VALUE OF INFORMATION AND FAIRNESS OPINIONS IN TAKEOVERS [1]

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We analyze the value of information in the market for corporate control. The raider and the shareholders are privately and imperfectly informed about the post-takeover value of the firm. We show that public information provision reduces the dispersion of the shareholders' beliefs resulting in a transfer of surplus from the raider to the shareholders. What is more, if the raider is privately informed all his private information is revealed through the price offer, hence he prefers not to acquire private information, provided that the shareholders do not engage in information acquisition themselves. The target shareholders, on the other hand, have incentives to acquire information—solicit a fairness opinion—after the raider makes a price offer. Finally, when both parties have access to an information market, they both have incentives to acquire information.

Keywords: takeovers, fairness opinion, tender offers, lemons problem, large shareholder.

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1. Introduction

Asymmetric information is a major friction that affects the performance and efficiency in the market for corporate control. Potential acquirers lack firm specific information, while target management and board lack information on the value created from the synergies after a takeover. The acquirers and their target shareholders, therefore, demand information about the potential post-takeover value of a firm, i.e. fairness opinions. Fairness opinions are a highly sought-after source of information in the mergers and acquisitions market. For example, in a study of mergers and acquisitions deals during 1994-2003, Kisgen et al. (2009) document that approximately 80% of target firms and 37% of acquiring firms in mergers and acquisitions obtained a fairness opinion.

A fundamental step in the analysis of fairness opinions is understanding the value of information in takeovers. We provide a systematic study of the latter here. Our model facilitates the analysis of the demand for information–fairness opinions–by the raider and the target shareholders under various configurations which differ by who has access to information markets.[1]

The market for corporate control distinguishes itself through two important features that create divergent incentives for the buyers and sellers to acquire information. First, in order to exchange the control the sellers need not transfer all post-takeover cash-flow rights. In particular, selling half of the total shares (the controlling stake) is sufficient to render the takeover or transfer of control successful. Second, the ownership of the controlling stake of the target company is typically dispersed, hence, the success of the takeover and the amount of shares that are transferred, is determined by the collective decisions of a number of shareholders that undertake actions to maximize their own wealth. This leads to market frictions emanating from free-riding and asymmetric information.

In our model, a raider wants to take over a company via an unconditional tender offer.[2] The target company's shares are widely dispersed across a large number of small shareholders and a finite number of large shareholders who even if put together own a minority stake.[3] The post-takeover value of the company is uncertain. Neither the raider nor the shareholders of the target company perfectly know the raider's ability to manage the company after the

[1] Similar studies have been conducted in other exchange markets in which the quality of a seller's good is unknown by the buyers, and the seller can choose to verifiably disclose his information (see for example Okuno-Fujiwara et al. (1990)).

[2] In an unconditional tender offer the raider makes a price offer, and is obligated to buy any share that is tendered at the offered price, regardless of the outcome of the takeover activity. We could have assumed that the raider makes a conditional offer, but as is shown in Marquez and Yılmaz (2007) and Ekmekci and Kos (2012a), both mechanisms yield identical equilibrium outcomes for the raider in a continuum shares model.

[3] We present the model with one large shareholder, however in Ekmekci and Kos (2012b) we show how these results can be extended to an environment with several large shareholders.

takeover. In particular, the shareholders have private and imperfect information regarding the post-takeover value.

Our analysis provides two main insights for the information provision market. First, when the raider is the only party with access to information markets, he has no incentive to acquire information. Therefore there would be no demand in a market that serves only the acquirers. Second, if both sides have access to the information market, then the demand for information emerges on both sides of the market.

We start our analysis by characterizing the value of *public information* for the raider and the shareholders, the value of *private information* for the raider and the value of *private information* for the board. We first show that a raider is averse to the release of public information regarding the post takeover value of the firm. This is partly due to the fact that the ability of the raider to make profits comes from the dispersion of beliefs among the small shareholders (see Ekmekci and Kos (2012b)). While public information helps the raider make a better informed price offer, it reduces the dispersion of shareholders' beliefs, thereby decreasing the extent to which the raider can extract the surplus. The overall effect of the public information is unambiguously bad for the raider, and beneficial for the shareholders.

Next we examine the raider's incentives to acquire private information. In the equilibria that survive the intuitive criterion a privately informed raider reveals his information through his price offer. Low price offers are interpreted by the shareholders as evidence of the raider being pessimistic about the value of the takeover, and lead to occasional takeover failures.[4] The raider's private information is completely reflected in his price offer. Therefore, he cannot utilize his private information to extract surplus from the shareholders. In fact, he is strictly worse off than if he were to credibly disclose his information, which subsequently is worse for him than committing not to acquire information at all. We conclude that in the absence of information acquisition from the shareholders, the raider has no incentives to acquire information himself. This result is in line with the results obtained by Admati and Pfleiderer (1990) and Grossman and Stiglitz (1980) in the context of financial markets where if the prices reveal information obtained by the market participants, the incentives to acquire information diminish drastically. The shareholders, on the other hand, would strictly prefer the raider to inform himself before initiating a takeover.

We also explore the incentives of a shareholder-value maximizing board to acquire information. The key issue is that the raider realizes that the board's ability to acquire information exposes him to a lemons problem. Hence, without any commitment to hide information, the existence of the information market causes a complete market breakdown, and leads the raider

[4] Note that here the collective action feature of the takeovers leads to the possibility that the takeover success is random.

to make the lowest possible price offer. However, if the sellers of information (i.e., investment banks) can commit to disclose information with some noise, then the shareholders can extract the full surplus of the takeover activity. Moreover, if the raider enjoys private benefits from controlling the company, the shareholders can even extract some of these private benefits through an optimal information disclosure policy by the banks.

These insights have direct implications for the regulation of information provision by investment banks. Most importantly, mandatory disclosure policies, or policies that require full transparency of the investment banks' information may be detrimental to the efficiency of the takeover market. In particular, such policies can lead to a severe lemons problem that causes a complete market breakdown.

Finally, we reconcile our findings with what we observe in the real world, namely that the market is active for both the raiders and the boards. We show that, if both the raider and the board have access to the information market, and if the raider enjoys private benefits, then there is an equilibrium in which both parties acquire information. Thereby, the board's access to information provides incentives for the raider to acquire information.

While we believe that information acquisition and disclosure is the central issue for fairness opinions, we should point out that other explanations for the role of fairness opinions have been proposed upon which our model has little bearing. For example, by eliciting fairness opinions the parties involved in the transaction satisfy their judicial liabilities. In 1985, Delaware Supreme Court ruled that the board of directors of the Trans Union Corporation (the target company) had been negligent and had not made an informed decision when approving the transaction. Since then, the widely held belief is that the court's decision made the use of fairness opinions, which are not required by law, a practical way to avoid liability. However, empirical evidence provided by Bowers (2002) suggests that the impact of the Delaware Court's decision on the frequency of fairness opinion solicitation is insignificant.

1.1. Related Literature The role of fairness opinions in mergers and acquisitions has been discussed extensively in the law literature where views on why they are solicited seem to differ. An excellent survey of the legal and financial literature is provided by Davidoff et al. (2011).

Despite the existence of empirical work and the size of the fairness opinion market, to the best of our knowledge, Ohta and Yee (2008) is the only other paper that proposes a model of fairness opinions. In that model, the board has a conflict of interest with the shareholders, and is privately informed about the future value of the firm. The paper interprets the messages of the board to the shareholders as a fairness opinion, and argues that the coarseness of the information transmitted in equilibrium relates to the imprecision of fairness opinions.

The main difference between their paper and ours is that they choose to forgo modeling of the collective choice problem and in exchange focus on the intertemporal incentives of a self-interested board. We instead retain a large number of small shareholders in the model, which as shown in Grossman and Hart (1980) leads to the central problem of the literature on takeovers - free-riding by the small shareholders. Moreover, the focus of our paper is on the asymmetry of information that the fairness opinion creates between the shareholders and the raider, which leads to a lemons problem. Such considerations are absent from the model of Ohta and Yee. In particular, the coarseness of the information disclosure through fairness opinions in our model emanates from the market's endogenous choice of the extent of the asymmetric information.

The core of our paper is a model of takeovers. The literature on takeovers, by and large initiated by Grossman and Hart (1980), is too large to be given full justice here. For an illuminating overview the reader is advised to see Burkart and Panunzi (2006). In what follows we outline a handful of related papers. Our model builds on the work developed in Marquez and Yılmaz (2008). Marquez and Yilmaz introduced a model of takeovers in which a continuum of shareholders obtains dispersed information about the post-takeover value of the firm while the raider remains uninformed. Among other things, they show that the dispersed information pins down a unique equilibrium of the takeover game in which the shareholders use threshold strategies and the raider cannot make a profit unless endowed with private benefits. Ekmekci and Kos (2012b) showed that in a similar model the presence of a large shareholder with a minority stake can result in profit for the raider even when he has no private benefits. In both of these models, unlike in the present paper, the raider is uninformed. The models where the raider is privately informed, and therefore signals his information through the offer, are to the best of our knowledge few and far apart; presumably due to technical problems brought about by signaling. Shleifer and Vishny (1986) consider a model in which the raider has a starting share in the company and private information about the post-takeover value before he makes an offer but the shareholders are uninformed. They show that a starting share results in profits for the raider. Hirshleifer and Titman (1990) extend the model of Shleifer and Vishny to allow for probabilistic success and failure of the takeover bids. They show the existence of an equilibrium in which raider's price offer reveals his private information perfectly, and the takeover fails with positive probability. In our paper, unlike in the two above mentioned the raider does not own a starting share. In addition, we show that the the equilibria satisfying the intuitive criterion are a subset of separating equilibria, even after allowing for probabilistic takeovers, which enables us to analyze the value of information in the model. In the above mentioned literature it is commonly assumed that the raider has all the bargaining power. Cornelli and Li (1997) and Ekmekci et al. (2014)

explore a model in which the owner of the firm has all the bargaining power.

In a recent paper Burkart and Lee (2010) provide a comprehensive study of signaling in takeovers. Our model differs from theirs in that in our case both the shareholders and the raider have private information. Moreover, while Burkart and Lee (2010) focus on the signaling mechanisms our paper explores the value of information in markets for corporate control.

2. Model

A firm is owned by a continuum of shareholders of measure one. A fraction $1-x$ of the firm is held by small shareholders each of whom owns a single share. The remaining fraction x is held by one large shareholder. The value of the firm under the current management is commonly known and normalized to 0.

The raider's ability to increase the value of the firm is uncertain. There are two states of the world, $\omega \in \{l, h\}$, over which the common prior belief that $\omega = h$ is $\lambda \in (0,1)$. If the true state is l, the firm's value (expected discounted cash flows, or security benefits) cannot be increased even with the new management. However, if the true state is h and the raider takes over the firm the value of the firm becomes 1. The value of the firm under current management is 0 regardless of the state of the world. To take over the firm the raider needs to acquire at least half of the shares.

Each small shareholder observes a signal $s \in S = [0,1]$ drawn independently from the distribution F_ω on S, where ω is the true state of the world. The large shareholder observes a signal s drawn from a distribution H_ω on S; the unconditional distribution is H. We assume that the shareholders' signals are conditionally independently distributed. We assume that the distributions have densities and that the strict version of monotone likelihood ration property (henceforth MLRP) holds: both $f(s|h)/f(s|l)$ and $h(s|h)/h(s|l)$ are strictly increasing in s. The function $\beta : [0,1] \to [0,1]$ denotes the posterior belief of a small shareholder that the state is h after observing signal s, and is calculated using Bayes' rule

$$\beta(s) = \frac{\lambda f(s|h)}{\lambda f(s|h) + (1-\lambda) f(s|l)}.$$

The strict MLRP implies that β is strictly increasing in the signal. Similarly, the function $\beta_L : [0,1] \to [0,1]$ denotes the posterior belief of the large shareholder that the state is h after observing signal s. For now we assume that the raider has no information beyond the prior.

The takeover starts with the raider making an unconditional price offer $p \in [0, \infty)$. The price offer p induces a tender subgame in which the shareholders make their tender decisions. In such a subgame, a symmetric pure strategy for the small shareholders specifies for every signal s whether they sell their share or not; i.e. $\sigma : S \to \{sell, keep\}$ denotes a pure tender subgame strategy for a small shareholder. The set of all pure tender subgame strategies for the small shareholders is Σ. A *tender strategy* for the small shareholders is $\tilde{\sigma} : [0,\infty) \to \Sigma$. That is, a tender strategy specifies one tender subgame strategy after every price offer the raider makes. A distributional tender subgame strategy for the large shareholder is a right continuous and increasing function $\sigma_L : S \times [0,1] \to [0,1]$, with the restriction that $\sigma_L(s,1) = H(s)$ for every $s \in S$. The term $\sigma_L(s,r)$ represents the joint probability that the large shareholder observes a signal no larger than s and sells a fraction of shares no larger than r. The set of all tender subgame strategies for the large shareholder is Σ_L, and a tender game strategy for the large shareholder is $\tilde{\sigma}_L : [0,\infty) \to \Sigma_L$. A symmetric tender subgame strategy σ for the small shareholders is a threshold strategy if there is an $\hat{s} \in S$ such that $\sigma(s) = \{sell\}$ for every $s < \hat{s}$ and $\sigma(s) = \{keep\}$ for every $s > \hat{s}$. Later we will argue that the small shareholders' best replies are threshold strategies.

In a tender subgame after the price offer p, a small shareholder's payoff depends on the price p, his signal, s, the probability of the takeover success in the high state, q, and his action. If he sells his share, his payoff is p. If he keeps it, his payoff is $\beta(s)q$. Similarly, the large shareholder with a signal s who sells fraction r of his shares and believes the takeover will succeed in the high state with probability $q(r)$ obtains a payoff

$$x[rp + (1-r)q(r)\beta_L(s)].$$

Namely, if the large shareholder sells fraction r of his shares at the price p, he receives a payment of xrp. Since he attaches probability $q(r)$ to the takeover succeeding in the high state and attaches the probability $\beta_L(s)$ to the high state occurring, his remaining shares are worth $x(1-r)q(r)\beta_L(s)$.

The raider's payoff is the value of the shares he purchases less the price he pays for them. The raider's payoff is augmented due to commonly known and state independent private benefits $b \geq 0$ in the case he acquires control. These private benefits are not reflected in the value of the firm.[5] More precisely, the raider's payoff when he offers the price p, the share-

[5]Private benefits are by now a well accepted concept in corporate finance literature. Beginning with Berle and Means (1932), there has been a substantial literature arguing that control of the firm allows the controller to enjoy benefits not shared with minority shareholders (see Jensen and Meckling (1976), Dodd and Warner (1983) and Johnson et al. (2000)). These benefits can be monetary, such as excess salary, or non-monetary, such as amenities like professional sports teams and newspapers. For example, Dyck and Zingales (2004) estimate that on average the private benefit of control is worth 14% of the equity value of a firm. See Barak and Lauterbach (2011) for a brief summary of the empirical literature devoted to estimating the magnitude of the private benefit. There are plenty of other reasons why an acquiring company may have private benefits of control that are not appropriated by the target company's shareholders. For example, the target company may have good distribution capabilities in new areas, which the acquiring company can use for its own products as well. Alternatively, the target company may allow the acquiring

holders use the symmetric subgame strategy profile (σ, σ_L), and he believes the probability of takeover success as a function of the large shareholder's behavior is determined by the collection $\{q(\cdot)\}_{r\in[0,1]}$, is given by:

$$\begin{aligned} U_R(p,\sigma,\sigma_L,\{q(\cdot)\}_{r\in[0,1]}) &= \lambda \int_{r,s} q(r)\left(xr + (1-x)\int_{s\in[0,1]} \sigma(s)f(s|h)ds\right) d\sigma_L(s,r|h) \\ &- p\left[(1-x)\int_{s\in[0,1]} \sigma(s)[\lambda f(s|h) + (1-\lambda)f(s|l)]ds + x\int_{s,r} r d\sigma_L(s,r)\right] \\ &+ b\left[\lambda \int_{s\in[0,1],r\in[0,1]} q(r)d\sigma_L(s,r) + (1-\lambda)\right]. \end{aligned}$$

The first term captures the value of the shares the raider purchases. The second, the total payment the raider makes to acquire the shares, and the third, the private benefits he obtains in the case the takeover succeeds.[6]

The equilibrium concept we use is the *tender equilibrium* which we proceed to define. The first step is the definition of a *tender subgame equilibrium*.

Definition 1 *A tuple $T(p) = (\sigma, \sigma_L, q, \{q(r)\}_{r\in[0,1]})$ consisting of a pair of tender subgame strategies (σ, σ_L), a probability q, and a probability function $\{q(r)\}_{r\in[0,1]}$ is a tender subgame equilibrium after a price offer p if*

- *small shareholders' strategy is* optimal *given p and q,*
- *large shareholder's strategy is* optimal *given price p and the function $\{q(\cdot)\}_{r\in[0,1]}$,*
- *$q(r)$ and q are obtained from shareholders' strategy:*

$$q(r) = \begin{cases} 0, & \text{if } (1-x)\int_0^1 \sigma(s)dF(s|h) + xr < 1/2 \\ 1, & \text{if } (1-x)\int_0^1 \sigma(s)dF(s|h) + xr > 1/2 \\ \in [0,1], & \text{if } (1-x)\int_0^1 \sigma(s)dF(s|h) + xr = 1/2, \end{cases}$$

and

$$q = \int_{s\in[0,1],r\in[0,1]} q(r)d\sigma_L(s,r|h).$$

The above equilibrium concept was proposed by Ekmekci and Kos (2012b), and builds on a concept introduced for a complete information framework in Tirole (2010) (see Section

company to enter a new market without having to take on the risk, time and expense of starting a new division. Finally, a takeover can facilitate the acquiring company to reduce its redundant functions.

[6]The raider's payoff as written here anticipates some of the result about the takeover success rate in state l. Namely, that when the private benefits b are strictly positive the takeover succeeds with probability 1 in the low state for any price. Hence the extra coefficient $1-\lambda$ on b. Adding the notation for probability of success in the low state would significantly complicate the notation while not adding in new insights.

11.5.1); see also Dekel and Wolinsky (2012). The fact that the small shareholders and the large shareholder respond optimally given their belief is standard. As is that the large shareholder assigns the probability $q(r) = 1$ (0) to the takeover succeeding in the high state when more (less) than half of the shares are sold in that state. Somewhat atypical is the assumption that $q(r)$ is left to be determined in equilibrium when the large shareholder expects precisely half of the shares to be sold in the high state. However, Ekmekci and Kos show that this equilibrium concept captures the limiting behavior in takeover games with a large but finite number of small shareholders and a large shareholder.

A tender subgame equilibrium $T(p)$ leaves the raider a payoff $\pi(T(p))$. When there is a unique tender subgame equilibrium after price offer p, we slightly abuse notation and denote the raider's payoff by $\pi(p)$. In what follows we restrict attention to price offers $p \leq 1$. Given that the shares are never worth more than 1, all shareholders sell their shares at any price offer larger than 1, hence prices strictly above 1 are never optimal for the raider. A *tender equilibrium* is a collection of tender subgame equilibria $\{T(p)\}_{p\in[0,1]}$ and a price $\hat{p}$ such $\pi(T(\hat{p}))$ is a maximum of $\pi(T(p))$.

2.1. Initial Analysis We call the type s^* uniquely defined by the equality

$$x + (1-x)F_h(s^*) = 0.5,$$

the *pivotal type*. Let

$$\phi := x + (1-x)F_l(s^*). \tag{1}$$

The pivotal type s^* is a type such that if the small shareholders use threshold strategy s^* (selling when their signal is below the threshold) and the large shareholder sells all of his shares, precisely half of the shares are sold in the high state. Fraction ϕ of shares is then sold in the low state.

Strict MLRP implies $\phi > 1/2$. Theorem 2 in Ekmekci and Kos (2012b) establishes that, the raider's equilibrium price offer is either zero or

$$\bar{p} := \beta(s^*).$$

Which of the two prices the raider offers depends on whether he can make a profit by offering the price $\bar{p}$. In the following paragraph we summarize some properties of the tender subgame equilibria that will be important in the following analysis. Lemma 2 in the Appendix provides a complete characterization of tender subgame equilibria after any price offer p; for the proof see Theorem 2 in Ekmekci and Kos (2012b).

For any $p > 0$, there is a unique equilibrium of the tender subgame.

(i) If $p \leq \bar{p}$, then:

a) Small shareholders' equilibrium threshold is the pivotal type, s^*.

b) The large shareholder sells all his shares regardless of his signal.

c) The probability of a successful takeover in the high state, q, is proportional to the price p, and is equal to 1 at $p = \bar{p}$.

d) The raider's profit is given by:

$$\pi(p) = \lambda \frac{p}{2\beta(s^*)} - p\left(\lambda\frac{1}{2} + (1-\lambda)\phi\right) + \left(\lambda\frac{p}{\beta(s^*)} + (1-\lambda)\right) b. \tag{2}$$

(ii) If $p > \bar{p}$, then

a) The takeover succeeds with probability 1 in both states.

b) The raider's payoff in the unique tender subgame equilibrium, $\pi(p)$, is decreasing in p.

Two properties of the raider's profit function stand out. First, the raider's profit is linear in the price offer p in the range $p \leq \bar{p}$. Second, price offers above $\bar{p}$ always yield strictly smaller profits than the price offer $\bar{p}$, and hence are not offered by the raider. Thus, the raider either offers $\bar{p}$, or 0. Let $\bar{\pi}$ be the raider's expected payoff if he offers $\bar{p}$. Lemma 2 implies:

$$\bar{\pi} = \lambda\frac{1}{2} - \beta(s^*)[\lambda\frac{1}{2} + (1-\lambda)\phi] + b,$$

which after rearranging and using the expression for $\beta(s^*)$ in the raider's payoff equation yields

$$\bar{\pi} = \left(\frac{f_h(s^*)}{1-\lambda} + \frac{f_l(s^*)}{\lambda}\right)^{-1}\left[\frac{1}{2}f_l(s^*) - \phi f_h(s^*)\right] + b. \tag{3}$$

First, notice that if $\lambda \in (0,1)$ and $b = 0$, then $\pi > 0$ if and only if $\frac{1}{2}f_l(s^*) - \phi f_h(s^*) > 0$. This term is independent of λ, it depends only on the distribution function F and the size of the large shareholder, x. In particular, when $b = 0$, for any F, there is an $\bar{x} < 1/2$ such that whenever $x > \bar{x}$, the raider makes a positive price offer in equilibrium, and earns a positive profit. Throughout the paper we assume that $x > \bar{x}$.[7] Therefore, as long as $\lambda \in (0,1)$, the

[7]The raider's inability to make a profit has been a major concern in the takeovers literature. In the absence of profits the raider has no incentives to undertake the takeover if he has some cost of initiating it. The assumption of a large enough shareholder and incomplete information enables the raider to make profits even when the private benefits are small or nil; see Ekmekci and Kos (2012b)

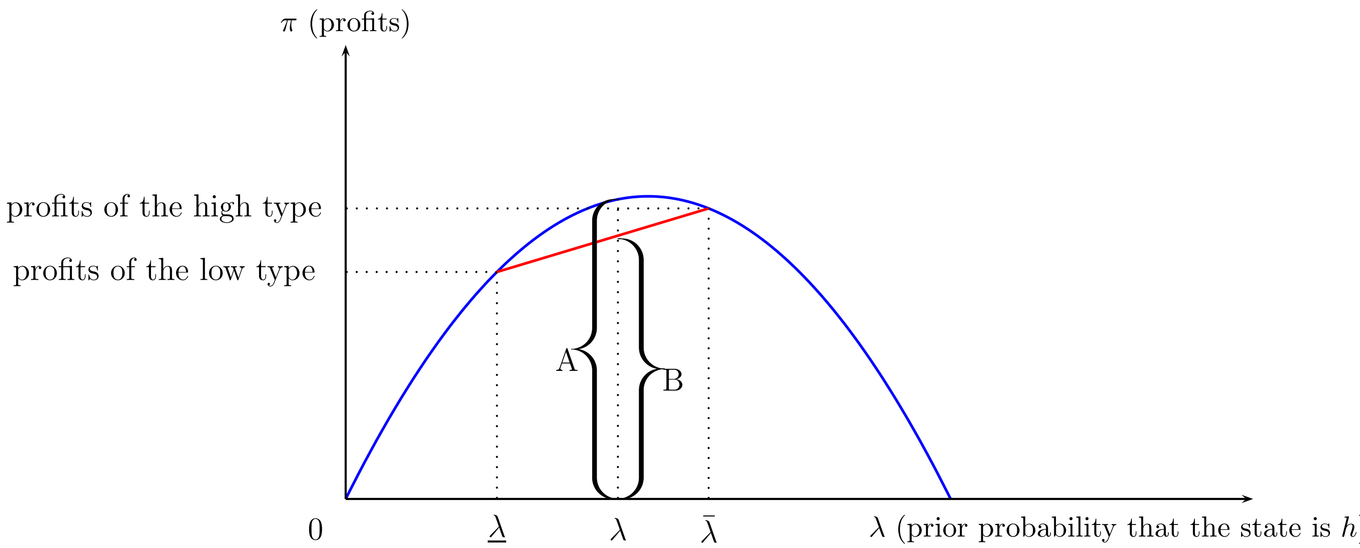

Figure 1: This figure shows the raider's equilibrium profits as a function of the prior probability that the state is h. This function is concave, and equal to 0 when λ is 0 or 1. When there is public information with two partially informative signals, the ex-ante expected profit of the raider is equal to the length of the line segment B. Without any public information, his profit is equal to the length of the line segment A. Therefore, the difference between the length of A and B is the amount of surplus that the public information transfers from the raider to the shareholders.

raider's optimal price offer is positive and equal to

$$\frac{\lambda f_h(s^*)}{\lambda f_h(s^*) + (1-\lambda) f_l(s^*)}.$$

From hereon, $\bar{\pi}(\lambda)$ denotes the raider's equilibrium profits when the common prior that the state is h is λ.

The function of λ in the brackets of the first term in (3) is convex. Therefore the inverse of the function is concave. Consequently the raider's ex-ante expected profit is concave in the prior belief. See Figure 1 for a graphical representation of the profit function in terms of the prior belief that the state if h, for $b = 0$. For later reference we state the above argued result as a lemma.

LEMMA 1 *The raider's equilibrium profit function, $\bar{\pi}(\lambda)$, is concave in λ.*

An immediate consequence of the finding that the profits are concave in the prior belief λ

is that the raider is averse to public release of information. We analyze this formally next.

2.2. Benchmark: Public Signal Here we examine consequences of arrival of public information on takeovers. We model the public information by assuming an arrival of a publicly observable signal before the takeover activity. The public signal, z belongs to a finite set Z, and is governed by a joint distribution $Pr(z, \omega)$. We assume that the public signal is informative, i.e., there exist $z_1, z_2 \in Z$ such that

$$\frac{Pr(z_1|h)}{Pr(z_1|l)} \neq \frac{Pr(z_2|h)}{Pr(z_2|l)}.$$

The timing is as follows:

- $t = 0$: Each shareholder observes a private signal.
- $t = 1$: The public signal is observed.
- $t = 2$: The raider makes an unconditional price offer for all the shares.
- $t = 3$: Shareholders simultaneously decide whether and, in the case of the large shareholder, how many shares to sell.
- $t = 4$: The outcome of the takeover is observed.

After observing the public signal z, the raider's posterior is $\lambda(z)$. In particular

$$\lambda(z) = \frac{\lambda Pr(z|h)}{\lambda Pr(z|h) + (1-\lambda)Pr(z|l)}.$$

Each shareholder, in addition to the public signal, privately observes a signal s. Agent's posterior after observing the pair of signals (s, z) is denoted $\beta(s, z)$. In particular:

$$\beta(s, z) = \frac{\beta(s)Pr(z|h)}{\beta(s)Pr(z|h) + (1-\beta(s))Pr(z|l)},$$

where $\beta(s)$ is the shareholder's belief about the high state after observing only the private signal s, but before observing the public signal.

The raider's pure strategy is a function from the public signal to a non-negative price. However, since the raider has no information that the shareholders do not have themselves, the game after each public signal z corresponds to the previously analyzed game without public information in which the common prior is $\lambda(z)$. Notice that the identity of the pivotal type, s^*, is independent of the prior belief, and therefore of $\lambda(z)$. This in conjunction with the observation that the raider's profits are concave in λ, and that by Bayes' rule $E[\lambda(z)] = \lambda$, implies that the raider is averse to public information. We provide the intuition for the theorem after the proof.

Theorem 1 *Public signal has no effect on welfare. However, the raider is better off without a public signal. Conversely, the shareholders are collectively better of with a public signal.*

Proof: The first part of the statement, the welfare considerations, follows from the above developed analysis. Indeed, in our model the welfare is affected only by the probability of success of the takeover in the high state. Equation (3) implies that whether the raider can make a profit, and therefore whether the takeover will succeed or not is independent of the common belief, $\lambda(z)$.[8]

On the other hand, the raider's profit is concave in his belief by Lemma 1. Therefore, Jensen's inequality delivers the result. □

We assume that takeovers increase the value of the company. Moreover, public information does not change takeover's success rate. Therefore, public information in our model does not have any impact on welfare. However, public information has an impact on the distribution of the surplus between the raider and the shareholders. Public information release unambiguously transfers surplus from the raider to the shareholders. It decreases the extent to which the shareholders' beliefs are spread out. Since one of the forces enabling the raider to make profits is the shareholders' dispersion of beliefs, their compression decreases the raider's profit.

3. Demand for Fairness Opinion by the Raider

In this section we analyze the value of private information—solicitation of a fairness opinion—by the raider. In order to do so, we analyze the raider's equilibrium use of private information. We show that in equilibria surviving the intuitive criterion the raider reveals his private information through the price offer, and obtains a payoff which is strictly lower than in the situation in which he is not privately informed. Therefore, having access to a fairness opinion has *no value* for him.

To make the point as simply as possible, we look at the case in which the raider observes an outcome of a binary signal. The environment is as in the previous section, with the addition that the raider observes a signal $z \in \{z_l, z_h\}$, from a joint distribution $Pr(z, \omega)$. Moreover,

[8] Note that, as $\lambda \to 0$ or as $\lambda \to 1$, $\pi \to b$. In other words, the raider's payoff in a complete information environment is equal to his private benefits. Although Equation (3) is not well defined at $\lambda = 0$ or $\lambda = 1$, the raider's payoff in these instances is identical to the limit of his payoffs as λ approaches these limit values.

we assume that the signal z_ω is more indicative of the state ω. We also assume that none of the signals is fully revealing.[9]

The raider's pure strategy is a mapping from his signal to a price between 0 and 1, denoted by ζ.[10] A belief function $\mu : [0,1] \to [0,1]$ denotes the belief that shareholders attach to the raider observing the signal z_h, after having observed the price p. A symmetric strategy for the small shareholders is $\tilde{\sigma} : [0,1] \to \Sigma$, i.e., a mapping from the set of all prices to the set of tender subgame strategies. Similarly, a strategy for the large shareholder is $\tilde{\sigma}_L : [0,1] \to \Sigma_L$.

Definition 2 *A* ***tender signaling equilibrium*** $(\tilde{\sigma}, \tilde{\sigma}_L, \{q(p,r)\}_{p\in[0,1],r\in[0,1]}, \{q(p)\}_{p\in[0,1]}, \zeta, \mu)$ *consists of a profile of strategies* $(\tilde{\sigma}, \tilde{\sigma}_L, \zeta)$ *and belief functions* $(\{q(p,r)\}_{p\in[0,1],r\in[0,1]}, \{q(p)\}_{p\in[0,1]}, \mu)$ *such that:*

(i) ζ *is optimal given* $(\tilde{\sigma}, \tilde{\sigma}_L, \{q(p,r)\}_{p\in[0,1],r\in[0,1]}, \{q(p)\}_{p\in[0,1]})$.

(ii) For any $p \in [0,1]$, $(\tilde{\sigma}(p), \tilde{\sigma}_L(p), \{q(p,r)\}_{r\in[0,1]}, q(p))$ *constitutes a tender subgame equilibrium of the continuation game after* p *when the shareholders' common part of the belief that the state is* h *is* $\lambda(p) := \mu(p)Pr(h|z_h) + (1-\mu(p))Pr(h|z_l)$.

(iii) $\mu(p)$ *is derived using Bayesian updating for each* p *in the support of* ζ.

We call an equilibrium *separating* if $\zeta(z_l) \neq \zeta(z_h)$. Clearly, in any separating equilibrium $1 - \mu(\zeta(z_l)) = \mu(\zeta(z_h)) = 1$. We denote by $\beta(s^*, z)$ the pivotal type's posterior belief about the state being h had he also observed signal z. Put differently, the shareholder's belief is $\beta(s^*, z)$ when he observes the signal s^* and believes with probability one that the raider observed signal z.

Theorem 2 *All tender signaling equilibria that survive the intuitive criterion are separating.*[11] *Hence, raider's information is fully reflected in his price offer. Among the equilibria that survive the intuitive criterion, the following equilibrium yields the highest payoff for both types of the raider:*

- *The raider with signal* z_h *offers* $p_h := \beta(s^*, z_h)$, *and the takeover succeeds with probability* 1 *in the high state.*
- *The raider with signal* z_l *offers a price* p_l *such that the high type of the raider is indifferent between offering* p_h *and being thought of as high type, and offering* p_l *and being*

[9]The raider is unable to make profits in models with complete information. We make this assumption to rule out the knife edge case where the raider does not even attempt to make a tender offer.

[10]Our results extend to the case where the raider uses mixed strategy. Such extension would significantly complicate the notation without adding substance. In fact, all of the arguments we use in the proof of Theorem 2 are valid when we consider equilibria in which the raider uses mixed strategies. Also note again that prices above 1 are never optimal for either type of the raider, and hence we restrict attention to prices less than 1.

[11]Strictly speaking, the intuitive criterion of Cho and Kreps (1987) was provided for the games with a finite number of signals, whereas in our case the shareholders' signal space is an interval. Such extension is rather innocuous.

thought of as the low type. In particular, $p_l < \beta(s^*, z_l)$*, and the takeover succeeds with probability strictly less than* 1 *in the high state.*

In what follows we call the raider observing the signal z_h the high type and the raider observing the signal z_l the low type. In the separating equilibrium that maximizes the payoff of both types the high type raider makes the price offer that would be optimal if his signal was publicly observed: $p_h^* = \beta(s^*, z_h)$. The low type, on the other hand, offers a price p_l that is strictly lower than the price offer he would have made, if his signal was publicly observed $p_l^* = \beta(s^*, z_l)$. Since p_l^* is the smallest price at which the takeover succeeds in both states when the raider is believed to be the low type and $p_l < p_l^*$, the takeover fails with positive probability when the raider is the low type.

The above equilibrium is supported by the out of equilibrium beliefs that attach probability one to the raider being the low type after the price offers $p \leq p_l$ and probability one to the raider being the high type for the prices $p > p_l$. First we verify that the high type does not have an incentive to deviate. When believed to be the high type, the high type's optimal price offer is p_h^*. Since he is believed to be the high type for all the prices $p > p_l$ he will not have a profitable deviation among those prices. On the other hand, when believed to be the low type his profit is increasing on $p \leq p_l^*$ and therefore on $p \leq p_l$. By construction he is indifferent between being thought of as the high type while offering p_h^* and being thought of as the low type while offering p_l. Therefore he also does not have an incentive to deviate to prices $p \leq p_l$. The low type's profit is also increasing for $p \leq p_l^*$ when he is believed to be the low type. Therefore he will not deviate to a price below p_l. When the low type is believed to be the high type his preferred price is p_h^*. We then show that low type strictly prefers the price p_l and the belief that he is the low type to the price p_h^* and the belief that he is the high type, which completes the argument. The last part is reminiscent of the standard single crossing condition. Namely, if the high type is indifferent between a high price p_h and being believed to be the high type, and the low price p_l along with being believed to be the low type, then the low type strictly prefers the second of the two options.

Here we briefly sketch the idea behind the result that pooling cannot be sustained as a part of an equilibrium that satisfies the intuitive criterion. We first argue that, in a candidate equilibrium where both types offer the same price, the high type's equilibrium payoff cannot exceed the payoff he would get by offering the price p_l^* and being believed to be the low type. We denote the high type's payoff in the latter case by π_h^*. If π_h^* was below the high type's equilibrium payoff π_h, the price offer p_l^* would be equilibrium dominated by the high type, but not by the low type. Out of equilibrium beliefs in any equilibrium surviving an intuitive criterion would therefore have to assign probability one to the deviations by the low type after prices around p_l^*. This, however, would create a profitable deviation for the low type who would have liked to deviate to p_l^* if believed to be the low type. Namely, the price p_l^* together with the belief that the raider is the low type gives the low type his highest payoff across all price offers and beliefs; see Lemma 4 in the Appendix.

The set of payoffs up to π_h^* for the high type can be attained by offers at or below p_l^*, under the restriction that the shareholders play a tender equilibrium with the belief that the raider is the low type. In particular, provided that he is believed to be the low type, the high type's payoff is strictly increasing in $p \leq p_l^*$. Therefore, for any pooling equilibrium that survives the intuitive criterion, there is a price $p^* \leq p_l^*$ such that the high type would never make a bid below p^* under any belief that the shareholders may hold. Moreover, the high type raider is indifferent between offering price p^* and his equilibrium offer if p^* induces a belief that the raider is the low type. As mentioned before, the following single-crossing like property holds in our environment: If the high type weakly prefers p^* along with the belief to be the low type to his pooling equilibrium price offer, then the low type strictly prefers p^*. This, in turn, means that the low type strictly prefers offers to prices just slightly below p^* if he is also believed to be the low type. Since the high type does not prefer these prices the intuitive criterion prescribes that for prices just below p^* the raider should be believed to be the low type. But then the pooling cannot be supported as an equilibrium since the low type would want to deviate to such a price.

A useful comparison is with the case in which the raider's signal was publicly observed before he made his price offer. In such a case, our conclusions from the previous section yields that the raider would offer p_h^* if the signal was high and p_l^* otherwise, and the takeover would succeed in both states. Therefore the raider would have been better off if his information was publicly observed. We summarize this in the following corollary. For an illustration see Figure 2.

Corollary 1 *The total welfare when the raider has access to private information is strictly smaller than the total welfare when the information is public, or when there is no access to information. Moreover, the raider is strictly worse of with private information than with public information, and strictly worse of with public information than no information. In sum, the raider is averse to access to private information, while the shareholders benefit from an informed raider.*

The reason why the raider prefers public information to private information is as follows. In equilibrium, he reveals his information through his bid, so he gains no further informational advantage against the shareholders when the latter make a decision. Moreover, the incentive constraints that are implied by the separation of types result in the low type burning some

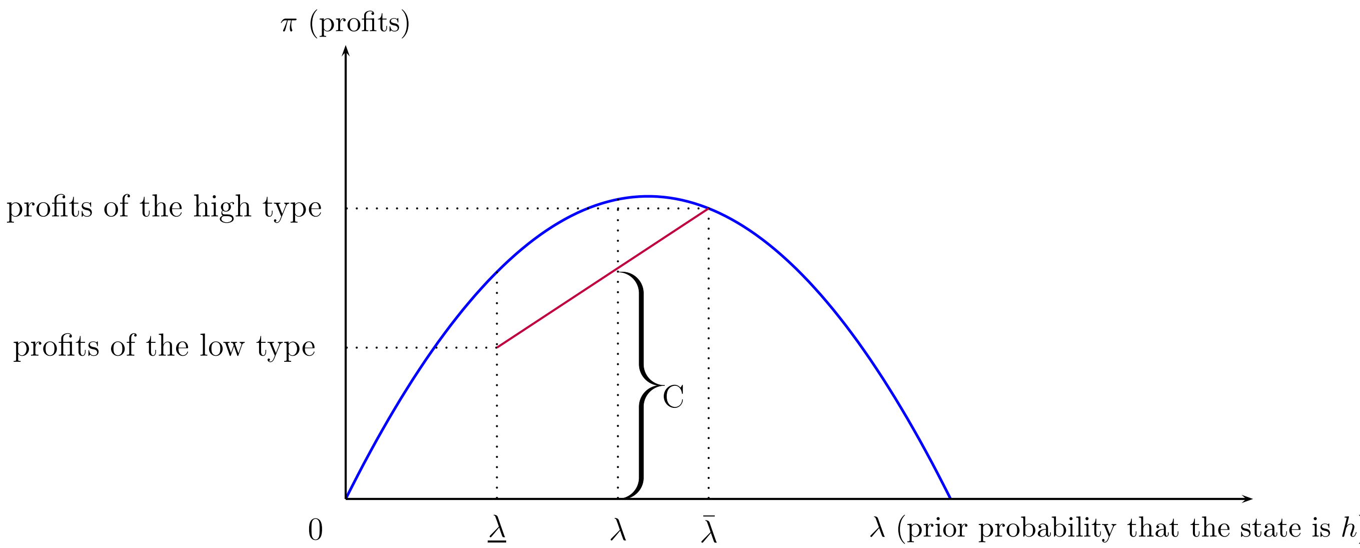

Figure 2: This figure shows equilibrium profits for each of the raider's types as a function of the interim probability that the state is h. Note that, the low type $\underline{\lambda}$ makes an equilibrium price offer after which the takeover fails with a positive probability, therefore the low type's profits are lower than $\pi(\underline{\lambda})$. Moreover, the equilibrium profits with private information is C, where $C < B < A$, and is summarized in Corollary 1.

payoffs. Had the low type offered a price p_l such that the takeover succeeded with certainty, the high type would have deviated and not offered p_h. Therefore, the payoff burnt by the low type makes the raider strictly worse of.

The equilibrium is fully separating, thus reflecting the raider's information in the price offer. In particular, the raider is able to signal his information merely by using cash bid offers, without alluding to the use of securities or contingent claims on the ex-post valuation of the firm. Raider's information is revealed because equilibrium price offers allow the takeover to succeed or fail with a probability between zero and one. The high type's profits as a function of the takeover probability are steeper than those of the low type. In other words, the high type's marginal profits as a function of takeover probability are higher than that of the low type, and therefore, lower prices that induce higher takeover failures deter the high type from making such low price offers. On the other hand, the low type makes such offers instead of higher price offers because roughly speaking, at any higher price offer, the raider is believed to be the high type, and the takeover fails with an even higher probability.

Consider now that the raider could verifiably disclose his signal to the shareholders. In such a case, standard unraveling arguments deliver a unique equilibrium in which the raider discloses all his information and the price offers coincide with those in the public information case. Therefore, from a policy perspective, encouraging communication between the raider and the board may result in voluntary disclosure of private information held by the raider, and avoid the low type from burning some surplus.

4. Two-Sided Demand for Fairness Opinions

So far we explored the raider's incentive to acquire information when the shareholders' information was given. We showed that the raider has no incentive to acquire information although the shareholders would strictly benefit from it. In this section we show that a raider who has private benefits from a takeover does have incentives to acquire information if he knows that the shareholders, or the board, can hire an investment bank to provide information about the desirability of the takeover (fairness opinion). To facilitate the analysis we start by exploring the case where the shareholders can solicit a fairness opinion and the raider has no information. Then we move to the general case, where both parties can solicit information.

4.1. Demand for fairness opinion by the board. The board can hire an investment bank to provide further information about the state of the world to the shareholders. We interpret this information as fairness opinion provided to the shareholders after the raider makes the price offer. The raider, on the other hand, has no information beyond the prior.

We model the investment bank's information revelation activity as a game of disclosure. The investment bank is in possession of hard pieces of information that it can decide to either disclose or not. But it can not forge the information. Formally, the investment bank observes the state ω, and commits to an information revelation strategy (here on IRS).[12] An information revelation strategy consists of a finite set of messages Z, and a distribution function $\{Pr(z|\omega)\}_{\omega\in\{h,l\},z\in Z}$ such that $\sum_z Pr(z|\omega) = 1$. Note that this formulation is equivalent to the alternative formulation in which the investment bank chooses a set of posterior beliefs $\lambda(z)$ and a set of weights $\alpha(z)$ such that $\sum_z \alpha(z) = 1$ and $\sum_z \alpha(z)\lambda(z) = \lambda$. At $t = 0$, the investment bank commits to an information revelation strategy. The raider and the shareholders play according to the following timing:

- $t = 1$: The raider makes a price offer.
- $t = 2$: The investment bank reveals a signal according to the information revelation strategy chosen at t=0.

[12] We do assume that the investment bank can commit to adhering to the information revelation strategy it picked at the beginning, even after it observes the state of the world. In this vein, our disclosure game is similar to that analyzed in Rayo and Segal (2010) and Kamenica and Gentzkow (2011).

- $t = 3$: Shareholders simultaneously decide whether and, in the case of the large shareholder, how many shares to sell.
- $t = 4$: Outcome of the takeover is observed.

The raider is facing a lemons problem. At the time he makes the price offer, he does not know what the investment bank's recommendation will be. He runs the risk of suffering considerable losses if the bank releases information in favor of state l and the shareholders dump their shares. In fact, if the investment bank did not have commitment to conceal some information, the size of potential losses could force the raider to make price offer zero if his private benefits are not large enough to cover the losses he makes on the shares. In such a case no surplus would be created in equilibrium. However, with the commitment there is an information revelation strategy that guarantees the shareholders a payoff larger than λ when the raider has strictly positive private benefits.

THEOREM 3 *The shareholder value can be maximized by an IRS with only two messages. Under such an optimal IRS:*

- *If $b > \frac{1-\lambda}{\lambda}$, then the shareholder value is equal to 1, and the raider's payoff is $\lambda + b - 1$.*
- *If $b \leq \frac{1-\lambda}{\lambda}$, then the shareholder value is equal to $\lambda + \lambda b$, and the raider's payoff is $(1 - \lambda)b$.*

An optimal information revelation strategy consists of two messages, "low" and "high"; for the details see the proof in the Appendix. The low message reveals that the state is low. The high message, on the other hand, leaves some uncertainty while signaling that the high state is more likely than it was ex ante. In equilibrium, the raider's price offer is strictly positive, his payoff is positive if the message is "high", and he suffers losses if the message is "low". The probability that the investment bank delivers message "low" is chosen in such a way that the raider breaks-even in equilibrium.

Two messages in an information revelation strategy are consistent with what we observe in fairness opinions, or board advices. If the raider is unable to make big improvement in the firm, then the advice is that the price offer is fair, otherwise that it is not fair. However, in equilibrium the takeover is successful in both states albeit the number of shares tendered depends on the message of the investment bank.

We obtain the result by first characterizing the optimal IRS with two messages. Such an IRS yields a payoff $\max\{b(1-\lambda), b+\lambda-1\}$ to the raider, and $\min\{\lambda+\lambda b, 1\}$ to the shareholders. Notice that in either case, as long as the raider's private benefits are strictly positive, his expected payoff is smaller than b. With other words, the investment bank has enough leeway in choosing the information revelation strategy to help the shareholders extract some of the raider's private benefits.

Next we argue that the optimal IRS with two messages is optimal among all IRS. A raider can ensure a successful takeover in the low state by making an arbitrarily low—but non-negative—price offer, hence enjoys a payoff of at least $(1-\lambda)b$. On the other hand, the value of the company is at most 1, implying that the raider can obtain all the shares, irrespective of the IRS, by offering price 1. This nets him a payoff $b+\lambda-1$. The two bounds imply that the raider cannot be held to a payoff lower than $\max\{(1-\lambda)b, b+\lambda-1\}$, which is precisely the payoff he obtains under the optimal IRS with two messages. Since the latter is constructed so that the takeover always succeeds in the high state, and therefore the welfare is maximized, it is optimal among all IRS. In the proof we show a stronger result. When the raider has no private benefits, the above described IRS with two messages is uniquely optimal.

The investment bank in our model is not modeled as a player. However, under the commitment assumption, the fact that it is maximizing shareholder value can be justified by specifying a particular extensive form game in which the board hires the investment bank by giving it a fee that is proportional to the shareholder value. In particular, a monopolist investment bank can charge a fee that is equal to the increment the bank brings to the shareholder value. Alternatively, a competitive investment bank industry results in information revelation strategies that maximize shareholder value. Another channel that may enable the investment bank to commit to a particular information disclosure rule is reputation concerns. An investment bank can build a reputation among the raiders that it is concealing some amount of information. A further source of commitment can arise due to observable bonus schemes that are used in the bank which aligns the incentives of the bankers to produce the desired information disclosure.

4.2. Two-sided demand for information. So far we have shown that the raider has no incentives to acquire private information, for the sake of information itself. Furthermore, we have shown that if the firm acquires private information the raider might loose some of the private benefits to the shareholders. In what follows we show that the raider has incentives to acquire information to inoculate himself against the information provided from the investment bank hired by the shareholders.[13]

The timing is as follows: At time $t = 0$, the raider and the board simultaneously decide the IRS that will provide them information about the state. At $t = 1$, after observing the

[13]Our model resembles the model of competition in persuasion in Gentzkow and Kamenica (2011). However, in our model the two sides (shareholders and the raider) receive information separately and then move sequentially, which creates novel complications.

IRS chosen by the board, the raider observes the message delivered by his IRS and makes a price offer p. At $t = 2$, the shareholders observe the price offer and the message delivered by their IRS and decide whether to sell or keep their shares.

Theorem 4 *There is an equilibrium in which both the board and the raider pick the fully revealing IRS. The raider's equilibrium payoff is b, and the shareholders enjoy the full surplus from share value increase.*

Theorem 3 shows that when the shareholders have access to fairness opinions, they obtain all the surplus accruing from security benefits. Therefore, the raider's ability to acquire fairness opinions does not allow him to recoup any part of the surplus generated by security benefits. Moreover, the same result implies that the shareholders may enjoy even some of the surplus from the private benefits if the raider does not acquire information. Theorem 4 shows that, by acquiring information, the raider insures himself against the information that the investment bank would reveal to shareholders. In particular, by acquiring information, the raider can salvage the private benefits, but not the surplus emanating from the security benefits.

5. Conclusion

We develop a systematic analysis of the information market within the market for corporate control. Our analysis shows that a market that serves only the acquirers would not have any demand. If the market served only the shareholders, then there would be demand, but for information that is imprecise or noisy. Such an information market would transfer some of the raider's private benefits to the shareholders. Finally, and most importantly, if both sides have access to the information market then the demand for information emerges on both sides of the market.

Insights from our paper have direct implications for the regulation of information provision by investment banks. For example, mandatory disclosure policies, or policies that require full transparency of the investment banks' information may have detrimental consequences for the efficiency of the takeover market. In particular, it may lead to a severe lemons problem that causes a complete market breakdown.

A. Proofs

We start the Appendix with a Lemma that summarizes the characterization of the tender subgame equilibria. Its proof can be found in Ekmekci and Kos (2012b): proof of Theorem 2 in Appendix B.

Lemma 2 *For any $p > 0$, there is a unique equilibrium of the tender subgame, $T = (\sigma, \sigma_L, q, q(r)_{r\in[0,1]})$.*

(i) If $p \leq \bar{p}$, then

a) σ is a threshold strategy with a threshold s^*.

b) $\sigma_L(s) = 1$ for every $s \in [0,1]$.

c) $q = \frac{p}{\beta(s^*)}$, $q(1) = \frac{p}{\beta(s^*)}$ and $q(r) = 0$ for all $r < 1$.

Moreover, the raider's profit is given by:

$$\pi(p) = \lambda \frac{p}{2\beta(s^*)} - p\left(\lambda \frac{1}{2} + (1-\lambda)\phi\right) + \left(1 - \lambda + \lambda \frac{p}{\beta(s^*)}\right) b. \tag{4}$$

(ii) If $p > \bar{p}$, then

a) σ is a threshold strategy with a threshold $\hat{\sigma}$ where $\hat{\sigma} = 1$ if $p \geq \beta(1)$, and otherwise is the unique solution to the equality $\beta(\hat{\sigma}) = p$.

b) There is a signal $s_L \in [0,1]$ and a fraction $a < 1$ such that, if the large shareholder's signal $s > s_L$, then he tenders fraction a of his shares; and if $s < s_L$, then he tenders all of his shares.

c) $q = 1$, $q(r) = 0$ for $r < a$, and $q(r) = 1$ for $r \geq a$.

Moreover, $\pi(p)$ is decreasing in p.

Before we proceed with the proof of Theorem 2 we introduce some notation and provide two intermediate lemmata.

Let $p_l^* = \beta(s^*, z_l)$. That is, p_l^* is the low type raider's preferred price offer, when he is believed to be the low type; consequence of Lemma 2 and $x \geq \bar{x}$. In the following Lemma we establish some facts about the dependence of the raider's profits on the shareholders' beliefs at the low prices. The shareholders' beliefs are for the purpose of this exercise taken as exogenous rather than derived from the raider's behavior.

Lemma 3 *For any price $p \leq p_l^*$ the raider's profit is maximized when the shareholders' believe him to be the low type, i.e., when $\mu = 0$.*

Proof: Let, $p \leq p_l^*$ and suppose the shareholders attach the belief μ to the raider having observed the high signal. Let $\beta(s^*, \mu)$ denote the probability that the pivotal type attaches to state h if he believes the raider received signal z_h with probability μ. Note that $\beta(s^*, \mu)$ is increasing in μ, and is at least p_l^*, since p_l^* is by definition $\beta(s^*, \mu = 0)$.

Lemma 2 implies that, given the shareholders' belief μ, for any price $p \leq \beta(s^*, \mu)$ precisely half of the shares are sold in the high state and

$$\phi \equiv (1 - x)F_l(s^*) + x > 0.5$$

in the low state. Consequently this will be the case for prices $p \leq p_l^* = \beta(s^*, 0)$, irrespective of the beliefs μ. Therefore for the prices considered in this lemma the amount of the shares sold in each state is independent of the shareholders' beliefs μ about the raider. The takeover succeeds with probability one in the low state and probability

$$q = \frac{p}{\beta(s^*, \mu)} \tag{5}$$

in the high state; due to Lemma 2. The above condition is the indifference condition of the threshold type s^*. If he sells the share he obtains p, whereas if he does not sell he reaps the benefits only if the takeover succeeds in the high state, which he expects to happen with the probability $q\beta(s^*, \mu)$. Now, (5) implies that q is decreasing in μ.

The raider's profit can be written as

$$\pi = \lambda_i q \frac{1}{2} - p[\lambda_i \frac{1}{2} + (1 - \lambda_i)\phi] + [\lambda_i q + (1 - \lambda_i)]b \tag{6}$$

$$= q\left(\lambda_i \frac{1}{2} - \beta(s^*, \mu)[\lambda_i \frac{1}{2} + (1 - \lambda_i)((1 - x)F_l(s^*) + x)]\right) + [\lambda_i q + (1 - \lambda_i)]b, \tag{7}$$

for $i \in \{l, h\}$, where λ_i is the raider's belief about the state of the world after observing the signal z_i. The raider's profit consists of the benefits he receives from the shares and the private benefits minus the price he pays for those shares. His shares are only worth something in the high state. The benefit is, therefore, the amount of the shares he obtains in the high state, which is as argued above 0.5, times the belief he attaches to the high state, λ_i, times the probability with which the takeover succeeds in the high state, q. His payment is the price times the expected amount of the shares bought. Similarly, the last term represents the value of private benefits he obtains.

Next we argue that $x > \bar{x}$ implies the first term in the brackets of equation (7) is strictly positive for the high type (i.e., when the term is calculated with λ_h). Indeed, remember that for $x > \bar{x}$ each type of the raider can make a profit when the shareholders have the correct beliefs about his type and he offers the optimal price, given the beliefs; see equation (3) and the discussion following it. In particular, the high type of the raider makes a non-negative profit on the shares (excluding the private benefits)

$$\lambda_h \frac{1}{2} - \beta(s^*, 1)[\lambda_h \frac{1}{2} + (1-\lambda_h)(xF_l(s^*) + (1-x))],$$

when offering the price $\beta(s^*, 1)$. Since the term in the brackets of equation (7), when evaluated at λ_h, is at least as large as this, it is positive. The total profit in (7) is therefore increasing in q and decreasing in μ. Similarly, for the low type, the term in the brackets is maximized when $\mu = 0$, in which case it is positive, so the total expression is maximized at $\mu = 0$ and $q = 1$. We have established that in the price range $[0, p_l^*]$, the profits of the raider are maximized when he is believed to be the low type. □

Next we show that even if the low type raider could alongside with the price choose shareholders' beliefs he would be best of by having the shareholders correctly believe that he is the low type.

Lemma 4 *The low type raider's total profit is maximized when he offers p_l^* and the shareholders believe him to be the low type.*

Proof: First, note that when the price offer is p_l^* and the raider is believed to be the low type, then takeover succeeds in both states of the world, and the raider receives private benefits b, i.e., the term in the profit function, $(\lambda_l q + (1-\lambda_l))b$ is equal to b. Because b is an upper bound for the private benefits in the raider's profit function, and the upper bound is attained in this scenario, it suffices to show that the profits from the shares are maximized when he offers p_l^* and is believed to be the low type. In the rest of the proof, we show that the security benefits are maximized in this scenario. In what follows, the term π represent the raider's profits only from the shares, i.e., excluding the profits coming from the private benefits.

We will denote by $\hat{\lambda}$ the shareholders' beliefs after updating their prior using the information contained in μ but not in their private signals. In particular, when the shareholders are certain that the raider observed the low signal, $\mu = 0$, their "common belief" (without also updating on the basis of their own signal) is equal to the raider's posterior when he observes the low signal $\hat{\lambda} = \lambda_l$. Analogously, when $\mu = 1$, $\hat{\lambda} = \lambda_h$. Therefore, given the information structure, $\hat{\lambda} \in [\lambda_l, \lambda_h]$.

Let $\pi(p, \hat{\lambda})$ be *low type* raider's equilibrium profits if he makes the price offer p and the shareholders' belief μ is such that the common part of the belief that the state is high is

$\hat{\lambda} \in [\lambda_l, 1]$. Although the information structure and Bayesian updating imply $\hat{\lambda} \leq \lambda_h$, the function π is defined also for $\lambda > \lambda_h$ as a technical construct that will prove useful later.

In what follows we will maximize the low type raider's profit over the pairs of prices and beliefs $\langle p, \hat{\lambda} \rangle$.

Step 1: Among the pairs $\langle p, \hat{\lambda} \rangle$ such that $p \leq p_l^*$, $\langle p_l^*, \lambda_l \rangle$ maximizes the low type raider's profit. Indeed, Lemma 3 established that if price p is restricted not to be above p_l^*, then $\pi(p, \hat{\lambda})$ is maximized uniquely at $\hat{\lambda} = \lambda_l$ and $p = p_l^*$, with a strictly positive profit. Therefore we are left to verify that by offering a price $p > p_l^*$ the raider cannot do better.

Step 2: Pairs $\langle p, \hat{\lambda} \rangle$ with $p \geq \lambda_l$ yield the expected profit of at most zero for the low type raider. Let $\psi(p)$ be the expected amount of shares sold in the good state and $\phi(p)$ the expected amount of shares sold in the low state after a price offer p given the fixed beliefs $\hat{\lambda}$. Given that $p \geq \lambda_l$ the expected fraction of the shares sold in low state is weakly higher than expected fraction of the shares sold in the high state for any $\hat{\lambda}$. This is a simple consequence of Lemma 2, namely shareholders use threshold strategies and the environments satisfies the MLRP. Consequently, the low type raider's profit is

$$\begin{aligned}
\pi_l(p) &= \lambda_l q(p)\psi(p) - p[\lambda_l \psi(p) + (1-\lambda_l)\phi(p)] \\
&\leq \psi(p)\left[\lambda_l q(p) - p\right] \\
&\leq 0,
\end{aligned}$$

where the first inequality follows due to $\psi(p) \leq \phi(p)$, as argued above, and the last from the assumption that $p \geq \lambda_l$. Therefore the low type raider's profit is at most zero at this price range, consequently less than $\pi(p_l^*, \lambda_l)$. Notice that at these prices the raider would be offering a price at or above what he believes a share is worth, thus clearly he cannot not make a profit in expectation.

Step 3: Here we consider the pairs $\langle p, \hat{\lambda} \rangle$ with $p \in (p_l^*, \lambda_l)$. Before we proceed we state the following implications of Lemma 2.

For each $p \in (p_l^*, \lambda_l)$ there exists a $\bar{\lambda}_p \in (\lambda_l, 1)$ such that if $\hat{\lambda} \leq \bar{\lambda}_p$, then the takeover succeeds in both states with probability one. Moreover, the small shareholders use a threshold strategy with a threshold σ such that $\beta(\sigma, \hat{\lambda}) = p$. The large shareholder uses the threshold s_L satisfying $\beta_L(s_L, \hat{\lambda}) = p$ or $\beta_L(0, \hat{\lambda}) > p$. He sells all of his shares if he receives the signal below his threshold and the amount such that exactly half of the shares are sold in the high state (together with what small shareholders sell) if he observes a signal above s_L. For any $\hat{\lambda} \in (\bar{\lambda}_p, 1)$, the takeover succeeds in the high state with probability $q < 1$ ($q = 1$ if $\hat{\lambda} = \bar{\lambda}_p$). Moreover, exactly half of the shares are sold in the high state, and fraction ϕ, defined in (1), is sold in the low state. If $\hat{\lambda} \leq \bar{\lambda}_p$, then the raider's profits are:

$$\pi(p, \hat{\lambda}) = \lambda_l \psi(\hat{\lambda}) - p[\lambda_l \psi(\hat{\lambda}) + (1 - \lambda_l)\phi(\hat{\lambda})] \tag{8}$$

where,

$$\psi(\hat{\lambda}) = H(s_L(\hat{\lambda})|h)[x + (1-x)F(\sigma(\hat{\lambda})|h)] + (1 - H(s_L(\hat{\lambda})|h))\frac{1}{2},$$

is the expected amount of the shares sold in the high state and

$$\begin{aligned}\phi(\hat{\lambda}) = &H(s_L(\hat{\lambda})|l)[x + (1-x)F(\sigma(\hat{\lambda})|l)] \\ &+ (1 - H(s_L(\hat{\lambda})|l))[1/2 + (1-x)(F(\sigma(\hat{\lambda})|l) - F(\sigma(\hat{\lambda})|h))]\end{aligned}$$

is the expected amount of the shares sold in the low state. The last two equalities are computed using the equilibrium strategies of shareholders. The small shareholders tender only if they observe a signal below σ, while the large shareholder sells all of his shares if his signal is below s_L and just enough so that exactly half of the shares are sold in the high state if he observes a signal above s_L.

In the remainder of the proof we will show that if $p \in (p_l^*, \lambda_l)$ and $\hat{\lambda} \in [\lambda_l, \bar{\lambda}_p]$, then $\frac{\partial \pi(p,\hat{\lambda})}{\partial \hat{\lambda}} \geq 0$. This implies that for the prices and beliefs in question the low type raider's profit is maximized at $\hat{\lambda} = \bar{\lambda}_p$.

Partial derivatives of $\psi(\cdot)$ and $\phi(\cdot)$ are

$$\frac{\partial \psi(\hat{\lambda})}{\partial \hat{\lambda}} = h(s_L(\hat{\lambda})|h)\left(x + (1-x)F(\sigma(\hat{\lambda})|h) - 1/2\right)\frac{ds_L(\hat{\lambda})}{d\hat{\lambda}} + (1-x)f(\sigma(\hat{\lambda})|h)H(s_L(\hat{\lambda})|h)\frac{d\sigma(\hat{\lambda})}{d\hat{\lambda}}$$

and

$$\begin{aligned}\frac{\partial \phi(\hat{\lambda})}{\partial \hat{\lambda}} =&h(s_L(\hat{\lambda})|l)\left(x + (1-x)F(\sigma(\hat{\lambda})|h) - 1/2\right)\frac{ds_L(\hat{\lambda})}{d\hat{\lambda}} \\ &+ (1-x)\left[-f(\sigma(\hat{\lambda})|h)(1 - H(s_L(\hat{\lambda})|l)) + f(\sigma(\hat{\lambda})|l)\right]\frac{d\sigma(\hat{\lambda})}{d\hat{\lambda}}.\end{aligned}$$

Note also that,

$$p = \beta(\sigma(\hat{\lambda})) = \beta_L(s_L(\hat{\lambda})),$$

since both the threshold type of the large shareholder, $s_L(\hat{\lambda})$, as well as the threshold type of the small shareholders $\sigma(\hat{\lambda})$ need to be indifferent between tendering and not tendering.

For example, in the case of the small shareholders tendering gives them the price p while not tendering gives them the value of the share, 1, in the case the state is high (for considered parameters the takeover succeeds with probability 1 in the high state). As a reminder, the belief of the small shareholder threshold type (given $\hat{\lambda}$) is

$$\beta(\sigma(\hat{\lambda})) = \frac{\hat{\lambda} f(\sigma(\hat{\lambda})|h)}{\hat{\lambda} f(\sigma(\hat{\lambda})|h) + (1-\hat{\lambda}) f(\sigma(\hat{\lambda})|l)},$$

while the large shareholder threshold type's belief is

$$\beta_L(s_L(\hat{\lambda})) = \frac{\hat{\lambda} h(s_L(\hat{\lambda})|h)}{\hat{\lambda} h(s_L(\hat{\lambda})|h) + (1-\hat{\lambda}) h(s_L(\hat{\lambda})|l)}.$$

Using the above equalities

$$1 - p = \beta(\sigma(\hat{\lambda})) \frac{(1-\hat{\lambda}) f(\sigma(\hat{\lambda})|l)}{\hat{\lambda} f(\sigma(\hat{\lambda})|h)} = \beta_L(s_L(\hat{\lambda})) \frac{(1-\hat{\lambda}) h(s_L(\hat{\lambda})|l)}{\hat{\lambda} h(s_L(\hat{\lambda})|h)}.$$

Partially differentiating (8) with respect to $\hat{\lambda}$ using the above equalities, rearranging and collecting terms yields:

$$\begin{aligned}
\frac{\partial \pi(p,\hat{\lambda})}{\partial \hat{\lambda}} &= \frac{d s_L(\hat{\lambda})}{d\hat{\lambda}} \frac{\beta_L(s_L(\hat{\lambda}))}{\hat{\lambda}} h(s_L(\hat{\lambda})|l) \left[x + (1-x)F(\sigma(\hat{\lambda})|h) - 1/2\right] \left(\lambda_l - \hat{\lambda}\right) \\
&\quad + (1-x) \frac{d\sigma(\hat{\lambda})}{d\hat{\lambda}} \frac{\beta(\sigma(\hat{\lambda}))}{\hat{\lambda}} f(\sigma(\hat{\lambda})|l) \\
&\quad \times \left(\lambda_l (1-\hat{\lambda}) H(s_L(\hat{\lambda})|h) - \hat{\lambda}(1-\lambda_l) \left[1 - \frac{f(\sigma(\hat{\lambda})|h)}{f(\sigma(\hat{\lambda})|l)} (1 - H(s_L(\hat{\lambda})|l))\right]\right).
\end{aligned}$$

Note that the signs of $\frac{ds_L(\hat{\lambda})}{d\hat{\lambda}}$ and $\frac{d\sigma(\hat{\lambda})}{d\hat{\lambda}}$ are both non positive, while $\lambda_l - \hat{\lambda} \leq 0$ by assumption, therefore the first term is nonnegative. In the second term,

$$\left(\lambda_l (1-\hat{\lambda}) H(s_L(\hat{\lambda})|h) - \hat{\lambda}(1-\lambda_l)(1 - \frac{f(\sigma(\hat{\lambda})|h)}{f(\sigma(\hat{\lambda})|l)} [1 - H(s_L(\hat{\lambda})|l)]) \right)$$

is also nonpositive. This is because, $\lambda_l(1-\hat{\lambda}) \leq \hat{\lambda}(1-\lambda_l)$ and $\frac{f(\sigma(\hat{\lambda})|h)}{f(\sigma(\hat{\lambda})|l)} < 1$ (due to $p < \lambda_l$) imply $H(s_L(\hat{\lambda})|h) \leq 1 - [1 - H(s_L(\hat{\lambda})|l)] \frac{f(\sigma(\hat{\lambda})|h)}{f(\sigma(\hat{\lambda})|l)}$. So overall the derivative is nonnegative. Therefore, the raider's profits after price offer $p \in (p_l^*, \lambda_l)$ are maximized at belief $\hat{\lambda} = \bar{\lambda}_p$.

If $p \in (p_l^*, \lambda_l)$ and $\hat{\lambda}$ is restricted to be in $[\bar{\lambda}_p, 1]$, then the raider's profits are clearly maximized at $\bar{\lambda}_p$. This is because, in the range $[\bar{\lambda}_p, 1]$, the takeover succeeds in high state with probability less than one if $\hat{\lambda} > \bar{\lambda}_p$, and the raider's expected payment is constant whereas the takeover probability in the high state decreases with $\hat{\lambda}$. Therefore, for any price $p \in (p_l^*, \lambda_l)$ the low type raider's profits are maximized when the agents hold the belief $\bar{\lambda}_p$. Moreover, at $\bar{\lambda}_p$ the takeover is succeeding with probability 1, half of the shares are sold in the high state and ϕ of the shares are sold in the low state.

Step 4. So far we have shown that $\pi(p_l^*, \lambda_l) \geq \pi(p, \lambda)$ for any $p \geq \lambda_l$ and that for prices $p \in (p_l^*, \lambda_l)$ the profit is maximized along the ray $\langle p, \bar{\lambda}_p \rangle$. Since $\bar{\lambda}_{p_l^*} = \lambda_l$, we are left to maximize $\pi(p, \lambda)$ over the ray $\langle p, \bar{\lambda}_p \rangle$ with $p \in [p_l^*, \lambda_l)$. Along this ray the takeover succeeds in both state with probability 1, precisely half of the shares are sold in the high state and precisely ϕ in the low state. Therefore the raider clearly prefers the lowest price at which this happens. I.e., $\pi(p, \lambda)$ is maximized at $\langle p_l^*, \lambda_l \rangle$. $\square$

Proof of Theorem 2: We establish the proof through a sequence of steps. In the first step we show that no pooling equilibrium survives the intuitive criterion. In the second step we argue that in all the separating equilibria that survive the intuitive criterion the high type raider is indifferent between his price offer and the low type's. Then we show that the equilibrium specified in the statement of the Theorem indeed maximizes the payoffs of both types of the raider.

Step 1: Pooling cannot be sustained. Fix an equilibrium in which both types of the raider offer a price p. First, notice that at p both types of the raider must be making a non-negative profit, otherwise they would be better off by offering the price 0. Moreover, 0 cannot be a pooling equilibrium since high type would be strictly better off by deviating to a price slightly above 0, regardless of the shareholders' beliefs after such a deviation. Since the price p is offered by both types the equilibrium beliefs must attach positive probability to both types, i.e., $\mu = \lambda \in (0, 1)$.

Let π_h be the equilibrium profit of the high type raider in the fixed pooling equilibrium. Then, either π_h is higher than the payoff he would make with the price offer p_l^* and belief $\mu = 0$, let's call this $\pi(p_l^*, 0, z_h)$, or less than or equal to this payoff. If $\pi_h > \pi(p_l^*, 0, z_h)$, then by Lemma 3 all prices $[0, p_l^*]$ are equilibrium dominated for the high type. Now let π_l be the equilibrium payoff of the low type. As we showed in Lemma 4, low type's highest payoff across all prices and beliefs is uniquely attained at price p_l^* and the belief $\mu = 0$. Therefore, $\pi(p_l^*, 0, z_l) > \pi_l$, and the price p_l^* is not equilibrium dominated for the low type. Consequently, the shareholders' equilibrium belief at price p_l^*, in an equilibrium surviving the intuitive criterion, has to be $\mu = 0$. But this contradicts the initial hypothesis that the low type offers p with a positive probability, because offering p_l^* gives him a higher payoff than offering p (which gives him a payoff of π_l).

Now suppose that $\pi_h \leq \pi(p_l^*, 0, z_h)$. Using the reasoning developed in the proof of Lemma 3, it is easy to see that the raider's profit, given $\mu = 0$ is linear in the price for prices $p' \leq p_l^*$ and attains value $(1 - \lambda_h)b$ for the price 0. Therefore there exists a price $\hat{p}$ such that the high type of the raider is indifferent between the equilibrium price p coupled with the equilibrium belief, and the price $\hat{p}$, coupled with the belief $\mu = 0$.

Next we show that given that the high type raider is indifferent between the price $\hat{p}$ coupled with the belief μ and the price p coupled with the equilibrium belief, it has to be the case that the low type of the raider strictly prefers the price $\hat{p}$. This result resembles the standard argument used in environments with single-crossing. Let $\psi(\hat{p})$ be the amount of shares sold in the high state and $\phi(\hat{p})$ be the amount of the shares sold in the low state after the price offer $\hat{p}$ when the shareholders' beliefs are $\mu = 0$. Likewise, let $\psi(p)$ and $\phi(p)$ be the amounts of the shares sold in the high and low state respectively after the price offer p and the equilibrium belief, $\mu(p)$. The high type raider's profit at price $\tilde{p} \in \{p, \hat{p}\}$ can be written

$$\lambda_h \psi(\tilde{p}) q(\tilde{p}) - \tilde{p}[\lambda_h \psi(\tilde{p}) + (1 - \lambda_h)\phi(\tilde{p})] + [\lambda_h q(\tilde{p}) + (1 - \lambda_h)]b,$$

where the first term represents the expected value of the purchased shares after the takeover, the second term the cost of purchased shares, and the third the expected value of private benefits. Since the high type raider is indifferent between the two prices

$$\lambda_h \left(\psi(\hat{p})q(\hat{p}) - \hat{p}[\psi(\hat{p}) - \phi(\hat{p})] + q(\hat{p})b\right) - \hat{p}\phi(\hat{p}) = \lambda_h \left(\psi(p)q(p) - \hat{p}[\psi(p) - \phi(p)] + q(p)b\right) - p\phi(p), \tag{9}$$

where $q(p')$, for $p' \in \{p, \hat{p}\}$ is the probability of success in the high state, coupled with the corresponding shareholders' beliefs. Lemma 2 implies that the fraction of shares sold in the low state ϕ is weakly monotonic in the price; it is flat at one half for low prices and starts strictly increasing at some point. Since $\hat{p} < p$ by the definition of $\hat{p}$, and since $\phi(\hat{p}) = 1/2$ is the smallest fraction of shares that can be sold by the shareholders in any equilibrium, we have $\hat{p}\phi(\hat{p}) < p\phi(p)$. Consequently the coefficient on λ_h in the left-hand side of equation (9) is smaller than the corresponding coefficient on the right-hand side. But then

$$\lambda_l \left(\psi(\hat{p})q(\hat{p}) - \hat{p}[\psi(\hat{p}) - \phi(\hat{p})] + q(\hat{p})b\right) - \hat{p}\phi(\hat{p}) > \lambda_l \left(\psi(p)q(p) - \hat{p}[\psi(p) - \phi(p)] + q(p)b\right) - p\phi(p),$$

meaning that the low type raider is better off by offering $\hat{p}$ and being believed to be the low type than offering p coupled with the equilibrium belief.

Since the high type is indifferent between p and $\hat{p}$ this means that he is strictly worse off by offering a price $p' < p$ and being believed to be the low type. The low type would strictly prefer p to $\hat{p}$, which implies there exists an $\epsilon > 0$ such that the low type would strictly prefer p to any $p' \in (\hat{p} - \epsilon, \hat{p}]$. This, in turn, implies that a pooling equilibrium that survives the intuitive criterion has to have out of equilibrium beliefs that attach positive probability to the high type having deviated to $(\hat{p} - \epsilon, \hat{p}]$, otherwise the low type would have a profitable deviation. On the other hand, the prices in the mentioned interval are equilibrium dominated for the high type, and are not equilibrium dominated for the low type. Therefore, if such a pooling equilibrium exists it does not satisfy the intuitive criterion.

Step 2: In this step, we analyze the properties of separating equilibria. Fix a separating equilibrium, and let p_h and p_l be prices that the high type and the low type offer respectively. Similarly, let π_h, π_l be the high and the low type's equilibrium profits. Since the equilibrium is separating $\mu(p_h) = 1$ and $\mu(p_l) = 0$.

First we argue that $\pi_h \leq \pi(p_l^*, 0, z_h)$. That is, the high type's equilibrium profit is not above the profit he would obtain if he offered the price p_l^* and was believed to be the low type. $\mu(p_h) = 1$ implies the high type's profits are bounded above by $\pi(p_h^*, 1, z_h)$ which, in turn, is strictly smaller than $\pi(p_l^*, 0, z_h)$.

In the rest of the proof we first establish that in any equilibrium surviving the intuitive criterion, the high type must be indifferent between p_h, coupled with $\mu = 1$, and p_l coupled with $\mu = 0$.

As in Step 1, let p be the price for which $\pi(p, 0, z_h) = \pi_h$. That is, p is the price such that the high type raider is indifferent between offering p_h together with being thought of as the high type and offering p while being thought of as the low type. We argue that in any equilibrium surviving the intuitive criterion $\pi_l = \pi(p, 0, z_l)$.

If $\pi_l < \pi(p, 0, z_l)$, then there exists an $\epsilon > 0$ such that prices $[p - \epsilon, p)$ are equilibrium dominated for the high type but not for the low types. Since the high type is just indifferent between p and his equilibrium price, he is strictly better off by offering the equilibrium price than offering any price in $[p - \epsilon, p)$ regardless of what the shareholder's beliefs after such a price offer. Hence any reasonable beliefs attached to prices in $[p - \epsilon, p)$ should be that the raider is the low type. However, if that was the case then the low type would have a profitable deviation to such prices.

Next we will show that if $\pi_l > \pi(p, 0, z_l)$, then the high type has a profitable deviation to p_l. Namely, $\pi_l > \pi(p, 0, z_l)$ implies $p_l > p$, otherwise the low type raider would be making a profit smaller than $\pi(p, 0, z_l)$ regardless of the shareholders' beliefs about him. The profit inequality can be written as:

$$\lambda_l q(p_l)\psi(p_l)-p_l(\lambda_l\psi(p_l)+(1-\lambda_l)\phi(p_l))+(\lambda_l q(p_l)+1-\lambda_l)b > \lambda_l q\frac{1}{2}-p(\lambda_l\frac{1}{2}+(1-\lambda_l)\phi)+(\lambda_l q+1-\lambda_l)b$$

or more conveniently,

$$\lambda_l\left(q(p_l)\psi(p_l)-p_l\psi(p_l)-q\frac{1}{2}-p\frac{1}{2}+q(p_l)b-qb\right) > (1-\lambda_l)\left(\phi(p_l)p_l-p\phi\right).$$

Note here that, $p_l > p$ and $\mu(p_l) = 0$ imply $\phi(p_l)p_l > p\phi$. Therefore the term in the brackets is positive. Since $\lambda_h > \lambda_l$, the above inequality also holds for the high type, i.e., if believed to be the low type, the high type strictly prefers p_l to p. But this cannot be the case in equilibrium, because π_h—the payoff from offering p coupled with belief $\mu = 0$—must be less than the payoff from deviating to p_l, otherwise the high type would have a profitable deviation. We have shown that $\pi_l > \pi(p, 0, z_l)$ leads to a contradiction. The only possibility for an equilibrium surviving the intuitive criterion is therefore $\pi_l = \pi(p, 0, z_l)$.

There are at most two prices that give the low type precisely the payoff π_l, when he is believed to be the low type. Indeed, Lemma 2 implies that as long as the low type raider is believed to be the low type, his payoff is single peaked with the peak at π_l^*. His payoff is also continuous, $(1-\lambda_l)b$ at the price 0 and negative for very large prices. Therefore, one price at which he obtains the payoff π_l, when believed to be the low type, is $p < p_l^*$ while another $p' > p_l^*$. But a separating equilibrium in which the low type offers p' cannot exist. At such a price the high type would have a profitable deviation to p'. This last claim follows from the exact same argument as used in the previous paragraph. Therefore, in any separating equilibrium, the low type offers p.

Now we argue that the separating equilibrium in which $p_h = \beta(s^*, z_h)$ is the one that gives both types of the raider the highest equilibrium payoff. Obviously, this is the equilibrium that the high type prefers the most among the separating equilibria. As for the low type. His equilibrium offer $\hat{p}$ is such that the high type is indifferent between his equilibrium offer and $\hat{p}$. Therefore, the larger the high type's payoff is, the higher is $\hat{p}$, and hence the higher is the low type's payoff.

To complete the proof, we argue that there is a separating equilibrium in which high type offers p_h and low type offer $\hat{p}$. For that, consider the beliefs $\mu(p) = 0$ for $p \leq \hat{p}$, and $\mu(p) = 1$ for $p > \hat{p}$. In words, after any price offer up to $\hat{p}$, the shareholders believe that such an offer was made by the low type raider. For any price above $\hat{p}$, they believe that the offer was made by the high type raider. It is now straightforward to check that this belief function supports a separating equilibrium that survives the intuitive criterion, and the high type offers p_h and the low type offers price $\hat{p}$. $\square$

Proof of Theorem 3: We start the proof by characterizing the optimal information revelation strategy with two messages. This gives us the lower bound on the welfare the shareholders can achieve under an optimal information revelation strategy. Then we argue that the optimal IRS with two messages is optimal among all IRS. In the last part of the proof we show the unique optimality of the two message structure when $b = 0$.

We focus on an information revelation strategy in which the message space is $Z = \{l, h\}$, the belief after message l is $\lambda(l) = 0$, the belief after message h is $\lambda(h) > \lambda$, message h occurs with probability $\alpha(h) = \frac{\lambda}{\lambda(h)}$ and l with probability $\alpha(l) = 1 - \alpha(h)$. The last part of the proof, showing uniqueness of the optimal strategy when $b = 0$, implies that an optimal IRS with two messages will have one message that induces the belief 0. Focus on such an IRS is therefore without loss of generality.

Let $\pi(p, \alpha)$ denote the raider's profits when he makes a price offer p and the information revelation strategy has $\alpha(h) = \alpha$. Furthermore, let p^*_α denote the optimal raider's price offer if he was to make an offer after the signal h was observed. Alternatively, p^*_α is the price the raider would offer in an environment without information revelation but with the common prior $\lambda(h)$. Using Lemma 2 (and the relevant probability of the takeover success in state h), the raider's expected profit for any price $p \leq p^*_\alpha$ can be written as

$$\pi(p, \alpha) = \lambda q(p, \alpha) 1/2 - p \left[1 - \alpha + \alpha \left(\frac{\lambda}{\alpha}\frac{1}{2} + \frac{\alpha - \lambda}{\alpha}\phi\right)\right] + b\left[1 - \lambda + \lambda q(p, \alpha)\right].$$

The seller's revenue—the first term in the above equation—equals the value of the acquired shares in the high state when the takeover succeeds. The high state occurs with probability λ, the raider acquires precisely half of the shares in the high state, and the takeover succeeds in the high state with probability $q(p, \alpha)$. The term $q(p, \alpha)$ is defined through the indifference condition of the threshold type of the small shareholder, s^*. That is, $\beta_\alpha(s^*) q(p, \alpha) = p$. The second term in the equation for $\pi(p, \alpha)$ represents the raider's costs arising from the purchases of the shares. After the price announcement $p > 0$ the bank reports message l with probability $1 - \alpha$ in which case it is revealed to the shareholder that the state is low and the value of the shares 0. Consequently the shareholders sell all their shares after the message l. Message h is reported with probability α. Conditionally on the message h the high state occurs with probability λ/α in which case precisely half of the shares are sold. The low state, conditionally on the message h occurs with probability $(\alpha - \lambda)/\alpha$ in which case $\phi > 0.5$. Notice that ϕ is independent of the price, as long as the price is not above p^*_α. Similarly, the coefficient on b captures the probability of the takeover success. Using the indifference condition of the threshold type of small shareholders, and the equality

$$\beta_\alpha(s^*) = \frac{\lambda(h) f_h(s^*)}{\lambda(h) f_h(s^*) + (1-\lambda_h) f_l(s^*)},$$

the raider's profit from offering a price $p \leq p_\alpha^*$ can be rewritten as

$$\pi(p,\alpha) = p\left[(\alpha-\lambda)\left(\frac{f(s^*|l)}{2f(s^*|h)} - \phi\right) - (1-\alpha)\right] + b(1-\lambda) + b\lambda \frac{p}{\beta_\alpha(s^*)}.$$

Note that π is linear in p. At the shareholder value maximizing α the derivative $d\pi/dp$ should not be negative. If it was, the raider's optimal price offer, and therefore the shareholder value, would be zero. The derivative can be written as

$$\frac{d\pi(p,\alpha)}{dp} = \left[(\alpha-\lambda)\left(\frac{f(s^*|l)}{2f(s^*|h)} - \phi\right) - (1-\alpha)\right] + \frac{\lambda}{\beta_\alpha(s^*)} b.$$

Due to the assumption on the size of the large shareholder, $\frac{f(s^*|l)}{2f(s^*|h)} - \phi$ is non-negative, making the term in the square brackets increasing in α. The second term is also increasing in α.

We now argue that for $b > \frac{1-\lambda}{\lambda}$ the derivative $\frac{d\pi(p,\alpha)}{dp}$ is strictly positive. Since we verified it is increasing in α it is enough to verify that the derivative is positive at $\alpha = \lambda$, because by the construction of the messages, $\alpha \geq \lambda$. The case in which $\alpha = \lambda$ corresponds to the full information disclosure. Therefore, the shareholders are certain it is the high state when they observe signal h. In particular $\beta_\alpha(s^*) = 1$, implying $\frac{d\pi(p,\lambda)}{dp} = -(1-\lambda) + \lambda b > 0$. The raider's optimal strategy is then to offer price p_α^*. The optimal IRS maximizes this price, which results in $\alpha^* = \lambda$ and $p_\alpha^* = 1$. The shareholders sell all of their shares, and receive a payoff 1, which is larger than the ex ante expected value of the shares λ. The raider buys all the shares which are worth λ in total and receives private benefits. His payoff is therefore $b+\lambda-1$. In this case, i.e., when $b > \frac{1-\lambda}{\lambda}$, the raider can guarantee himself a payoff of $\lambda + b - 1$ by making a price offer arbitrarily close to 1, no matter what the IRS chosen by the shareholders is. Because the maximum total surplus is $\lambda + b$, the maximum shareholder value is bounded above by $\lambda + b - (\lambda + b - 1) = 1$. Since the IRS with two messages attains the shareholder value equal to this upper bound, it is an optimal IRS.

Instead, suppose $b \leq \frac{1-\lambda}{\lambda}$. Since $d\pi/dp$ is continuous in α, non-positive at $\alpha = \lambda$, and positive at $\alpha = 1$ (due to $\frac{f(s^*|l)}{2f(s^*|h)} - \phi \geq 0$), there exists an α^* such that $\frac{d\pi(p,\alpha^*)}{dp} = 0$. For $\alpha = \alpha^*$ the raider is indifferent between all the prices $p \in [0, p_{\alpha^*}^*]$. The welfare, however, is maximized when he offers the price $p = p_{\alpha^*}^*$ in which case the takeover succeeds in both states and the expected welfare is $\lambda + b$. Since $\frac{d\pi(p^*_{\alpha^*},\alpha^*)}{dp} = 0$, the raider's profit is identical to his payoff from offering price 0 and the takeover succeeding in state l only, i.e., $\pi(p^*_{\alpha^*}, \alpha^*) = b(1-\lambda)$. Finally, the shareholders' welfare is $\lambda + b - b(1-\lambda) = \lambda + b\lambda$. In this case, note that the raider can guarantee himself a payoff equal to $(1-\lambda)b$, by making a price offer that is arbitrarily close to zero, and ensuring the takeover succeeds in state l, the equilibrium shareholder value is bounded above by $\lambda + b - (1-\lambda)b = \lambda + \lambda b$, which is attained by the IRS with two messages.

The second part of the proof shows that the above presented information revelation strategy is the unique optimum among all information revelation strategies with two messages when $b = 0$. The uniqueness here is up to compounding of messages that result in the same posterior.

Let $Z, \{Pr(\omega, z)\}_{\omega,z}$ be an IRS. Let $z_1 = argmin_{z\in Z}\lambda(z)$, and $p(z)$ be the optimal price offer the raider would have made if the message z was publicly announced before his offer. If $\lambda(z_1) > 0$, then the raider can secure himself a strictly positive payoff by making an offer $p(z_1)$. Indeed, Lemma 2 implies that $p(z)$ is increasing in $\lambda(z)$, and that the raider's profit is linear in price for prices in $[0, p(z)]$. Moreover equation (3) implies that if profit is positive for some z with $\lambda(z) > 0$ then it is positive for all such z. Hence, the raider can guarantee himself positive profit by offering $p(z_1)$. Since the total welfare is bounded above by λ, the shareholder value is therefore strictly smaller than λ. Therefore any shareholder value maximizing information revelation strategy must have $\lambda(z_1) = 0$.

Now let $z_2 = argmax_{z\in Z}\lambda(z)$. In equilibrium, the raider will never offer $p > p(z_2)$. Indeed, Theorem 2 in Ekmekci and Kos (2012b) shows that given the common prior λ, and if the large shareholder is large enough, the raider optimally offers the smallest price at which the takeover succeeds in both states. Moreover from that price upward the raider's profit is decreasing in the price. $p(z_2)$ is the least price under which the takeover succeeds in both states of the world when the belief is $\lambda(z_2)$. Therefore, the raider is strictly better off offering $p(z_2)$ than offering any higher price if the message is z_2 and consequently also after any other message. On the other hand, if offering a $p \le p(z_2)$ gives the raider a strictly positive payoff, then this information revelation strategy does not maximize shareholder value. Namely, given that the total welfare is at most λ this would mean that the shareholders obtain less than λ. If offering $p(z_2)$ gives him a negative payoff, then he would have offered a price strictly less than $p(z_2)$, and the takeover would fail with positive probability, rendering the total welfare and thus the shareholder value strictly smaller than λ. In short, any shareholder value maximizing IRS must yield zero expected profits for the raider after offering $p(z_2)$.

With two signals the above findings uniquely pin down the IRS to be the one we constructed in the first part of the proof. It remains to be argued that if there are more than two messages, and $b = 0$, then the shareholder value is not maximized. Above discussion implies that the shareholder value maximization demands $\lambda(z_1) = 0$, and $p(z_1) = 0$. Let $z_3 = argmin_{z\in Z\setminus\{z_1\}}\lambda(z)$, and remember that $\pi(0) = 0$. Consider $p = 0$. If $\frac{d\pi(p)}{dp}|_{p=0} > 0$, then the shareholder value is not maximized because the raider's equilibrium payoff is strictly positive. Namely, the raider can guarantee himself a strictly positive profit by offering a price just slightly above 0. Therefore the derivative is nonpositive. Next we will argue that the nonpositivity of the derivative together with having more than 2 possible messages implies $\pi(p(z_2)) < 0$. Since we showed in the previous paragraph that a shareholder value maximizing scheme must have $\pi(p(z_2)) = 0$, the shareholder value thus cannot be maximized using an IRS with more than two messages.

To show that $\pi(p(z_2)) < 0$ if $\frac{d\pi(p)}{dp}|_{p=0} \leq 0$, we calculate $\frac{d\pi(p)}{dp}$ in the interval $[0, p(z_3))$ as follows:

$$\pi(p) = p\left(\sum_{z\in Z\setminus\{z_1\}}\alpha(z)\left(\frac{\lambda(z)}{2p(z)} - \frac{\lambda(z)}{2} - (1-\lambda(z))\phi\right) - \alpha(z_1)\right),$$
$$\frac{d\pi(p)}{dp} = \sum_{z\in Z\setminus\{z_1\}}\alpha(z)\left(\frac{\lambda(z)}{2p(z)} - \frac{\lambda(z)}{2} - (1-\lambda(z))\phi\right) - \alpha(z_1).$$

Note that $\frac{d\pi(p)}{dp}$ is constant in the interval $[0, p(z_3)]$. Moreover, $\frac{d\pi(p)}{dp}|_{p\leq p(z_3)} > \frac{d\pi(p)}{dp}|_{p>p(z_3)}$, because for any z, $\frac{d\pi(p|z)}{dp}$ is constant for $p < p(z)$ and $\frac{d\pi(p|z)}{dp}|_{p_1} > \frac{d\pi(p|z)}{dp}|_{p_2}$ for any pair of prices (p_1, p_2) such that $p_1 < p(z)$ and $p_2 > p(z)$. Hence, $0 \geq \frac{d\pi(p)}{dp}|_{p\leq p(z_3)} > \frac{d\pi(p)}{dp}|_{p>p(z_3)}$. Therefore, if $z_2 \neq z_3$, then $\pi(z_2) < 0$. □

Proof of Theorem 4: The proposed strategies for the equilibrium are as follows: Both players acquire fully revealing IRS, and the raider offers price 1 if the state is high and 0 otherwise. The shareholders sell all shares if the price is the value of the share in the corresponding state. If the price is p and the state is high, then they sell in such a way that half of the shares are sold, and takeover succeeds with probability p. The raider's deviation in the information acquisition stage are ignored by the shareholders, i.e., because the board acquires fully revealing information, the shareholders' belief about the raider is irrelevant. If the board deviates in information acquisition stage, then in the continuation play, the shareholders ignore the message delivered by their IRS and play according to the separating equilibrium play in the environment in which the raider is privately informed.

We will first check the optimality of the raider's strategy. Fixing the shareholders' behavior, suppose the raider adopts an IRS and chooses a pricing strategy p that induces a distribution of prices conditional on the state, i.e., let $Pr(p|\omega)$ denote the probability that the raider offers price p in state ω. Then, if the state is high the takeover succeeds with probability p and the raider's payoff is $b\sum(p \times Pr(p|h))$. That is, the shareholders are randomizing, therefore the raider must be paying precisely the expected value of the shares. But then his payoff is only the private benefit times the probability that the takeover succeeds in the high state. The raider's payoff in the low state is $b - \sum(p \times Pr(p|l))$. In the low state the shareholders dump their shares and the takeover succeeds with certainty for any price offer $p > 0$. The raider receives his private benefits with certainty but also has to pay for the worthless shares. The payoffs in both states are bounded above by b, which is precisely what the raider obtains under the equilibrium strategy.

Finally, we check that the board does not have a profitable deviation. Obtaining an IRS different than fully revealing gives the shareholders a payoff equal to λ, which is not more than (actually equal to) their payoff from not deviating. $\square$

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