Testing the CAPM

Carlo Favero

Testing the CAPM

- The time-series performance of the CAPM is questioned by the significance of the alpha coefficients. (consider for example the case of the 25 FF protfolios)
- We also know that other factors beside the market are significant in explaining excess returns
- Fama-Mac Beth propose to exploit cross sectional information to produce a different test of the CAPM

Fama-MacBeth(1973)

- Consider the 25 portfolios and run for each of them the CAPM regression over the sample 1962:1 2014:6. These regressions deliver 25 betas.

$$\left(r_t^i - r_t^f\right) = \alpha_i + \beta_i \left(r_t^m - r_t^f\right) + u_t^i$$

- Take now a second-step regression in which the cross-section of the average (over the sample 1962:1-2014:6) monthly returns on the 25 portfolios are projected on the 25 betas:

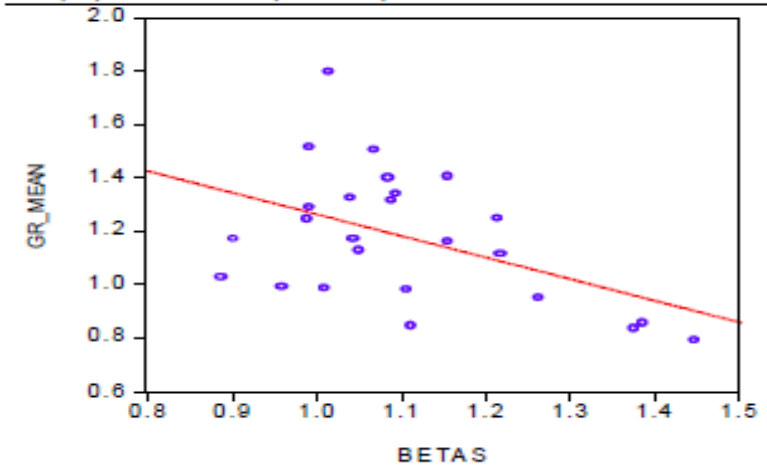
$$r_i = \gamma_0 + \gamma_1 \beta_i + u_i$$

Under the null of the CAPM i) residuals should be randomly distributed around the regression line, ii) $\gamma_0 = E\left(r^f\right), \gamma_1 = E\left(r^m - r^f\right)$

Fama-MacBeth(1973)

TABLE 3.4: CAPM in the cross-section of 25 portfolios

Dependent Variable r_i $(i = 1, ...25)$					
Variable	Coefficient	Std. Error	t-ratio	HAC Std.Err.	Prob.
C	2.07	0.35	5.94	0.451	0.3069
β_i	-0.80	0.31	-2.57	0.364	0.0169
R^2 0.22	S.E. of reg 0.22	S.E. dep.var 0.25	1926:7-2014:6		
$E\left(r^f\right) = 0.40$, $E\left(r^m - r^f\right) = 0.89$					



Fama-MacBeth(1973)

- The cross-sectional regression strongly rejects the CAPM.
- Note however that this regression is affected by an inference problem caused by the correlation of residuals in the cross-section regression.
- Fama and MacBeth (1973) address this problem by estimating month-by-month cross-section regressions of monthly returns on the betas obtained on the full sample.
- The time series means of the monthly slopes and intercepts, along with the standard errors of the means, are then used to run the appropriate tests

Fama-MacBeth(1973)

- The application of the Fama-MacBeth on the sample 1962:1 2014:6 delivers the following results

TABLE 3.5: Statistics on the distribution of coefficients from Fama-MacBeth

	γ_0	γ_1
Mean	2.07	-0.807
St.Dev	9.56	10.06
Obs	630	630
t-stat	5.437	-2.01

An alternative route would be to construct Heteroscedasticity and Autocorrelation Consistent(HAC) estimators.

A Multifactor model

- The testing procedure could be easily extended to multifactor models. In which the first stage regression in general can be written as

$$\left(r_t^i - r_t^f\right) = \alpha_i + \beta_{1,i}\left(r_t^m - r_t^f\right) + \sum_{j=1}^{n}\beta_{j,i}F_{j,t} + u_t^i$$

- and the second stage regression:

$$r_i = \gamma_0 + \gamma_1\beta_{1,i} + \sum_{j=2}^{n}\gamma_j\beta_{j,i} + u_i$$

Under the null of the CAPM i) residuals should be randomly distributed around the regression line, ii)
$\gamma_0 = E\left(r^f\right), \gamma_1 = E\left(r^m - r^f\right), \gamma_j = E\left(F_j\right)$