INTRO FINANCIAL ECONOMETRICS - (20185)

Entry Test - September 2017

Family Name (Surname)	First Name	Student Number (Matr)	Average in the Exams

Please answer all the questions by choosing the most appropriate alternative(s). There might be more than one correct answer(s) for each question. Each selected alternative which is a correct answer will be awarded one point. Wrong answers will be penalized with minus one point. Correct answers not selected will receive zero points. Only answers explicitly reported in the appropriate box will be considered.

Question 1

Investors A and B bought at the end of the year 1975 for 1000 dollars the same asset that gave a constant returns of 5 per cent per year over the period 1976-2015. A reinvested in each year the interest earned , while B did not. State which of the following is correct

a. the capital of investor A in 2015 is worth 10 per cent more than the capital of investor B

b. the capital of investor A and B in 1976 was 1050 dollars

c. the capital of investor A in 1977 was 1102,5 dollars, while that of investor B was 1100,

d. the capital of investor A in 2015 was worth 1.5 the capital of investor B

e. given that we do not know the data on inflation we cannot tell if real value of the capital of investor A was higher or not than that of investor B in 2015

f. the capital of investor A in 2015 is worth twice the capital of investor B

g. the capital of investor A is worth 7040 in 2015, while that of investor B is 3000

Answers
b,c,g

the Capital of A is worth 7040, the Capital of B is worth 3000 in 2015, it is the same only after the first year of investment.

Question 2

Consider the CAPM estimated for the asset i:

$$r_t^i = \underset{(0.006)}{0.08} + \underset{(0.039)}{0.612} r_t^m + \hat{u}_t$$

Where r_t^m are the returns in excess of the risk free at time t for a proxy of the Market Portfolio and r_t^i are the returns in excess for the risk free for asset i.

Indicate which of the following statements is correct:

a. CAPM assumptions are confirmed by empirical results

b. the volatility of asset i is smaller than the one of the Market Portfolio

c. you would be willing to add asset i to a well diversified portfolio

d. you would be willing to buy asset i anyway

e. the correlation between the returns of asset i and the ones of the market portfolio is positive

Answers
e

Question 3

Consider the Markowitz mean-variance model for asset allocation, in the presence of a risk free asset:

a) every efficient frontier portfolio can be generated as a linear combination of efficient portfolios

b) the correlation between returns of efficient portfolios is one

c) two efficient portfolios may have two different sharpe ratios

d) in general, it is impossible to identify the efficient frontier if there is not a risk free asset in the market

Answers
b

"efficient portfolio" refers to a portfolio lying on the CML, as opposed to those lying on a Markowitz efficient frontier.

Question 4
State which of the following is correct

a) In the Mean Variance framework weights of different assets do not depend on risk aversion if the portfolio is entirely allocated to risky assets

b) in the Mean-Variance framework all agents will hold the same portfolio, the so called world portfolio

c) the efficient frontier is always upward sloping

d) the CAPM is not consistent with the mean-variance framework

e) the points on the efficient frontier can be indifferently derived by finding the weigths that minimize the variance of portfolio for a target level of return, or by finding the weigths that maximize returns for a target level of the variance.

Answers
a,c,e

Denote with $\mathbf{r}$ the random vector of linear total returns from time t to time T from a given menu of N risky assets for interval $[t, T]$, $\mathbf{r} \sim \mathcal{D}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.[1] The investor can also invest at time t in a security the price of which at T is known at t (typically, a non-defaultable Aaa bond), called risk-free security. Let r^f be the discretely compounded non-random return from this investment over every single period. Short sales are admitted without any constraints. For simplicity, we ignore transaction costs and any other frictions. Given a degree of risk aversion λ, a standard *mean-variance* description of this allocation problem is the following:

$$\max_{\mathbf{w}} (1 - \mathbf{w}'\mathbf{e})\, r^f + \mathbf{w}'\boldsymbol{\mu} - \frac{1}{2}\lambda(\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w})$$

where $E[\mathbf{r}] = (1 - \mathbf{w}'\mathbf{e})\, r^f + \mathbf{w}'\boldsymbol{\mu} = r^f + \mathbf{w}'(\boldsymbol{\mu} - r^f\mathbf{e})$ and $Var[\mathbf{r}] = \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$. The solution of this problems determines the portfolio weights in terms of the preferences of the investor, as captured by the parameter λ, and the (known) mean and the covariance matrix describing the joint distribution of returns. Because this is an unconstrained convex problem, the first-order conditions (FOCs) are necessary and sufficient and define the following system of N linear equations in N unknowns, the portfolio weights $\mathbf{w} \in \mathcal{R}^N$:

$$(\boldsymbol{\mu} - r^f\mathbf{e}) - \lambda\boldsymbol{\Sigma}\mathbf{w} = \mathbf{0}.$$

Solving the FOCs yields:

$$\hat{\mathbf{w}} = \frac{1}{\lambda}\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu} - r^f\mathbf{e}\right), \tag{1}$$

[1] Notice that in $\mathcal{D}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\mathcal{D}$ is not necessarily multivariate normal.

where $\left(\boldsymbol{\mu}-r^f\mathbf{e}\right)$ defines the vector of risk premia for the N risky assets. (1) defines the solution to a standard mean-variance portfolio program and it is one of the most crucial and commonly used results in all of financial economics. Of course, in order to make this approach to portfolio allocation operational, knowledge of λ needs to be paired with estimates (better, forecasts of future values) of $\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$ (or $\boldsymbol{\mu}-r^f\mathbf{e}$, when more convenient or appropriate).

Consider now the special case in which $\hat{\mathbf{w}}'\mathbf{e} = 1$, that is no investment in the riskfree bond is allowed. The optimal portfolio in this case is the famous *tangency portfolio*:

$$\mathbf{e}'\hat{\mathbf{w}} = \frac{1}{\lambda}\mathbf{e}'\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu}-r^f\mathbf{e}\right) = 1 \Longrightarrow \lambda = \mathbf{e}'\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu}-r^f\mathbf{e}\right)$$

so that (1) becomes in this case

$$\hat{\mathbf{w}}^T = \frac{\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu}-r^f\mathbf{e}\right)}{\mathbf{e}'\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu}-r^f\mathbf{e}\right)}, \tag{2}$$

where the T in $\hat{\mathbf{w}}^T$ stands for tangency.

Question 5

The risk-free rate is 6%, the expected return on the market portfolio is 14%, and the standard deviation of the return on the market portfolio is 25%. Consider a portfolio P with expected return of 16% and assume that it is an efficient portfolio lying on the CML. State which of the following is correct

a) The Beta of this portfolio is 1.25

b) The standard deviation on the return of the portfolio is 31.25%

c) The correlation between the return on the market portfolio and the return of the portfolio P is 1

d) the weight of the risk free asset in the portfolio is negative

e) the weight of risky assets in the portfolio is negative

Answers
a,b,c,d

Question 6

Suppose that in year t you invest € 1000 in 5-year zero coupon bonds with annualized yield to maturity of 10%. In year t+1 rates increase to 15%. Which is the value of your investment in year t+1?

a. 1100

b. 1010

c. 1150

d. none of the above

e. 900

Answers
d

Question 7

Consider a Utility Function $u(W)$ (W stands for wealth, and $W > 0$) such that $u'(W) > 0$ and $u''(W) < 0$.

a. the shape of the utility function entails decreasing absolute risk aversion

b. the agent represented by this Utility Function is risk averse

c. a Utility Function compatible with the conditions stated above is $u(W) = \log(W)$

d. a Utility Function compatible with the conditions stated above is $u(W) = W^2$

Answers
b,c

Question 8

Consider a variance-covariance matrix Σ:

a. the matrix has to be semi-definite positive to be a valid variance-covariance matrix

b. the variance-covariance matrix can be written as $\Sigma = D\Gamma D'$ only under very restrictive conditions

c. consider the decomposition $\Sigma = D\Gamma D'$, and assume that Γ is the correlation matrix; if this is the case, D is a diagonal matrix with variances on its main diagonal

d. the correlation matrix Γ might not be symmetric

Answers
a

Question 9

Consider two identically and independently distributed variables x and y. Given that z= x-y , state which of the following is true:

a. z is a normally distributed random variable

b. $\sigma_z^2 = \sigma_y^2 + \sigma_x^2$

c. $\sigma_z^2 = \sigma_y^2 - \sigma_x^2$

d. $\sigma_z^2 = \sigma_y^2 + \sigma_x^2 - 2\sigma_{xy}$

e. $\sigma_z^2 = \sigma_y^2 + \sigma_x^2 + 2\sigma_{xy}$

Answers
b,d

Question 10

Type I and Type II errors are important concepts in statistical hypothesis testing. State which of the following is true:

a. The probability of Type I error is the probabability of rejecting an hypothesis when it is true.

b. The probability of Type II error is the probability of not rejecting an hypothesis when it is false

c. Type I error occurs when the test it is not normally distributed

d. Type II error occurs when the test it is not normally distributed

e. The probability of Type I error is also called the size of the test

f. The probability of Type II error is also called the power of the test

Answers
a,b,e

Question 11

A common blood test indicates the presence of a disease 95% of the time when the disease is actually present in an individual. Joe's doctor draws some of Joe's blood, and performs the test on his drawn blood. The results indicate that the disease is present in Joe. Here's the information that Joe's doctor knows about the disease and the diagnostic blood test:

One-percent (that is, 1 in 100) people have the disease. That is, if D is the event that a randomly selected individual has the disease, then P(D) = 0.01.If H is the event that a randomly selected individual is disease-free, that is, healthy, then P(H) = 1-P(D) = 0.99.

The sensitivity of the test is 0.95. If a person has the disease, then the probability that the diagnostic blood test comes back positive is 0.95. That is, P(T+ | D) = 0.95.

The specificity of the test is 0.95. That is, if a person is free of the disease, then the probability that the diagnostic test comes back negative is 0.95. That is, P(T- | H) = 0.95. That is, P(T+ | H) = 0.05.

- State which of the following is correct:

  a. The probability that a randomly selected individual tests positively for the disease is 0.059

  b. The probability that,given that the blood test is positive for the disease, Joe actually has the disease is 0,161

  c. The probability that, given that the blood test is positive for the disease, Joe actually has the disease is 0,95

  d. The probability that a randomly selected individual tests positively for the disease is 0.05

Answers
a,b

P(D)=0.01, P(H)=1-P(D)=0.99,P(T+ | D) = 0.95,P(T- | H) = 0.95,P(T+ | H) = 0.05.

- P(T+)=P(T+ | D) P(D)+P(T+ | H) P(H)=0.95*0.01+0.05*0.99=0.059
- P(D | T+) $=\frac{P(T+|D)P(D)}{P(T+)}$ = 0.161

Question 12

A desk lamp produced by The Luminar Company was found to be defective (D). There are three factories (A, B, C) where such desk lamps are manufactured. A Quality Control Manager (QCM) is responsible for investigating the source of found defects. This is what the QCM knows about the company's desk lamp production and the possible source of defects:

Factory	% of total production	Probability of defective lamps
A	0.35 = P(A)	0.015 = P(D | A)
B	0.35 = P(B)	0.010 = P(D | B)
C	0.30 = P(C)	0.020 = P(D | C)

Please state which of the following is correct:

a. in a random sample of 10 lamps from the Luminar Company less than 2 will be defective

b. in a random sample of 1000 lamps from the Luminar Company less than 20 will be defective

c. If a randomly selected lamp is defective, the probability that the lamp was manufactured in factory C is 0.5.

d. If a randomly selected lamp is defective, the probability that the lamp was manufactured in factory B is 0.237.

e. If a randomly selected lamp is defective, the probability that the lamp was manufactured in factory A is 0.356.

Answers
b,e,d

Question 13

Statistical Research Group (SRG), a statistial support unit to the US army during WWII, were faced with a problem

to prevent planes from being shot down by enemy fighters your armor them.

but armour makes the plane heavier ...

armoring the planes too much is a problem and armoring them too little is a problem

which is the optimum level of armoring ?

When American planes came back from engagements over Europe, they were covered with bullet holes. Here are the data

Section of the Plane	Bullet Holes per sq. f.
Engine	1.11
Fuselage	1.73
Fuel System	1.55
Rest of the plane	1.8

So in the light of the data where do you put the armoring ?

a. Engine

b. Fuselage

c. Fuel System

d. Rest of the plane

Answers
a