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Investment Certificates: A Developing Frontier for Structured Products

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1. Introduction

Investment certificates belong to the category of structured products, i.e. financial securities consisting of various elementary components which are combined together to generate a specific risk-return profile to satisfy the needs of an investor. In the academic literature, there are no consistently used and widely accepted definitions of structured product. According to Wohlwend, *"a structured product combines the payout structures of at least two financial assets into one financial product, where at least one of these components has to be a derivative financial instrument" (Wohlwend, 2001; p.5).* The aim of structured products is to set the return distribution of an underlying asset at one or more specific points, or to change the distribution shape as compared to their underlying asset altogether.

Investment Certificates are securitized derivatives, combining financial derivatives (mainly options), risk-free zero-coupon bonds, and/or underlying asset(s). They represent a vast class of financial products, characterized by a rich variety of risk-return profiles that can be selected in the implementation of many investment strategies. Certificates can be linked to many different underlying assets, from equity indexes to stocks, from currencies to commodities, including interest rates.

Certificates can be divided in two main categories: Investment Certificates and Leveraged Certificates. Investment Certificates can be used by investors for risk management and yield enhancement purposes, while Leverage Certificates allow the investor to be exposed to the performance of the underlying, in a magnified way thanks to the leverage effect included in this category of financial products.

An investor can buy a certificate on the primary market during the offer period (usually three weeks) from the distributor/manager/private banker or directly on the secondary market if the product has been listed. In Italy for instance, certificates can be traded either on the SeDeX of Borsa Italiana SpA or on the Cert-X of EuroTLX SIM. A distinctive element characterizing those products is a low investment requirement as certificates have a minimum size, in general, equal to 100 or 1000 euros and an investor can acquire an amount as low as one unit of the issue.

In general, the price of a certificate is determined by its characteristics (type, payoff, protection, maturity, etc.). In particular, the payoff of a structured product is equal to the algebraic sum of the payoff of its financial components and in no-arbitrage conditions the price of the product must be equal to the algebraic sum of the prices of the components.

However, during the offer period, the price of a certificate is not equal to the algebraic sum of the price of each single component because the issuer includes in the issue price the costs for the structuring, distribution and listing of the product. There is a further implicit cost component for an investor who purchases a certificate: he does not receive any dividend distributed by the underlying asset during the life of the product. Dividends are exploited by the issuer to buy the basic elements that generate the desired characteristics of an Investment Certificate. For instance, the dividends paid by the stock can be employed by the issuer to "buy" the capital protection at maturity for a certificate offering capital protection, or they can be used to grant the investor a remuneration even in case of a moderate market decline as it happens for a Bonus Certificate. Hence, the forgone dividends must not be understood as a profitability reduction but rather as a necessary condition for the structuring of the financial product.

Investment Certificates are characterized by high flexibility: the great variety of financial structures and maturities (in general from one to five years) allows investors to select the certificate which best matches his/her portfolio needs. Besides, thanks to a wide range of underlying assets, an investor can be exposed to an asset otherwise not available or only available at very high costs. Moreover, despite being considered complex products, they have an easy payoff structure and their offering must contain a final term-sheet to facilitate their understanding. Plus, they can be listed on an exchange, and they are issued by an institution through a specific Base Prospectus, previously approved by the CONSOB (in Italy) or another National Authority.[1] Indeed, after their distribution, almost all certificates are admitted to trading on the secondary market. In this market, liquidity is guaranteed by market makers and/or specialists, who commit to quote on a continuous base bids and offers, with a maximum bid-ask spread defined by law.[2,3] Independently from the chosen structure and by the underlying asset, all certificates can be bought and sold on a per-share basis anytime during market hours. It is important to highlight that by

[1] See also "Consultation paper on the new reporting obligations for intermediaries (June 19, 2009)".

[2] For every product traded on the secondary market there exists a trading book, with offers to buy (bid) and sell (ask). When there is a match between the bid price and ask price, the two orders are matched, and a transaction occurs. All other orders placed are shown in the book in order of price, from the highest to the lowest for bids and from the lowest to the highest for asks (best bid and offer criterion).

[3] There are different maximum spreads depending on the reference price of the product. However, since most of investment certificates have a price higher than 30 euros, the maximum spread allowed is 3.50% in this case.

investing in certificates, an investor may be able to recover losses linked to previous investment made in other financial assets. Indeed, in Italy the taxation of certificates is easy and more convenient than that of other investment products.[4]

However, by investing in certificates an investor bears many risks. Indeed, certificates are a direct, unsubordinated, unconditional and unsecured obligation issued by a bank or a financial intermediary, and rank equally with all other direct, unconditional and unsecured obligations of the issuer. Therefore, the redemption of the invested capital and the solvency of an Investment Certificate is determined by the capital solidity of the issuer. The probability that the issuer would not be able to fulfill its obligations, that is to liquidate at maturity the proceeds owed to the buyer of the financial instrument, is not zero.[5] Moreover, the value of these products is essentially linked to the performance of the underlying asset(s). The return cannot be predetermined and in case of a negative performance of the underlying(s), the value of the derivative can go to zero (market risk). Besides, the underlying asset(s) may be denominated in a difference currency than that of the certificate and hence, the value of the instrument may be subject to changes in the value of the exchange rate between the two currencies. Eventually, the price of a certificate can be affected by the liquidity on the market. The existence of a regulation imposed by Borsa Italiana SpA in terms of quantity and maximum bid-ask spread lowers this risk, but it does not exclude possible divestment issues before maturity.

The rest of the thesis has the following structure. Section 2 analyzes the recent evolution of the market for Investment Certificates in Europe and Italy. Section 3 describes analytically the Investment Certificates types, grouping them in three main categories: Capital Protection, Yield Enhancement, and Participation. Section 4 examines the behavior of Investment Certificates during their lifetime: first it presents the most common types of exotic options and describes the main risk factors taken into consideration for the analysis and then it focuses on the behavior of an Equity Protection and a Bonus Certificate. Section 5 continues with a synoptic comparison of different investment products and Section 6 concludes.

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[4] For a detailed analysis please see section 5.

[5] From January 1st, 2016, there is another risk that needs to be considered. This risk is linked to the application of the bail in, a mechanism, introduced by the 2014/59/EU Directive (BRDD), that allows BankItalia to activate a preventive measure if the financial institution is deemed to be in trouble, in order to preserve its business continuity.

2. Recent Evolution and Current Framework

2.1 The Evolution of the Market in Europe

Structured products experienced a great success before the financial crisis of 2008 but then lost the trust of investors. According to the Federation of European Securities Exchanges (FESE) the traded volumes of securitized derivatives reached its maximum in 2008, at around 200 billion of euros and 19 million of trades. Since then, those numbers have decreased. Nevertheless, Exchange turnover in structured investment and leveraged products on Europe's financial markets stood at 27.6 billion euros at the end of September 2017, representing a significant 6.0% increase year on year. At the end of September, exchanges located in Italy, Germany, France, Belgium, Holland, Switzerland, Sweden, UK, and Austria (all EUSIPA member countries) were offering 565,533 investment products and 1,127,562 leverage products (a 7.0% increase quarter on quarter, and a significant 23.0% year on year).[6]

At present, certificates are listed in at least fourteen European Exchanges, even if most of the European contracts are negotiated on the SeDeX (the Italian market for Covered Warrants and Certificates), Euwax (European Warrant Exchange) which is Europe's largest platform for exchange trading in securitized derivatives, Scoach and Nyse Euronext (a pan-European exchange, spanning Belgium, France, the Netherlands, Portugal and the UK).

All these exchanges share the following features:

- types of listed certificates: Bonus, Express, Discount, Outperformance, Equity Protection and others
- main issuers: BNP Paribas, Société Générale, Deutsche Bank, Royal Bank of Scotland, and UniCredit
- the European regulation: in particular the Markets in Financial Instruments Directive 2014/65/EU (MiFID II), The Prospectus Directive 2003/71/EC (PD), and The Market Abuse Regulation (EU) 596/2014 (MAR)

2.2 The Evolution of the Market in Italy

Investment Certificates were introduced on the market many years ago. The first certificate was issued in Germany in 1989, whilst in Italy some certificates were already

[6] Source: European Structured Investment Products Association (EUSIPA)

traded on the secondary market at the end of the 90s.[7] Nevertheless, it is only in the last decade that these products have gained in importance, with a strong acceleration in recent years. This can be seen in Figure 1 and Figure 2 that show the evolution of the number and the turnover of certificates issued every year by banks in Italy, starting from 2008.

The number of new ISIN on the market has shown a progressive increase until 2015: after a first sharp increase in 2009 and a consolidation in the three following years, from 2013 to 2015 there has been a boom in the offering of certificates, followed by a drop in 2016. The numbers for 2017 are better than that of 2016, and in line with those recorded in 2015. Indeed, according to ACEPI, 473 new certificates were issued last year (a 29% increase year on year).

By looking at the issued, the growth process is even more noticeable. We had a first significant raise between 2008 and 2010, when volumes recorded a 332% increase (for an annual rate of 82%), going from 1.21 billion euros in 2008 to 4.03 billion in 2010. However, it is from 2013 that we recorded an incredible jump in the issued, moving from 2.5 to 10.6 billion of euros in 2015, with an average annual growth rate of 62%. The drop recorded in 2016, in line with the reduction, in the same year, in the numbers of issues, could have been suggested a crisis for the market of certificates. However, first, it must be noticed that, after three years of significant growth, a period of correction in the market is not uncommon. Indeed, a similar path was already recorded after the first peak of 2010. As a matter of fact, data from 2016 consolidate the growth seen in the previous three years: 2016 volumes are more than double that of 2012. What is more, the recent figures for 2017 shows a new rise in the issued (4.7 billion euros) even if the number is still below the peak reached in 2015.

A different analysis can be performed using the statistics on certificates turnover on the SeDeX, one of the two markets where certificates are regularly traded, and provided by Borsa Italiana SpA, the manager of the market for securitized derivatives in Italy[8].

Figure 3 shows the evolution of turnover in millions of euros on the trading platform since 2011. The numbers, which comprise both investment and leveraged certificates, confirm the trend shown in the issue of certificates: in the three years coinciding with the boost of the former, the trading volume increased sevenfold.

[7] See Camelia (2009), page 24.

[8] Data are publicly available on Borsa Italiana SpA website. Figures are not exhaustive since trading volumes on the other available platform, EuroTLX, are not included.

Finally, the decomposition between the two macro categories of certificates highlights that the main contribution to the growth of these products is mainly given by leveraged certificates, which since 2013 have overtaken in terms of turnover the other category, above all thanks to the success of the fixed leverage certificates. If it is natural that leveraged certificates, more suitable for speculative short-term operations, are much more traded, it is desirable that Investment Certificates, more suitable instead for a medium-long term investment horizon, would return to take a larger market share (Figure 4).

There are many reasons at the basis of the success of certificates in recent years. First, the transparency of these products is considerably improved thanks to the detailed documentation provided by financial intermediaries. This allows a better understanding of the product for the investor, enhancing his/her trust in the instrument. Second, market efficiency contributes to improve the liquidity of the market, making it easy to manage investments. Therefore, always more investors approach these products independently, and acquire consciousness of their potentials. Last, but not least, financial intermediaries are focusing on the development of Investment Certificates to expand the range of offered products and provide a valid alternative to other financial products. In recent years, the main driving factor has been the low interest rate environment (we have experienced even negative interest rates). In this phase, the investment in other instruments is penalized. For instance, the return on short-term Italian government bonds (BOT and CCT), historically appreciated by Italian households, is negative at the time of writing, close to the -0.5% region, and even by extending the maturity (and hence risk), returns remain unsatisfactory anyway.

Investment certificates represent an optimum alternative solution for putting capital at work.

2.3 Primary and Secondary Markets

When an issuer structures an issue of a new certificate, it can opt for a direct listing on the Exchange (secondary market) or for a distribution to clients through an offer (primary market), usually followed by the listing on the Exchange (SeDeX or Cert-X). In the last ten years, the market has progressively moved from the first to the second alternative and today the offering activity preceding the issue represents a considerable part of the job of a certificates’ issuer.

Following the example of the issue of BTP Italia, which recorded a great success, since December 2013 the primary market has strengthened itself with the introduction of the direct distribution of certificates through the market platform. This new distribution method allows issuer banks to list certificates directly through the trading platform of Borsa Italiana, ensuring a continuous link between the "primary" market (of the offer) and the following listing on the SeDeX. This new offering method of certificates has many advantages for investors in terms of transparency and simplification of the subscription method. Besides, it generates less costs for intermediaries (which do not pay trading fees), with a likely costs reduction for the certificate holder as well.

SeDeX is the acronym of Securitized Derivative Exchange. It is a non-regulated market where certificates and covered warrants can be traded. It was established in 2004 to combine the trading of securitized derivatives. From November 27th SeDeX changed its status as a regulated market, becoming a Multilateral Trading Facility (MTF). This regulatory move was put in place to comply with a new provision in MiFID II requiring the mandatory introduction of a Central Counterparty (CC&G), from January 3rd, 2018. However, not only the name of the market remained unchanged, but it follows the same set of rules of the regulated one.

It is divided into four segments: the first two are the ones dedicated to plain vanilla and exotic covered warrants, while the other two to the trading of investment and leveraged certificates. Leverage certificates are then divided into variable leverage (Class A) and fixed leverage (Class B). A similar distinction is made for Investment Certificates: the products whose payoff replicates linearly the underlying (e.g. tracker or open-end) belong to the Class A, whilst all the other products are clustered in Class B.

Borsa Italiana S.p.A defines with its own rule book the issuers admitted to present listing proposal, as well as the characteristics of the certificates, in terms of maturities, underlying asset(s), and liquidation terms. SeDeX is an order-driven market, where price formation is driven by the offers existing on the market. The orders are matched according to the best bid and offer criterion and, in case of offers with the same price, the time of the offer prevails, without opening and closing auctions (only continuous trading from 9.00 to 17.25). It is foreseen the mandatory figure of the Specialist, i.e. a financial intermediary, most of the times coinciding with the intermediary that has issued the product, with the obligation to quote bid and ask prices, with a minimum quantity and a maximum spread defined by the market itself.

EuroTLX, on the other hand, has always been a MTF. MTFs were introduced in 2004 by MiFID I and the main differentiation point with respect to regulated markets is the quality of the subject enabled to the management of the exchange. A regulated market can be managed by a management company, as for example is Borsa Italiana SpA, whilst a MTF can be managed by an investment firm as well. As a matter of fact, EuroTLX was managed by Intesa Sanpaolo and UniCredit, both owning a 50% share until September 23rd, 2013, date in which the control of the platform passed to Borsa Italiana, which now holds 70% of the shares.[9]

From an operating point of view, however, few are the changes with respect to the organization and functioning of a regulated market (continuous trading from 9.0 to 17.30, order-driven market with price and time criteria). As for the SeDeX, the presence of a liquidity provider is mandatory, with the former obliged to continuously quote bid and ask prices with minimum quantities and maximum spread limits. Within the platform, the Cert-X segment is where the trading of Investment Certificates takes place (together with that of covered warrants).

3. The Different Categories

In the following pages I describe the different categories of certificates. Every certificate is analyzed with the following method: first, I give a brief description of the instrument. Then I define the profit function and then I show which is the option strategy at the basis of the certificate's structure. I decided to postpone the analysis of the behavior of the mark-to-market of Investment Certificates during their lifetime to the next chapter because this analysis is at the heart of the identification process of the risks and opportunities linked to an investment in these products.

The order followed in the presentation of the different categories follows the European Product Categorization sponsored by EUSIPA. The association defines four categories for structured products: Capital Protection, Yield Enhancement, Participation and Leverage. I have decided to focus my analysis on the products with no leverage and hence defined, *stricto sensu*, Investment Certificates.

The categorization proposed by EUSIPA differentiates the products by their risk-return profile and in Figure 5 their risk is compared with that of familiar asset classes.

[9] Source: EuroTLX website

However, it is important to remember that structured products in general and investment certificates in particular are characterized by asymmetric return distributions. The risk of each certificate will depend on its construction features. Therefore, the representation in figure 5 is incomplete and should consider other parameters (described in section 4 of this work) to become more accurate.

3.1 Capital Protection

Capital Protection instruments guarantee the redemption of the capital at maturity, in addition to positive returns if the price of the underlying asset(s) touches or breaches a certain level. Those products, having a strong prudential attitude that perfectly fits the hedging needs of risk-averse investors, are viewed with a positive light by supervisors. They have understandable payoff and are widespread among retail clients. Capital guaranteed products guarantee the redemption of the invested capital at maturity in addition to participating to a certain degree in the performance of an underlying risky asset (Bluemke, 2009). In this category we find Equity Protection, Digital, Double Win, Butterfly and Express Protection.

Equity Protection

Equity Protection are the simplest among capital protection products. They can be suitable for an investor willing to participate in the positive performance of the underlying asset, receiving at the same time a hedge if the price of the underlying drops badly. Indeed, there is a protection level below which the liquidation amount cannot fall, even in case of negative performances of the underlying.

To better understand the structure and the profit and loss profile of any type of certificate it is worth knowing the elements characterizing them. In general, the date from which the gain (or loss) of the underlying asset starts to be computed is known as the fixing date (or pricing date) while the ending date of the period is known as the expiration date (or valuation date). For the Equity Protection, the key elements are:

- Initial Level (S_0): the value of the underlying asset on the fixing date. Usually the strike price is set at the initial level and it is expressed as a percentage of it

- Protection Level (PL): minimum amount received at maturity, regardless of the underlying value, expressed as a percentage of the invested capital[10]
- Participation Factor (PF): expressed in percentage points, it defines to what extent the certificate participates in the performance, if positive, of the underlying asset
- Multiple (M): the quantity of the underlying asset controlled by each certificate. It is obtained by dividing the issue price of the certificate by the market price of the underlying asset

At maturity, two different scenarios can be observed: if the price of the underlying asset (S_T) is above the protection level, the investor receives the capital originally invested to buy the certificate at issue plus the positive performance, weighted for the participation factor. If instead S_T is below the protection level, the liquidation value will be equal to the minimum guaranteed. In analytical terms:

$$Payoff_T = \begin{cases} NV \times PL\,, & S_T < PL \\ \left\{PL + \left\{\{PF \times [(S_T - S_0)]/S_0\}\right\}\right\} \times NV, & S_T \geq PL \end{cases} \tag{1}$$

Where NV means Nominal Value. An Equity Protection is a combination of European options, i.e. it incorporates options which can be exercised only at maturity. By buying this certificate an investor is long a zero-strike call, which is equivalent to an investment in the underlying asset, and a put option. The exercise price of the put coincides with the protection level. This ensures capital protection at maturity in case of a negative performance of the underlying. Moreover, if the participation factor is not equal to one, the purchase or sale (depending on whether the it is greater or smaller than one, respectively) of a quantity of calls is required. The quantity to be bought is equal to the difference between the PF and 100% (in the opposite case the quantity to be sold is given by the difference between 100% and the PF).

Two variants of the Equity Protection are the Equity Protection Cap (aka Collar Certificate) and the Equity Protection Short. The former is characterized by the presence of a cap, i.e. the participation to the positive performance of the underlying asset is capped at a predefined level. The cap gives the structurer the possibility to increase the participation factor or the protection level.[11] The equity protection short instead allows

[10] As invested capital is intended here and in what follows the amount of money spend by an investor to buy one unit of a certificate at the issue, i.e. the issue price.

[11] The Cap is obtained by selling a call option whose strike coincides with the cap level. The sale proceed can then be reinvested for increasing the participation to the upside or the protection of the capital at maturity.

to participate in the downside performance of the underlying asset. In other words, if the price of the underlying closes below the protection level the investor receives the capital originally invested plus the negative performance (changed in sign), else he/she gets the minimum guaranteed.[12]

It is worth noting that thanks to the put-call parity the financial structure behind an equity protection can be obtained in different ways, all leading to the same result. Let's first illustrate this fundamental relationship of the option theory. The put-call states that the value of an European call option with a given exercise price and maturity can be obtained from the value of an European put option with the same strike and maturity and vice versa. Two portfolios are considered to analyze this relationship: a portfolio made of an European call option (C) and a zero-coupon bond with liquidation value corresponding to the exercise price of the options (X), the same maturity of the options (T) and interest rate r (portfolio A), and another built with an European put option (P) and the purchase at the price (S) of the underlying stock (portfolio B).[13] At maturity, both portfolios have the same payoff, corresponding to the higher between the stock price (S_T) and the exercise price (X).[14] If the value of the two portfolios is the same at maturity, they must have the same value at the beginning. Consequently, the following relationship must hold:

$$S + P = C + Xe^{-rT} \tag{2}$$

In the equation, Xe^{-rT} represents the actuarial value of the zero-coupon bond, with maturity T and yield r. Starting from equation (2), through some easy steps we can obtain the formulae defining the value of each financial instrument as a function of the others, as illustrated in table 1. For instance, the purchase of a put option can be replicated through the short-sale of the underlying stock and the contemporaneous reinvestment of the proceeds in a zero-coupon bond and a call option.

The put-call parity is fundamental in the financial analysis of certificates, both at maturity and in any instant of the life of these products. An Equity Protection can then be obtained as a combination of a zero-coupon bond (fiscally more efficient) and a call option (synthetic hedge). In normal market conditions, the value of this certificate incorporates

[12] An Equity Protection Short is constructed with a long zero-strike call option, a short call option, and two long put options. and a possible purchase/sale of several put options if the participation factor is not one.
[13] In the following illustration we consider options on non-dividend paying stocks.
[14] The options are European and hence they cannot be exercised before maturity.

the participation to the upside of the underlying and/or the expectation of a future participation. Suppose, for instance, that during the lifetime of the certificate the price of the underlying asset drops well below the protection level in a way that it would be highly unlikely for the underling to close at maturity above this level. The value of the certificate will then be a function of the yield remunerating the capital at maturity. For a better understanding of what just said please consider the following example. Let's consider a certificate granting 100% capital protection with a nominal value of 100 euros and one-year maturity. If the underlying asset value drops 50% below the protection level, next year the certificate will probably pay just the protection level, that is 100 euros. If the interest rate p.a. is 4% then the value of the certificate would have a market value of approximately 96 euros and if we acquire it today, we would make a capital gain that will be considered for tax purposes as "*redditi diversi*" and not "*redditi da capitale*". In highly volatile markets and bear (for long certificates) or bull (for short certificates) market phases, the example is recurrent, and it holds for all certificates belonging to the capital protection category but Double Win.

Digital

Digital certificates pay a coupon, usually annual, in case the underlying is above, on predetermined observation dates, a predefined level and a premium at maturity if the underlying is greater or equal to the initial level. If this is not the case, the product provides partial or full capital protection. They can be considered a capital protection variation of another certificate, the Cash Collect: that is why they are also known as Cash Collect Protection. Other commercial definitions are Target *Cedola* and Protection Premium.

The main features of this certificate are:

- Strike/Initial Level (K): underlying level, defined before the issue, from which intermediate levels are computed
- Protection Level (PL): minimum amount that the investor receives at maturity, regardless of the value of the underlying asset, expressed as a percentage of the invested capital
- Digital Level (DL): predefined underlying level recorded on the observation dates
- Digital Coupons (DC): coupon paid when, on the observation dates, the price of the underlying is above the digital level

- Premium (P): an amount of money paid at maturity if the underlying is at least equal to the strike on the valuation date
- Multiple (M): the amount of the underlying asset(s) controlled by each certificate

At maturity we can observe three different scenarios: if the price of the underlying is greater or equal to the strike, the certificate pays the nominal value and the Premium (P). If the price is below the strike but above the protection level the certificate pays an amount of money equal to the underlying price multiplied by the multiple. If instead the underlying is below the protection level, the certificate ensures capital protection.

In analytical terms:

$$Payoff_T = \begin{cases} NV \times PL, & S_T < PL \\ S \times M, & PL \leq S_T < K \\ NV \times M + P, & S_T \geq K \end{cases} \tag{3}$$

If the certificate offers full capital protection, Scenario two does not take place.

This payoff can be obtained with a combination of different options: the purchase of a zero-strike call option and that of a put option with exercise price corresponding to the protection level, the sale of a call option with strike K (to delete upside participation), and the purchase of a stream of digital call options, all with digital level as strike, the coupon as cash payment, and all with different expiration dates. The different maturities of these digital options must coincide with the various observation dates of the certificate.

Double Win

Double Win certificates give the possibility to participate in the performance of the underlying asset, regardless of its sign, and ensure at the same time capital protection.

The features characterizing the Double Win certificates are:

- Initial Level (S_0): the value of the underlying asset on the fixing date and coinciding with the strike of the certificate (K)
- Protection Level (PL): minimum amount received by the investor at maturity, regardless of the value of the underlying asset, expressed in percentage of the invested capital
- Participation Level UP (LU): percentage difference, on the upside, of the price of the underlying from which the Participation Up begins
- Participation Level Down (LD): percentage difference, on the downside, of the price of the underlying from which Down Participation begins

- Participation Factor Up (PU): percentage measure of the participation of the certificate to the upside performance of the underlying asset
- Participation Factor Down (PD): percentage measure of the participation of the certificate to the downside performance of the underlying asset
- Multiple (M): amount of the underlying asset controlled by the certificate

At maturity the investor will benefit both from a positive and a negative performance of the underlying if the price of the underlying is above or below the participation level up and the participation level down, respectively. If instead the underlying value is between those two values the investor will receive the minimum guaranteed.

In analytical terms:

$$Payoff_T = \begin{cases} \{PL + PD \times [(K - S_T)/K - LD]\} \times NV, & S_T < LD \\ PL \times VN, & LD \leq S_T < LU \\ \{PL + PU \times [(S - K)/K - LD]\} \times NV, & S_T \geq LU \end{cases} \quad (4)$$

If participation level up and down coincides with the initial level, the investor receives at maturity the capital originally invested and benefits of any performance of the underlying asset (positive or negative) with respect to its initial value.

The payoff at maturity can be constructed with a long zero strike call option, a long put option with exercise price equal to the protection level, a short call option with strike coinciding with protection level, long an amount of call options equal to the participation factor up and with strike equal to the participation level up, and long an amount of put options equal to the participation factor down and with strike coinciding with the participation level down.

Butterfly

This type of certificate gives an investor the opportunity to benefit from moderate positive and negative performances of the underlying asset, provided that during its life the underlying asset remains in a predefined range. It is basically a Double Win with the addition of two elements: an Up Barrier (UB) and a Down Barrier (DB), whose upward or downward violation, respectively, during the life of the certificate determines the loss of the participation to the performance of the underlying asset.

The financial structure of a Butterfly is more elaborated and combines plain vanilla and exotic options. It is obtained by combining the purchase of a zero-strike call option, the purchase of a put option with strike equal to the protection level, the sale of a call option with strike equal to the protection level, the purchase of a quantity (equal to the PU) of up and out call options, with strike equal to the participation level, barrier equal to UB and a possible rebate, and the purchase of a quantity (equal to PD) of down-and-out put options, with strike equal to the initial level, barrier equal to DB and a possible rebate.

Express Protection

The Express Protection represents a capital protected variation of the Express Certificate. This product allows to receive the payment of a premium, that increases over time (a feature know as memory effect), upon occurrence of a predetermined event (trigger). In such a case, the early redemption of the capital takes place (autocallability).

The product is characterized by:

- Initial Level (S_0)/Strike (K): the value of the underlying asset on the fixing date and coinciding with the strike of the certificate
- Protection Level (PL): minimum amount guaranteed at maturity, regardless of the price of the underlying asset and expressed as a percentage of the invested capital
- Trigger (T): a predefined level of the underlying, usually coinciding with the strike, whose achievement on one of the observation dates activates the payment of the premium and the early redemption clause
- Premium (P): a predefined amount payable if the trigger is activated. The memory effect ensures that the premium paid at every observation date is increasing, depending on the number of observations already carried out, hence at the nth observation the premium paid amount to *n x P*
- Multiple (M): quantity of the underlying asset controlled by each certificate

Differently from other types of securities, an investor in an Express Protection must not consider the payoff at maturity but he/she will need to consider intermediate observations as well. At the first observation date if the price of the underlying asset is above the Trigger, the premium will be paid, and the certificate expires, i.e. the capital will be redeemed because of to the autocallability feature. In all other cases, no premium is paid, and the certificate goes on to the next observation date when the same evaluation is made. At maturity if the underlying level is above the trigger, the investor receives the invested capital and an amount comprising all the unpaid premiums. If the underlying is below the protection level, the certificate pays a minimum guarantee level, whilst if it is between the trigger and the protection level, the certificates pays an amount equal to the value of the underlying multiplied by M.

In analytical terms:

$$Payoff_T = \begin{cases} PL \times VN, & S_T < PL \\ S_T \times M, & PL \leq S_T < T \\ PL \times VN + \ n \times p, & S_T \geq T \end{cases} \tag{5}$$

The Express Protection payoff can be obtained with the following combination: the purchase of a zero-strike call option and a put option both with exercise price equal to the protection level, the sale of a call option with strike at K and the purchase of a stream of European binary options, with maturity matching the observation dates of the certificate and payoff equal to the Premium (which is different at each date because of the memory effect).

3.2 Yield Enhancement

Yield enhancement products are designed without unconditional capital protection and with capped upside participation. They offer periodical returns strengthened by conditional or unconditional coupons. They represent prudential products that aim to generate a high return relative to bond yields; the risk may become comparable to their underlying asset(s) in case of adverse market conditions (Bluemke, 2009). In this category we find Discount, Express, Cash Collect and the variants with the cap of Bonus and Outperformance certificates.

Discount

Discount Certificates belong to the category of yield enhancement products and are financially equivalent to the purchase of the underlying asset at a discount, that is at a lower price with respect to price available on the market. This implies a higher performance of the certificate than that of the underlying asset. And it allows to obtain positive returns in cases of small negative performances of the underlying. This advantage is compensated by the presence of a cap, i.e. the investor does not participate to the upside performance above a predefined level.

The key elements of this certificate are the Strike/Initial Level (K), the Cap (C), representing the level of the underlying above which upside participation is capped, and the Multiple (M).

At maturity we can have two possible scenarios, depending on whether the underlying is above or below the cap. If it is below, the certificate linearly follows the performance of the underlying, even if the discount provides a partial hedge. In particular, between the discount price and the cap, the direct investment brings a loss which instead corresponds to a lower profit for the buyer of the certificate.

The payoff function is the following:

$$Payoff_T = \begin{cases} S_T \times M, & S_T < C \\ C \times M, & S_T \geq C \end{cases} \tag{6}$$

An investment in this type of certificate generates losses only below the strike level, although they are lower than those born by an investor who investment in the underlying, as in the previous case. On the other hand, in a bullish market, the certificate is penalized above the break-even price, obtained by summing to the cap the discount offered to the buyer of the certificate.

The Discount certificate is the result of the following combination of European call options: the purchase of a zero-strike call option (equivalent to an investment in the underlying asset) and the sale of a call option with strike price equal to the cap, to delate the upside participation to the underlying performance above this level.

Cash Collect

Cash Collect certificates are characterized by a stream of conditional periodical coupons and a capital protection which depends on the level of the underlying at maturity (it should be above a predefined barrier level).[15] The coupons are conditional because the payments are made only if the underlying asset is above a predefined level. In the autocallable variation of the Cash Collect the certificate expires before maturity if on the early observation dates the value of the underlying is greater or equal than a predetermined trigger level, usually coinciding with the initial level.

The main elements of a Cash Collect certificate are:

- Initial Level/Strike (K): underlying level, defined before the issue, from which intermediate levels are defined
- Additional Amount Level (AL): level of the underlying (usually the same as K), if exceeded on the valuation dates determines the payment of the additional amount

[15] Usually Cash Collect certificates are built with an European style barrier, i.e. the barrier is observed only at maturity. However, there are some variations with the American barrier feature, in which the barrier is monitored for the whole life of the certificate.

- Additional Amount (A): periodic cash stream payable, on the valuation dates, if the underlying is above the additional amount level
- Barrier Level (B): underlying level that if not violated at maturity ensures the repayment of the capital
- Multiple (M): amount of the underlying controlled by each certificate

At maturity the possible outcomes are two: if the barrier is not violated the certificate repays the capital originally invested, whilst if the underlying closes below the barrier the investor participates to the negative performance of the underlying. What is more, the investor has the possibility to receive periodical coupons on the intermediate observation dates and at maturity. Those amounts concur to define the overall performance of the product.

In analytical terms the profit function at maturity is the following (assuming B=AL):

$$Payoff_T = \begin{cases} S_T \times M, & S_T < B \\ A + PL \times VN, & S_T \geq B \end{cases} \tag{7}$$

The Cash Collect can be structured with a combination of a long zero-strike call option, a long European down-and-out put option, knocking out at the barrier level and with exercise price equal to the strike (for conditional capital protection), short a call option with strike equal to K (capping the potential upside) and long a stream of digital calls. These exotic options are needed to generate the conditional coupons offered to investors and hence they must have maturity matching the different valuation dates (and maturity) and an exercise price equal to the additional amount level.

Depending on the maturity and the payment frequency of the coupons, a Cash Collect can used for a short or medium-term investment strategy. Indeed, the certificate can be structured with a maturity of twelve months and monthly or quarterly coupons, or with a 2 or 3-year maturity and semiannual or annual coupons.

Cash Collect certificate can have as underlying a single asset or a basket of securities. If a certificate has as underlying a basket of stocks, on the observation dates for the conditional coupons, for the early redemption, and on the final valuation date the valuations of the underlying will be made considering as underlying reference value the weighted average of the values of all the stocks comprised in the basket, as it is shown in the following equation:

$$\sum_{k=1}^{n} w_k \frac{S_{k,T}}{S_{k,0}} \tag{8}$$

Where n is the number of stocks included in the basket, w_k the weight of the kth component, $S_{k,T}$ the price of the kth component at date T and $S_{k,T}$ the price of the kth component at the initial valuation date. The most used types of certificates on a basket of stocks are the constant weights, worst of and best of. In the constant weight version, the weights are determined before the issue and remain constant for the whole life of the certificate. The worst of version instead gives a weight of 100% to the stock with the worst performance in the timeframe considered, whilst the other components are not considered. The best of version gives to the top performer a weight of 100% and to all others a weight of zero.

Express

Express Certificates are like Cash Collect, except from two elements: the autocallability feature and the memory effect. This product indeed pays a premium if the trigger event is verified on the observation date. Differently from the Cash Collect however, the occurrence of the trigger event determines the early redemption of the certificate. Besides, if the trigger event is not verified on the first observation date, on the nth observation date the additional amount will be equal to n-times the first premium (memory effect). The main advantage of this certificate is given by the opportunity to make an interesting profit in a short period of time. Indeed, it suffices that the underlying price remains stable around the level it had at the fixing of the certificate to obtain the early redemption together with a very attractive coupon.

The elements characterizing this type of certificate are:

- Initial Level/Strike (K): underlying level, defined before the issue
- the Trigger (TR): predefined underlying level, usually coinciding with the strike, activating the payment of the premium if reached on the observation dates
- the Premium (P): predefined amount payable on the premium payment dates if the trigger is activated. Thanks to the memory effect, there is the possibility to recover the premiums not paid during the life of the certificate.
- the Barrier (B): level of the underlying that if not reached allows to receive the invested capital at maturity
- the Multiple (M): quantity of the underlying asset controlled by the certificate

In the case of Express certificates, the redemption does not occur necessarily at maturity. To analyze the profit function, it is useful to start from the first observation date. On this date if the underlying level is above the trigger the certificate pays the premium and it ceases to exist, repaying the nominal value. If this is not the case, the certificate does not pay any premium and continues to the next observation date. If no early redemption occurs, at maturity we can have three different scenarios: if the underlying is above the trigger the certificate pays the nominal value plus the n premiums. If instead the underlying level is between the trigger and the barrier, the investor sees its capital protected as he/she receives the nominal value originally paid. However, if the price of the underlying asset is below the barrier, the investor lost his/her capital protection and participate in the negative performance of the underlying.

In analytical terms:

$$Payoff_\tau = \begin{cases} 0, & S_\tau < TR \\ VN + \tau \times p, & S_\tau \geq TR \end{cases} \quad with\, \tau = 1, 2, \ldots, n \tag{9}$$

If no early redemption occurs, at maturity we have:

$$Payoff_T = \begin{cases} S_T \times M, & S_T < B \\ NV, & B \leq S_T < TR \\ NV + n \times p, & S_T \geq TR \end{cases} \tag{10}$$

The financial structure of an Express is similar to that of the Express Protection, except from the fact that the put option purchased is not a plain vanilla. Indeed, it is a down-and-out put option, which is cheaper than a plain vanilla put and therefore it allows, *ceteris paribus*, to acquire digital calls that pay a greater sum of money.

3.3 Participation

Certificates belonging to this category participate in the positive performance of their underlying asset(s) and can have no, or only conditional, capital protection.[16] All the products analyzed in this thesis are mostly equity based, however it must be said that they can be based on any underlying asset. This category includes products with a more speculative feature than those belonging to the categories previously described, and therefore, they are less common among retail clients. The most common are: Bonus, Airbag, Twin Win, Outperformance and Tracker Certificates.

[16] Please note that this feature is different to the capital protection granted to the first category analyzed before, for which the protection is unconditional.

Bonus

Bonus Certificates are one of the most common participation products. The certificate does not offer full capital protection, even if some technical measures allow to limit the probability of losses. However, in the worst scenario the performance is equivalent to that obtained by the underlying asset.

In their standard version, Bonus certificates give the possibility to participate linearly and without any cap to the upside performance of the underlying asset, allowing at the same time, in cases of poor performances or even slightly negative ones, the payment of a bonus (from which the name of the instrument), provided that, during the life of the certificate, the underlying asset does not violate on the downside a predefined level (also known as Barrier Level). In this case, the certificate loses its benefits instantaneously when the barrier is touched (Barrier Event) and from that moment onward it linearly replicates the performance, positive or negative, of the underlying. The barrier can be of two types: continuous barrier (American) or discrete at maturity (European). The first relevant difference between the two different monitoring methods of the barrier is observed during the structuring phase. When structuring a bonus, the choice of the type of the barrier has a significant impact on the characteristics of the certificate. For instance, let's consider a Bonus with an American style barrier, with a given price and bonus. The same certificate with discrete monitoring of the barrier at maturity would be more expensive. To obtain the same price we need to reduce the bonus or increase the barrier level. Moreover, the choice of the type of the barrier generates differences in the pricing of the strategy before maturity (delta and vega) as well.

The key elements of the certificate are the Initial Level/ Strike (K), the Barrier Level (BL), the Bonus (B) (expressed in percentage points), paid at maturity if the asset has not touched the barrier, and the Multiple (M). Considering the payoff function of a Bonus Certificate, at maturity, if the barrier has not been touched, there are two possible scenarios: if the performance of the underlying is greater than the Bonus, the certificate linearly replicates the underlying, whilst if the underlying performance is greater than the barrier but smaller than the bonus level, the certificate pays the bonus and the capital originally invested.

If the barrier is touched during the life of the certificate, the Bonus becomes a Tracker certificate (analyzed in what follows), losing its bonus and capital protection. It is important to highlight that a barrier event occurring does not necessarily imply a negative performance for the certificate, since from that event to maturity the asset could recover and close above the strike.

In analytical terms, at maturity if the barrier has not been violated during the lifetime of the certificate we have:

$$Payoff_T = \begin{cases} S_T \times M, & S_T < BL \\ NV \times (1+B), & BL \le S_T < NV \times (1+B) \\ S_T \times M, & S_T \ge NV \times (1+B) \end{cases} \tag{11}$$

whereas if the barrier has been touched before maturity the certificate pays $S_T \times M$.

To structure a Bonus the combination of a zero-strike call option and a down-and-out put option is required.[17] Besides, if the upside participation is higher or lower than 100%, the purchase or sale, respectively, of a number of call options with exercise price K would be required. From the point of view of an investor, a Bonus certificate on stocks or indexes should have a short maturity, ranging between one and two years.

The great success of Bonus certificates is demonstrated by the number of offered products. On the market there are different version, the most common are:

- Bonus Cap: in this version the participation of the certificate to the positive performance of the underlying asset is capped thanks to the sale of a call option. The strike of this option is set at the level that we want to "cap".
- Top Bonus: differently from the standard version of the Bonus the barrier is discrete at maturity, that is the essential condition to receive the Bonus is that the underlying, at maturity and only on this date, is above the Barrier Level. The certificate is not path-dependent because the evolution of the price of the underlying is not relevant for the payment of the Bonus. Indeed, in this version if the underlying drops below the barrier before maturity and then recovers, the right to receive the Bonus does not vanish.
- Bonus Plus: a Bonus certificate with a discrete monitoring of the barrier at maturity that allows to receive an unconditional bonus paid at maturity. The issue price (usually 100 euros) is redeemed at maturity if on the final valuation date, the underlying value is above the barrier level. Otherwise the certificate replicates the negative performance of the underlying (the bonus is paid in any case).

[17] Another way to structure the certificate is to go short a down-and-in put option and long a call option with a contemporaneous money-market investment. The strike of the two options would be set at the same value equal to $1 + B$.

- Reverse Bonus Cap: this version allows to participate to the negative performance of the underlying stock or index up to a maximum level (Cap), paying a premium, Bonus, if the underlying value has been stable or has dropped slightly. The condition to receive the bonus is that the underlying price does not touch the barrier level during the life of the certificate. If the appreciation of the underlying is above this level, then the redemption value will be inversely proportional to the performance of the underlying asset.
- Bonus Cap Worst Of: this version has the same functioning of a classical Bonus Cap, with the only difference that the underlying is represented by a basket of stocks. To receive the Bonus, none of the underlying asset must reach its own barrier level during the life of the product. If one of the underlying stock drops below its barrier, the redemption at maturity will be linked to the worst performing stock comprised in the basket.

Twin Win

Twin Win certificates allow an investor to participate in the positive and negative performance, within certain limits, of the underlying asset, hence the name, twin-win. The differentiation from the Bonus certificate lies in the fact that instead of a bonus level, a positive participation to the downside of the underlying asset is bought. They are also like Double Win with capital protection, with the only difference being the presence of a barrier, whose violation determines the loss of the relative benefits.

An investor should be willing to buy a Twin Win when on the market there exists a version with an unconditional capital protection because the lower costs of an exotic put option (for the Twin Win) with respect to a plain vanilla put option (for the Double Win) implies a saving that can be reinvested in the purchase of other contracts, allowing to increase the participation level, hence the expected performance of the Twin Win.

The certificate can be identified by the following essential elements:

- Initial Level/ Strike (K): predetermined level of the underlying asset before the issue
- Barrier Level (B): predefined level of the underlying whose missed breach ensures the liquidation at maturity of the absolute performance of the underlying, i.e. negative performances are turned into positive ones.
- Upside Participation Factor (UF): measure of the participation in the positive performance of the underlying, expressed in percentage terms

- Downside Participation Factor (DF): measure of the participation in the negative performance of the underlying, expressed in percentage terms
- Multiple: quantity of the underlying asset controlled by each certificate

The structure of the Twin Win implies three possible different scenarios at maturity: if the underlying level is above the Strike (K), the payoff is equal to the Nominal Value (NV) augmented by the positive performance of the underlying, which could be weighted by the upside participation factor. In a neutral scenario, when the underlying is between the Strike (K) and the barrier (B), we can have two possible cases: if the barrier was not violated, the investor receives negative performance in absolute value weighted for the downside participation factor, whilst if the barrier was breached, the certificates becomes a Tracker certificate, linearly replicating the underlying. If instead the underlying is below the barrier level, the certificate records the same negative performance of the underlying asset, repaying $S_T \times M$.

In analytical terms, if the barrier is not breached during the lifetime of the product, at maturity we have:

$$Payoff_T = \begin{cases} S_T \times M, & S_T < B \\ NV \times |[(S \ - K\ )/\ K\ ]| \times UF + NV, & B \leq S_T < K \\ NV \times [(S \ - K\ )/\ K\ ] \times UF + NV, & S_T \geq K \end{cases} \tag{12}$$

The structure of a Twin Win requires the purchase of two down-and-out put options, rather than one as in the Bonus, in addition to a zero-strike call option.

A variation of the classical Twin Win certificate is represented by the Twin Win Autocallable that has, at maturity, the same functioning of the former, with an extra autocallable clause, that allows the early redemption of the certificate in predefined dates (early redemption dates) if certain conditions are met and at a determined liquidation amount. Thanks to the callability option the investor has the opportunity to cash-in the initial investment before the natural maturity of the certificate, receiving the capital invested plus a premium that is already known at the moment of the subscription. In particular, these certificates early redeem if on one early observation date the value of the underlying is above its initial value; in case of early redemption the liquidation value is equal to the invested capital, augmented by an amount defined since the issue for each early redemption date. Therefore, the occurrence or not of an early redemption determines the amount and the terms of the liquidation for the certificate.

Airbag

As the Bonus, the Airbag Certificate has an exposure to the performance of an underlying asset with a certain amount of downside protection. Differently from the Bonus, at maturity they do not have any discontinuity in their profit and loss function: the protection continues below the protection level thanks to the airbag. What is more, the amount to be repaid is defined at maturity, hence the path followed by the underlying asset during the life of the certificate is not considered. Those considerations make the Airbag Certificate an efficient investment option, above all if considered as an alternative to a direct investment in stocks or stock indexes.

Airbag certificate are characterized by the following elements:

- Initial Level/ Strike (K): predefined initial level of the underlying
- Protection Level (PL): level of the underlying defining the protection at maturity
- Airbag Factor (AF): obtained dividing the strike (K) by the protection level (PL)
- Participation Factor (PF): measure of the participation of the certificate in the upside of the underlying, expressed in percentage terms
- Multiple (M): the quantity of the underlying asset controlled by each certificate

The profit and loss function of an Airbag is simple. At maturity we can have three scenarios: if the underlying ends above its initial level (K), the certificate replicates the performance of the former. If the underlying value is below the strike but above the protection level, the investor does not lose any of his capital regardless of the negative performance of the underlying asset. If instead the underlying is below the protection level, the certificate records a negative performance, mitigated by the existence of the airbag.

In analytical terms:

$$Payoff_T = \begin{cases} S_T \times M \times AF & S_T < PL \\ NV & PL \leq S_T < K \\ S_T \times M, & S_T \geq K \end{cases} \tag{13}$$

It is worth mentioning that, as the airbag factor shows values always greater or equal than one, the liquidation value, in the negative scenario will be always greater than what we would get by investing directly in the underlying asset.[18]

[18] If the Protection Level (PL) is below the Strike (K), then the fraction K/LP will always have values greater than one.

As stated above, airbags are different from Bonus or Twin Win as their structure does not include barriers that could be knocked-out in case of adverse market conditions. The product includes plain vanilla European-style options only. It is structured by a long zero-strike call option, a long put option with exercise price equal to K, short a quantity (equal to the Airbag Factor) of put options with strike equal to the PL.[19] Plus, if the upside participation is higher or lower than 100%, the purchase or sale, respectively, of call options with exercise price K would be required.

Tracker

Tracker certificates are structures that linearly replicate an underlying asset, usually stock or stock indexes. In practice, trackers have as underlying excess return assets, like a basket of stocks, or an index like the Eurostoxx50.[20] In this case they represent a valid option to invest in underlying assets, often characterized by high volatility, exploiting the diversification effect incorporated in the notion of Tracker: this investment is not exposed to the idiosyncratic risk associated with the purchase of a single asset, exposing to systemic risk only.

Conceptually it is close to an investment in an Exchange Traded Fund (ETF), from which it differentiates itself for a different exposure to the issuer risk and for the greater fiscal efficiency, other than, in the specific, for the absence of periodic flows (dividends or yields), found instead in an ETF. A Tracker is characterized by two factors only: Initial Level/Strike (K) and Multiple (M) and can be structured with a long zero-strike call option.

Please note that during its life a Tracker having as underlying an excess return index tends to trade at a discount, that is at lower values than those it has at maturity. The value of the certificate is discounted at an intermediate date of a value equal to the expected dividends from that day until maturity, dividends that the holder of the certificate is not going to receive and that however, contributes to decrease the value of the index at each ex-dividend date.

[19] if the upside participation is higher or lower than 100%, the purchase or sale, respectively, of calls with exercise price K would be required.

[20] A total return index includes any dividends or yield, whilst an excess return index is ex-dividends, i.e. it does not include any dividends or yield. Please note that a total return index always overperforms an excess return index on the same underlying asset.

Eventually, it is worth noting that a change in realized or expected dividends has an impact on all certificates having as underlying dividend paying stocks. Before the issue, the structurer first discounts the future dividends expected to be paid during the life of the product and then subtracts this value from the spot price. Therefore, the expected dividends are comprised in the issue price of the certificate that will redeem at maturity 100% of the issue price. This means that the issuer, not the investor, is exposed to the dividend risk, i.e. if the realized dividends will be lower than what expected the issuer will make a loss and vice versa if dividends will be higher than what previously expected the issuer will make a profit.

Outperformance

Outperformance certificates have no capital protection and offer the possibility to participate more than proportionally to the positive performance of the underlying, thanks to the so-called leverage effect, whilst replicating exactly the trend of the asset in case of negative ones. Hence, they allow to amplify a positive move of the underlying without an increase of losses in case of negative performance of the underlying asset. Indeed, in negative market conditions an investor bears the same losses of those born by a direct investment in the underlying.

The elements characterizing these certificates are:

- Initial Level/Strike (K): value of the underlying on the strike date, starting from which upside participation begins to be computed
- Outperformance (Up Participation Factor) (UP): percentage measure of the upside participation of the certificate
- Multiple (M): underlying quantity controlled by the certificate

At maturity, depending on the price of the underlying asset with respect to its initial value (K) we can have two different scenarios.

In analytical terms:

$$Payoff_T = \begin{cases} S_T \times M, & S_T < K \\ NV + (S - K) \times UP \times M\,, & S_T \geq K \end{cases} \tag{14}$$

The financial structure of an Outperformance is not complicated, comprising only European call options exercisable at maturity. It is a combination of a long zero-strike call option and a ratio of long at-the-money call options, the ratio depends on the difference between the outperformance (UP) and 100%. This portfolio is financed by the level of dividends of the underlying asset, and therefore the certificate is used with dividend paying stocks or excess return indexes.

Two variations of this certificate are the Outperformance Cap and the Outperformance Conditional Protection. The former has a higher upside participation than the standard Outperformance, together with a cap to the positive performance. The latter instead provides capital protection unless a barrier is touched during its life.[21]

4. The Behavior of Investment Certificates During their Lifetime

4.1 Options: An Essential Component of Investment Certificates

Options are at the heart of investment certificates. As stated before, investment certificates are obtained as a portfolio of options and their price depends on the value of each option included in this portfolio. A depth knowledge of the elements that characterize every single option allows both to understand the choices made by an issuer in terms of offered products, and to know in which way and to what extent the price of the certificate is going to be affected by a change in one of the drivers of options valuation. Options are derivative contracts which incorporate the right to buy (call) or sell (put) a predetermined underlying asset at a predefined price (strike). Depending on when the right can be exercised we can distinguish between American or European options. If the right can be exercised every moment until maturity we are dealing with American options, whilst if the right can be exercised only at maturity the option is European.[22]

With the term exotic options instead, we identify all options with a more sophisticated payoff than that of plain vanilla options: this can refer both to the formation process of the payoff over time and the configuration of the same at maturity. In general, these options are traded over the counter between qualified counterparties. Their extensive use in the construction of investment certificates can be explained by the need of higher flexibility in the operational strategy of the issuers that thanks to these contracts can satisfy the different preferences of investors, and by the possibility to trade certain types of exotic

[21] The financial structure of an Outperformance Cap is the same to that of the standard version, plus short a call option with exercise price equal to the cap. The Outperformance Conditional Protection is obtained by adding to the standard version an at-the-money down-and-out put option.

[22] Please note that a third type of option is the so-called Bermudan option. It is exercisable only on predetermined dates, typically every month.

options at a lower price than that of plain vanilla options. In the structuring of certificates, the most used exotic options are barrier options and binary options.

Barrier Options

Barrier options are used in the structuring of many certificates, as for instance Bonus, Express and Twin Win. They are characterized by two reference levels: the first is equal to the traditional exercise price, at which it is possible to buy or sell the underlying asset, the second is the barrier level: the option come into existence (knock-in) or ceases to exist (knock-out) if this level is violated before maturity. Barrier options are options whose payoff depends on whether the underlying asset's price touches a predetermined level during a certain time period (Hull, 2018). As seen in section 3, the most used barrier option among the investment certificates structures is the down-and-out put option. This option allows to guarantee the conditional protection of some typologies of certificates. In fact, it protects the investor when the underlying asset records a negative performance up to the barrier, but once this level is reached, the option "dies" and with it the capital protection offered to the investor.

Barrier options are cheaper than plain vanilla options because after a predetermined level of the underlying asset, their value becomes zero. This explains why we buy a down-and-out-put rather than a plain vanilla put option that provides full capital protection in case the underlying asset drops below the strike. If the probability of reaching the barrier were equal to zero, the value of the two options would be the same.

Among the properties of barrier options there is one that makes them uncommon and useful: in certain cases, the vega of these options is negative. When there is a barrier, an increase in volatility does not represent only an advantage, because the higher variability of the price of the underlying increases the probability of a knock-out as well. In addition to the basic forms of barrier options a number of more exotic barrier options exist on the market. Examples of such options are options with partial barriers, double barriers, time-dependent barriers and compound barriers (Brockhaus et al., 1999).

Binary Options

Also known as digital options, they are the easiest class of exotic options. They present a discontinuous payoff at maturity: they represent real bets on the fact that the price of the underlying would reach or be above the strike. They differ from plain vanilla options for the discontinuity in the payoff (in plain vanilla options the payoff is a continuous function of the price of the underlying).

There exist different typologies of binary options: the most common are the cash or nothing and the asset or nothing. Cash or nothing option represents the easiest type of digital options: if the price of the underlying is above (below) the strike for a call (put), the investor receives the payment of a predefined amount (X), otherwise the option expires worthless.

The payoff function of a cash or nothing can be described as follow:

$$\text{cash or nothing call} = \begin{cases} 0 \; if \; S \; \leq K \\ X \; if \; S \; > K \end{cases} \tag{15}$$

$$\text{cash or nothing put} = \begin{cases} X \; if \; S \; \leq K \\ 0 \; if \; S \; > K \end{cases} \tag{16}$$

4.2 Option Pricing Models: A Primer

The correct pricing of options allows ex-ante to give a right price to the certificate, while providing an efficient hedging ex-post. To this aim, the structurer can use different option pricing models. The most used in practice are: Black & Scholes, Local Volatility and Stochastic Volatility.

The model of Black & Scholes (1973) is the easiest and the most famous among option pricing models. This model allows to derive the value of an European option starting from the following inputs: implied volatility, spot price of the underlying asset, strike price, maturity, expected dividends, and risk-free interest rate. All these factors will be described in the following paragraph. This model is usually implemented for the valuation of plain vanilla options, whilst not very common in the valuation of many exotic options. Indeed, the hypothesis at the basis of this model are in contrast with the characteristics of these options. For instance, the Black & Scholes approach does not directly suit barrier option. Merton (1973) modified the model and derived a first relationship to calculate the price of a down-and-out European call option. Later Rubinstein and Reiner (1991) used the Merton formula on eight types of barrier options, while Haug (1998) on all sixteen single types. Besides, Nishiba (2012) discusses a new method for pricing exotic options in his work "Pricing Exotic Options and American Options: A Multidimensional Asymptotic Expansion Approach".

What is more, the Black & Scholes model prescribes that the volatility of the underlying asset is constant over time and over all the strike levels, hypothesis that represents a theoretical simplification. Empirically, we observe that among options with different strike or maturity there is instead a different implied volatility. In the past, to solve this inconvenient it was tried to introduce a variant to the model, using implied volatility as a variable, but with poor results. Therefore, other models have been developed.

The first step in this direction is the Local Volatility model. It allows to consider the skewness without adding variability elements and it is the easiest way to take into account the volatility skew and term structure.[23] In this model, volatility is neither constant nor a random variable, rather it is a function of the price of the underlying asset and time (Gatheral, 2006). The process that allows to find this function is called calibration: it consists of the analysis of implied volatilities of vanilla options to find a "rule" (the function mentioned above), that makes consistent all the prices observed on the market as described in Derman and Kani (1994). The model, once calibrated, can be used to value different exotic options. However, Dupire (1993) comes up with an explicit formula that derives the local volatility function, meaning that the model does not have to be calibrated to market prices. This is the approach that is generally implemented by issuing banks for the pricing of path-dependent options.

Stochastic Volatility models, Heston (1993) being the most famous, are the most articulated because they start from assumptions closer to reality than the models described before. In particular, in Stochastic Volatility models, volatility is neither constant, as in Black & Scholes, nor a function of other parameters, as in Local Volatility. It is considered instead, like the price of the underlying asset, as a random variable. This approach has two advantages. Firstly, if volatility is a random variable, it must have its own volatility that the model allows to compute. We are talking in this case of volatility of volatility or vol-of-vol. Some derivatives show a non-linear sensitivity to volatility, i.e. there is a non-zero second-order price sensitivity (or convexity) to a change in volatility. We are talking in this case of Vega Convexity. Vol-of-vol allows to consider Vega convexity. (Bouzoubaa and Osseiran, 2010). Secondly, stochastic models do generate forward skew, that is the skew of an option that starts on a future date $t_{(1)}$ and ends at $t_{(2)}$. The options of this type, that have a high sensitivity to the skewness, would not be correctly priced with

[23] The term structure shows how the maturity date of an option will change the implied volatility over time, while volatility skew refers to fact that options on the same underlying asset, with different strike prices, but which expire at the same time, have a different implied volatility.

the two models previously described. Stochastic Volatility models, instead, are able to generate a coherent computation of the forward skew, allowing consequently to value in a more accurate way these types of options.

4.3 Variables at Stake

As described by Black and Scholes (1973), if at maturity the value of an option coincides with its intrinsic value, in every moment before this date, instead, the price of an option is a function of different variables: underlying asset price, strike price, volatility of the underlying, maturity, risk-free interest rate, and dividends.[24] All these factors (except the strike) represent a potential source of risk (and return) and to each of them synthetic indicators, also known as sensitivity coefficients or *Greeks*, are associated (Figure 6 provides a definition of the most common greeks). These parameters allow to measure the variation of the value of an option corresponding to a change in one of the risk factors, keeping all the others constant.[25] The theoretical value of a Certificate is computed by banks with pricing models that have as inputs these factors.

The price of the underlying asset is the main risk factor influencing the price of an option (hence a certificate). The delta of an option measures the variation of the option price for a given change in the price of the underlying asset, and from a mathematical point of view it is computed as the first derivative of the option price with respect to the underlying asset price.[26] As it can be seen in Figure 8, the delta of a call option is a positive number between 0 and 1, whilst that of a put is a negative number, ranging from -1 and 0. The value of delta is affected mainly by the spot level and time to maturity. Figure 7 plots

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[24] The premium of an option has two components: intrinsic value (IV) and time value (TV). The former is the value obtainable today from the implicit right of the option. If the option is American: IV = Max [S - E, 0] for the call, whilst IV = Max [E - S, 0] for the put. Time value is the second component of the premium: for out of the money options it coincides with the value of the "hope of exercise", while for in the money options it coincides with the value of the right to abandon. Time value is maximum for at the money options.

[25] *Greeks* can be computed both with an analytical procedure and with a numerical procedure (for both methods we need an option pricing model). The former approach consists in the computation of the first derivative of the option price (given by the model equation) with respect to the parameter whose sensitivity is being measured. It is standard practice however to compute the *greeks* by changing the considered risk factor in the pricing method of a certain amount and then computing the difference between the new value of the option and the initial one.

[26] The delta of an option is represented by the steepness of the slope: a payoff line of 45° implies a delta of 1. To compute the delta of a certificate, we need to draw a tangent line to the desired point on the payoff line: the delta is the slope at that point. A horizontal line indicates a point where participation is zero whilst a vertical line implies an infinite delta. We have infinite participation where the barrier of a in or out option is located or where the strike of a digital option is set.

the delta of an in-the-money call (assuming a 100% strike and asset's price of 110%) and the delta of an out-of-the-money call (assuming 100% strike and asset's price of 90%) as a function of time to maturity. The delta of the ITM call is 73% at the beginning and that of the OTM call is 43%. Both figures are stable at the beginning, and only when around 40% time to maturity is left they start to move to their final values. This behavior of the delta explains why the mark-to-market of investment certificates do not behave as implied by their final payoff diagrams before expiration. Indeed, at maturity, the delta of an option can be either 1 (or −1 for puts) if the option ends up in-the-money or zero (for both calls and puts) if it is out-of-the-money. And therefore, at maturity the final payoff diagrams of investment certificates is made of straight lines with sharp angles, whilst before maturity we see instead smooth lines. The risk-free interest rate and dividends play a minor role in determining the value of delta, even if they are needed to compute the forward price of the underlying asset.[27] The value of delta is also considered as the probability that the option ends in the money ($N(d_1)$ in the Black & Scholes model).

The second sensitivity coefficient linked to the underlying price is *gamma*. It measures how delta changes for a given change in the price of the underlying. It is important to compute it because there exists a risk factor defined of second order and determined by the presence of a non-linear relationship between the price of the option and that of the underlying and hence non-perfectly detectable by delta that assumes instead a linearity between these two values. We say that gamma increases the knowledge of delta, avoiding a static view of the latter. It is computed as the second derivative of the option price with respect to price of the underlying and it is positive for an option buyer.[28]

Volatility is a statistical indicator measuring the variability of the returns of the underlying with respect to their average value. This factor constitutes one of the indicators employed to measure the risk of an investment. We can measure volatility ex-post, computing the standard deviation of the daily returns recorded in the past (historical

[27] The formula for the forward price in case the underlying pays a continuous dividend yield is the following: $F = Se^{(r-q)\tau}$.

[28] As a measure of risk, gamma is influenced by the variation of the other parameters as well, in particular by the time to maturity and volatility. For at the money options, gamma sharply increases with the passing of time because the delta becomes discontinuous close to maturity and jumps from 0 to 1 (call) just because of a small movement in the underlying price; if the option is out the money or in the money instead gamma decreases progressively with the passing of time, converging rapidly to 0. If the option is at the money, gamma decreases if volatility increases and increases up to infinite if volatility decreases, differently from what happens for out and in the money options.

volatility), or we can measure that expected by the market in a determined time horizon by finding the value that makes the theoretical price (obtained through an option pricing model) equal to the market price of an option. Empirically, it has been shown that volatility is not constant over time, even though, as stated before, in the Black & Scholes option pricing model it is assumed to be constant. The price of an option is linked in a positive way to volatility, in the sense that an increase in the latter determines an increase in the value of the option. Vega measures the sensitivity of the value of the option to changes in implied volatility. In analytical terms it is computed as the first derivative of the option price with respect to implied volatility and as stated before, it has a positive value for plain vanilla options. Therefore, if an equity investor does not see volatility as a positive concept, one who buys plain vanilla options sees volatility has one of his/her best allies. Vega is maximum for at-the-money options and it is positively correlated to the maturity of the option, i.e. the longer the maturity the greater the variation of the option value following a change in implied volatility.

For an investment in certificates, without caps and barriers, an increase in implied volatility determines an increase in the value of the selected product, because, in this case, the investor has bought volatility. The presence of barriers in the structure of the certificate modifies what has been just said: the possible volatility increase produces a higher probability of hitting the barriers and hence of losing the capital protection foreseen at its issue, if the barriers are set below the strike price. Generally, an increase in the volatility of the underlying generates a reduction in the value of the certificate for products such as Bonus, Airbag, Express, Cash Collect, and Twin Win. However, the departure from the barriers and the approximation of the maturity of the certificate contribute to decrease this type of behavior. Vice versa, a reduction in volatility determines an increase in the market value of these certificates.

Even if all the inputs of the pricing method remain constant from a moment of time to another, the price of an option changes. Theta expresses the effect of the passing of time on the option value. It is positive for an option seller and negative for an options buyer, because the passing of time, *ceteris paribus*, reduces the probability associated to a change in the underlying and volatility. The configuration of theta for a certificate can be very different depending on the type of contract. The problem lies in the fact that the passing of time can affect: time value only (e.g. options on Bund Future), time value and intrinsic value (e.g. options on non-dividend paying stock). In the first case the effect is more symmetric. The passing of time can generate, depending on the product and the underlying level, a reduction or an increase in the value of the certificate. The presence of barriers and caps allow to be positively exposed to time decay.

The risk-free interest rate (usually identified with the Euribor/Swap rate on the maturity of the option) is used to compute the forward price of the underlying asset. An increase in the interest rate, *ceteris paribus,* tends to increase the forward price and hence the value of a call option, whilst decreasing the value of a put option. Therefore, *rho*, that measure the sensitivity of the option price to changes in interest rates, is positive for call options and negative for put options. However, the variations of interest rates affect the value of certificates not always in an intuitive way and to a limited extent compared to what happens for volatility. What is more, if the certificate has the quanto feature, that is if it offers protection to the exchange rate risk, the combined dynamic of the interest rates of the Euro Zone and of that in which the underlying is denominated must be considered to compute the value of the certificate. [29]

The stream of dividends always constitutes a source of income relevant for equity investors. The exposure to this asset class in an indirect way, i.e. through derivatives, deprives of these streams the investor. Besides, dividends distribution causes a (theoretical) reduction in the price of the underlying stock/index on the ex-dividend date and this event naturally affects the price of the options: a higher dividend implies a reduction of the call option price and an increase of the put option.

For basket options, the implied correlation between the underlying risky assets must be considered as it influences the value of those options.[30] The linear relationship characterizing the movements of the returns of the various underlying asset included in the basket is important. If two assets have a similar price behavior their correlation will be close to 1, while if their prices diverge in a symmetric way the correlation will be equal to -1. This variable is not stable over time and changes with the implied volatility. A higher correlation can determine an increase in the value of multi-asset certificates with the "worst-of" feature and on the other hand a lower correlation decreases their value. In

[29] The term "quanto" derives from "quantity adjusted option". A quanto option is an option in which the underlying asset is denominated in one currency (e.g. US Dollar) but the derivative itself is settled in another currency (e.g Euro) at a predefined exchange rate.
[30] Options whose payoff depends on the value of a portfolio, that is a basket of assets.

these cases, the value in the implied correlation of two assets is transformed into higher coupons or bigger discounts.[31]

In the following paragraphs the behavior of the mark-to-market of an equity protection and a bonus certificate during their lifetime is presented. The examples have been obtained using the closed form equations of Black & Scholes for plain vanilla options and the formula proposed by Haug (1999) for the down-and-out put option.

4.4 Equity Protection

As stated before, thanks to the put-call parity, the payoff of an equity protection at maturity can be obtained either with a long zero-strike call option and a long put option or with a zero-coupon bond and a long call option. For illustration purposes I analyze an Equity Protection with 100% participation and 100% capital protection, using as underlying the EuroStoxx 50 Index. The implied volatility and the dividend yield of the former are obtained from Bloomberg (13% and 4% respectively) while the risk-free rate is assumed to be 2%. Figure 9 plots the mark-to-market of the certificate as a function of time to maturity and price of the underlying asset. There are five lines, each line corresponds to a different period during the time of the certificate: at issue (green line), with 70% time remaining (violet line), with 50% time remaining (yellow line), with 25% time remaining (red line) and maturity (blue line).

It can be observed that the delta for the certificate is positive, implying a positive relationship between the price of the underlying and that of the certificate. However, the behavior of the mark-to-market of the equity protection over time depends on the moneyness of the product. When the product is out-of-the-money, the call is practically worthless, and the certificate tends to its bond floor. In this area, the passing of time has a positive impact on the certificate's price (theta is positive). Around the at-the-money point we observe a similar behavior at the beginning of the product's life because the value lost for the call is smaller than the interest gained on the zero-coupon bond. However, this behavior changes after a point in time, i.e. theta becomes negative and the payoff lines move back towards the blue line. This can be explained by the fact that from that moment onward the time value of the call decreases at a faster rate than the interest gained on the bond.[32]

[31] These conclusions are inversely valid for "best-of" options.

[32] Please note that this is true for the great majority of certificates where the interest rate level of the zero-coupon bond is medium to high but not for extremely low levels (e.g JPY bonds).

An Equity Protection is long volatility. This means that an increase in implied volatility increases the value of the certificate. Figure 10 is obtained by shifting volatility from 13% to 21%. Compared to figure 9, certificate's value is increased, and the lines are now steeper. Besides, it is worth mentioning that now the index can fall by over 10% before the certificate's value declines below the strike. This is relevant for investors because usually the implied volatility spikes in times of crisis, i.e. when prices fall: in such a case, the decrease of the underlying asset price is partly offset by a gain in the implied volatility of the option.

The interest rate level influences both the price of the embedded zero-coupon bond and that of the call. However, the impact on the option price is smaller than that it can have on the bond value. In general, when interest rates rise, bond prices fall and vice versa. Plus, zero-coupon bonds have the greatest sensitivity to changes in interest rates (duration coincides with remaining time to maturity).[33] Considering that usually the zero-coupon bond has a weight of 80% or more on the product's value, any change in interest rate has a great impact on the certificate. In figure 11 the effects of an increase in interest rates can be observed: by shifting the interest rate level from 2% to 4%, the whole certificate's value has dropped significantly. Whilst the bond price decreases linearly in all points of the graph (horizontal shift of the bond component), the effect on the call value depends on its moneyness. Indeed, when the call is far out-of-the-money the impact of a change in interest rates on the equity protection is greatest, while when the call is deep in-the-money, an interest rate shift is less effective. Eventually, when there is a change in expected dividends, the call option is negatively affected. Hence, the overall effect on the certificate value is negative.

4.5 Bonus

The Bonus certificate considered is constructed with a 10% bonus and an 80% barrier, while all the other inputs coincides with those of the equity protection described above. These data are used for the payoff diagrams explained in what follows. The variables with a higher influence on the Bonus are the price of the underlying, implied volatility and dividends. It is important to note that the Bonus is the certificate with the greatest difference between the mark-to-market value and its payoff at maturity: at the issue the

[33] The duration of a bond is the weighted average maturity of a bond's cash flows. Since a zero-coupon bond pays only one cash-flow at maturity, its duration always equals its maturity.

delta of the product is almost 1. Indeed, in Figure 12 the mark-to-market of the certificate at the issue date (green line) is almost a 45° straight line. This means that the down-and-out put option, providing the bonus and the protection to the certificate, gives a small contribution to the price development of the certificate at inception. However, it can be seen in Figure 12 that as times goes by the intermediate lines move closer to the blue line, representing the final payoff at maturity, that is the certificate starts behaving like the final payoff line only after most of its lifetime has passed. This behavior can also be explained in a different way. At the issue date, variations in the price of the underlying do not have a significant effect on the down-and-out option value. Indeed, when the underlying asset's price increases, the down-and-out put loses in intrinsic value but at the same time it is less probable that the barrier will be touched. On the other hand, when the underlying value decreases, the put intrinsic value increases but the underlying is closer to the barrier, and the chances that the it will be touched are higher. This behavior is better exemplified in Figure 14, that shows the delta of a down-and-out put option as a function of the underlying price. Although we are dealing with a put option, close to the barrier the delta is positive because if the price of the underlying asset increases it is true that we are less in-the-money, but it is also true that we are moving away from the barrier, decreasing the chances that the option will be exercised worthless. The higher the price of the underlying, the lower the influence of the barrier on the price of the option and delta turns negative as with plain vanilla put options.

Besides, it is worth noting the behavior of the delta of the certificate around the strike. In Figure 12, all the payoff lines become steeper around the knock-out level (80% of spot) and the delta assumes values greater than 100% as it is clearly verifiable in Figure 15. Therefore, for a given change in the underlying price the certificate value increases or declines more than proportionally: there exists a leverage effect that depends on the remaining time to maturity and the distance between the barrier and the underlying value (the payoff lines tend to a vertical asymptote the lower those two values become).

The impact of a change in implied volatility is not easy to analyze because of the presence of the down-and-out put option. We need to consider many variables, but generally, the longer the maturity, the greater the impact of the implied volatility on the certificate's value. Usually an increase in implied volatility corresponds to a reduction for the mark-to-market of the certificate, but this is not always true as it will be explained in what follows. Figure 13 has been obtained by increasing volatility from 13% to 21%. As it can be seen, the payoff lines are less sinuous, i.e. their curvatures are less pronounced. Between the barrier and the strike all payoff lines are lower because the probability of violating the barrier is higher, in other words, the vega of the certificate is negative around the barrier. Indeed, in this area vega represents a great risk because an increase in implied volatility can determine a violation of the barrier. Above the strike of the certificate instead the payoff lines of one-year and 0.5 year remaining to maturity seem to be higher, because considering the time left to maturity and the distance of the underlying price to the barrier the probability of a knock-out decreases and the model assumes that the barrier will not be violated, Hence, an increase in volatility can only have positive effects on the value of the certificate. Eventually, below the barrier the situation is easier because once the barrier has been touched the down-and-out option ceases to exist and the certificate is not affected anymore by volatility (only the zero-strike call remains), transforming itself in a delta 1 product.

Dividends are essential because in their absence the structuring of some certificates would not be possible, bonus included. Dividend amount and payment date have a strong effect on the price of the zero-strike call: an increase in the dividend yield decreases the forward price and the zero-strike call becomes cheaper, allowing to invest more money on the purchase of the put option. A more expensive put with a lower barrier can be bought. In the extreme case, when the dividend yield is so high (19% in our example) that the barrier can be set at zero, the bonus becomes an equity protection. After the issue, an increase in dividends has a negative impact on the certificate value.

To conclude, it is important to note that the two examples above are two of many possibilities to structure an equity protection and a bonus certificate. I decided to illustrate two “classical” examples that have a similar structure to those most seen in the market place. Therefore, what stated above may not hold for certificates structured in a different way and with different features.

5. A Comparison with other Financial Assets

After having analyzed all the main characteristics of Investment Certificates, I proceed with a useful comparison with other financial securities typically selected in the implementation of a portfolio by investors or asset managers. The selected assets are: bonds, stocks, ETFs, and mutual funds. The purpose of this analysis is to highlight the main advantages and drawbacks of Investment Certificates with respect to the other financial assets considered. To perform this analysis, I use the following four drivers:

- Diversification
- Management fees
- Transaction costs
- Taxation

5.1 Certificates, Stocks and Bonds

The issuer of stocks and bonds establishes with the underwriter a relationship that enables the investor to receive an interest agreed by contract (bonds) or dividends in case of profits (stocks).

Stocks or bonds are financial asset issued by a company to raise funds. They qualify themselves as an investment alternative by nature non-diversifiable: to invest in a single stock or bond means on one side to be exposed to the volatility of the stock or interest rate, without the benefit of compensatory mechanisms, and on the other to concentrate the exposure risk to negative events related to the issuer. A person investing exclusively in these instruments will have to build a portfolio with many different assets, each representing a small percentage of the whole portfolio, and with a low correlation level. Besides, if many instruments are issued by the same entity, an investor needs to avoid concentrating his/her investments in those assets.

Investment Certificates share with stocks and bonds this source of risk, i.e. issuer risk. On the other side, the possibility for certificates to link their performance to an index or a basket of financial assets makes them a more efficient alternative in terms of diversification.

Management costs are not an issue for a stock or bond holder (since there is no need of a professional asset manager to manage those kind of investments), whilst with respect to transaction costs, a lot depends on two factors: costs charged by the intermediary and the bid-ask spread. The first factor, usually does not differentiate financial assets issued by a firm and certificates, because the fees charged to trade these securities are similar, being around 0.2% and 0.5%. The second factor instead depends on the type of instrument that is being traded. The most liquid securities (for instance the stocks of companies with a large market capitalization or government bonds) have a negligible spread, which is higher for corporate bonds, especially for high yield.[34] The bid-ask spread is not fixed in time: it depends on the liquidity conditions of the market (liquidity not always granted by market makers or liquidity providers), and usually it tends to elapse during financial turmoil. In general, however, the spread of a stock or bond is on average lower than that empirically observed for Investment Certificates (Candia, 2017), expect for high yield bonds.

By looking at the taxation, earnings generated by stocks or bonds can be of two types: periodical stream of cash flows (dividends and coupons) or capital gains and losses. In Italy, as stated before, the periodical streams of cash flows are fiscally treated as "*redditi di capitale*", whilst the potential capital gain or loss are considered instead "*redditi diversi*", which can be used for netting purposes. Hence, although a portion of the earnings associated with stocks and bonds can be labeled as "*redditi diversi*", these instruments are less efficient than Investment Certificates, since the latter generate "*redditi diversi*" only.

5.2 Certificates and Mutual Funds

Mutual funds are investment vehicles established to manage the savings of investors. They provide liquidity and diversification for investors, who know that their money is managed by a professional investor. A fund can be associated with passive or active management. A fund's manager with an active style of management attempts to beat the reference market. To assess the ability of the manager, his/her performance is usually compared with a benchmark. However, there exists funds without benchmark, with "flexible" investment strategies, that try to achieve positive performances in every possible market condition.

Mutual funds provide a high degree of diversification as they give the possibility to buy a basket of securities with only one instrument. What is more, they are not exposed to the issuer risk, differently from Investment Certificates. In Italy there exists a separation between the assets of the fund and those of the management company: if the latter goes bankrupt, the capital invested by the fund would not be touched.

An investment in a fund is subject to management fees (measured by the TER).[35] Those fees can range from an average 0.5% of money market funds, to 2.5% for equity

[34] High yield bonds are bonds issued by entities which are deemed to be risky, with a low rating and high coupons.
[35] The Total Expense Ratio (TER), is a synthetic indicator of costs, and it is computed as the sum of management and administrative costs, divided by the net asset value of the fund.

funds, with some exceptions above this threshold. Performance fees must be added, if foreseen by the contract, to the management fees. The former are fees recognized to the portfolio manager in case of positive excess returns of the fund with respect to a predefined benchmark. Given that there are no standard criteria to establish these fees, it can prove useful to know in advance the amount and the application criteria. Eventually, since investment funds are underwritten directly through an authorized intermediary, there are no direct transaction costs. However, in certain categories of funds (also known as load) entry fees are charged by the management company.

From a fiscal point of view, investment funds are basically inefficient. Indeed, the profits generated by a fund are considered as "*redditi da capitale",* both if they are periodical payments and if they generate from the redemption at a price above the purchase price. In case of losses (sale at a price below the purchase price) instead, they are treated as "*redditi diversi".* An investor holding a fund-only portfolio would not be able to compensate the possible losses with gains of the same amount.

5.3 Certificates and ETFs

Exchange Traded Funds (ETFs), have gained in importance in recent years for their peculiar characteristic of being a mutual investment fund or SICAV freely exchangeable on the market, combining the possibility to make a diversified investment in a single instrument, with the comfortability of acquiring or selling its shares in every moment.

An ETF shares with a traditional mutual fund the legal form (both are structured as mutual fund, or SICAV), but differently from the latter, it has the peculiarity of being traded on a regulated market. Another difference between the two alternatives is the management philosophy: mutual funds are actively managed, with the aim of beating a reference market, whilst ETFs are mostly passively managed, that is they passively mimic the return of a predetermined benchmark.[36]

ETFs replicate a series of equity, fixed income or commodities indexes, sometimes ad hoc built by specialized companies (index providers). To replicate the index different procedures have emerged over time: the most relevant are physical and synthetic replication.

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[36] A debate on active versus passive investing has existed since the inception of these investment structures. For a detailed analysis see Grima, Hili, Pace, 2016.

Physical replication consists in the purchase of the underlying assets of the index that the manager wants to track. There are two methods: full or stratified sampling. In the first case 100% of the assets included in the index are purchased, in the same proportion as their weighting on the index being replicated. In the second method instead, a sample of securities is selected whose behavior is as similar as the one of the index as a whole. Synthetic replication is obtained using derivatives contracts. In particular, the ETF manager enters a swap contract with an investment bank. The latter agrees to pay the index return in exchange for a small fee, plus any returns on collateral held in the portfolio of the ETF.

The physical replication is the one that allows to benefit completely from the diversification effects, as it happens for traditional mutual funds, because the investor purchase with just one trade the ownership of each of the underlying assets of the tracked index. With synthetic replication there is the counterparty risk. Indeed, in the ETF case there is the risk that the counterparty of the swap contract would not be able to fulfill its obligation of payment with respect to the ETF manager. Therefore, some adjustments are foreseen to mitigate this risk, even though whoever invests in a synthetic replication ETF must consider this negative event.

ETFs have relatively low management costs, above all by virtue of the passive management, enabling the ETF manager to minimize the costs associated with the market analysis and the trades. The annual fee of the most common ETFs, tracking the most known indexes, is around 0.20% whilst the most expensive ETFs, usually linked to indexes built ad hoc, almost never exceed 1.1%. For the ETF, as well as for stocks and bonds by virtue of their trading on the market, there exists implicit and explicit trading costs. The first, i.e. the ones applied directly by the financial intermediary are in general similar to what has been already seen for stocks and bonds (many intermediaries associate ETFs to stocks for fees purposes). Instead, the bid-ask spread depends on the category of the ETF: the most common have a minimum spread of 0.20% or 0.30%, whilst less traded ETF can reach spread of 1% or even greater ones.

Considering all the financial instruments traded on the market, ETFs have a substantial fiscal inefficiency, inefficiency shared with the traditional mutual funds. Even ETFs, in fact, generate *"redditi diversi"* only in case of losses, whilst they generate exclusively "*redditi da capitale*" both in case of payment of periodical proceeds, and in case of capital gains.[37]

5.4 A Synoptic Framework

At the end of this chapter I briefly summarize the key aspects of all the financial instruments described to highlight in a synthetic and immediate way the strengths and weaknesses of each of them.

With respect to diversification, mutual funds ensure the greatest guarantees, as on one side they enable the purchase of a diversified basket of securities and on the other the investor is not exposed to the counterparty risk, thanks to the balance sheet separation imposed to the management company. The same guarantees are offered by ETFs, but only by those with physical replication. Synthetic replication ETFs, instead, while guaranteeing the same results of a diversified basket, expose an investor to counterparty risk. Investment Certificates lies in between, because while they can have as underlying an index, linking their returns to that of a diversified basket, there exists a counterparty risk related to the issuer of the certificate. Stocks and bonds instead do not provide any type of diversification.

However, if we look at management fees, different considerations must be made. Generally, Investment Certificates, single stocks, and bonds are not associated with any type of management fee. ETFs instead have relatively low management fees (no more than 1%), while for mutual funds, as already said, higher fees are charged by the management company.

For transaction costs it is more difficult to make a classification which can hold as a rule of thumb. Certainly, no load mutual funds have an advantage as they do not have any entry and exit fees even though they have on average higher management fees than load funds. For the remaining assets I define an average total cost. Trading fees applied by an intermediary are almost the same for all the traded financial products, whilst the bid-ask

[37] The legislative decree n.44/2014 has finally equated ETFs to traditional mutual funds. Before the decree, earnings generated by ETFs were divided into Delta Nav (the difference between the Nav in the acquisition day and the Nav in the day of the sale), considered "*redditi da capitale*", and differential on the Delta Nav (obtained from the difference between acquisition price and sale price, minus the Delta Nav), considered "*redditi diversi*". There was hence a residual possibility to determine nettable capital gains. The stated decree delated this eventuality. From April 9th, 2014 the difference between the two prices is considered only (independently from the Nav of the relevant days) and, in case of positive difference this is considered "*redditi da capitale*".

spread can be wider or tighter depending on whether we consider a certificate, ETF, stock or bond. If we assume an average cost of 0.30% for a transaction, depending on the spread of the various instruments, the total costs of an operation on the market, whether it is for buying or selling a security, can be approximately 0.35% for a stock, 0.50% for a liquid bond, up to 0.80% or 1.00% for a high yield bond; for a widely traded ETF this can be around 0.40%, but also in this case it can raise up to 0.80% or 1.00% for the less liquid products; for Investment Certificates, the total costs would be around 0.80%. Eventually, load mutual funds have on average entry fees around 1.00% but differently from the securities traded on the market, they do not have exit fees.

Last, but not least, the taxation. By looking at this aspect, Investment Certificates are the most efficient among all products considered in the analysis. They are followed by stocks and bonds, whilst mutual funds and ETFs are the worst performers.

All the considerations made are summarized in table 4, that compares all these financial products with respect to diversification, management fees, transaction costs, and taxation.

6. Concluding Remarks

The purpose of our thesis was to investigate the world of Investment Certificates. They are a large family of financial products. Since they were introduced in Italy in 1999 (2002 for leveraged certificate), certificates have grown significantly, and they represent now a relevant, and increasing, share of the holding of investors. Indeed, these financial products are characterized by a high flexibility: new asset(s) can be selected as underlying and new investment strategies can be realized. In this way, issuers are put in the perfect condition to propose to investors new innovative products for their portfolio optimization. Besides, for an investor who autonomously manages his/her savings, the market of Investment Certificates offers multiple financial solutions designed to satisfy every investment need.

Starting from the categorization realized by EUSIPA (European Structured Investment Products Association), this thesis analyzes the main types of certificates, describing their features and how they can be structured as a combination of different plain vanilla and exotic options. Then, we proceed with a brief description of the most used option pricing models and we examine the main risk factors affecting the value of an option and therefore of an Investment Certificate to get to the analysis of the behavior of these products during their lifetime. Finally, we conclude with a comparison of Investment Certificates with other asset classes.

There is much more to know and learn about this fascinating world. There are many more structures, where small variations can make big differences. As stated above, the segment of modern Investment Certificates has been gaining in popularity and is still going through a continuous development. Our analysis offers the possibility to understand the structure of these complicated investment instruments. The methodology adopted in this thesis can be used as an inspiration for an analysis of other structured products. Besides, further studies are needed to compare the analyzed Investment Certificates with other structured products existent on the market.

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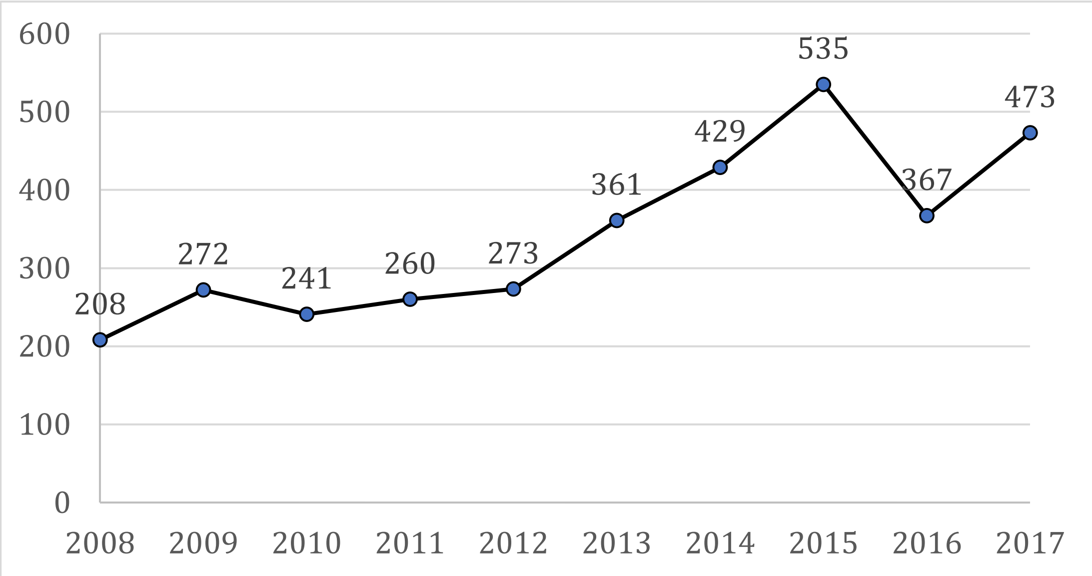
Figure 1
The evolution of the number of issued certificates in Italy

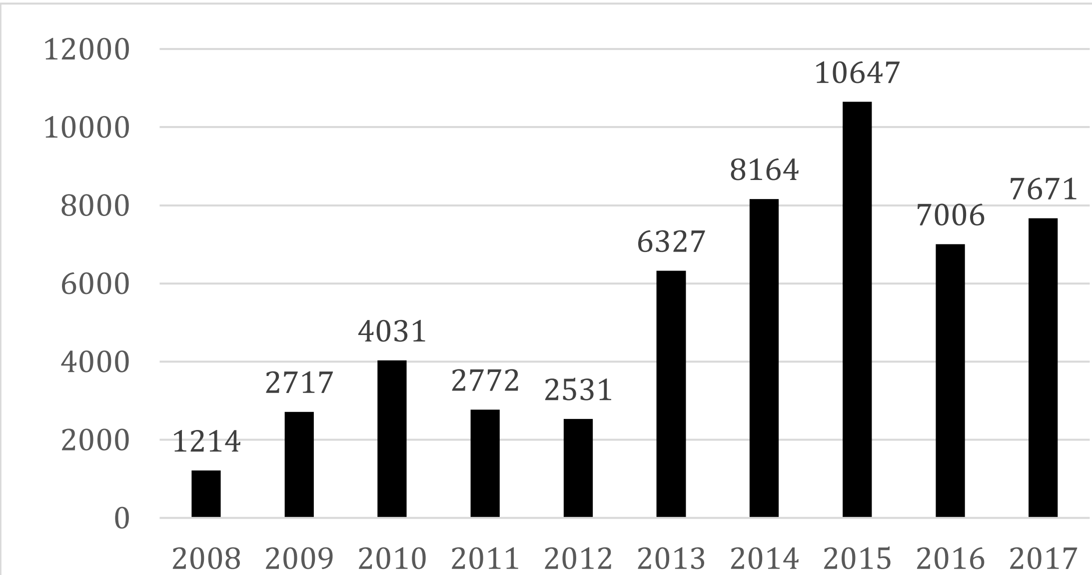
The chart reports the number of new certificates issued by financial intermediaries in the last ten years in Italy.



Source: Acepi (2018)

Figure 2
The evolution of turnover (€ m) in Italy

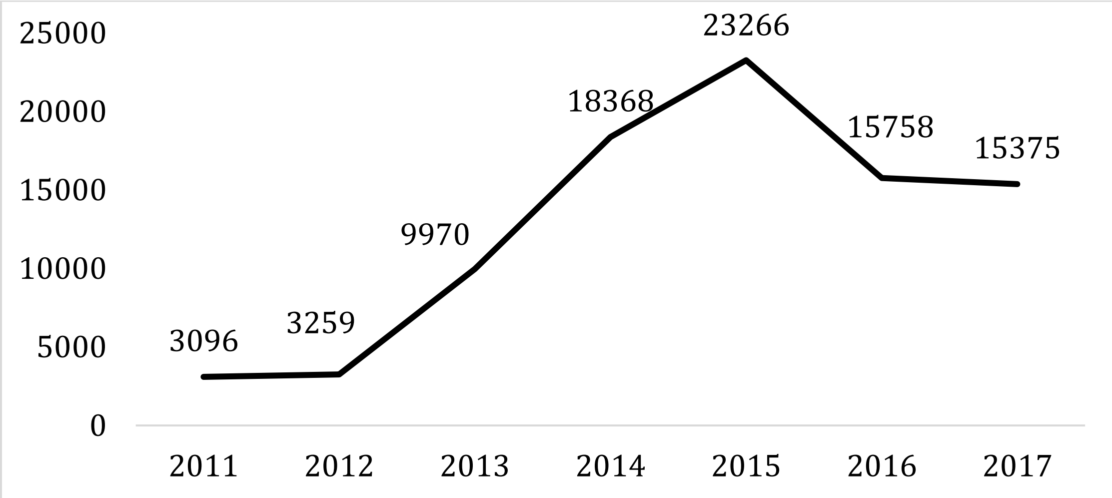
The chart shows the euro value (in million) of the issued in Italy between 2008 and 2017.



Source: Acepi (2018)

Figure 3
The evolution of certificates turnover (€ m) traded on the SeDeX

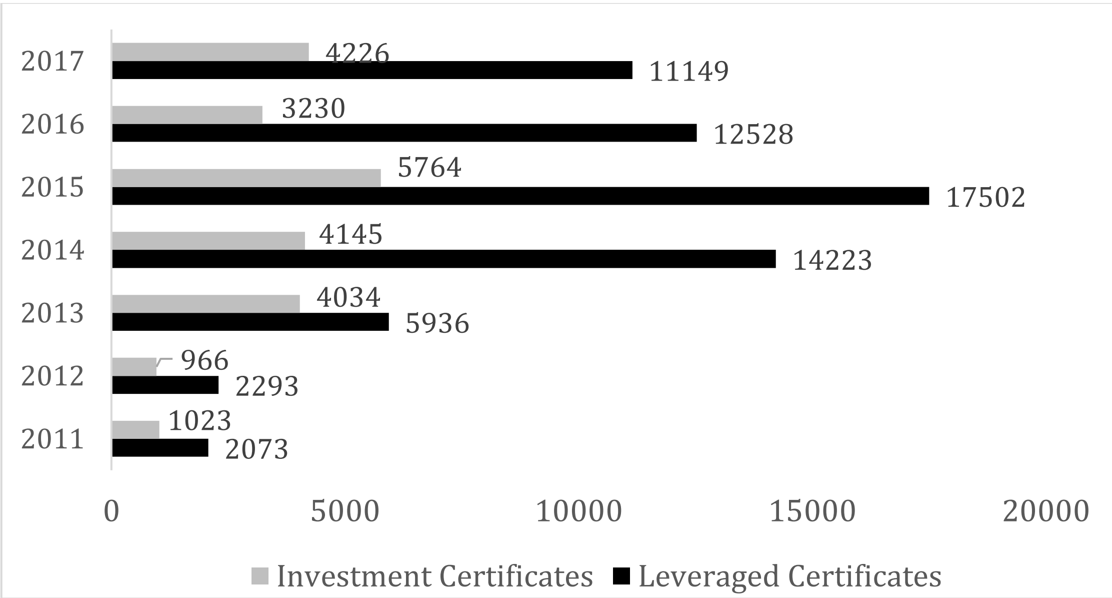
The figure shows the euro value (in million) of the traded volumes of certificates on the SeDeX between 2011 and 2017.



Source: Borsa Italiana SpA

Figure 4
The decomposition between Leveraged and Investment Certificates

The chart plots the number of listed certificates on the SeDeX divided for Investment and Leveraged Certificates between 2011 and 2017.



Source: Borsa Italiana SpA

Figure 5
Classical Certificates representation (Bluemke 2009)

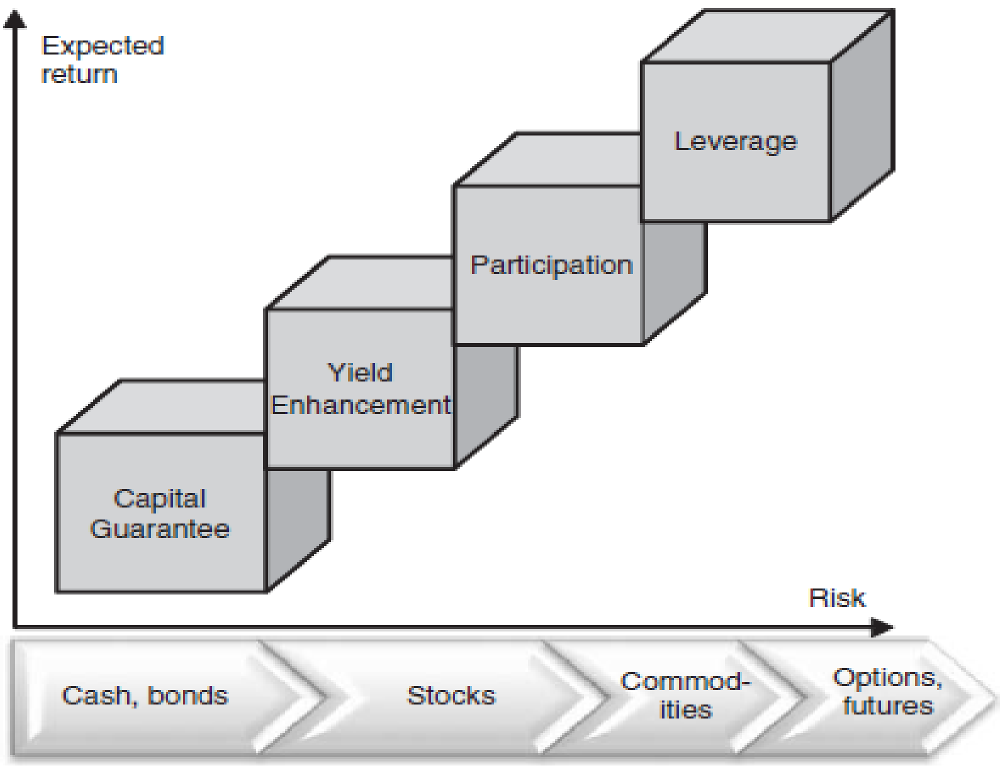


Figure 6
The Greeks Letters (Chris 1997)

Greek letter	Symbol	Meaning
Delta	Δ	The rate of change of the value of an option with respect to changes in the stock price.
Gamma	Γ	The rate of change of the delta with respect to changes in the stock price.
Theta	Θ	The rate of change of the value of an option with respect to time.
Rho	ρ	The rate of change of the value of an option with respect to the risk-free rate of interest.
Vega	υ	The rate of change of the value of an option with respect to volatility.

Figure 7
Delta as a function of time and strike

The figure shows the delta of an in-the-money call (assuming a 100% strike and asset's price of 110%) and the delta of an out-of-the-money call (assuming 100% strike and asset's price of 90%) as a function of time to maturity.

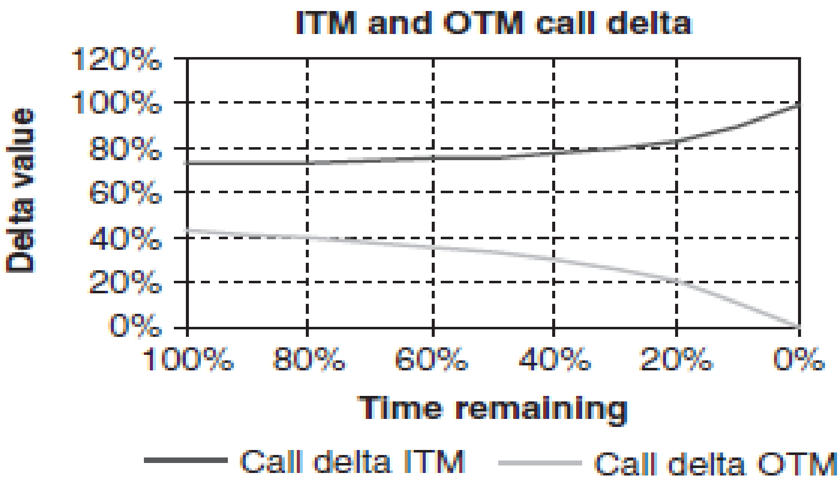


Figure 8
Delta of an option

The chart plots the delta of a call and put option on the Eurostoxx50 Index as a function of its price and with a maturity of one year.

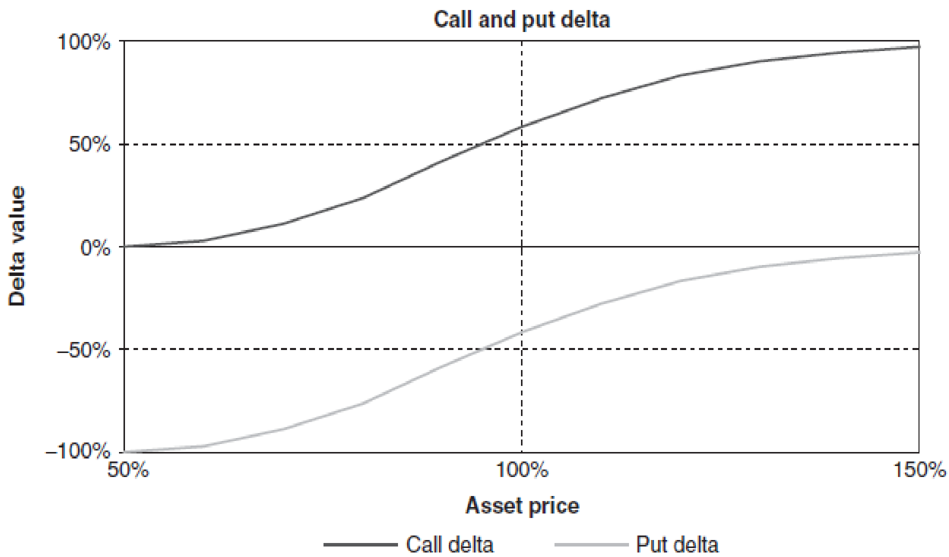


Figure 9

Payoff 1: Equity protection price as a function of spot price

The figure shows the mark-to-market of the certificate as a function of time to maturity and price of the underlying asset. There are five lines, each line corresponds to a different period during the time of the certificate: at issue (green line), with 70% time remaining (violet line), with 50% time remaining (yellow line), with 25% time remaining (red line) and maturity (blue line).

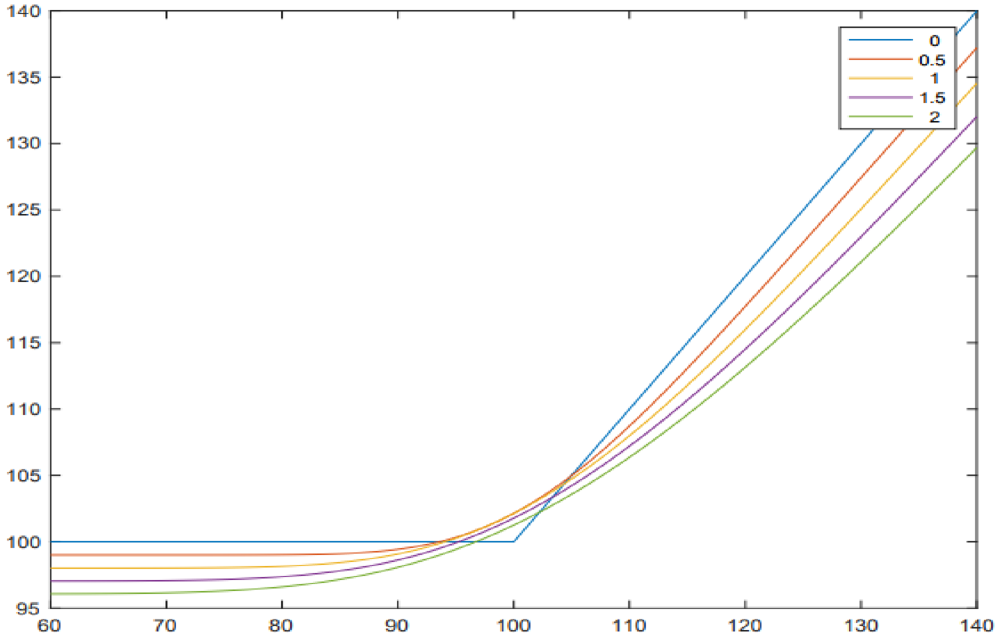


Figure 10

Payoff 2: Implied Volatility increase

The figure shows the impact of a volatility increase on the price of the equity protection. It is obtained by shifting volatility from 13% to 21%. Compared to figure 8, certificate's value is increased, and the lines are now steeper.

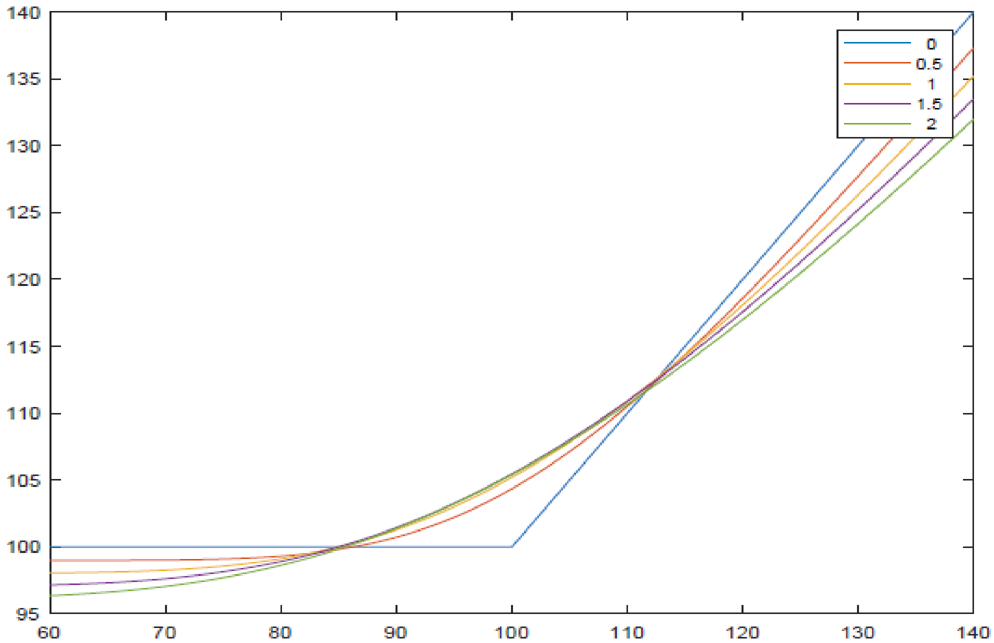


Figure 11
Payoff 3: Interest rate increase

The figure shows the effects of an increase in interest rates on the mark-to-market of an equity protection and it is obtained by shifting the interest rate level from 2% to 4%. Compared to Figure 9, it is clear that the certificate's value has dropped.

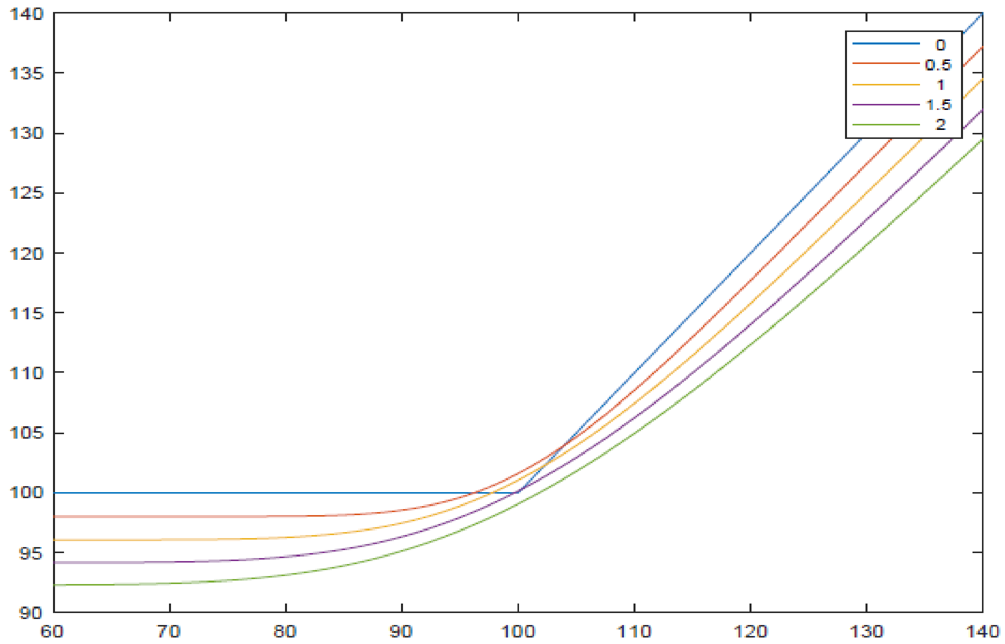


Figure 12
Payoff 4: Bonus price as a function of spot price

The figure shows the price of the Bonus as a function of time to maturity and price of the underlying asset. There are five lines, each line corresponds to a different period during the time of the certificate: at issue (green line), with 70% time remaining (violet line), with 50% time remaining (yellow line), with 25% time remaining (red line) and maturity (blue line).

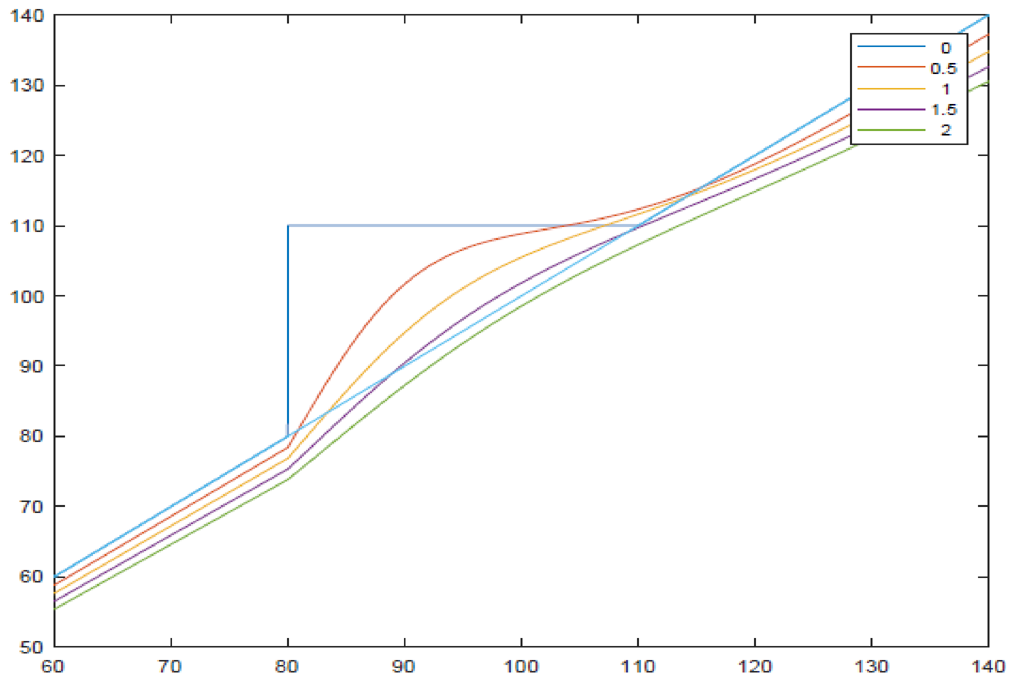


Figure 13
Payoff 5: Implied volatility increase

The figure shows the impact of a volatility increase on the price of the equity protection. It is obtained by increasing volatility from 13% to 21%.

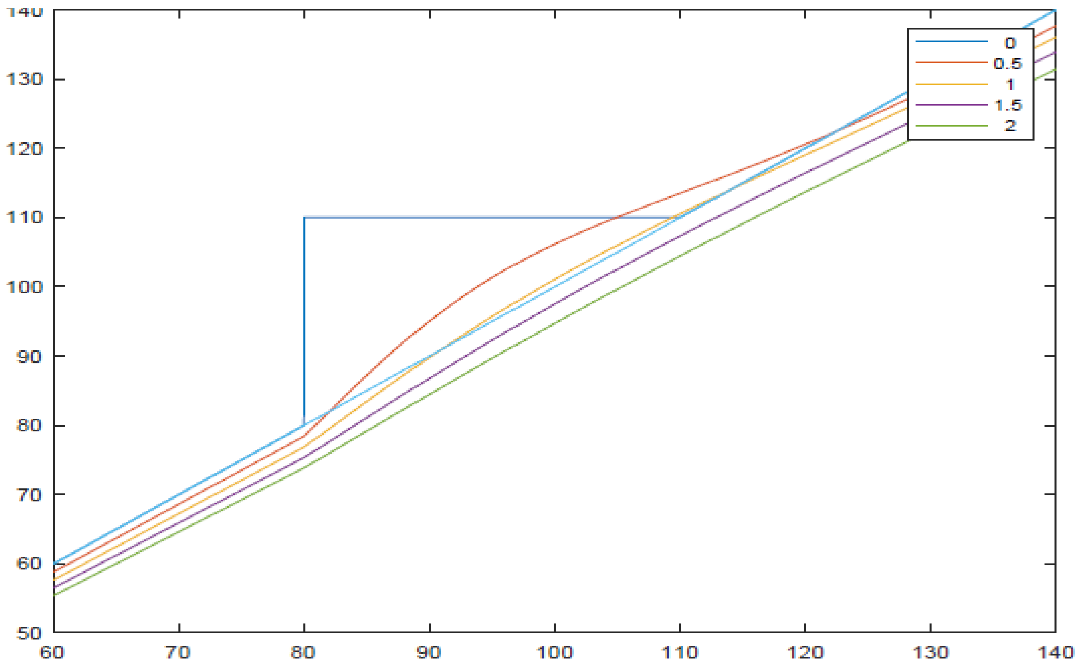


Figure 14
Delta of a down-and-out put

The figure shows the delta of a down-and-out put option as a function of the underlying price and for different time to maturity. Close to the barrier the delta is positive while the higher the price of the underlying, the lower the influence of the barrier and delta turns negative as with a plain vanilla put option.

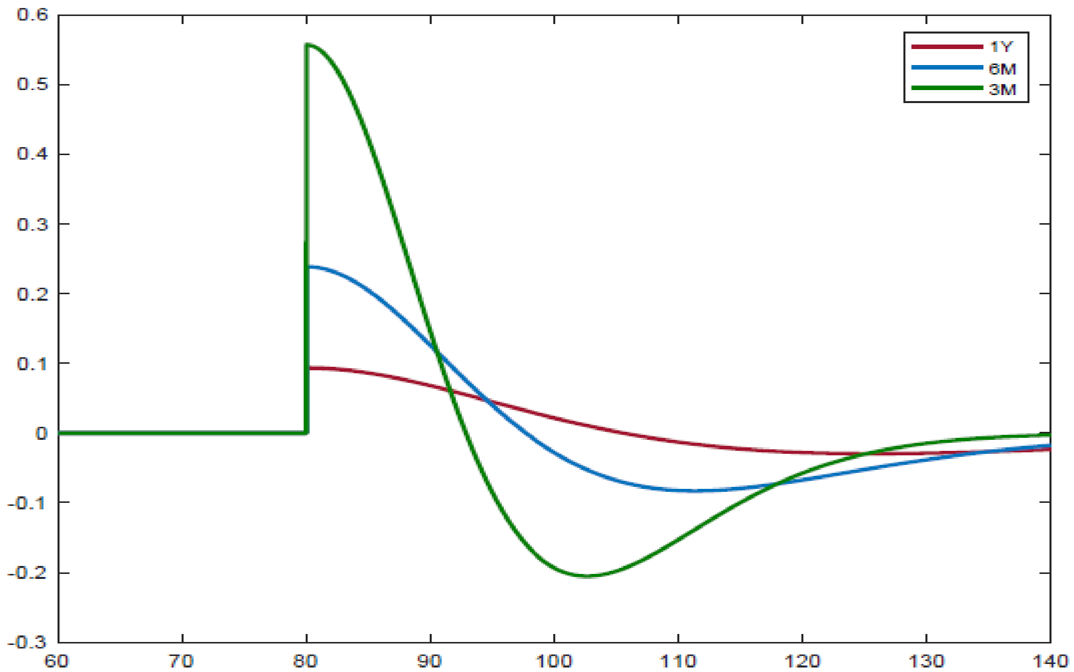


Figure 15
Delta of the Bonus Certificate

The figure shows the delta of a Bonus Certificate. Below the barrier, the Bonus becomes a delta 1 product. Just above the barrier, delta is large and positive- if price moves slightly higher the probability of knock-out is reduced. At much higher price levels, the barrier's impact is lighter and delta is lower than one because the delta of the put is slightly negative while that of the call is one.

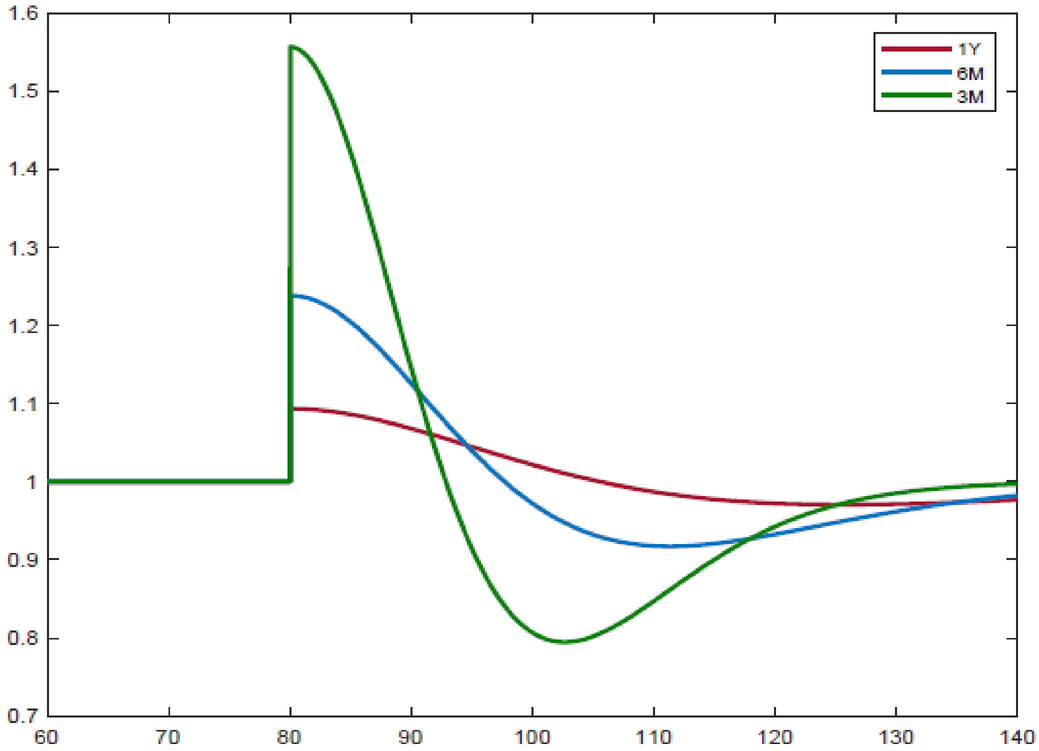


Table 1

Fundamental relationship derived from the put-call parity

Position	*Relationship*
Long Stock (S) =	$\boldsymbol{C - P + Xe^{-rT}}$
Long Zero-coupon bond ($X\boldsymbol{e}^{-rT}$) =	$\boldsymbol{S + P - C}$
Long call option (C) =	$\boldsymbol{S + P - Xe^{-rT}}$
Long put option(P) =	$\boldsymbol{C - S + Xe^{-rT}}$

Table 2

Variables influencing the Equity Protection

		Variables		
Variables' tendency	**Spot Price**	**Implied Volatility**	**Interest rate**	**Dividend Yield**
Up	↑	↑	↓	↓
Down	↓	↓	↑	↑
Impact	Maximum	Medium-high	Low	Low

Table 3

Variables influencing the Bonus

	Variables			
Variables' tendency	**Spot Price**	**Implied Volatility**	**Dividend Yield**	**Interest rate**
Up	⬆	Depends	⬇	⬇
Down	⬇	Depends	⬆	⬆
Impact	Maximum	Depends-high	Medium	Low

Table 4

A comparison of financial instruments with respect to diversification, management fees, transaction costs, and taxation.
Legend = "+" efficient; "++" = very efficient; "=" = neutral; "-" = inefficient; "/" = not pertinent

	Diversification	Management fees	Transaction Costs	Taxation
Stock or Bond	-	/	+	=
No load mutual fund	++	-	++	-
Load mutual fund	++	=	=	-
ETF	+	+	+	-
Certificate	+	/	=	+