\varphi (= \bar{\varphi} \circ (-id)$ where $\bar{\varphi}(t) = \varphi(-t)$ for all t) and continuous decreasing functions φ (= id $\circ \varphi$) are clearly continuously decreasing, while decreasing step functions are not (unless they are constant).

¹⁶We denote by $\overline{c}E\underline{c}$ the binary act that gives \overline{c} if E obtains, and \underline{c} otherwise.

8.1 Social Loss Aversion

An outcome profile where your peers get a socially better outcome than yours can be viewed as social loss; conversely, a profile where you get more than them can be viewed as a social gain. This taxonomy is important because individuals might well have different attitudes toward such social gains and losses, similarly to what happens for standard private gains and losses.

We say that a preference \succeq is more envious than provide (or averse to social losses), relative to an ethically neutral event E, a convex set $D \subseteq C$, and a given $x_o \in D$, if

$$(x_o, x_o) \succeq (x_o, x_i E y_i) \tag{11}$$

for all $x_i, y_i \in D$ such that $(1/2) x_i + (1/2) y_i \sim x_o$. The intuition is that agent o tends to be more frustrated by envy than satisfied by pride (or, assuming w.l.o.g. $x_i \succeq y_i$, he is more scared by the social loss (x_o, x_i) than lured by the social gain (x_o, y_i)).

Proposition 5 If \succeq admits a representation (10), then \succeq is more envious than proud, relative to an ethically neutral event E, a convex $D \subseteq C$, and $x_o \in D$ if and only if

$$\varrho(v(x_o), v(x_o) + h) \le -\varrho(v(x_o), v(x_o) - h)$$
(12)

for all $h \ge 0$ such that $v(x_o) \pm h \in v(D)$. In particular,¹⁷

$$D_{+}\varrho\left(v\left(x_{o}\right), v\left(x_{o}\right)\right) \leq D_{-}\varrho\left(v\left(x_{o}\right), v\left(x_{o}\right)\right)$$

$$\tag{13}$$

provided $v(x_o) \in int(v(D))$.

An immediate implication of Proposition 5 is that, given D and x_o , \succeq is more envious than proud relatively to an ethically neutral event E if and only if it is more envious than proud relatively to any other ethically neutral event. In other words, the choice of E is immaterial in the definition of social loss aversion.

8.2 Social Risk Aversion

More generally, decision makers may dislike uncertainty about their peers' social standing. This suggests to strengthen the notion that we just discussed as follows. Say that a preference \succeq is *averse to social risk*, relatively to an ethically neutral event E, a convex set $D \subseteq C$, and a given $x_o \in C$, if

$$(x_o, w_i) \succeq (x_o, x_i E y_i) \tag{14}$$

for all $x_i, y_i, w_i \in D$ such that $(1/2) x_i + (1/2) y_i \sim w_i$. Notice that the previous definition of being more envious than proud requires that (14) holds only for $w_i = x_o$.¹⁸

The next result characterizes social risk aversion in terms of concavity of ρ .

Proposition 6 If \succeq admits a representation (10), then \succeq is averse to social risk, relative to an ethically neutral event E, a convex $D \subseteq C$, and $x_o \in C$ if and only if $\varrho(v(x_o), \cdot)$ is concave on v(D).

Propensity to social risk is defined analogously, and characterized by convexity of $\rho(v(x_o), \cdot)$ on v(D). More importantly, the standard analysis of risk attitudes applies to our more general "social" setting: for example, coefficients of social risk aversion can be studied and compared.

Similarly to what happened for social loss aversion, also here it is immediate to see that the choice of E in the definition of social risk aversion is immaterial.

Finally, observe that for the special case $\rho(z,t) = \gamma(z-t)$ at the end of Section 7, Proposition 6 characterizes the concavity of the function γ and thus provides a behavioral foundation for the comparison-concave utility functions of Clark and Oswald (1998).

¹⁷Here $D_{+}\varrho\left(r,r\right) = \liminf_{h\downarrow 0} h^{-1}\left[\varrho(r,r+h) - \varrho(r,r)\right]$ and $D_{-}\varrho\left(r,r\right) = \liminf_{h\uparrow 0} h^{-1}\left[\varrho(r,r+h) - \varrho(r,r)\right]$.

 $^{^{18}}$ A more general definition of social risk aversion can be actually given, without requiring that E is ethically neutral, but just essential.

9 Comparative Interdependence

In this section we show how comparative attitudes are determined by the externality function ρ in the basic representation (4) of Theorem 1, which includes all representations considered so far and is based on axioms A.1-A.6. Specifically, we consider two preferences \gtrsim_1 and \gtrsim_2 on \mathcal{F} both satisfying A.1-A.6, and for n = 1, 2 we denote by $u_n : C \to \mathbb{R}$ and $\rho_n : \text{pim}(C) \to \mathbb{R}$ the two functions representing \gtrsim_n in the sense of the representation (4).

9.1 Social Ranking Aversion

A decision maker is more averse to social ranking than another one if he has more to lose (in subjective terms) from social comparisons. Formally, say that \succeq_1 more ranking averse than \succeq_2 if for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $c \in C$

$$\left(x_o, (x_i)_{i \in I}\right) \succeq_1 c_{I_o} \Longrightarrow \left(x_o, (x_i)_{i \in I}\right) \succeq_2 c_{I_o}.$$
(15)

In other words, \succeq_1 is more ranking averse than \succeq_2 if, whenever \succeq_1 prefers a possibly unequal social profile to an egalitarian one, then the same is true for \succeq_2 .

Proposition 7 Given two preferences \succeq_1 and \succeq_2 on \mathcal{F} that satisfy Axioms A.1-A.6, the following conditions are equivalent:¹⁹

- (i) \succeq_1 is more ranking averse than \succeq_2 ,
- (ii) $u_1 \approx u_2$ and (provided $u_1 = u_2$) $\varrho_1 \leq \varrho_2$.

This result thus behaviorally characterizes the ρ function as an index of rank aversion.

Let us have a closer look at ranking aversion. First observe that, by the first part of (ii) of Proposition 7, if two preferences \succeq_1 and \succeq_2 can be ordered by ranking aversion, then they are *outcome* equivalent; that is, they agree on the set C (precisely, on $\{(c) : c \in C\}$).

If we consider the preferences on the set of all outcome profiles, we can then see that comparability according to ranking aversion can be decomposed in two components:

- (a) $x_o \succ_2 y_o \succeq_2 (x_o, (x_i)_{i \in I})$ implies $x_o \succ_1 y_o \succeq_1 (x_o, (x_i)_{i \in I})$, and
- (b) $(x_o, (x_i)_{i \in I}) \succeq_1 y_o \succ_1 x_o$ implies $(x_o, (x_i)_{i \in I}) \succeq_2 y_o \succ_2 x_o$.

Condition (a) says that, if a society $(x_i)_{i\in I}$ makes the decision maker 2 dissatisfied of his outcome x_o , then it makes 1 dissatisfied too. In this case we say that \succeq_1 is more envious than \succeq_2 . Similarly, (b) means that every time the decision maker 1 prefers to have the intrinsically inferior outcome x_o in a society $(x_i)_{i\in I}$ than the superior y_o in solitude (or in an egalitarian society), then the same is true for 2. In this case we say that \succeq_1 is less proved than \succeq_2 .

The next result shows how ranking aversion can be expressed in terms of the two behavioral traits we just described.

Proposition 8 Given two preferences \succeq_1 and \succeq_2 on \mathcal{F} that satisfy Axioms A.1-A.6, the following conditions are equivalent:

- (i) \succeq_1 is more ranking averse than \succeq_2 ,
- (ii) \succeq_1 is outcome equivalent to \succeq_2 , more envious, and less proud.

¹⁹Recall that $u_1 \approx u_2$ means that there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $u_1 = \alpha u_2 + \beta$.

9.2 Social Sensitivity

Decision makers are more socially sensitive when they have more at stake, in subjective terms, from social comparisons; intuitively, they are at the same time more envious and more proud.²⁰ We show that this notion of social sensitivity is characterized in the representation through a ranking of the absolute values of ρ .

Proposition 9 Given two preferences \succeq_1 and \succeq_2 on \mathcal{F} that satisfy Axioms A.1-A.6, the following conditions are equivalent:

- (i) \succeq_1 is outcome equivalent to \succeq_2 , more envious, and more proud,
- (ii) $u_1 \approx u_2$ and (provided $u_1 = u_2$) $|\varrho_1| \ge |\varrho_2|$ and $\varrho_1 \varrho_2 \ge 0$.

10 Inequity Aversion

We apply the conceptual and formal structure that we have developed so far to provide an easy and transparent characterization of social preferences that are based on a separation of peers into those that are above and those that are below the decision maker: these two subsets of peers affect differently the welfare of the decision maker. In the analysis that follows, higher or lower positions are defined in terms of the utility scale (a similar analysis is possible when the order is determined by social value). The leading example of preferences based on this separation are inequity averse preferences. Inequity Aversion is based on fairness considerations: we refer the interested reader to Fehr and Schmidt (1999) for a thorough presentation. In Sections 10.1 and 10.2, we put this concept in perspective by considering two different ways that attitudes toward peers with higher and lower status can take.

10.1 Characterization of Inequity Aversion

The starting point are the basic Axioms A.1-A.6. The first additional assumption we make is that agent o evaluates peers' outcomes via his own preference:

Axiom F. 1 Let $(x_o, (x_i)_{i \in I}), (y_o, (y_i)_{i \in I}) \in \mathcal{X}$. If $x_i \sim y_i$ for all $i \in I_o$, then $(x_o, (x_i)_{i \in I}) \sim (y_o, (y_i)_{i \in I})$.

It is easy to see that this axiom is satisfied by preferences that have the private utility representation (6), that is, preferences that satisfy both the basic axioms and Axioms B.1-B.2. The next axiom is, instead, peculiar to inequity aversion and can be regarded as the inequity averse counterpart of the envy/pride Axiom B.1, which is clearly violated by inequity averse decision makers.

As Fehr and Schmidt (1999, p. 822) write, "... [players] experience inequity if they are worse off in material terms than the other players in the experiment, and they also feel inequity if they are better off." This translates into the behavioral assumption F.2. We write the assumption by specifying two cases to make the comparison with the next case easier.

Axiom F. 2 Let $(x_o, (x_i)_{i \in I}) \in \mathcal{X}, j \in I, and c \in C.$

(i) If $c \succeq x_j \succeq x_o$, then $(x_o, (x_i)_{i \in I}) \succeq (x_o, (x_i)_{i \in I - \{j\}}, c_{\{j\}})$. (ii) If $x_o \succ x_j \succeq c$, then $(x_o, (x_i)_{i \in I}) \succeq (x_o, (x_i)_{i \in I - \{j\}}, c_{\{j\}})$.

 $^{2^{0} \}succeq_{1}$ is more proud than \succeq_{2} when the implication in the above point (b) is reversed, i.e. $(x_{o}, (x_{i})_{i \in I}) \succeq_{2} y_{o} \succ_{2} x_{o}$ implies $(x_{o}, (x_{i})_{i \in I}) \succeq_{1} y_{o} \succ_{1} x_{o}$.

In other words, agent o dislikes any change in the outcome of a given peer j that in his view increases inequity, either by improving an already better outcome (i.e., $c \succeq x_j \succeq x_o$) or by impairing a worse one (i.e., $x_o \succ x_j \succeq c$).

We can now state our basic inequity aversion representation result.

Theorem 5 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.6, F.1 and F.2 if and only if there exist a non-constant affine function $u: C \to \mathbb{R}$, a diago-null function $\xi : \operatorname{pid}(u(C)) \to \mathbb{R}$ increasing in the second component and decreasing in the third one (w.r.t. stochastic dominance), and a probability P on Σ such that

$$V(f_{o}, (f_{i})_{i \in I}) = \int_{S} u(f_{o}(s)) dP(s)$$

$$+ \int_{S} \xi\left(u(f_{o}(s)), \sum_{i:u(f_{i}(s)) < u(f_{o}(s))} \delta_{u(f_{i}(s))}, \sum_{i:u(f_{i}(s)) \ge u(f_{o}(s))} \delta_{u(f_{i}(s))}\right) dP(s)$$
(16)

represents \succeq on \mathcal{F} and satisfies $V(\mathcal{F}) = u(C)$.

The uniqueness properties of the representation of inequity averse preferences are similar to the ones we obtained so far.

If there is no uncertainty and outcomes are monetary, an important specification of (16) is:

$$V(x_{o}, (x_{i})_{i \in I}) = x_{o} + \frac{1}{|I|} \sum_{i \in I} \gamma(x_{o} - x_{i}), \qquad (17)$$

where $\gamma : \mathbb{R} \to \mathbb{R}$ is such that $\gamma(0) = 0.^{21}$ Taking

$$\gamma(t) = \begin{cases} -\beta_o t & \text{if } t \ge 0, \\ \alpha_o t & \text{if } t < 0, \end{cases}$$

delivers

$$V\left(x_{o}, (x_{i})_{i \in I}\right) = x_{o} - \alpha_{o} \frac{1}{|I|} \sum_{i \in I} \max\left\{x_{i} - x_{o}, 0\right\} - \beta_{o} \frac{1}{|I|} \sum_{i \in I} \max\left\{x_{o} - x_{i}, 0\right\},$$
(18)

which is the specification adopted by Fehr and Schmidt (1999). The behavioral nature of our derivation allows to use behavioral data to test in a subject the relevance of fairness/inequity considerations, as opposed to, say, envy/pride ones. In fact, it is enough to check experimentally, through choice behavior, whether for example a subject tends to satisfy Axiom B.1 rather than F.2. This is a key dividend of our behavioral analysis.

Finally, observe that in the representation (18), Axiom F.2 is violated and Axiom B.1 is satisfied when $\beta_o < 0 \le \alpha_o$. In this case (18) becomes a simple and tractable example of the private utility representation (6).²² This is a possibility mentioned by Fehr and Schmidt (1999), who on p. 824 of their paper observe "... we believe that there are subjects with $\beta_o < 0$..." that is, as Veblen (1899, p. 31) wrote long time ago, there are subjects for whom "... it is extremely gratifying to possess something more than others." These subjects experience envy/pride, and so violate Axiom F.2 and satisfy B.1.

²¹See Maccheroni, Marinacci, Rustichini (2009) for details.

²²This happens, more generally, in (17) when γ is an increasing function.

10.2 Inequity Loving

The specific characteristic of inequity aversion is the different attitude to people with lower and larger outcome. If the effect of a worsening of those with lower outcome is changed into its opposite ($\beta_o < 0$ in the words of Fehr and Schmidt) then we have a different representation.

Axiom F. 3 Let $(x_o, (x_i)_{i \in I}) \in \mathcal{X}, j \in I, and c \in C$.

(i) If
$$c \succeq x_j \succeq x_o$$
, then $(x_o, (x_i)_{i \in I}) \succeq (x_o, (x_i)_{i \in I - \{j\}}, c_{\{j\}})$.
(ii) If $x_o \succ x_j \succeq c$, then $(x_o, (x_i)_{i \in I - \{j\}}, c_{\{j\}}) \succeq (x_o, (x_i)_{i \in I})$.

Agent o dislikes any improvement in the outcome of a given peer j that is above him: this is identical to the first condition of the inequity aversion Axiom F.2. He also likes a worsening of peer j below him, and this is the opposite of what is the case of the inequity aversion agent described by Axiom F. 2.

Naturally, the symmetric version of the inequity aversion representation result is:

Theorem 6 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.6, F.1, and F.3 if and only if there exist a non-constant affine function $u: C \to \mathbb{R}$, a diago-null function $\xi : \text{pid}(u(C)) \to \mathbb{R}$ decreasing in the second and third component (w.r.t. stochastic dominance), and a probability P on Σ such that

$$V(f_{o}, (f_{i})_{i \in I}) = \int_{S} u(f_{o}(s)) dP(s)$$

$$+ \int_{S} \xi\left(u(f_{o}(s)), \sum_{i:u(f_{i}(s)) < u(f_{o}(s))} \delta_{u(f_{i}(s))}, \sum_{i:u(f_{i}(s)) \ge u(f_{o}(s))} \delta_{u(f_{i}(s))}\right) dP(s)$$
(19)

represents \succeq on \mathcal{F} and satisfies $V(\mathcal{F}) = u(C)$.

Both representations, in Theorem 5 and 6, are based on the idea that the decision maker considers separately and differently individuals with better outcomes than his own from those who, instead, have worse outcomes. Inequity aversion assumes that any increase in inequity is disliked, whereas inequity loving is based on the idea that any improvement in the outcome of others is disliked. Neither of these two formulations seems very convincing in its pure form. The attitude toward people with lower outcomes is likely to be non monotonic, the result of the interaction of two factors: when the distance is large, compassion prevails and fear of competition is weak, while the opposite occurs when the distance is small. But, the two cases represent a potential, although extreme, attitude to the outcome of others. In any case, our framework allows to model and contrast them through behavioral (and so testable) assumptions.

11 Conclusions

We have developed an axiomatic analysis of preferences of decision makers that take into account the outcome of others. These social preferences are defined on profiles of acts, which include both the decision makers' acts and those of their peers. The representation we establish has a simple additive form: the subjective value for a decision maker of an acts' profile is equal to the expected utility of his own act, plus the expected value of the externality created by the peers' outcomes. This representation is arguably the most parsimonious extension of standard theory that is able to accommodate relative outcome concerns.

We provided a behavioral foundation for two different, but complementary, views on the nature of this externality: a private one, akin to regret and motivated by counterfactual thinking, and a social one, determined by the symbolic nature of outcomes.

On this basis we have carried out a systematic analysis of the intra and inter-personal comparative statics of these preferences, giving a rigorous behavioral foundation to the different social attitudes that characterize them. This analysis extends insights of prospect theory from the private to the social domain, where social gains and losses are determined by the relation between the social value of the decision makers' own outcomes and those of their peers.

This characterization has allowed us to establish in Maccheroni, Marinacci, and Rustichini (2009b) broad features of economies where agents exhibiting our social preferences interact. Fundamental characteristics of the equilibrium, for example the income distribution, are shown to depend on simple properties of the externality term in the representation, that is, on agents' social attitudes. This work may be considered as a further step in the line of investigation initiated long ago by Friedman and Savage (1948) and Friedman (1953).

An interesting direction for future research is to try to bridge the gap between the axiomatic approach adopted here and other approaches that are trying to provide explanations of social preferences. In particular, it might be interesting to study which properties of the preferences that we have defined are predicted by these other models.

A Proofs and Related Material

A.1 Preliminaries

Distribution Functions

Let $n, m \in \mathbb{N}$, $I = \{i_1, ..., i_n\}$, $J = \{j_1, ..., j_m\}$, $\mathbf{a} = (a_{i_1}, a_{i_2}, ..., a_{i_n}) \in \mathbb{R}^I$, and $\mathbf{b} = (b_{j_1}, ..., b_{j_m}) \in \mathbb{R}^J$. In this subsection, we regroup some useful results on stochastic dominance.

Lemma 1 If $a_{i_1} \leq a_{i_2} \leq \ldots \leq a_{i_n}$ and $b_{j_1} \leq b_{j_2} \ldots \leq b_{j_m}$, then the following facts are equivalent:

- (i) $F_{\mathbf{a}}(t) \leq F_{\mathbf{b}}(t)$ for all $t \in \mathbb{R}$.
- (ii) $n \leq m$ and $F_{\mathbf{a}}(t) \leq F_{(b_{i_1},\dots,b_{i_n})}(t)$ for all $t \in \mathbb{R}$.
- (iii) $n \leq m$ and $a_{i_k} \geq b_{j_k}$ for all k = 1, ..., n.

A corresponding result holds for decreasing distribution functions G^{23}

Lemma 2 The following statements are equivalent:

- (i) $\mu_{\mathbf{a}}$ stochastically dominates $\mu_{\mathbf{b}}$.
- (ii) n = m and if σ and τ are permutations of $\{1, ..., n\}$ such that $a_{i_{\sigma(1)}} \leq a_{i_{\sigma(2)}} \leq ... \leq a_{i_{\sigma(n)}}$ and $b_{j_{\tau(1)}} \leq b_{j_{\tau(2)}} \leq ... \leq b_{j_{\tau(n)}}$, then $a_{i_{\sigma(k)}} \geq b_{j_{\tau(k)}}$ for all k = 1, ..., n.
- (iii) n = m and there exists a permutation ζ of $\{1, ..., n\}$ such that $a_{i_{\zeta(k)}} \geq b_{j_k}$ for all k = 1, ..., n.
- (iv) There exists a bijection $\pi: I \to J$ such that $a_i \ge b_{\pi(i)}$ for all $i \in I$.
- (v) |I| = |J| and $F_{\mathbf{a}}(t) \leq F_{\mathbf{b}}(t)$ for all $t \in \mathbb{R}$.
- (vi) |I| = |J| and $G_{\mathbf{a}}(t) \ge G_{\mathbf{b}}(t)$ for all $t \in \mathbb{R}$.

²³See Maccheroni, Marinacci, and Rustichini (2008, Lemma 6) for details.

Moreover, if I = J and $a_i \ge b_i$ for all $i \in I$, then for each $z \in \mathbb{R}$ and all $t \in \mathbb{R}$, $G_{(a_i)_{i \in I:a_i \ge z}}(t) \ge G_{(b_j)_{j \in J:b_j \ge z}}(t)$, and $F_{(a_i)_{i \in I:a_i < z}}(t) \le F_{(b_j)_{j \in J:b_j < z}}(t)$.

In particular, if $\mu_{\mathbf{a}}$ stochastically dominates $\mu_{\mathbf{b}}$, then $\mu_{\mathbf{a}}(K) = \mu_{\mathbf{b}}(K)$ for all $K \subseteq \mathbb{R}$ containing the supports of $\mu_{\mathbf{a}}$ and $\mu_{\mathbf{b}}$ (i.e., they have the same total mass). On the other hand if $\mu_{\mathbf{e}} = 0$ (that is $\mathbf{e} = (e_i)_{i \in \emptyset}$), then $F_{\mathbf{e}} = 0 \leq F_{\mathbf{d}}$ and $G_{\mathbf{d}} \geq 0 = G_{\mathbf{e}}$ for all \mathbf{d} , that is $\mu_{\mathbf{e}}$ lower dominates and is upper dominated by every measure $\mu_{\mathbf{d}}$. Therefore, if $\mu_{\mathbf{d}}$ stochastically dominates or is stochastically dominated by $\mu_{\mathbf{e}}$, it follows that $\mu_{\mathbf{d}} = 0$ (from $0 \leq F_{\mathbf{d}} \leq F_{\mathbf{e}} = 0$ and $0 \leq G_{\mathbf{d}} \leq G_{\mathbf{e}} = 0$, respectively). This allows to conclude that in any case stochastic dominance between $\mu_{\mathbf{a}}$ and $\mu_{\mathbf{b}}$ implies that they have the same total mass.

A.1.1 Weakly Increasing Transformations of Expected Values

Let K be a nontrivial interval in the real line, I a non-empty finite set, and \succeq be a binary relation on the hypercube K^{I} .

Axiom 1 \succeq *is complete and transitive.*

Axiom 2 Let $x, y \in K^I$. If $x_i \ge y_i$ for all i in I, then $x \succeq y$.

Axiom 3 For all $x, y, z \in K^I$, the sets $\{\alpha \in [0,1] : \alpha x + (1-\alpha) z \succeq y\}$ and $\{\alpha \in [0,1] : \alpha x + (1-\alpha) z \preceq y\}$ are closed.

Axiom 4 Let $x, y \in K^I$. If $\alpha x + (1 - \alpha) z \succ \alpha y + (1 - \alpha) z$ for some α in (0, 1] and z in K^I , then $\beta x + (1 - \beta) w \succeq \beta y + (1 - \beta) w$ for all β in (0, 1] and w in K^I .

Axiom 5 Let $x, y \in K^I$. If $x \succeq y$, then $\alpha x + (1 - \alpha) z \succeq \alpha y + (1 - \alpha) z$ for all α in (0, 1] and z in K^I .

Passing to the contrapositive shows that the classical independence Axiom 5 implies Axiom 4 (under completeness).

Denote by $\Pi(I)$ the set of all permutations of I.

Axiom 6 $x \sim x \circ \pi$, for all $x \in K^{I}$ and each $\pi \in \Pi(I)$.

Lemma 3 A binary relation \succeq on K^I satisfies Axioms 1-4 if and only if there exist a probability measure m on I and a continuous and (weakly) increasing function $\psi : K \to \mathbb{R}$ such that

$$x \succeq y \Leftrightarrow \psi (m \cdot x) \ge \psi (m \cdot y).$$
⁽²⁰⁾

In this case, \succeq satisfies Axiom 6 if and only if (20) holds for the uniform m (i.e. $m_i \equiv 1/|I|$ for all $i \in I$).

Proof of Lemma 3. If \succeq is trivial take any *m* and any constant ψ (in particular, the uniform *m* will do).

If \succeq is not trivial, set $x \succeq^* y \Leftrightarrow \alpha x + (1 - \alpha) z \succeq \alpha y + (1 - \alpha) z$ for all $\alpha \in (0, 1]$ and $z \in K^I$. Notice that (taking $\alpha = 1$) this definition guarantees that $x \succeq^* y$ implies $x \succeq y$.

Next we show that \succeq^* is complete. In fact, $x \not\geq^* y$ implies $\alpha x + (1 - \alpha) z \prec \alpha y + (1 - \alpha) z$ for some $\alpha \in (0,1]$ and $z \in K^I$, but \succeq satisfies Axiom 4, thus $\alpha x + (1 - \alpha) z \preceq \alpha y + (1 - \alpha) z$ for all $\alpha \in (0,1]$ and $z \in K^I$, that is $y \succeq^* x$. Moreover, \succeq^* is transitive. In fact, $x \succeq^* y$ and $y \succeq^* w$ implies $\alpha x + (1 - \alpha) z \succeq \alpha y + (1 - \alpha) z$ and $\alpha y + (1 - \alpha) z \succeq \alpha w + (1 - \alpha) z$ for all $\alpha \in (0,1]$ and $z \in K^I$, then $\alpha x + (1 - \alpha) z \succeq \alpha w + (1 - \alpha) z$ for all $\alpha \in (0, 1]$ and $z \in K^{I}$, thus $x \succeq^{*} w$. Then \succeq^{*} satisfies Axiom 1.

Next we show that \succeq^* satisfies Axiom 2. Let $x, y \in K^I$. If $x_i \ge y_i$ for all i in I, then $\alpha x_i + (1 - \alpha) z_i \ge \alpha y_i + (1 - \alpha) z_i$ for all $i \in I$, $\alpha \in (0, 1]$, and $z \in K^I$, but \succeq satisfies Axiom 2, thus $\alpha x + (1 - \alpha) z \succeq \alpha y + (1 - \alpha) z$ for all $\alpha \in (0, 1]$ and $z \in K^I$, that is $x \succeq^* y$.

Next we show that \succeq^* satisfies Axiom 3. Let $x, y, w \in K^I$, $\{\beta_k\}_{k \in \mathbb{N}} \subseteq [0, 1]$ be such that $\beta_k x + (1 - \beta_k) y \succeq^* w$ for all $k \in \mathbb{N}$, and $\beta_k \to \beta$ as $k \to \infty$. Arbitrarily choose $\alpha \in (0, 1]$ and $z \in K^I$, then $\alpha (\beta_k x + (1 - \beta_k) y) + (1 - \alpha) z \succeq \alpha w + (1 - \alpha) z$ for all $k \in \mathbb{N}$, but $\alpha (\beta_k x + (1 - \beta_k) y) + (1 - \alpha) z = \beta_k (\alpha x + (1 - \alpha) z) + (1 - \beta_k) (\alpha y + (1 - \alpha) z)$, hence $\beta_k (\alpha x + (1 - \alpha) z) + (1 - \beta_k) (\alpha y + (1 - \alpha) z) \gtrsim \alpha w + (1 - \alpha) z$ for all $k \in \mathbb{N}$. Since \succeq satisfies Axiom 3, pass to the limit as $k \to \infty$ and find $\beta (\alpha x + (1 - \alpha) z) + (1 - \beta) (\alpha y + (1 - \alpha) z) \succeq \alpha w + (1 - \alpha) z$, that is $\alpha (\beta x + (1 - \beta) y) + (1 - \alpha) z \succeq \alpha w + (1 - \alpha) z$. Since this is true for all $\alpha \in (0, 1]$ and $z \in K^I$, it implies $\beta x + (1 - \beta) y \succeq^* w$. Therefore $\{\gamma \in [0, 1] : \gamma x + (1 - \gamma) y \succeq^* w\}$ is closed. Replacing \succeq^* with \preceq^* (and \succeq with \preceq), the same can be proved for the set $\{\gamma \in [0, 1] : \gamma x + (1 - \gamma) y \preceq^* w\}$.

Next we show that \succeq^* satisfies Axiom 5. Let $x \succeq^* y$, α, β in (0, 1], and w, z in K^I .

$$\alpha \left(\beta x + (1-\beta)w\right) + (1-\alpha)z = \begin{cases} x & \text{if } \alpha\beta = 1 \text{ (i.e. } \alpha = \beta = 1), \\ \alpha\beta x + (1-\alpha\beta)\left(\frac{\alpha(1-\beta)}{1-\alpha\beta}w + \frac{(1-\alpha)}{1-\alpha\beta}z\right) \text{ else} \end{cases}$$

Notice that, in the second case, $\frac{\alpha(1-\beta)}{1-\alpha\beta}w + \frac{(1-\alpha)}{1-\alpha\beta}z \in K^I$ is a *bona fide* convex combination. Thus, if $\alpha\beta \neq 1$, since $x \succeq^* y$, $\alpha\beta x + (1-\alpha\beta)\left(\frac{\alpha(1-\beta)}{1-\alpha\beta}w + \frac{(1-\alpha)}{1-\alpha\beta}z\right) \succeq \alpha\beta y + (1-\alpha\beta)\left(\frac{\alpha(1-\beta)}{1-\alpha\beta}w + \frac{(1-\alpha)}{1-\alpha\beta}z\right)$ follows, that is,

$$\alpha \left(\beta x + (1-\beta)w\right) + (1-\alpha) z \succeq \alpha \left(\beta y + (1-\beta)w\right) + (1-\alpha) z.$$
(21)

Clearly, (21) descends from $x \succeq^* y$ also if $\alpha\beta = 1$. Therefore $x \succeq^* y$ implies (21) for all α, β in (0,1] and w, z in K^I ; a fortiori it implies $\beta x + (1 - \beta) w \succeq^* \beta y + (1 - \beta) w$ for all β in (0,1] and w in K^I .

Finally, since $x \succeq^* y$ implies $x \succeq y$ and both relations are complete, nontriviality of \succeq implies nontriviality of \succeq^* .

By the Anscombe-Aumann Theorem there exists a (unique) probability measure m on I such that $x \succeq^* y$ if and only if $m \cdot x \ge m \cdot y$; in particular,

$$m \cdot x \ge m \cdot y \Rightarrow x \succeq y. \tag{22}$$

Consider the restriction of \succeq to the set of all constant elements of K^I and the usual identification of this set with K.²⁴ Such restriction is clearly complete, transitive, and monotonic. Next we show that it is also topologically continuous. Let $t_n, t, r \in K$ be such that $t_n \to t$ as $n \to \infty$ and $t_n \succeq r$ (resp. $t_n \preceq r$) for all $n \in \mathbb{N}$. Since t_n is converging to $t \in K$, there exist $\tau, T \in K$ ($\tau < T$) such that $t_n, t \in [\tau, T]$ for all $n \in \mathbb{N}$. Let $\alpha_n = (T - \tau)^{-1} (t_n - \tau)$ for all $n \in \mathbb{N}$. Clearly $\{\alpha_n\}_{n \in \mathbb{N}} \subseteq [0, 1],$ $\alpha_n \to (T - \tau)^{-1} (t - \tau) = \alpha$ as $n \to \infty$, $t_n = \alpha_n T + (1 - \alpha_n) \tau$ and $t = \alpha T + (1 - \alpha) \tau$. Axiom 3 and $\alpha_n T + (1 - \alpha_n) \tau = t_n \succeq r$ (resp. $t_n \preceq r$) imply $t = \alpha T + (1 - \alpha) \tau \succeq r$ (resp. $t \preceq r$).

Therefore, there exists a continuous and increasing function $\psi : K \to \mathbb{R}$ such that $\vec{t} \succeq \vec{r}$ if and only if $\psi(t) \ge \psi(r)$. Let m be any probability measure that satisfies (22), then $x \sim \overrightarrow{m \cdot x}$ for every $x \in K^I$, and $x \succeq y$ if and only if $\overrightarrow{m \cdot x} \succeq \overrightarrow{m \cdot y}$ if and only if $\psi(m \cdot x) \ge \psi(m \cdot y)$. This proves that Axioms 1-4 are sufficient for representation (20). The converse is trivial.

Assume that ψ and m represent \succeq in the sense of (20). Notice that the set O of all probabilities p such that ψ and p represent \succeq in the sense of (20) coincides with the set of all probabilities q is such that

²⁴With the usual convention of denoting by t both the real number $t \in K$ and the constant element \vec{t} of K^I taking value t for all $i \in I$.

 $q \cdot x \ge q \cdot y$ implies $x \succeq y$.²⁵ Let $p, q \in O$ and α in [0, 1], then $(\alpha p + (1 - \alpha) q) \cdot x \ge (\alpha p + (1 - \alpha) q) \cdot y$ implies $\alpha (p \cdot x) + (1 - \alpha) (q \cdot x) \ge \alpha (p \cdot y) + (1 - \alpha) (q \cdot y)$, hence either $p \cdot x \ge p \cdot y$ or $q \cdot x \ge q \cdot y$, in any case $x \succeq y$. Therefore O is convex.

Assume \succeq satisfies Axiom 6, and let $m \in O$. For each $\pi \in \Pi(I)$ and each x in K^{I} , $x \sim x \circ \pi$ implies $\psi(m \cdot x) = \psi(m \cdot (x \circ \pi)), \text{ but } m \cdot (x \circ \pi) = \sum_{i \in I} m_i x_{\pi(i)} = \sum_{i \in I} m_{\pi^{-1}(i)} x_{\pi(\pi^{-1}(i))} = (m \circ \pi^{-1}) \cdot x.$ Therefore $\psi(m \cdot x) = \psi((m \circ \pi^{-1}) \cdot x)$ for all $x \in K^{I}$ and each $\pi \in \Pi(I)$. Then, for each $\sigma \in \Pi(I)$, $x \succeq y$ if and only if $\psi((m \circ \sigma) \cdot x) \ge \psi((m \circ \sigma) \cdot y)$, that is $m \circ \sigma \in O$. But O is convex, thus the uniform probability $(1/|I|) \vec{1} = \sum_{\sigma \in \Pi(I)} (1/|I|!) m \circ \sigma$ belongs to O. The converse is trivial.

A.2**Basic Axioms and Representation**

Lemma 4 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.5 if and only if there exist a non-constant affine function $u: C \to \mathbb{R}$, a function $r: \mathcal{X} \to \mathbb{R}$, with $r(c_{I_0}) = 0$ for all $c \in C$ and $I \in \wp(N)$, and a probability P on Σ such that the functional $V: \mathcal{F} \to \mathbb{R}$ defined by

$$V(f) = \int_{S} u(f_{o}(s)) dP(s) + \int_{S} r(f_{o}(s), (f_{i}(s))_{i \in I}) dP(s)$$
(23)

represents \succeq and satisfies $V(\mathcal{F}) = u(C)$. Moreover, $(\hat{u}, \hat{r}, \hat{P})$ is another representation of \succeq in the above sense if and only if $\hat{P} = P$ and there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$ and $\hat{r} = \alpha r$.

Proof. The von Neumann-Morgenstern Theorem guarantees that there exists an affine function $u: C \to \mathbb{R}$ such that $(c) \succeq (\overline{c}) \Leftrightarrow u(c) \ge u(\overline{c})$, provided $c, \overline{c} \in C$.

Claim 4.1. For all $f \in \mathcal{F}$ there is $c^f \in C$ such that $f \sim (c^f)$.

Proof. First observe that for all $c \in C$ and all $I \in \wp(N)$, iterated application of Axiom A.5 and transitivity deliver $c_{I_o} \sim (c)$. Hence by Axiom A.3 there exist $\underline{c}, \overline{c} \in C$ such that $\underline{c} \preceq (f_o, (f_i)_{i \in I})$ and $(f_o, (f_i)_{i \in I}) \preceq \overline{c}$. If one of the two relations is an equivalence the proof is finished. Otherwise, the above relations are strict, and, by Axiom A.2, there exist $\alpha, \beta \in (0,1)$ such that $(1-\alpha)\underline{c} + \alpha \overline{c} \prec (f_o, (f_i)_{i \in I}) \prec$ $(1-\beta)\underline{c}+\beta\overline{c}$, and it must be $\alpha < \beta$ (*u* is affine on *C* and it represents \succeq on *C*). By Axiom A.3 again, there exist $\lambda, \mu \in (0, 1)$ such that $(1 - \lambda) ((1 - \alpha)\underline{c} + \alpha \overline{c}) + \lambda ((1 - \beta)\underline{c} + \beta \overline{c}) \prec (f_o, (f_i)_{i \in I})$ and $(f_o, (f_i)_{i \in I}) \prec (1-\mu) ((1-\alpha)\underline{c} + \alpha \overline{c}) + \mu ((1-\beta)\underline{c} + \beta \overline{c}).$ In particular, there exist $\alpha^* = (1-\lambda) \alpha + \lambda \beta$, $\alpha^* > \alpha$, and $\beta^* = (1-\mu)\alpha + \mu\beta$, $\beta^* < \beta$, such that, denoting $(1-\alpha)\underline{c} + \alpha\overline{c}$ by $\underline{c}\alpha\overline{c}$, we have $\underline{c}\alpha \overline{c} \prec \underline{c}\alpha^* \overline{c} \prec f \prec \underline{c}\beta^* \overline{c} \prec \underline{c}\beta \overline{c}$ and $\alpha < \alpha^* < \beta^* < \beta$. (Call this argument: "shrinking".)

Set $\gamma \equiv \sup \{\delta \in [0,1] : \underline{c}\delta \overline{c} \prec f\}$. If $\delta \geq \beta^*$, then $f \prec \underline{c}\beta^* \overline{c} \preceq \underline{c}\delta \overline{c}$, and thus $\gamma \leq \beta^* < \beta$. Obviously $\gamma \ge \alpha^* > \alpha.$

Suppose $f \prec \underline{c}\gamma \overline{c}$, then $\underline{c}\alpha \overline{c} \prec f \prec \underline{c}\gamma \overline{c}$ and (shrinking) there exists $\gamma^* < \gamma$ such that $f \prec \underline{c}\gamma^* \overline{c} \prec \underline{c}\gamma \overline{c}$. Therefore sup $\{\delta \in [0,1] : c\delta \overline{c} \prec f\} \leq \gamma^* < \gamma$, which is absurd.

Suppose $\underline{c}\gamma \overline{c} \prec f$, then $\underline{c}\gamma \overline{c} \prec f \prec \underline{c}\beta \overline{c}$ and (shrinking) there exists $\gamma^* > \gamma$ such that $\underline{c}\gamma \overline{c} \prec \underline{c}\gamma^* \overline{c} \prec f$. Therefore sup $\{\delta \in [0,1] : x\delta y \prec f\} \ge \gamma^* > \gamma$, which is absurd.

Conclude that $f \sim \underline{c}\gamma \overline{c} \in C$.

Claim 4.2. For all $f = (f_o, (f_i)_{i \in I}) \in \mathcal{F}$ there exists $a^f \in \mathcal{A}$ such that $f(s) \sim a^f(s)$ for all $s \in S$.

Proof. Given $f = (f_o, (f_i)_{i \in I}) \in \mathcal{F}$, denote by $\{A^k\}_{k=1}^n$ a finite partition of S in Σ that makes f_i measurable for all $i \in I_o$. For all k = 1, ...n, if $s, \bar{s} \in A^k$, then $(f_o(s), (f_i(s))_{i \in I}) = (f_o(\bar{s}), (f_i(\bar{s}))_{i \in I});$

²⁵ If $p \in O$, then $p \cdot x \ge p \cdot y$ implies $\psi(p \cdot x) \ge \psi(p \cdot y)$, because ψ is increasing, and then $x \succeq y$. Conversely, observe that ψ represents \succeq on K (in fact, $\vec{t} \succeq \vec{r}$ if and only if $\psi(m \cdot \vec{t}) \ge \psi(m \cdot \vec{r})$ if and only if $\psi(t) \ge \psi(r)$). If q is such that $q \cdot x \ge q \cdot y$ implies $x \succeq y$, then $x \sim \overrightarrow{q \cdot x}$ for every $x \in K^I$, and $x \succeq y$ if and only if $\overrightarrow{q \cdot x} \succeq \overrightarrow{q \cdot y}$ if and only if $\psi(q \cdot x) \ge \psi(q \cdot y)$; that is $q \in O$.

take $c^{f,k} \in C$ such that $(c^{f,k}) \sim f(s) = f(\bar{s})$. Define $a^f(s) = c^{f,k}$ if $s \in A^k$ (for k = 1, ..., n). The map $a: S \to C$ is a simple act, and

$$\left(a^{f}\left(s\right)\right) \sim f\left(s\right) \tag{24}$$

for all $s \in S$.²⁶

In particular, Axiom A.2 implies $(a^f) \sim f$.

By Axiom A.1, there exist $f, g \in \mathcal{F}$ such that $f \succ g$. It follows from Claim 4.2 that $(a^f) \succ (a^g)$. Thus, the restriction of \succeq to \mathcal{A} (or more precisely to the subset of \mathcal{F} consisting of elements of the form f = (a) for some $a \in \mathcal{A} = \mathcal{A}_o$) satisfies the assumptions of the Anscombe-Aumann Theorem. Then there exist a probability P on Σ and a non-constant affine function $u : C \to \mathbb{R}$ such that $(a) \succeq (b) \Leftrightarrow \int_S u(a(s)) dP(s) \ge \int_S u(b(s)) dP(s)$, provided $a, b \in \mathcal{A}^{.27}$

For all $x \in \mathcal{X}$ set $U(x) \equiv u(c^x)$ provided $c^x \in C$ and $x \sim (c^x)$, clearly, U is well defined (on \mathcal{X}). Moreover, as observed, $c_{I_o} \sim (c)$ for all $c \in C$ and $I \in \wp(N)$, thus $U(c_{I_o}) = u(c)$.

Let f, g in \mathcal{F} and take a^f and a^g in \mathcal{A} such that $(a^f(s)) \sim f(s)$ and $(a^g(s)) \sim g(s)$ for every s in S (see Claim 4.2). Then $(f_o, (f_i)_{i\in I}) \succeq (g_o, (g_j)_{j\in J}) \Leftrightarrow (a^f) \succeq (a^g) \Leftrightarrow \int_S u(a^f(s)) dP(s) \ge \int_S u(a^g(s)) dP(s) \Leftrightarrow \int_S U(f_o(s), (f_i(s))_{i\in I}) dP(s) \ge \int_S U(g_o(s), (g_j(s))_{j\in J}) dP(s)$. That is, the function defined by $V(f_o, (f_i)_{i\in I}) \equiv \int_S U(f_o(s), (f_i(s))_{i\in I}) dP(s)$, for all $(f_o, (f_i)_{i\in I}) \in \mathcal{F}$, represents \succeq on \mathcal{F} . Notice that $V(x_o, (x_i)_{i\in I}) = U(x_o, (x_i)_{i\in I})$ for all $(x_o, (x_i)_{i\in I}) \in \mathcal{X}$. Set $r(x_o, (x_i)_{i\in I}) \equiv U(x_o, (x_i)_{i\in I}) = U(c_{I_o}) - u(c) = 0$ for all $I \in \wp(N)$ and $c \in C$, and

$$V(f_{o},(f_{i})_{i\in I}) = \int_{S} \left[u(f_{o}(s)) + r(f_{o}(s),(f_{i}(s))_{i\in I}) \right] dP(s)$$
(25)

for all $(f_o, (f_i)_{i \in I}) \in \mathcal{F}$. Which delivers representation (23). Moreover, for all $c \in C$, $u(c) = V(c) \in V(\mathcal{F})$ and conversely, for all $f \in \mathcal{F}$, $V(f) = V(c^f) = u(c^f) \in u(C)$; i.e. $V(\mathcal{F}) = u(C)$.

Conversely, assume that there exist a non-constant affine function $u: C \to \mathbb{R}$, a function $r: \mathcal{X} \to \mathbb{R}$ with $r(c_{I_o}) = 0$ for all $c \in C$ and $I \in \wp(N)$, and a probability P on Σ , such that representation (23) holds and $V(\mathcal{F}) = u(C)$.²⁸ Then:

- (i) $V(x_o, (x_i)_{i \in I}) = u(x_o) + r(x_o, (x_i)_{i \in I})$ for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$,
- (ii) r(c) = 0 for all $c \in C$,
- (iii) $V(a) = \int_{S} u(a(s)) dP(s)$ for all $a \in \mathcal{A}$,
- (iv) $V(c_{I_o}) = u(c)$ for all $c \in C$ and $I \in \wp(N)$.

Proving necessity of the axioms for the representation is a standard exercise. We report it just for the sake of completeness. Completeness and transitivity of \succeq are obvious, nontriviality descends from (iv) above and the fact that u is not constant: Axiom A.1 holds. Let $f, g \in \mathcal{F}$ be such that $f(s) \succeq g(s)$ for all $s \in S$, then, by (i), $u(f_o(s)) + r(f_o(s), (f_i(s))_{i \in I}) \ge u(g_o(s)) + r(g_o(s), (g_j(s))_{j \in J})$, therefore $\int_S [u(f_o(s)) + r(f(s))] dP(s) \ge \int_S [u(g_o(s)) + r(g(s))] dP(s)$, which together with representation (23) delivers $f \succeq g$: Axiom A.2 holds. Axiom A.3 holds because of (iv), $V(\mathcal{F}) = u(C)$, and affinity of u. Axiom A.4 holds because of (iii). Finally, for all $I \in \wp(N)$, $j \in N \setminus I$, and $c \in C$, by (iv), $V(c_{I_o}) = u(c) = V(c_{I_o \cup \{j\}})$ and Axiom A.5 holds.

Let $\hat{u} : C \to \mathbb{R}$ a non-constant affine function, $\hat{r} : \mathcal{X} \to \mathbb{R}$ a function with $\hat{r}(c_{I_o}) = 0$ for all $I \in \wp(N)$ and $c \in C$, and \hat{P} be a probability on Σ , such that the functional $\hat{V} : \mathcal{F} \to \mathbb{R}$, defined

²⁶In fact, $(a^{f}(s)) = (c^{f,k}) \sim f(s)$ provided $s \in A^{k}$.

²⁷Notice that u represents \succeq on C, hence w.l.o.g. this u is the same u we considered at the very beginning of this proof.

²⁸Notice that $r \circ f : S \to \mathbb{R}$ is a simple and measurable function for all $f \in \mathcal{F}$, hence the integral in (23) is well defined.

by $\hat{V}(f) = \int_{S} \left[\hat{u}(f_{o}(s)) + \hat{r}(f_{o}(s), (f_{i}(s))_{i \in I}) \right] dP(s)$, for all $f \in \mathcal{F}$, represents \succeq and satisfies $\hat{V}(\mathcal{F}) = \hat{u}(C)$. The above point (iii) implies that $\hat{V}(a) = \int_{S} \hat{u}(a(s)) d\hat{P}(s)$, for all $a \in \mathcal{A}$, is an Anscombe-Aumann representation of \succeq on \mathcal{A} . Therefore $\hat{P} = P$, and there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$. For all $x \in \mathcal{X}$, take $c \in C$ such that $\hat{V}(x) = \hat{u}(c)$, then, by (iv), $x \sim (c)$ and, by (iv) again, V(x) = u(c). Points (i) and (iv) imply $\hat{r}(x) = \hat{V}(x) - \hat{u}(x_{o}) = \hat{u}(c) - \hat{u}(x_{o}) = \alpha (u(c) - u(x_{o})) = \alpha (V(x) - u(x_{o})) = \alpha r(x)$, that is, $\hat{r} = \alpha r$. Conversely, if there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$, $\hat{r} = \alpha r$, and $\hat{P} = P$, then $\hat{u} : C \to \mathbb{R}$ is a non-constant affine function, $\hat{r} : \mathcal{X} \to \mathbb{R}$ is a function with $\hat{r}(c_{I_{o}}) = 0$ for all $I \in \wp(N)$ and $c \in C$, \hat{P} is a probability on Σ , and $\hat{V}(f) = \int_{S} [\hat{u}(f_{o}(s)) + \hat{r}(f(s))] d\hat{P}(s) = \int_{S} [\alpha u(f_{o}(s)) + \beta + \alpha r(f(s))] dP(s) = \alpha V(f) + \beta$ obviously represents \succeq on \mathcal{F} ; finally $\hat{V}(\mathcal{F}) = \alpha V(\mathcal{F}) + \beta = \alpha u(C) + \beta = \hat{u}(C)$.

Lemma 5 Let \succeq be a binary relation on \mathcal{F} that satisfy Axiom A.1. The following conditions are equivalent:

- (i) \succeq satisfies Axioms A.6 and B.1.
- (ii) If $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J}) \in \mathcal{X}$ and there is a bijection $\pi : J \to I$ such that $y_j \succeq x_{\pi(j)}$ for all $j \in J$, then $(x_o, (x_i)_{i \in I}) \succeq (x_o, (y_j)_{j \in J})$.

Proof of Lemma 5. (i) \Rightarrow (ii). Assume $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J})$ in \mathcal{X} are such that there is a bijection $\pi: J \to I$ with $y_j \gtrsim x_{\pi(j)}$. Set $w_j = x_{\pi(j)}$ for all $j \in J$, by Axiom A.6, $(x_o, (x_i)_{i \in I}) \sim (x_o, (w_j)_{j \in J})$.

If $I = \emptyset$, then $J = \emptyset$ and $(x_o, (x_i)_{i \in I}) = x_o \succeq x_o = (x_o, (y_j)_{j \in J})$. Else, we can assume $J = \{j_1, j_2, ..., j_n\}$ and, observing that $y_j \succeq w_j$ for all $j \in J$, repeated applications of Axiom B.1 deliver that $(x_o, (x_i)_{i \in I}) \sim (x_o, w_{j_1}, w_{j_2}, ..., w_{j_n}) \succeq (x_o, y_{j_1}, w_{j_2}, ..., w_{j_n}) \succeq (x_o, y_{j_1}, y_{j_2}, ..., w_{j_n}) \models (x_o, (y_j)_{j \in J})$, as wanted.

(ii) \Rightarrow (i). Assume $(x_o, (x_i)_{i \in I})$, $(x_o, (y_j)_{j \in J})$ in \mathcal{X} are such that there is a bijection $\pi : J \to I$ such that $y_j = x_{\pi(j)}$ for all $j \in J$. Then a fortiori, $y_j \succeq x_{\pi(j)}$ and by (ii) $(x_o, (x_i)_{i \in I}) \succeq (x_o, (y_j)_{j \in J})$. Moreover, $\pi^{-1} : I \to J$ is such that $x_i = x_{\pi(\pi^{-1}(i))} = y_{\pi^{-1}(i)}$ for all $i \in I$, in particular $x_i \succeq y_{\pi^{-1}(i)}$ for all $i \in J$, and by (ii) $(x_o, (x_i)_{i \in I}) \precsim (x_o, (y_j)_{j \in J})$. Therefore $(x_o, (x_i)_{i \in I}) \sim (x_o, (y_j)_{j \in J})$ and Axiom A.6 holds.

Assume $(x_o, (x_i)_{i \in I}) \in \mathcal{X}, j \notin I$, and $\overline{c} \succeq \underline{c}$. Consider $(x_o, (x_i)_{i \in I}, \overline{c}_{\{j\}}), (x_o, (x_i)_{i \in I}, \underline{c}_{\{j\}})$, and consider the identity $\pi : I \cup \{j\} \to I \cup \{j\}$, (ii) implies $(x_o, (x_i)_{i \in I}, \underline{c}_{\{j\}}) \succeq (x_o, (x_i)_{i \in I}, \overline{c}_{\{j\}})$, Axiom B.1 holds.

Proof of Theorem 1 First observe that for all $I, J \in \wp(N), (x_i)_{i \in I} \in C^I, (y_j)_{j \in J} \in C^J$ the following facts are equivalent:

- There is a bijection $\pi: J \to I$ such that $y_j = x_{\pi(j)}$.
- $\mu_{(x_i)_{i\in I}} = \mu_{(y_j)_{i\in J}}.$

By Lemma 4, there exist a non-constant affine function $u : C \to \mathbb{R}$, a function $r : \mathcal{X} \to \mathbb{R}$ with $r(c_{I_o}) = 0$ for all $c \in C$ and $I \in \wp(N)$, and a probability P on Σ , such that the functional $V : \mathcal{F} \to \mathbb{R}$, defined by (23), represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

If $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J}) \in \mathcal{X}$ and $\mu_{(x_i)_{i \in I}} = \mu_{(y_j)_{j \in J}}$, then there is a bijection $\pi : J \to I$ such that $y_j = x_{\pi(j)}$. By Axiom A.6, $(x_o, (x_i)_{i \in I}) \sim (x_o, (y_j)_{j \in J})$, thus $u(x_o) + r(x_o, (x_i)_{i \in I}) =$ $u(x_o) + r\left(x_o, (y_j)_{j \in J}\right)$ and $r\left(x_o, (x_i)_{i \in I}\right) = r\left(x_o, (y_j)_{j \in J}\right)$. Therefore, for $(x_o, \mu) \in C \times \mathcal{M}(C)$ it is well posed to define $\varrho(x_o, \mu) = r\left(x_o, (x_i)_{i \in I}\right)$, provided $\left(x_o, (x_i)_{i \in I}\right) \in \mathcal{X}$ and $\mu = \mu_{(x_i)_{i \in I}}$.

Finally, let $c \in C$ and $0 \leq n \leq |N|$. Choose $I \in \wp(N)$ with |I| = n, then $\wp(c, n\delta_c) = r(c_{I_o}) = 0$. That is, ϱ is diagonull. This concludes the proof of the sufficiency part.

For the proof of necessity, set $r(x_o, (x_i)_{i \in I}) = \rho(x_o, \mu_{(x_i)_{i \in I}})$ for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ to obtain that \succeq satisfies Axioms A.1-A.5 (Lemma 4). Moreover, if $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J}) \in \mathcal{X}$ and there is a bijection $\pi : J \to I$ such that $y_j = x_{\pi(j)}$ for all $j \in J$, then $\mu_{(x_i)_{i \in I}} = \mu_{(y_j)_{j \in J}}$, and hence $u(x_o) + \rho(x_o, \mu_{(x_i)_{i \in I}}) = u(x_o) + \rho(x_o, \mu_{(y_j)_{j \in J}})$, that is $(x_o, (x_i)_{i \in I}) \sim (x_o, (y_j)_{j \in J})$. Therefore Axiom A.6 holds too.

Proof of Proposition 1 This Proposition immediately follows from Lemma 4.

A.3 Private Utility Representation

Proof of Theorem 2. By Lemma 4, there exist a non-constant affine function $u: C \to \mathbb{R}$, a function $r: \mathcal{X} \to \mathbb{R}$ with $r(c_{I_o}) = 0$ for all $c \in C$ and $I \in \wp(N)$, and a probability P on Σ , such that the functional $V: \mathcal{F} \to \mathbb{R}$, defined by (23), represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

Next we show that if $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J}) \in \mathcal{X}$ and $\mu_{(u(x_i))_{i \in I}}$ stochastically dominates $\mu_{(u(y_j))_{j \in J}}$, then $r(x_o, (x_i)_{i \in I}) \leq r(x_o, (y_j)_{j \in J})$. Therefore, for $(x_o, \mu) \in C \times \mathcal{M}(u(C))$ it is well posed to define $\theta(x_o, \mu) = r(x_o, (x_i)_{i \in I})$, provided $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $\mu = \mu_{(u(x_i))_{i \in I}}$. The obtained function θ is decreasing in the second component with respect to stochastic dominance.

If $\mu_{(u(x_i))_{i\in I}}$ stochastically dominates $\mu_{(u(y_j))_{j\in J}}$, then Lemma 2 guarantees that there exists a bijection $\pi : I \to J$ such that $u(x_i) \ge u(y_{\pi(i)})$ for all $i \in I$, therefore $x_i \succeq y_{\pi(i)}$ for all $i \in I$. Axioms A.6 and B.1 and Lemma 5 yield $(x_o, (x_i)_{i\in I}) \preceq (x_o, (y_j)_{j\in J})$. Then $u(x_o) + r(x_o, (x_i)_{i\in I}) \le u(x_o) + r(x_o, (y_j)_{j\in J})$ and $r(x_o, (x_i)_{i\in I}) \le r(x_o, (y_j)_{j\in J})$.

Next we show that if $(x_o, \mu), (y_o, \mu) \in C \times \dot{\mathcal{M}}(u(C))$ and $u(x_o) \geq u(y_o)$, then $\theta(x_o, \mu) \geq \theta(y_o, \mu)$. Therefore, for $(z, \mu) \in \text{pim}(u(C))$ it is well posed to define $\varrho(z, \mu) = \theta(x_o, \mu)$, provided $z = u(x_o)$, and ϱ is increasing in the first component and decreasing in the second component with respect to stochastic dominance.

Let $(x_o, \mu), (y_o, \mu) \in C \times \mathcal{M}(u(C))$ with $u(x_o) \geq u(y_o)$, and choose $(x_i)_{i \in I}$ such that $\mu = \mu_{(u(x_i))_{i \in I}}$. Axiom B.2 implies $2^{-1}c(x_o, (x_i)_{i \in I}) + 2^{-1}y_o \succeq 2^{-1}x_o + 2^{-1}c(y_o, (x_i)_{i \in I})$. That is, $2^{-1}u(c(x_o, (x_i)_{i \in I})) + 2^{-1}u(y_o) \geq 2^{-1}u(x_o) + 2^{-1}u(c(y_o, (x_i)_{i \in I}))$, then $V(x_o, (x_i)) = u(c(x_o, (x_i)))$ delivers $2^{-1}u(x_o) + 2^{-1}\theta(x_o, \mu) + 2^{-1}u(y_o) \geq 2^{-1}u(x_o) + 2^{-1}u(y_o) + 2^{-1}\theta(y_o, \mu)$, as wanted.

Finally, let $z \in u(C)$ and $0 \leq n \leq |N|$. Choose $c \in C$ such that u(c) = z and $I \in \wp(N)$ with |I| = n, then $\varrho(z, n\delta_z) = \varrho(u(c), \sum_{i \in I} \delta_{u(c)}) = \theta(c, \sum_{i \in I} \delta_{u(c)}) = r(c_{I_o}) = 0$. That is ϱ is diago-null. This concludes the proof of the sufficiency part, since $r(x_o, (x_i)_{i \in I}) = \theta(x_o, \mu_{(u(x_i))_{i \in I}}) = \varrho(u(x_o), \mu_{(u(x_i))_{i \in I}})$ for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$.

For the proof of necessity, set $r(x_o, (x_i)_{i \in I}) = \varrho(u(x_o), \mu_{(u(x_i))_{i \in I}})$ for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ to obtain that \succeq satisfies Axioms A.1-A.5 (Lemma 4).

Let $(x_o, (x_i)_{i \in I})$ and $(x_o, (y_j)_{j \in J})$ in \mathcal{X} be such that there exists a bijection $\pi : J \to I$ such that $y_j \succeq x_{\pi(j)}$ for all $j \in J$. Then $u(y_j) \ge u(x_{\pi(j)})$ for all $j \in J$, and by Lemma 2, $\sum_{j \in J} \delta_{u(y_j)}$ stochastically dominates $\sum_{i \in I} \delta_{u(x_i)}$ and $V(x_o, (x_i)_{i \in I}) \ge V(x_o, (y_j)_{j \in J})$, thus $(x_o, (x_i)_{i \in I}) \ge (x_o, (y_j)_{j \in J})$. By Lemma 5, \succeq satisfies Axiom A.6 and Axiom B.1.

Let $(x_o, (x_i)_{i \in I}), (y_o, (x_i)_{i \in I}) \in \mathcal{X}$ be such that $x_o \succeq y_o$ then $u(x_o) \ge u(y_o)$ and moreover $\varrho\left(u(x_o), \mu_{(u(x_i))_{i \in I}}\right) \ge \varrho\left(u(y_o), \mu_{(u(x_i))_{i \in I}}\right)$. Hence $V\left(x_o, (x_i)_{i \in I}\right) - u(x_o) \ge V\left(y_o, (x_i)_{i \in I}\right) - u(y_o)$

and $2^{-1}u\left(c\left(x_{o}, (x_{i})_{i\in I}\right)\right) + 2^{-1}u\left(y_{o}\right) \geq 2^{-1}u\left(x_{o}\right) + 2^{-1}u\left(c\left(y_{o}, (x_{i})_{i\in I}\right)\right)$. Finally, affinity of u delivers $2^{-1}c\left(x_{o}, (x_{i})_{i\in I}\right) + 2^{-1}y_{o} \gtrsim 2^{-1}x_{o} + 2^{-1}c\left(y_{o}, (x_{i})_{i\in I}\right)$. That is, Axiom B.2 holds.

Proof of Proposition 2. Omitted (it is very similar to the one of Proposition 3).

A.4 Social Value Representation

Lemma 6 Let \succeq be a binary relation on \mathcal{F} that satisfy Axioms A.1-A.5 and A.7. The following conditions are equivalent:

- (i) \succeq satisfies Axiom A.6,
- (ii) If $(x_o, (x_i)_{i \in I})$, $(x_o, (y_j)_{j \in J}) \in \mathcal{X}$ and there is a bijection $\pi : J \to I$ such that $y_j \succeq x_{\pi(j)}$ for all $j \in J$, then $(x_o, (x_i)_{i \in I}) \succeq (x_o, (y_j)_{j \in J})$.

Proof of Lemma 6. (i) \Rightarrow (ii). Let $(x_o, (x_i)_{i \in I})$, $(x_o, (y_j)_{j \in J})$ in \mathcal{X} be such that there is a bijection $\pi: J \to I$ with $y_j \succeq x_{\pi(j)}$. Set $w_j = x_{\pi(j)}$ for all $j \in J$, by Axiom A.6, $(x_o, (x_i)_{i \in I}) \sim (x_o, (w_j)_{j \in J})$. If $I = \emptyset$, then $J = \emptyset$ and $(x_o, (x_i)_{i \in I}) = x_o \succeq x_o = (x_o, (y_j)_{j \in J})$. Else we can assume $J = \{j_1, j_2, ..., j_n\}$ and, observing that $y_j \succeq w_j$ for all $j \in J$, conclude $(x_o, (x_i)_{i \in I}) \sim (x_o, w_{j_1}, w_{j_2}, ..., w_{j_n}) \succeq (x_o, y_{j_1}, y_{j_2}, ..., y_{j_n}) = (x_o, (y_j)_{j \in J})$, as wanted.

(ii) \Rightarrow (i). Assume $(x_o, (x_i)_{i \in I})$, $(x_o, (y_j)_{j \in J})$ in \mathcal{X} are such that there is a bijection $\pi : J \to I$ such that $y_j = x_{\pi(j)}$ for all $j \in J$. Then a fortiori, $y_j \succeq x_{\pi(j)}$ and hence $(x_o, (x_i)_{i \in I}) \succeq (x_o, (y_j)_{j \in J})$. Moreover, $\pi^{-1} : I \to J$ is such that $x_i = x_{\pi(\pi^{-1}(i))} = y_{\pi^{-1}(i)}$ for all $i \in I$, in particular $x_i \succeq y_{\pi^{-1}(i)}$ for all $i \in J$, and hence $(x_o, (x_i)_{i \in I}) \precsim (x_o, (y_j)_{j \in J})$. Therefore $(x_o, (x_i)_{i \in I}) \sim (x_o, (y_j)_{j \in J})$ and Axiom A.6 holds.

Lemma 6 plays for the proof of Theorem 3 the role that Lemma 5 plays for the proof of Theorem 2, as we see in the next proof.

Proof of Theorem 3. By Lemma 4, there exist a non-constant affine function $u: C \to \mathbb{R}$, a function $r: \mathcal{X} \to \mathbb{R}$ with $r(c_{I_o}) = 0$ for all $c \in C$ and $I \in \wp(N)$, and a probability P on Σ , such that the functional $V: \mathcal{F} \to \mathbb{R}$, defined by (23), represents \succeq and satisfies $V(\mathcal{F}) = u(C)$. Moreover, by Axiom A.7, there exists $v: C \to \mathbb{R}$ that represents \succeq .

Next we show that if $(x_o, (x_i)_{i \in I}), (x_o, (y_j)_{j \in J}) \in \mathcal{X}$ and $\mu_{(v(x_i))_{i \in I}}$ stochastically dominates $\mu_{(v(y_j))_{j \in J}}$, then $r(x_o, (x_i)_{i \in I}) \leq r(x_o, (y_j)_{j \in J})$. Therefore, for $(x_o, \mu) \in C \times \mathcal{M}(v(C))$ it is well posed to define $\theta(x_o, \mu) = r(x_o, (x_i)_{i \in I})$, provided $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $\mu = \mu_{(v(x_i))_{i \in I}}$. The obtained function θ is decreasing in the second component with respect to stochastic dominance.

If $\mu_{(v(x_i))_{i\in I}}$ stochastically dominates $\mu_{(v(y_j))_{j\in J}}$, then Lemma 2 guarantees that there exists a bijection $\pi: I \to J$ such that $v(x_i) \ge v(y_{\pi(i)})$ for all $i \in I$, therefore $x_i \succeq y_{\pi(i)}$ for all $i \in I$. Since \succeq satisfies Axioms A.1-A.8, Lemma 6 yields $(x_o, (x_i)_{i\in I}) \precsim (x_o, (y_j)_{j\in J})$. Then $u(x_o) + r(x_o, (x_i)_{i\in I}) \le u(x_o) + r(x_o, (y_j)_{j\in J})$ and $r(x_o, (x_i)_{i\in I}) \le r(x_o, (y_j)_{j\in J})$.

Like in the Proof of Theorem 2, it can be shown that if $(x_o, \mu), (y_o, \mu) \in C \times \mathcal{M}(v(C))$ and $v(x_o) \geq v(y_o)$, then $\theta(x_o, \mu) \geq \theta(y_o, \mu)$. Therefore, for $(z, \mu) \in \text{pim}(v(C))$ it is well posed to define, $\varrho(z, \mu) = \theta(x_o, \mu)$, provided $z = v(x_o)$, and ϱ is increasing in the first component and decreasing in the second component with respect to stochastic dominance.

Also the proof of diago-nullity of ρ is similar to the one we detailed for Theorem 2.

For the proof of necessity, set $r(x_o, (x_i)_{i \in I}) = \varrho(v(x_o), \mu_{(v(x_i))_{i \in I}})$ for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ to obtain that \succeq satisfies Axioms A.1-A.5 (Lemma 4). Moreover, since v is non-constant affine and it represents \succeq , then \succeq satisfies Axiom A.7.

Let $(x_o, (x_i)_{i \in I})$ and $(x_o, (y_j)_{j \in J})$ in \mathcal{X} be such that there exists a bijection $\pi : J \to I$ such that $y_j = x_{\pi(j)}$ for all $j \in J$. Then $\sum_{j \in J} \delta_{v(y_j)} = \sum_{j \in J} \delta_{v(x_{\pi(j)})} = \sum_{i \in I} \delta_{v(x_i)}$ and $V(x_o, (x_i)_{i \in I}) = V(x_o, (y_j)_{j \in J})$, thus $(x_o, (x_i)_{i \in I}) \sim (x_o, (y_j)_{j \in J})$ and Axiom A.6 holds.

Let $(x_o, (x_i)_{i \in I})$, $(y_o, (x_i)_{i \in I}) \in \mathcal{X}$ be such that $x_o \succeq y_o$ then $v(x_o) \ge v(y_o)$ and moreover $\varrho\left(v(x_o), \mu_{(v(x_i))_{i \in I}}\right) \ge \varrho\left(v(y_o), \mu_{(v(x_i))_{i \in I}}\right)$. Hence $V\left(x_o, (x_i)_{i \in I}\right) - u(x_o) \ge V\left(y_o, (x_i)_{i \in I}\right) - u(y_o)$, and $2^{-1}u\left(c\left(x_o, (x_i)_{i \in I}\right)\right) + 2^{-1}u\left(y_o\right) \ge 2^{-1}u\left(x_o\right) + 2^{-1}u\left(c\left(y_o, (x_i)_{i \in I}\right)\right)$. Finally, affinity of u delivers $2^{-1}c\left(x_o, (x_i)_{i \in I}\right) + 2^{-1}y_o \succeq 2^{-1}x_o + 2^{-1}c\left(y_o, (x_i)_{i \in I}\right)$. That is, Axiom A.8 holds.

Proof of Proposition 3. Let $(\hat{u}, \hat{v}, \hat{\varrho}, \hat{P})$ be another representation of \succeq and \succeq in the sense of Theorem 3. Set $r(x_o, (x_i)_{i\in I}) = \varrho(v(x_o), \mu_{(v(x_i))_{i\in I}})$ and $\hat{r}(x_o, (x_i)_{i\in I}) = \hat{\varrho}(\hat{v}(x_o), \mu_{(\hat{v}(x_i))_{i\in I}})$ for all $(x_o, (x_i)_{i\in I}) \in \mathcal{X}$. By Lemma 4, there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$, $\hat{r} = \alpha r$, and $\hat{P} = P$. Moreover, since \hat{v} represents \succeq , there are $\dot{\alpha}, \dot{\beta} \in \mathbb{R}$ with $\dot{\alpha} > 0$ such that $\hat{v} = \dot{\alpha}v + \dot{\beta}$. Let $(z, \sum_{i\in I} \delta_{z_i}) \in pim(\hat{v}(C))$, then there exist $x = (x_o, (x_i)_{i\in I}) \in \mathcal{X}$ such that $(z, (z_i)_{i\in I}) = (\hat{v}(x_o), (\hat{v}(x_i))_{i\in I})$. Therefore $(z, \sum_{i\in I} \delta_{z_i}) = (\hat{v}(x_o), \mu_{(\hat{v}(x_i))_{i\in I}})$, and from $\hat{r} = \alpha r$ it follows that $\hat{\varrho}(z, \sum_{i\in I} \delta_{z_i}) = \hat{\varrho}(\hat{v}(x_o), \mu_{(\hat{v}(x_i))_{i\in I}}) = \hat{r}(x_o, (x_i)_{i\in I}) = \alpha r(x_o, (x_i)_{i\in I}) = \alpha \varrho(v(x_o), \mu_{(v(x_i))_{i\in I}}) = \alpha \varrho(\frac{z-\dot{\beta}}{\dot{\alpha}}, \sum_{i\in I} \delta_{\frac{z_i-\dot{\beta}}{\dot{\alpha}}})$, since $\hat{v} = \dot{\alpha}v + \dot{\beta}$ amounts to $v = \dot{\alpha}^{-1}(\hat{v} - \dot{\beta})$.

Conversely, if $\hat{P} = P$, and there exist $\alpha, \beta, \dot{\alpha}, \dot{\beta} \in \mathbb{R}$ with $\alpha, \dot{\alpha} > 0$ such that $\hat{u} = \alpha u + \beta, \hat{v} = \dot{\alpha}v + \dot{\beta}$, and $\hat{\varrho}\left(z, \sum_{i \in I} \delta_{z_i}\right) = \alpha \varrho\left(\dot{\alpha}^{-1}\left(z - \dot{\beta}\right), \sum_{i \in I} \delta_{\dot{\alpha}^{-1}\left(z_i - \dot{\beta}\right)}\right)$ for all $\left(z, \sum_{i \in I} \delta_{z_i}\right) \in \text{pim}\left(\hat{v}\left(C\right)\right)$, then $\hat{u}, \hat{v} : C \to \mathbb{R}$ are non-constant affine, it is easy to check that $\hat{\varrho} : \text{pim}\left(\hat{v}\left(C\right)\right) \to \mathbb{R}$ is well defined, diago-null, increasing in the first component and decreasing (w.r.t. stochastic dominance) in the second, \hat{P} is a probability on Σ, \hat{v} represents $\dot{\Sigma}$, and

$$\begin{split} \hat{V}\left(f\right) &= \int_{S} \left[\hat{u}\left(f_{o}\left(s\right)\right) + \hat{\varrho}\left(\hat{v}\left(f_{o}\left(s\right)\right), \sum_{i \in I} \delta_{\hat{v}\left(f_{i}\left(s\right)\right)}\right) \right] dP\left(s\right) \\ &= \int_{S} \left[\alpha u\left(f_{o}\left(s\right)\right) + \beta + \alpha \varrho\left(\frac{\hat{v}\left(f_{o}\left(s\right)\right) - \dot{\beta}}{\dot{\alpha}}, \sum_{i \in I} \delta_{\frac{\hat{v}\left(f_{i}\left(s\right)\right) - \dot{\beta}}{\dot{\alpha}}}\right) \right] dP\left(s\right) = \alpha V\left(f\right) + \beta, \end{split}$$

obviously represents \succeq on \mathcal{F} ; finally $\hat{V}(\mathcal{F}) = \alpha V(\mathcal{F}) + \beta = \alpha u(C) + \beta = \hat{u}(C)$.

A.5 Private versus Social

Proof of Proposition 4. (iii) \Rightarrow (i) and (ii). If \succeq coincides with \succeq on C, then A.7 is satisfied (Lemma 4 guarantees that \succeq , hence \succeq , is represented on C by an affine non-constant function $u: C \to \mathbb{R}$).

If $c \succeq c'$, then $c \succeq c'$, that is $(x_o, (x_i)_{i \in I}, c'_{\{j\}}) \succeq (x_o, (x_i)_{i \in I}, c_{\{j\}})$, for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$. Then Axiom B.1 is satisfied.

Moreover, $c \succ c'$ implies that $c \succeq c'$, thus (by definition of \succeq) there exist $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin C$ such that $(x_o, (x_i)_{i \in I}, c'_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, c_{\{j\}})$. That is, Axiom B.3 holds.

(ii) \Rightarrow (iii). By Axiom B.3, $c \succeq c'$ implies that $c \succeq c'$, for all $c, c' \in C$. Moreover, $c \succ c'$ implies $c \succeq c'$, for all $c, c' \in C$; that is, $c \preceq c'$ implies $c \preccurlyeq c'$.

(i) \Rightarrow (iii). By Axiom B.1, $c \succeq c'$ implies that $c \succeq c'$, for all $c, c' \in C$. Moreover, Lemma 4 guarantees that \succeq is represented by an affine non-constant function $u : C \to \mathbb{R}$, Axiom A.7 guarantees that \succeq

is represented by an affine non-constant function $v: C \to \mathbb{R}$, it follows that there are $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $v = \alpha u + \beta$, that is \succeq coincides with \succeq on C.

A.6 Average Payoff

Proof of Theorem 4. By Theorem 3 there exist two non-constant affine functions $u, v : C \to \mathbb{R}$, a diago-null function ρ : pim $(v(C)) \to \mathbb{R}$ increasing in the first component and decreasing (w.r.t. stochastic dominance) in the second, and a probability P on Σ , such that v represents \succeq and the function $V : \mathcal{F} \to \mathbb{R}$, defined by (9), represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

Fix $z \in v(C)$ and $I \in \wp(N) \setminus \emptyset$. Consider the relation on $v(C)^{I}$ defined by $(z_i)_{i \in I} \succeq_{z,I} (w_i)_{i \in I}$ if and only if there exist $(x_o, (x_i)_{i \in I}), (x_o, (y_i)_{i \in I}) \in \mathcal{X}$ such that $v(x_o) = z, v(x_i) = z_i, v(y_i) = w_i$ for all $i \in I$, and $(x_o, (x_i)_{i \in I}) \preceq (x_o, (y_i)_{i \in I})$. $\succeq_{z,I}$ is well defined, in fact, if there exist another pair $\left(x'_{o}, (x'_{i})_{i \in I}\right), \left(x'_{o}, (y'_{i})_{i \in I}\right) \in \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \precsim \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \rightrightarrows \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \rightrightarrows \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \rightrightarrows \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \rightrightarrows \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \rightrightarrows \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \text{ then } \left(x_{o}, (x_{i})_{i \in I}\right) \ x_{i} \in \mathcal{X} \text{ such that } v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(y'_{i}\right) = w_{i}, \ v\left(x'_{o}\right) = z, \ v\left(x'_{i}\right) = z_{i}, \ v\left(x'$ $\left(x_{o},\left(y_{i}\right)_{i\in I}\right)\Leftrightarrow u\left(x_{o}\right)+\varrho\left(v\left(x_{o}\right),\mu_{v\left(x_{i}\right)_{i\in I}}\right)\leq u\left(x_{o}\right)+\varrho\left(v\left(x_{o}\right),\mu_{v\left(y_{i}\right)_{i\in I}}\right)\Leftrightarrow \varrho\left(z,\mu_{\left(z_{i}\right)_{i\in I}}\right)\leq \varrho\left(z,\mu_{\left(z_{i}\right)_{i\in I}}\right)$ $\mu_{(w_i)_{i\in I}} \right) \Leftrightarrow u\left(x'_o\right) + \varrho\left(v\left(x'_o\right), \mu_{v\left(x'_o\right)_{i\in I}}\right) \leq u\left(x'_o\right) + \varrho\left(v\left(x'_o\right), \mu_{v\left(y'_i\right)_{i\in I}}\right) \Leftrightarrow \left(x'_o, (x'_i)_{i\in I}\right) \precsim \left(x'_o, (y'_i)_{i\in I}\right) = 0$ In particular, $(z_i)_{i \in I} \succeq_{z,I} (w_i)_{i \in I}$ if and only if $\rho\left(z, \mu_{(z_i)_{i \in I}}\right) \leq \rho\left(z, \mu_{(w_i)_{i \in I}}\right)$, thus $\succeq_{z,I}$ is complete, transitive, monotonic, symmetric (that is it satisfies Axioms 1, 2, 6).

Fix $z = v(x_o) \in v(C)$ and $I \in \wp(N) \setminus \emptyset$. Let $(z_i)_{i \in I}, (\bar{z}_i)_{i \in I} \in v(C)^I$. If there exist $\bar{\alpha} \in (0, 1]$ and $\bar{w} \in v\left(C\right)^{I} \text{ such that } \left(\bar{\alpha}z_{i}+\left(1-\bar{\alpha}\right)\bar{w}_{i}\right)_{i\in I} \succ_{z,I} \left(\bar{\alpha}\bar{z}_{i}+\left(1-\bar{\alpha}\right)\bar{w}_{i}\right)_{i\in I}, \text{ take } (x_{i})_{i\in I}, (\bar{x}_{i})_{i\in I}, (\bar{y}_{i})_{i\in I} \in I, (\bar{y}_{i})_{i\in I}, (\bar{y}_{i}), (\bar{y}_{i})_{i\in I}, (\bar{y}_{i})_{i\in I}, (\bar{y}_{i})_{i\in I}, (\bar{y}_{i})_{i\in I}, (\bar{y}_{i}), (\bar{y}_{i})_{i\in I}, (\bar{y}_{i}), (\bar{y}_{i})_{i\in I}, (\bar{y}_{i}), (\bar{y}_{i})_{i\in I}, (\bar{y}_{i}), (\bar{y}_{i}), (\bar{y}_{i}), (\bar{y}_{i}), (\bar{y}_{i}), (\bar{y}_{i}), (\bar$ C^{I} such that $v(x_{i}) = z_{i}, v(\bar{x}_{i}) = \bar{z}_{i}, \text{ and } v(\bar{y}_{i}) = \bar{w}_{i}, \text{ then it results } (x_{o}, (\bar{\alpha}x_{i} + (1 - \bar{\alpha})\bar{y}_{i})_{i \in I}) \prec$ $(x_o, (\alpha \bar{x}_i + (1 - \bar{\alpha}) \bar{y}_i)_{i \in I})$, by Axiom A.10, for all $(y_i)_{i \in I} \in C^I$ and $\alpha \in (0, 1], (x_o, (\alpha x_i + (1 - \alpha) y_i)_{i \in I})$ $\lesssim (x_o, (\alpha \bar{x}_i + (1 - \alpha) y_i)_{i \in I})$, that is, $(v (\alpha x_i + (1 - \alpha) y_i))_{i \in I} \succeq_{z,I} (v (\alpha \bar{x}_i + (1 - \alpha) y_i))_{i \in I}$, and $(\alpha z_i + (1 - \alpha) y_i)_{i \in I}$. $(1-\alpha) v(y_i)_{i \in I} \succeq_{z,I} (\alpha \bar{z}_i + (1-\alpha) v(y_i))_{i \in I}$. Thus $\succeq_{z,I}$ satisfies Axiom 4.²⁹

Fix $z = v(x_o) \in v(C)$ and $I \in \wp(N) \setminus \emptyset$. For all $(z_i)_{i \in I}, (\bar{z}_i)_{i \in I}, (w_i)_{i \in I} \in v(C)^I$, take $(x_i)_{i \in I}, (w_i)_{i \in I} \in v(C)^I$. $(\bar{x}_i)_{i \in I}, (y_i)_{i \in I} \in C$ such that $v(x_j) = z_j, v(\bar{x}_j) = \bar{z}_j$, and $v(y_j) = w_j$ for all $j \in I$, and notice that $\{\alpha \in I\}$ $[0,1] : (\alpha z_i + (1-\alpha) \, \bar{z}_i)_{i \in I} \succeq_{z,I} (w_i)_{i \in I} \} = \{ \alpha \in [0,1] : (x_o, (\alpha x_i + (1-\alpha) \, \bar{x}_i)_{i \in I}) \precsim (x_o, (y_i)_{i \in I}) \}$ and $\{\alpha \in [0,1] : (\alpha z_i + (1-\alpha) \bar{z}_i)_{i \in I} \preceq_{z,I} (w_i)_{i \in I}\} = \{\alpha \in [0,1] : (x_o, (\alpha x_i + (1-\alpha) \bar{x}_i)_{i \in I}) \succeq_{z,I}\}$ $(x_o, (y_i)_{i \in I})$ are closed sets because of Axiom A.11; thus $\succeq_{z,I}$ satisfies Axiom 3. By Lemma 3, there exists a (weakly) increasing and continuous function $\psi_{z,I}: v(C) \to \mathbb{R}$ such that

$$(z_i)_{i \in I} \succeq_{z,I} (w_i)_{i \in I} \Leftrightarrow \psi_{z,I} \left(\frac{1}{|I|} \sum_{i \in I} z_i \right) \ge \psi_{z,I} \left(\frac{1}{|I|} \sum_{i \in I} w_i \right)$$
(26)

Next we show that if $(z, \mu), (z, \mu') \in \text{pim}(v(C)) \setminus \{(z, 0)\}$ and $\mathcal{E}(\mu) = \mathcal{E}(\mu'),^{30}$ then $\varrho(z, \mu) = \varrho(z, \mu')$.

- If $\mu(v(C)) = \mu'(v(C)) = n$ (which must be positive), let I be an arbitrarily chosen subset of I with cardinality n. Then there exist $(z_i)_{i \in I}, (w_i)_{i \in I} \in v(C)^I$ such that $\mu = \mu_{(z_i)_{i \in I}}$ and $\mu' = \mu_{(w_i)_{i \in I}}$. $E(\mu) = E(\mu')$ and (26) imply that $(z_i)_{i \in I} \sim_{z,I} (w_i)_{i \in I}$ which amounts to $\varrho\left(z,\mu_{(z_i)_{i\in I}}\right) = \varrho\left(z,\mu_{(w_i)_{i\in I}}\right), \text{ i.e., } \varrho\left(z,\mu\right) = \varrho\left(z,\mu'\right).$
- If $\mu(v(C)) = n$ and $\mu'(v(C)) = m$, then there exist $x = (x_o, (x_i)_{i \in I})$ and $(x_o, (y_j)_{j \in J})$ with |I| = n and |J| = m such that $z = v(x_o), \ \mu = \mu_{(v(x_i))_{i \in I}}$ and $\mu' = \mu_{(v(y_j))_{j \in J}}$. Let $c \in C$ be such that $c \sim x$, then $c_{I_o} \sim x$, that is $(c, c_I) \sim (x_o, (x_i)_{i \in I})$ and by Axiom A.9, given any class $\{J_i\}_{i \in I}$ of disjoint subsets of N with $|J_i| = m$ for all $i \in I$, $(c, (c_{J_i})_{i \in I}) \sim$ $(x_o, (x_i J_i)_{i \in I})$, but $(c, (c_{J_i})_{i \in I}) = c_{(\cup_i J_i) \cup \{o\}} \sim c$, hence $(x_o, (x_i)_{i \in I}) \sim (x_o, (x_i J_i)_{i \in I})$ and,

²⁹Since $v(C)^{I} = \{(v(y_{i}))_{i \in I} : (y_{i})_{i \in I} \in C^{I}\}.$ ³⁰Here $E(\mu) = \mu(\mathbb{R})^{-1} \sum_{r \in \operatorname{supp}(\mu)} r\mu(r)$, that is $|I|^{-1} \sum_{i \in I} z_{i}$ if $\mu = \mu_{(z_{i})_{i \in I}}.$

setting $L \equiv \bigcup_i J_i$ and $\left(x_o, \left(x_i J_i\right)_{i \in I}\right) \equiv \left(x_o, \left(\bar{x}_l\right)_{l \in L}\right)$, obviously $\mathcal{E}(\mu) = \frac{1}{|I|} \sum_{i \in I} v(x_i) = \frac{1}{|I|} \sum_{i \in I} v(x_i)$ $\frac{1}{|I|} \sum_{i \in I} \left(\frac{1}{|J_i|} \sum_{j_i \in J_i} v(\bar{x}_{j_i})\right) = \frac{1}{|\bigcup_i J_i|} \sum_{i \in I} \left(\sum_{j_i \in J_i} v(\bar{x}_{j_i})\right) = \mathcal{E}\left(\mu_{(v(\bar{x}_l))_{l \in L}}\right)$. Summing up: There exists $L \in \wp(N)$ with |L| = mn and $\left(x_o, \left(\bar{x}_l\right)_{l \in L}\right) \in \mathcal{X}$ such that $\left(x_o, \left(x_i\right)_{i \in I}\right) \sim \left(x_o, \left(\bar{x}_l\right)_{l \in L}\right)$ and $\mathcal{E}(\mu) = \mathcal{E}\left(\mu_{(v(\bar{x}_l))_{l \in L}}\right)$. By an identical argument we can consider an *n*-replica $\left(x_o, \left(\bar{y}_l\right)_{l \in L}\right)$ of $\left(x_o, \left(y_j\right)_{j \in J}\right)$ (where *L* is the set define above) and show that $\left(x_o, \left(y_j\right)_{j \in J}\right) \sim \left(x_o, \left(\bar{y}_l\right)_{l \in L}\right)$ and $\mathcal{E}(\mu') = \mathcal{E}\left(\mu_{(v(\bar{y}_l))_{l \in L}}\right)$. Then $\varrho(z, \mu) = \varrho(z, \mu') \Leftrightarrow u(x_o) + \varrho\left(v(x_o), \mu_{(v(x_i))_{i \in I}}\right) = u(x_o) + \varrho\left(v(x_o), \mu_{(v(y_j))_{j \in J}}\right) \Leftrightarrow \left(x_o, \left(x_i\right)_{i \in I}\right) \sim \left(x_o, \left(\bar{y}_l\right)_{l \in L}\right) \leftrightarrow \left(v(\bar{x}_l)_{l \in L}\right) \to \left(x_o, \left(\bar{y}_l\right)_{l \in L}\right) = \mathcal{E}(\mu) = \mathcal{E}(\mu') = \mathcal{E}\left(\mu_{(v(\bar{y}_l))_{l \in L}}\right)$ and the last indifference descends from $\mathcal{E}\left(\mu_{(v(\bar{x}_l))_{l \in L}}\right) = \mathcal{E}(\mu) = \mathcal{E}(\mu') = \mathcal{E}\left(\mu_{(v(\bar{y}_l))_{l \in L}}\right)$ and (26).

Therefore $\rho(z,\mu) = \rho(z,\delta_{\mathrm{E}(\mu)})$, for all $(z,\mu) \in \mathrm{pim}(v(C)) \setminus \{(z,0)\}$. With the conventions $\mathrm{E}(0) = |\emptyset|^{-1} \sum_{i \in \emptyset} z_i = \infty$ and $\delta_{\infty} = 0$, we also have $\rho(z,0) = \rho(z,\delta_{\infty}) = \rho(z,\delta_{\mathrm{E}(0)})$. The function $\eta(z,t) = \rho(z,\delta_t)$ for all $(z,t) \in v(C) \times (v(C) \cup \{\infty\})$ is diago-null, increasing in the first component and decreasing in the second on v(C), and $\rho(z,\mu) = \eta(z,\mathrm{E}(\mu))$ for all $(z,\mu) \in \mathrm{pim}(v(C))$.

It only remains to show that η is continuously decreasing in the second component on v(C). Fix $z \in v(C)$, $i \in N$ and notice that $\eta(z,t) \geq \eta(z,\bar{t}) \Leftrightarrow \varrho(z,\delta_t) \geq \varrho(z,\delta_{\bar{t}}) \Leftrightarrow t \preceq_{z,\{i\}} \bar{t} \Leftrightarrow \psi_{z,\{i\}}(t) \leq \psi_{z,\{i\}}(\bar{t})$ for all $t, \bar{t} \in v(C) = v(C)^{\{i\}}$. Therefore, there exists a strictly increasing function $\vartheta : -\psi_{z,\{i\}}(v(C)) \to \mathbb{R}$ such that $\eta(z,t) = \vartheta(-\psi_{z,\{i\}}(t))$ for all $t \in v(C)$. The proof of sufficiency is concluded by renaming η into ϱ .

To prove necessity, assume there exist two non-constant affine functions $u, v : C \to \mathbb{R}$, a diago-null function $\eta : v(C) \times (v(C) \cup \{\infty\}) \to \mathbb{R}$ increasing in the first component and continuously decreasing in the second on v(C), and a probability P on Σ , such that v represents \succeq and the function defined by $V(f_o, (f_i)_{i \in I}) = \int_S \left[u(f_o(s)) + \eta \left(v(f_o(s)), \frac{1}{|I|} \sum_{i \in I} v(f_i(s)) \right) \right] dP(s)$, for all $(f_o, (f_i)_{i \in I}) \in \mathcal{F}$, represents \succeq and satisfies $V(\mathcal{F}) = u(C)$. Set $\varrho(z, \mu) = \eta(z, \mathrm{E}(\mu))$ for all $(z, \mu) \in \mathrm{pim}(v(C))$ (with the above convention $\mathrm{E}(0) = \infty$). It is clear that ϱ is diago-null, increasing in the first component and decreasing (w.r.t. stochastic dominance) in the second, and hence, by Theorem 3, \succeq on \mathcal{F} satisfies Axioms A.1-A.8. It remains to show that \succeq satisfies Axioms A.9, A.10, and A.11.

As observed, for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$, all $m \in \mathbb{N}$, and each *m*-replica $(x_o, (\bar{x}_l)_{l \in L})$ of $(x_o, (x_i)_{i \in I})$, $\operatorname{E}\left(\mu_{(v(x_i))_{i \in I}}\right) = \operatorname{E}\left(\mu_{(v(\bar{x}_l))_{l \in L}}\right)$, hence $V\left(x_o, (x_i)_{i \in I}\right) = V\left(x_o, (\bar{x}_l)_{l \in L}\right)$, which implies Axiom A.9.

As to Axiom A.10, let $(x_o, (x_i)_{i \in I})$, $(x_o, (y_i)_{i \in I}) \in \mathcal{X}$ and assume that $(x_o, (\bar{\alpha}x_i + (1 - \bar{\alpha})\bar{z}_i)_{i \in I})$ $\succ (x_o, (\bar{\alpha}y_i + (1 - \bar{\alpha})\bar{z}_i)_{i \in I})$ for some $\bar{\alpha}$ in (0, 1] and $(x_o, (\bar{z}_i)_{i \in I}) \in \mathcal{X}$,³¹ then $u(x_o) + \eta \left(v(x_o), \frac{1}{|I|} \sum_{i \in I} v(\bar{\alpha}x_i + (1 - \bar{\alpha})\bar{z}_i) \right) > u(x_o) + \eta \left(v(x_o), \frac{1}{|I|} \sum_{i \in I} v(\bar{\alpha}y_i + (1 - \bar{\alpha})\bar{z}_i) \right)$, that is

$$\eta\left(v\left(x_{o}\right), \bar{\alpha}\frac{1}{|I|}\sum_{i\in I}v\left(x_{i}\right) + (1-\bar{\alpha})\frac{1}{|I|}\sum_{i\in I}v\left(\bar{z}_{i}\right)\right) > \eta\left(v\left(x_{o}\right), \bar{\alpha}\frac{1}{|I|}\sum_{i\in I}v\left(y_{i}\right) + (1-\bar{\alpha})\frac{1}{|I|}\sum_{i\in I}v\left(\bar{z}_{i}\right)\right)$$

hence, since η is decreasing in the second component on v(C), $|I|^{-1} \sum_{i \in I} v(y_i) \ge |I|^{-1} \sum_{i \in I} v(x_i)$. Therefore for all α in (0, 1] and $(x_o, (z_i)_{i \in I}) \in \mathcal{X}$,

$$|I|^{-1} \sum_{i \in I} v \left(\alpha y_i + (1 - \alpha) z_i \right) \ge |I|^{-1} \sum_{i \in I} v \left(\alpha x_i + (1 - \alpha) z_i \right)$$

thus $u(x_o) + \eta \left(v(x_o), \frac{1}{|I|} \sum_{i \in I} v(\alpha x_i + (1 - \alpha) z_i) \right) \ge u(x_o) + \eta \left(v(x_o), \frac{1}{|I|} \sum_{i \in I} v(\alpha y_i + (1 - \alpha) z_i) \right)$ and $\left(x_o, (\alpha x_i + (1 - \alpha) z_i)_{i \in I} \right) \succeq \left(x_o, (\alpha y_i + (1 - \alpha) z_i)_{i \in I} \right)$, as wanted.

³¹This cannot be the case if I is empty.

Finally let $(x_o, (x_i)_{i \in I})$, $(x_o, (y_i)_{i \in I})$, $(x_o, (z_i)_{i \in I}) \in \mathcal{X}$ and assume $\{\alpha_n\}_{n \in \mathbb{N}} \subseteq [0, 1]$, $\alpha_n \to \alpha$, and $(x_o, (\alpha_n x_i + (1 - \alpha_n) z_i)_{i \in I}) \succeq (x_o, (y_i)_{i \in I})$ for all $n \in \mathbb{N}$. Clearly, if I is empty, $(x_o, (\alpha x_i + (1 - \alpha) z_i)_{i \in I}) = (x_o, (y_i)_{i \in I})$, hence $(x_o, (\alpha x_i + (1 - \alpha) z_i)_{i \in I}) \succeq (x_o, (y_i)_{i \in I})$. Else, let $\psi_{v(x_o)} : v(C) \to \mathbb{R}$ be a weakly decreasing and continuous function such that for all $t, \bar{t} \in v(C)$,

$$\eta\left(v\left(x_{o}\right),t\right) \geq \eta\left(v\left(x_{o}\right),\bar{t}\right) \Leftrightarrow \psi_{v\left(x_{o}\right)}\left(t\right) \geq \psi_{v\left(x_{o}\right)}\left(\bar{t}\right)$$

(which exists since $\eta(v(x_o), \cdot)$ is continuously decreasing on v(C)). Then, for all $n \in \mathbb{N}$, the preference $(x_o, (\alpha_n x_i + (1 - \alpha_n) z_i)_{i \in I}) \succeq (x_o, (y_i)_{i \in I})$ implies

$$\eta\left(v\left(x_{o}\right), |I|^{-1}\sum_{i\in I}v\left(y_{i}\right)\right) \leq \eta\left(v\left(x_{o}\right), |I|^{-1}\sum_{i\in I}v\left(\alpha_{n}x_{i}+(1-\alpha_{n})z_{i}\right)\right)$$

that is $\psi_{v(x_o)}\left(|I|^{-1}\sum_{i\in I}v(y_i)\right) \leq \psi_{v(x_o)}\left(|I|^{-1}\sum_{i\in I}\alpha_n v(x_i) + (1-\alpha_n)v(z_i)\right)$, and continuity of $\psi_{v(x_o)}$ delivers $\psi_{v(x_o)}\left(|I|^{-1}\sum_{i\in I}v(y_i)\right) \leq \psi_{v(x_o)}\left(|I|^{-1}\sum_{i\in I}\alpha v(x_i) + (1-\alpha)v(z_i)\right)$, which in turn implies $(x_o, (\alpha x_i + (1-\alpha)z_i)_{i\in I}) \succeq (x_o, (y_i)_{i\in I})$.

Then the set $\{\alpha \in [0,1] : (x_o, (\alpha x_i + (1-\alpha) z_i)_{i \in I}) \succeq (x_o, (y_i)_{i \in I})\}$ is closed, and analogous considerations hold for $\{\alpha \in [0,1] : (x_o, (\alpha x_i + (1-\alpha) z_i)_{i \in I}) \preceq (x_o, (y_i)_{i \in I})\}$.

Proposition 10 Two quadruples (u, v, ρ, P) and $(\hat{u}, \hat{v}, \hat{\rho}, \hat{P})$ represent the same relations \succeq and \succeq as in Theorem 4 if and only if $\hat{P} = P$ and there exist $\alpha, \beta, \dot{\alpha}, \dot{\beta} \in \mathbb{R}$ with $\alpha, \dot{\alpha} > 0$ such that $\hat{u} = \alpha u + \beta$, $\hat{v} = \dot{\alpha}v + \dot{\beta}$, and

$$\hat{\varrho}(z,r) = \alpha \varrho\left(\frac{z-\dot{\beta}}{\dot{\alpha}}, \frac{r-\dot{\beta}}{\dot{\alpha}}\right)$$

for all $(z,r) \in \hat{v}(C) \times (\hat{v}(C) \cup \{\infty\}).$

Proof. Omitted (it is very similar to the one of Proposition 3).

A.7 Attitude to Social Gains and Losses

Proof of Proposition 5. First, observe that for a real valued function ϕ defined on an interval $K \ni z$ the following statements are equivalent:

- (i) $\phi(z) \ge \phi(z+h) + \phi(z-h)$ for all $h \ge 0$ such that $z \pm h \in K$,
- (ii) $\phi(z) \ge \phi(t) + \phi(w)$ for all $t, w \in K$ such that t/2 + w/2 = z.³²

Assume \succeq is more envious than proud, relative to an ethically neutral event E, a convex $D \subseteq C$, and $x_o \in D$. Let $t, w \in v(D)$ be such that $t/2 + w/2 = v(x_o)$. Choose $x_i, y_i \in D$ such that $t = v(x_i)$ and $w = v(y_i)$. Then $v(2^{-1}x_i + 2^{-1}y_i) = 2^{-1}v(x_i) + 2^{-1}v(y_i) = v(x_o)$ implies $2^{-1}x_i + 2^{-1}y_i \stackrel{\sim}{\sim} x_o$, and the assumption of social loss aversion delivers $(x_o, x_o) \succeq (x_o, x_i E y_i) \Longrightarrow u(x_o) \ge \frac{1}{2}(u(x_o) + \varrho(v(x_o), v(x_i))) + \frac{1}{2}(u(x_o) + \varrho(v(x_o), v(y_i))) \Longrightarrow 0 \ge \varrho(v(x_o), v(x_i)) + \varrho(v(x_o), v(y_i))$ $\Longrightarrow \varrho(v(x_o), v(x_o)) \ge \varrho(v(x_o), t) + \varrho(v(x_o), w)$. Then $0 = \varrho(v(x_o), v(x_o)) \ge \varrho(v(x_o), v(x_o) + h) + \varrho(v(x_o), v(x_o) - h)$ for all $h \ge 0$ such that $v(x_o) \pm h \in v(D)$.

Conversely, if (12) holds, then $\varrho(v(x_o), v(x_o) + h) + \varrho(v(x_o), v(x_o) - h) \leq 0 = \varrho(v(x_o), v(x_o))$ for all $h \geq 0$ such that $z \pm h \in v(D)$, that is, $\varrho(v(x_o), v(x_o)) \geq \varrho(v(x_o), t) + \varrho(v(x_o), w)$ for all

³²(i) \Rightarrow (ii) If $t, w \in K$ are such that t/2 + w/2 = z, and $t \ge w$, set h = (t - w)/2, it follows that $h \ge 0$ and that $z + h = t/2 + w/2 + (t/2 - w/2) = t \in K$, $z - h = t/2 + w/2 - (t/2 - w/2) = w \in K$. By (i), $\phi(z) \ge \phi(z + h) + \phi(z - h) = \phi(t) + \phi(w)$. If $t \le w$, set h = (w - t)/2 and repeat the same argument.

⁽ii) \Rightarrow (i) If $h \ge 0$ is such that $z \pm h \in K$, then, from (z+h)/2 + (z-h)/2 = z and (ii), it follows that $\phi(z) \ge \phi(z+h) + \phi(z-h)$.

 $\begin{aligned} t, w \in v\left(D\right) \text{ such that } t/2 + w/2 &= v\left(x_o\right). \text{ If } x_i, y_i \in D \text{ are such that } (1/2) x_i + (1/2) y_i \sim x_o, \text{ then } \\ 2^{-1}v\left(x_i\right) + 2^{-1}v\left(y_i\right) &= v\left(2^{-1}x_i + 2^{-1}y_i\right) = v\left(x_o\right) \text{ and hence } 0 = \varrho(v\left(x_o\right), v\left(x_o\right)) \geq \varrho(v\left(x_o\right), v\left(x_i\right)) + \\ \varrho(v\left(x_o\right), v\left(y_i\right)) &\Longrightarrow 0 \geq \frac{1}{2}\varrho(v\left(x_o\right), v\left(x_i\right)) + \frac{1}{2}\varrho(v\left(x_o\right), v\left(y_i\right)) \Longrightarrow u\left(x_o\right) \geq \frac{1}{2}u\left(x_o\right) + \frac{1}{2}\varrho(v\left(x_o\right), v\left(x_i\right)) + \\ \frac{1}{2}u\left(x_o\right) + \frac{1}{2}\varrho(v\left(x_o\right), v\left(y_i\right)) \Longrightarrow \left(x_o, x_o\right) \succeq \left(x_o, x_i E y_i\right). \text{ Thus } \succeq \text{ is more envious than proud. Finally,} \\ \text{inequality (13) easily follows from (12). In fact, let } v\left(x_o\right) &= r \in \text{ int } (v\left(D\right)), \text{ since } \varrho(r, r) = 0, \\ \underline{D}_2^+ \varrho\left(r, r\right) &= \liminf_{h \downarrow 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} &= \lim_{\varepsilon \downarrow 0} \inf_{h \in (0,\varepsilon)} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} \leq \lim_{\varepsilon \downarrow 0} \inf_{h \in (-\varepsilon,0)} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} &= \lim_{\varepsilon \downarrow 0} \inf_{h \to (-\varepsilon,0)} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \downarrow 0} \inf_{h \to (-\varepsilon,0)} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \downarrow 0} \inf_{v \neq (-\varepsilon,0)} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \downarrow 0} \inf_{v \neq (-\varepsilon,0)} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r, r+h) - \varrho(r, r)}{h} = \lim_{\varepsilon \to 0} \frac{\varrho(r,$

Before entering the details of the proof of Proposition 6, recall that an event $E \in \Sigma$ is essential if $\overline{c} \approx \overline{c}E\underline{c} \approx \underline{c}$ for some \overline{c} and \underline{c} in C. Representation (10) guarantees that this amounts to say that $P(E) \in (0, 1)$, in particular, ethically neutral events are essential.

We say that a preference \succeq is averse to social risk, relatively to an essential event E, a convex set $D \subseteq C$, and a given $x_o \in C$, if $(x_o, w_i) \succeq (x_o, x_i E y_i)$ for all $x_i, y_i, w_i \in D$ such that $P(E) x_i + (1 - P(E)) y_i \sim w_i$. Notice that this definition is consistent with the previous one in which only ethically neutral events E where considered (thus P(E) = 1/2). Instead of **proving Proposition 6** we will prove the more general

Proposition 11 If \succeq admits a representation (10), then \succeq is averse to social risk, relative to an essential event E, a convex $D \subseteq C$, and $x_o \in C$ if and only if $\rho(v(x_o), \cdot)$ is concave on v(D).

Proof of Proposition 11. Assume \succeq is averse to social risk, relative to an essential event E, a convex $D \subseteq C$, and $x_o \in C$. Essentiality of E guarantees that $P(E) = p \in (0, 1)$. Therefore, for all $t = v(x_i), r = v(y_i) \in v(D)$, social risk aversion implies $(x_o, px_i + (1 - p)y_i) \succeq (x_o, x_i Ey_i)$, whence $u(x_o) + \varrho(v(x_o), v(px_i + (1 - p)y_i)) \ge p(u(x_o) + \varrho(v(x_o), v(x_i))) + (1 - p)(u(x_o) + \varrho(v(x_o), v(y_i)))$, thus $u(x_o) + \varrho(v(x_o), pt + (1 - p)r) \ge u(x_o) + p\varrho(v(x_o), t) + (1 - p)\varrho(v(x_o), r)$, and it follows that $\varrho(v(x_o), pt + (1 - p)r) \ge p\varrho(v(x_o), t) + (1 - p)\varrho(v(x_o), r)$. In turn, this (together with monotonicity of ϱ in the second component) can be shown to imply continuity of $\varrho(v(x_o), \cdot)$ on $v(D) \setminus \sup v(D)$. Theorem 88 of Hardy, Littlewood and Polya (1934) guarantees concavity on $v(D) \setminus \sup v(D)$. Again monotonicity delivers concavity of $\varrho(v(x_o), \cdot)$ on v(D). The converse is trivial.

A.8 Comparative Interdependence

Proof of Proposition 7. (i) \Rightarrow (ii) Taking $I = \emptyset$, since u_1 and u_2 are affine, non-constant, and represent \succeq_1 and \succeq_2 on C, we obtain $u_1 \approx u_2$. W.l.o.g. choose $u_1 = u_2 = u$. For all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ choose $c \in C$ such that $(x_o, (x_i)_{i \in I}) \sim_1 c$, then $(x_o, (x_i)_{i \in I}) \succeq_2 c$ and $u(x_o) + \varrho_2(x_o, \mu_{(x)_{i \in I}}) \ge u(c) = u(x_o) + \varrho_1(x_o, \mu_{(x)_{i \in I}})$, that is $\varrho_2 \ge \varrho_2$ on pim (C).

(ii) \Rightarrow (i) Take $u_1 = u_2 = u$. If $(x_o, (x_i)_{i \in I}) \succeq_1 c_{I_o}$, then $u(x_o) + \varrho_1(x_o, \mu_{(x)_{i \in I}}) \ge u(c)$ hence $u(x_o) + \varrho_2(x_o, \mu_{(x)_{i \in I}}) \ge u(c)$ and $(x_o, (x_i)_{i \in I}) \succeq_2 c_{I_o}$. As wanted.

Proof of Proposition 8. By Proposition 7, (i) is equivalent to $u_1 \approx u_2$ and, choosing $u_1 = u_2$, $\varrho_1 \leq \varrho_2$.

(i) \Rightarrow (ii) Intrinsic equivalence is obvious. Take $u_1 = u_2 = u$. If $x_o \succ_2 y_o \succeq_2 (x_o, (x_i)_{i \in I})$, then $u(x_o) > u(y_o) \ge u(x_o) + \varrho_2(x_o, \mu_{(x)_{i \in I}})$ amounts to $\varrho_2(x_o, \mu_{(x)_{i \in I}}) \le u(y_o) - u(x_o) < 0$ which implies $\varrho_1(x_o, \mu_{(x)_{i \in I}}) \le u(y_o) - u(x_o) < 0$ and $x_o \succ_1 y_o \succeq_1 (x_o, (x_i)_{i \in I})$. An analogous argument shows that, if $x_o \prec_1 y_o \precsim_1 (x_o, (x_i)_{i \in I})$, then $x_o \prec_2 y_o \precsim_2 (x_o, (x_i)_{i \in I})$.

(ii) \Rightarrow (i) Since u_1 and u_2 are affine, if \succeq_1 is intrinsically equivalent to \succeq_2 , then $u_1 \approx u_2$. W.l.o.g. choose $u_1 = u_2 = u$. For all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ choose c such that $c \sim_2 (x_o, (x_i)_{i \in I})$, i.e. $u(c) = u(x_o) + \varrho_2 (x_o, \mu_{(x)_{i \in I}})$, and \overline{c} such that $\overline{c} \sim_1 (x_o, (x_i)_{i \in I})$.

If $\varrho_2\left(x_o, \mu_{(x)_{i\in I}}\right) < 0$, then $u\left(x_o\right) + \varrho_2\left(x_o, \mu_{(x)_{i\in I}}\right) < u\left(x_o\right)$ and $x_o \succ_2 c \sim_2 \left(x_o, (x_i)_{i\in I}\right)$, which implies $x_o \succ_1 c \succeq_1 \left(x_o, (x_i)_{i\in I}\right)$ and $u\left(x_o\right) + \varrho_1\left(x_o, \mu_{(x)_{i\in I}}\right) \leq u\left(c\right) < u\left(x_o\right)$, thus $\varrho_1\left(x_o, \mu_{(x)_{i\in I}}\right) \leq u\left(c\right) - u\left(x_o\right) = \varrho_2\left(x_o, \mu_{(x)_{i\in I}}\right)$. Analogously, if $\varrho_1\left(x_o, \mu_{(x)_{i\in I}}\right) > 0$, then $\varrho_2\left(x_o, \mu_{(x)_{i\in I}}\right) \geq u\left(\overline{c}\right) - u\left(x_o\right) = \varrho_1\left(x_o, \mu_{(x)_{i\in I}}\right)$. Conclude that: if $\varrho_2\left(\cdot\right) < 0$ then $\varrho_1\left(\cdot\right) \leq \varrho_2\left(\cdot\right)$, if $\varrho_2\left(\cdot\right) \geq 0$, then either $\varrho_1\left(\cdot\right) > 0$ and $\varrho_1\left(\cdot\right) \leq \varrho_2\left(\cdot\right)$ or $\varrho_1\left(\cdot\right) \leq 0$ and $\varrho_1\left(\cdot\right) \leq 0 \leq \varrho_2\left(\cdot\right)$.

Proof of Proposition 9. Omitted (it is very similar to the previous ones).

A.9 Inequity Aversion

For the proof of the Theorem 5, we begin with a preliminary lemma

Lemma 7 A binary relation \succeq on \mathcal{F} satisfies Axioms A.1-A.6 and F.1 if and only if there exist a non-constant affine function $u: C \to \mathbb{R}$, a diago-null function $\varrho: \text{pim}(u(C)) \to \mathbb{R}$, and a probability P on Σ , such that the function $V: \mathcal{F} \to \mathbb{R}$, defined by

$$V\left(f_{o},\left(f_{i}\right)_{i\in I}\right) = \int_{S} \left[u\left(f_{o}\left(s\right)\right) + \varrho\left(u\left(f_{o}\left(s\right)\right),\mu_{\left(u\left(f_{i}\left(s\right)\right)\right)_{i\in I}}\right)\right]dP\left(s\right)$$

$$(27)$$

for all $(f_o, (f_i)_{i \in I}) \in \mathcal{F}$, represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

The triplet $(\hat{u}, \hat{\varrho}, \hat{P})$ is another representation of \succeq in the above sense if and only if $\hat{P} = P$ and there exist $\alpha, \beta \in \mathbb{R}$ with $\alpha > 0$ such that $\hat{u} = \alpha u + \beta$, and

$$\hat{\varrho}\left(z,\sum_{i\in I}\delta_{z_i}\right) = \alpha \varrho\left(\alpha^{-1}\left(z-\beta\right),\sum_{i\in I}\delta_{\alpha^{-1}\left(z_i-\beta\right)}\right)$$

for all $(z, \sum_{i \in I} \delta_{z_i}) \in pim(\hat{u}(C)).$

Proof. By Lemma 4, there exist a non-constant affine function $u : C \to \mathbb{R}$, a function $r : \mathcal{X} \to \mathbb{R}$ with $r(c_{I_o}) = 0$ for all $c \in C$ and $I \in \wp(N)$, and a probability P on Σ , such that the functional $V : \mathcal{F} \to \mathbb{R}$, defined by (23), represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

Next we show that if $(x_o, (x_i)_{i \in I}), (y_o, (y_j)_{j \in J}) \in \mathcal{X}, u(x_o) = u(y_o), \text{ and } \mu_{(u(x_i))_{i \in I}} = \mu_{(u(y_j))_{j \in J}},$ then $r(x_o, (x_i)_{i \in I}) = r(y_o, (y_j)_{j \in J})$. Therefore, for $(z, \mu) \in \text{pim}(u(C))$, it is well posed to define $\varrho(z, \mu) = r(x_o, (x_i)_{i \in I})$ provided $z = u(x_o)$ and $\mu = \mu_{(u(x_i))_{i \in I}}$. But first notice that at least one $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ such that $z = u(x_o)$ and $\mu = \mu_{(u(x_i))_{i \in I}}$ exists for every $(z, \mu) \in \text{pim}(u(C))$.³³ If $\mu_{(u(x_i))_{i \in I}} = \mu_{(u(y_j))_{j \in J}} = 0$, then $I = J = \emptyset$. In this case, $r(x_o, (x_i)_{i \in I}) = r(x_o) = 0 = r(y_o) = r(y_o) = r(y_o, (y_j)_{j \in J})$. Else if $\mu_{(u(x_i))_{i \in I}} = \mu_{(u(y_j))_{j \in J}} \neq 0$, then, by Lemma 2, there is a bijection $\pi : I \to J$ such that $u(x_i) = u(y_{\pi(i)})$ for all $i \in I$. Then $y_{\pi(i)} \sim x_i$ and $y_o \sim x_o$. Axiom F.1 guarantees that $(x_o, (x_i)_{i \in I}) \sim (y_o, (y_{\pi(i)})_{i \in I})$. Consider the inverse bijection $\pi^{-1} : J \to I$ and notice that $y_j = y_{\pi(\pi^{-1}(j))}$ for all $j \in J$, then Axiom A.6 delivers $(y_o, (y_{\pi(i)})_{i \in I}) \sim (y_o, (y_j)_{j \in J})$ and $u(x_o) + y_{\pi(i)} = y_{\pi(\pi^{-1}(j))}$.

 $[\]overline{{}^{33}\text{In any case take } x_o \in u^{-1}(z) \text{. If } \mu = 0 \text{ take } I = \emptyset. \text{ Else if } \mu = \sum_{k=1}^n \delta_{z_k} \text{ for some } n \ge 1, \text{ take a subset } I = \{i_1, ..., i_n\} \text{ of } N \text{ with cardinality } n \text{ and arbitrarily choose } x_{i_k} \in u^{-1}(z_k) \text{ for all } k = 1, ..., n.$

 $r\left(x_{o},(x_{i})_{i\in I}\right) = u\left(y_{o}\right) + r\left(y_{o},(y_{j})_{j\in J}\right), \text{ which, together with } u\left(x_{o}\right) = u\left(y_{o}\right), \text{ delivers } r\left(x_{o},(x_{i})_{i\in I}\right) = r\left(y_{o},(y_{j})_{j\in J}\right).$ As wanted.

If $z \in u(C)$ and $0 \le n \le |N|$, take $c \in C$ such that u(c) = z and $I \in \wp(N)$ such that |I| = n, then $\wp(z, n\delta_z) = r(c_{I_o}) = 0$.

Therefore ρ is diago-null and $V(f) = \int_{S} \left[u(f_{o}(s)) + \rho\left(u(f_{o}(s)), \mu_{(u(f_{i}(s)))_{i \in I}} \right) \right] dP(s)$ for all $f \in \mathcal{F}$. This completes the proof of the sufficiency part of the theorem.

As to the necessity part, we just have to show that a preference represented by (27) satisfies Axioms A.6 and F.1 (the rest descends from Lemma 4, by setting $r(x_o, (x_i)_{i \in I}) = \rho(u(x_o), \mu_{(u(x_i))_{i \in I}})$ for all $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and observing that, for all $I \in \wp(N)$ and $c \in C$, $r(c_{I_o}) = \rho(u(c), |I| \delta_{u(c)}) =$ 0). Let $(x_o, (x_i)_{i \in I}), (y_o, (y_j)_{j \in J}) \in \mathcal{X}$ be such that $x_o \sim y_o$ and there is a bijection $\sigma : J \to I$ such that for every $j \in J$, $y_j \sim x_{\sigma(j)}$. If $I = \emptyset$, then $J = \emptyset$ and $x = (x_o) \sim (y_o) = y$. Else $\mu_{(u(y_j))_{j \in J}} = \sum_{j \in J} \delta_{u(y_j)} = \sum_{j \in J} \delta_{u(x_{\sigma(j)})} = \sum_{i \in I} \delta_{u(x_i)} = \mu_{(u(x_i))_{i \in I}}$, since also $u(x_o) = u(y_o)$, then $\rho(u(x_o), \mu_{(u(x_i))_{i \in I}}) = \rho(u(y_o), \mu_{(u(y_j))_{j \in J}})$ and $(x_o, (x_i)_{i \in I}) \sim (y_o, (y_j)_{j \in J})$. From the special case in which $x_o = y_o$ and $y_j = x_{\sigma(j)}$ for all $j \in J$, it follows that Axiom A.6 holds. From the special case in which I = J and σ is the identity, it follows that Axiom F.1 holds.

The proof of the uniqueness part is very similar to that of Proposition 3.

Proof of Theorem 5. By Lemma 7 there exist a non-constant affine function $u: C \to \mathbb{R}$, a diago-null function $\varrho: \operatorname{pim}(u(C)) \to \mathbb{R}$, and a probability P on Σ , such that the function $V: \mathcal{F} \to \mathbb{R}$, defined by (27), represents \succeq and satisfies $V(\mathcal{F}) = u(C)$.

For all $(z, \mu, \nu) \in \text{pid}(u(C))$ set $\xi(z, \mu, \nu) = \varrho(z, \mu + \nu)$, clearly ξ is well defined and $\xi(z, 0, n\delta_z) = \varrho(z, n\delta_z) = 0$ for all $z \in u(C)$ and $0 \le n \le |N|$, that is ξ is diago-null. Next we show that ξ is increasing in the second component w.r.t. stochastic dominance.

Let $(z, \sum_{i \in I} \delta_{a_i}, \sum_{l \in L} \delta_{z_l}), (z, \sum_{j \in J} \delta_{b_j}, \sum_{l \in L} \delta_{z_l}) \in \text{pid}(u(C))$ and assume $\sum_{i \in I} \delta_{a_i}$ stochastically dominates $\sum_{j \in J} \delta_{b_j}$.³⁴

If
$$I = J = \emptyset$$
, then $\xi \left(z, \sum_{i \in I} \delta_{a_i}, \sum_{l \in L} \delta_{z_l} \right) = \xi \left(z, 0, \sum_{l \in L} \delta_{z_l} \right) = \xi \left(z, \sum_{j \in J} \delta_{b_j}, \sum_{l \in L} \delta_{z_l} \right)$.

Else if $I, J \neq \emptyset$, then |I| = |J|, and w.l.o.g. we can assume $I = \{i_1, ..., i_n\}$ and $J = \{j_1, ..., j_n\}$ with $a_{i_1} \leq ... \leq a_{i_n} < z$ and $b_{j_1} \leq ... \leq b_{j_n} < z$, and $F_{\mathbf{a}} \leq F_{\mathbf{b}}$, by Lemma 1, $a_{i_k} \geq b_{j_k}$ for all k = 1, ..., n. Let $(x_o, (y_{j_k})_{k=1}^n, (w_l)_{l \in L}), (x_o, (x_{j_k})_{k=1}^n, (w_l)_{l \in L}) \in \mathcal{X}$, be such that $u(x_o) = z, u(y_{j_k}) = b_{j_k}$ for all $k = 1, ..., n, u(w_l) = z_l$ for all $l \in L, u(x_{j_k}) = a_{i_k}$ for all k = 1, ..., n. Then $x_o \succ x_{j_k} \succeq y_{j_k}$ for k = 1, ..., n, and n applications of Axiom F.2 deliver $(x_o, x_{j_1}, x_{j_2}, ..., x_{j_n}, (w_l)_{l \in L}) \gtrsim (x_o, y_{j_1}, x_{j_2}, ..., x_{j_n}, (w_l)_{l \in L}) \gtrsim (x_o, y_{j_1}, x_{j_2}, ..., x_{j_n}, (w_l)_{l \in L}) \approx (x_o, (x_{j_k})_{k=1}^n, (w_l)_{l \in L}) \gtrsim (x_o, (y_{j_k})_{k=1}^n, (w_l)_{l \in L})$ so we have $u(x_o) + \varrho\left(u(x_o), \sum_{k=1}^n \delta_{u(x_{j_k})} + \sum_{l \in L} \delta_{u(w_l)}\right) \ge u(x_o) + \varrho\left(u(x_o), \sum_{k=1}^n \delta_{u(y_{j_k})} + \sum_{l \in L} \delta_{u(w_l)}\right)$ and $\xi\left(z, \sum_{i \in J} \delta_{a_i}, \sum_{i \in J} \delta_{a_i}, \sum_{i \in J} \delta_{a_i}\right) = \varrho\left(u(x_o), \sum_{k=1}^n \delta_{u(x_{j_k})} + \sum_{l \in L} \delta_{u(w_l)}\right) \ge \varrho\left(u(x_o), \sum_{k=1}^n \delta_{u(y_{j_k})} + \sum_{l \in L} \delta_{u(w_l)}\right) = \xi\left(z, \sum_{i \in J} \delta_{b_i}, \sum_{l \in L} \delta_{z_l}\right)$. A similar argument shows that Axiom F.2 also delivers decreasing monotonicity

of $\dot{\xi}$ in the third component w.r.t. stochastic dominance.

This completes the proof of the sufficiency part. As to the necessity part, notice that for all $(z,\mu) \in \text{pim}(u(C))$, $\sum_{r \in \text{supp}(\mu): r < z} \mu(r) \delta_r$ and $\sum_{r \in \text{supp}(\mu): r \geq z} \mu(r) \delta_r$ are positive integer measures finitely supported in $u(C) \cap (-\infty, z)$ and $u(C) \cap [z, \infty)$ respectively, and their sum μ has total mass bounded by |N|, that is $\left(z, \sum_{r \in \text{supp}(\mu): r < z} \mu(r) \delta_r, \sum_{r \in \text{supp}(\mu): r \geq z} \mu(r) \delta_r\right) \in \text{pid}(u(C))$.

³⁴Notice that since $(z, \sum_{i \in I} \delta_{a_i}, \sum_{l \in L} \delta_{z_l})$ and $(z, \sum_{j \in J} \delta_{b_j}, \sum_{l \in L} \delta_{z_l})$ belong to pid (u(C)) we can assume that I, J, L are finite subsets of N with $I \cap L = \emptyset$ and $J \cap L = \emptyset$.

Define $\rho(z,\mu) \equiv \xi\left(z, \sum_{r \in \text{supp}(\mu): r < z} \mu(r) \,\delta_r, \sum_{r \in \text{supp}(\mu): r \geq z} \mu(r) \,\delta_r\right)$ and notice that $\rho(z, n\delta_z) = \xi(z, 0, n\delta_z) = 0$ for all z in u(C) and all non-negative integers $n \leq |N|$. Thus $u: C \to \mathbb{R}$ is a non-constant affine function, $\rho: \text{pim}(u(C)) \to \mathbb{R}$ is a diago-null function, and P is a probability on Σ , such that the function $V: \mathcal{F} \to \mathbb{R}$, defined by

$$V\left(f_{o},(f_{i})_{i\in I}\right) = \int_{S} \left[u\left(f_{o}\left(s\right)\right) + \varrho\left(u\left(f_{o}\left(s\right)\right),\sum_{i\in I}\delta_{u\left(f_{i}\left(s\right)\right)}\right)\right]dP\left(s\right)$$
$$= \int_{S} \left[u\left(f_{o}\left(s\right)\right) + \xi\left(u\left(f_{o}\left(s\right)\right),\sum_{i\in I:u\left(f_{i}\left(s\right)\right) < u\left(f_{o}\left(s\right)\right)}\delta_{u\left(f_{i}\left(s\right)\right)},\sum_{i\in I:u\left(f_{i}\left(s\right)\right) \geq u\left(f_{o}\left(s\right)\right)}\delta_{u\left(f_{i}\left(s\right)\right)}\right)\right]dP\left(s\right)$$

for all $(f_o, (f_i)_{i \in I}) \in \mathcal{F}$, represents \succeq and satisfies $V(\mathcal{F}) = u(C)$. Lemma 7 guarantees that \succeq satisfies Axioms A.1-A.6 and F.1.

Next we show that \succeq satisfies Axiom F.2. Let $(x_o, (x_i)_{i \in I}) \in \mathcal{X}, j \in I$, and $c \in C$.

If $c \succeq x_j \succeq x_o$. Then $u(c) \ge u(x_j) \ge u(x_o)$, and, by Lemma 2, $\sum_{i \in I - \{j\}: u(x_i) \ge u(x_o)} \delta_{u(x_i)} + \delta_{u(c)}$ stochastically dominates $\sum_{i \in I: u(x_i) \ge u(x_o)} \delta_{u(x_i)}$.

Then, $\xi\left(u\left(x_{o}\right), \sum_{i \in I: u(x_{i}) < u(x_{o})} \delta_{u(x_{i})}, \sum_{i \in I: u(x_{i}) \geq u(x_{o})} \delta_{u(x_{i})}\right) \geq \xi\left(u\left(x_{o}\right), \sum_{i \in I: u(x_{i}) < u(x_{o})} \delta_{u(x_{i})}, \sum_{i \in I - \{j\}: u(x_{i}) \geq u(x_{o})} \delta_{u(x_{i})} + \delta_{u(c)}\right)$ and we conclude $\left(x_{o}, \left(x_{i}\right)_{i \in I}\right) \succeq \left(x_{o}, \left(x_{i}\right)_{i \in I \setminus \{j\}}, c_{\{j\}}\right)$. Else, if $c \preceq x_{j} \prec x_{o}$, then $u\left(c\right) \leq u\left(x_{j}\right) < u\left(x_{o}\right)$, and, by Lemma 2, $\sum_{i \in I: u(x_{i}) < u\left(x_{o}\right)} \delta_{u(x_{i})}$ stochastically dominates $\sum_{i \in I - \{j\}: u(x_{i}) < u\left(x_{o}\right)} \delta_{u(x_{i})} + \delta_{u(c)}$.

Then, $\xi\left(u\left(x_{o}\right), \sum_{i \in I: u(x_{i}) < u(x_{o})} \delta_{u(x_{i})}, \sum_{i \in I: u(x_{i}) \geq u(x_{o})} \delta_{u(x_{i})}\right) \geq \xi\left(u\left(x_{o}\right), \sum_{i \in I - \{j\}: u(x_{i}) < u(x_{o})} \delta_{u(x_{i})} + \delta_{u(c)}, \sum_{i \in I: u(x_{i}) \geq u(x_{o})} \delta_{u(x_{i})}\right)$ and we conclude $\left(x_{o}, (x_{i})_{i \in I}\right) \succeq \left(x_{o}, (x_{i})_{i \in I \setminus \{j\}}, c_{\{j\}}\right)$. This completes the proof.

Proof of Theorem 6. Omitted since it is very similar to that of Theorem 5.

A.10 The Social Value Order

In this section, we provide the behavioral versions of Axiom A.7 for the social value order \succeq introduced in Section 5. The first axiom requires that the decision maker be consistent across groups in his social ranking of outcomes.

Axiom A. 12 (Group Invariance) Given any $c, d \in C$, if

$$(x_o, (x_i)_{i \in I}, d_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, c_{\{j\}})$$
(28)

for some $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$, then there is no other $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$ such that $(x_o, (x_i)_{i \in I}, c_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, d_{\{j\}}).$

The ranking \succeq is thus group invariant, that is, it does not depend on the particular peers' group in which the decision maker happens to make the comparison (28). In terms of the representation, Axiom A.12 implies that the function v does not depend on I. Axiom A.12 can be regarded as a group anonymity axiom, that is, it does not matter the particular group where a choice is made. Like the anonymity Axiom A.6, this condition guarantees that only outcomes per se matter and it thus allows us to study in purity the relative outcomes effects, our main object of interest.

The following axiom guarantees that the preference $\stackrel{\cdot}{\succ}$ is nontrivial.

Axiom A. 13 (Non-triviality) There are $c, d \in C$, $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$ such that

$$(x_o, (x_i)_{i \in I}, d_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, c_{\{j\}}).$$
(29)

The next two axioms just require standard independence and Archimedean conditions with respect to a given peer j's outcome. To ease notation, $c\alpha d$ denotes $(1 - \alpha)c + \alpha d$.

Axiom A. 14 (Outcome Independence) Let $\alpha \in (0,1)$ and $c, d, e \in C$. If

 $(x_o, (x_i)_{i \in I}, c_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, d_{\{j\}})$

for some $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$, then

 $(x_o, (x_i)_{i \in I}, c\alpha e_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, d\alpha e_{\{j\}})$

for some $(x_o, (x_i)_{i \in I}) \in \mathcal{X}$ and $j \notin I$.

Axiom A. 15 (Outcome Archimedean) Let $c, d, e \in C$. If

 $(x_o, (x_i)_{i \in I}, c_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, d_{\{j\}}) \text{ and } (y_o, (y_h)_{h \in H}, d_{\{k\}}) \succ (y_o, (y_h)_{h \in H}, e_{\{k\}})$

for some $(x_o, (x_i)_{i \in I}), (y_o, (y_h)_{h \in H}) \in \mathcal{X}, j \notin I$, and $k \notin H$, then

 $(x_o, (x_i)_{i \in I}, c\alpha e_{\{j\}}) \succ (x_o, (x_i)_{i \in I}, d_{\{j\}}) \text{ and } (y_o, (y_h)_{h \in H}, d_{\{k\}}) \succ (y_o, (y_h)_{h \in H}, c\beta e_{\{k\}})$

for some $(x_o, (x_i)_{i \in I}), (y_o, (y_h)_{h \in H}) \in \mathcal{X}, j \notin I, k \notin H, \alpha, \beta \in (0, 1).$

Axioms A.12-A.15 correspond to Axiom A.7.

Lemma 8 Let \succeq satisfy A.1. Then A.12-A.15 are equivalent to Axiom A.7.

Proof of Lemma 8. Assume \succeq satisfy A.12-A.15. If not $d\succeq c$, then there are $(x_o, (x_i)_{i\in I}) \in \mathcal{X}$ and $j \notin I$ such that not $(x_o, (x_i)_{i\in I}, c_{\{j\}}) \succeq (x_o, (x_i)_{i\in I}, d_{\{j\}})$. By A.1, $(x_o, (x_i)_{i\in I}, c_{\{j\}}) \prec (x_o, (x_i)_{i\in I}, d_{\{j\}})$. By A.12 and A.1, for all $(x_o, (x_i)_{i\in I}) \in \mathcal{X}$ and $j \notin I$, $(x_o, (x_i)_{i\in I}, c_{\{j\}}) \preceq (x_o, (x_i)_{i\in I}, d_{\{j\}})$, by definition $c \succeq d$. Thus \succeq is complete. Transitivity follows from A.1, and \succeq is a weak order. It is readily checked that A.13, A.14, and A.15 imply that \succeq is nontrivial, independent, and Archimedean, respectively. The converse is trivial.

References

- Anscombe, Francis J., and Robert J. Aumann, A definition of subjective probability, Annals of Mathematical Statistics, 34, 199-205, 1963.
- [2] Bagwell, Laurie S., and B. Douglas Bernheim, Veblen effects in a theory of conspicuous consumption, American Economic Review, 86, 349-373, 1996.
- [3] Becker, Gary S., A theory of social interactions, Journal of Political Economy, 82, 1063-1093, 1974.
- [4] Clark, Andrew E., and Andrew J. Oswald, Satisfaction and comparison income, *Journal of Public Economics*, 61, 359-381, 1996.
- [5] Cole, Harold L., Goerge J. Mailath, and Andrew Postlewaite, Social norms, saving behavior, and growth, *Journal of Political Economy*, 100, 1092-1125, 1992.
- [6] Duesenberry, James S., Income, savings, and the theory of consumer behavior, Harvard University Press, Cambridge, 1949.

- [7] Easterlin, Richard A., Does economic growth improve the human lot? Some empirical evidence, in Nations and households in economic growth: essays in honor of Moses Abramowitz, (Paul A. David and Melvin W. Reder, eds.), Academic Press, New York, 1974.
- [8] Easterlin, Richard A., Will raising the incomes of all increase the happiness of all?, Journal of Economic Behavior and Organization, 27, 35-47, 1995.
- [9] Fehr, Ernst, and Klaus M. Schmidt, A theory of fairness, competition, and cooperation, Quarterly Journal of Economics, 114, 817-868, 1999.
- [10] Fershtman, Chaim, Economics and social status, The New Palgrave Dictionary of Economics, Palgrave Macmillan, 2008.
- [11] Festinger, Leon, A theory of social comparison processes, Human Relations, 7, 117-140, 1954a.
- [12] Festinger, Leon, Motivation leading to social behavior, Nebraska Symposium on Motivation, edited by Jones, M. R., Lincoln, University of Nevada Press, 1954b.
- [13] Frank, Robert H., Choosing the right pond, Oxford University Press, Oxford, 1985.
- [14] Friedman, Milton, and Leonard J. Savage, The utility analysis of choices involving risk, Journal of Political Economy, 56, 279-304, 1948
- [15] Friedman, Milton, Choice, chance and the personal distribution of income, Journal of Political Economy, 61, 277-290, 1953
- [16] Gul, Faruk, and Wolfgang Pesendorfer, The canonical type space for interdependent preferences, mimeo, Princeton University, 2005.
- [17] Hardy, George H., John E. Littlewood, and George Polya, *Inequalities*, Cambridge University Press, Cambridge, 1934.
- [18] Gilboa, Itzhak, and David Schmeidler, A cognitive model of individual well-being, Social Choice and Welfare, 18, 269-288, 2001.
- [19] Hawley, P., The ontogenesis of social dominance: a strategy-based evolutionary perspective, Devolopment Review, 19, 97-132, 1999.
- [20] Hopkins, Ed and Tatiana Kornienko, Running to keep in the same place: consumer choice as a game of status, American Economic Review, 94, 1085-1107, 2004.
- [21] Karni, Edi, and Zvi Safra, Individual sense of justice: a utility representation, *Econometrica*, 70, 263-284, 2002.
- [22] Ireland, Norman J., On limiting the market for status signals, *Journal of Public Economics*, 53, 91-110, 1994.
- [23] Luttmer, Erzo F. P., Neighbors as negatives: relative earnings and well-being, Quarterly Journal of Economics, 120, 963-1002, 2005.
- [24] Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini, Social decision theory, mimeo, 2009a.
- [25] Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini, Pride and diversity in social economics, mimeo, 2009b.

- [26] Mailath, G. and Postlewaite, A. Social assets, International Economic Review, 47, 1057-1091, 2006.
- [27] Maslow, Abraham H., The role of dominance in the social and sexual behavior of infra-human primates: I, *Journal of Genetic Psychology*, 48, 261-277, 1936.
- [28] Maslow, Abraham H., Dominance-feeling, behavior, and status, *Psychological Review*, 44, 404-429, 1937.
- [29] McBride, Michael, Relative-income effects on subjective well-being in the cross-section, Journal of Economic Behavior and Organization, 45, 251-278, 2001.
- [30] Michael, Robert T., and Gary S. Becker, On the new theory of consumer behavior, Swedish Journal of Economics, 75, 378-396, 1973.
- [31] Ok, Efe A., and Levent Kockesen, Negatively interdependent preferences, Social Choice and Welfare, 17, 533–558, 2000.
- [32] Postlewaite, Andrew, The social basis of interdependent preferences, European Economic Review, 42, 779-800, 1998.
- [33] Rayo, Luis, and Gary S. Becker, Evolutionary efficiency and happiness, *Journal of Political Econ*omy, 115, 302-37, 2007.
- [34] Samuelson, Larry, Information-based relative consumption effects, *Econometrica*, 72, 93-118, 2004.
- [35] Stigler, George J., and Gary S. Becker, De gustibus non est disputandum, American Economic Review, 67, 76-90, 1977.
- [36] Tomes, Nigel, Income distribution, happiness and satisfaction: a direct test of the interdependent preferences model, *Journal of Economic Psychology*, 7, 425-446, 1986.
- [37] Van de Stadt, Huib, Arie Kapteyn, and Sara van de Geer, The relativity of utility: evidence from panel data, *Review of Economics and Statistics*, 67, 179-187, 1985.
- [38] Veblen, Thorstein, The theory of the leisure class, Macmillan, New York, 1899.
- [39] Zizzo, Daniel J., and Andrew J. Oswald, Are people willing to pay to reduce others' incomes?, Annales d'Economie et de Statistique, 63, 39-62, 2001.