CLASS V: Horizontal Mergers, Entry, Foreclosure
SOLUTIONS

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Exercise 1

(a) According to the anti-trust agency's estimations, the profit of firm i ($i = 1, 2, 3$) is given by

$$\pi_i = (150 - Q) \cdot q_i - 50q_i - 100$$

Under Cournot competition, firm i's maximization program is

$$\max_{q_i} \left[150 - q_i - \sum_{j \neq i} q_j \right] \cdot q_i - 50q_i - 100$$

The first-order condition for q_i is

$$-q_i + 150 - q_i - \sum_{j \neq i} q_j - 50 = 0$$

from which we get

$$q_i (q_j) \equiv R_i (q_j) = \frac{1}{2} \left[100 - \sum_{j \neq i} q_j \right]$$

Symmetry implies

$$q_i = \frac{1}{2} (100 - 2q_i)$$

Hence, the quantity produced by each firm is

$$q_i^C = \frac{100}{4} = 25$$

Total output is

$$Q^C = 3q_i = 75$$

Finally, the market price is equal to

$$P^C = 150 - Q^C = 150 - 75 = 75$$

Total profit of each firm is

$$\pi_i^C = P^C \cdot q_i^C - cq_i - f = 75 \cdot 25 - 50 \cdot 25 - 100 = 525$$

(b) After the merger we have only two firms in the market, i.e. firm 3 and the new entity resulting from the merger between firm 1 and firm 2, which is labelled as firm M.

Assuming that the merger does not entail efficiency gains, firm M's maximization program is

$$\max_{q_M} (150 - q_M - q_3) \cdot q_M - 50q_M - 200$$

The first-order condition for q_M is

$$-q_M + 150 - q_M - q_3 - 50 = 0$$

from which we get

$$q_M (q_3) \equiv R_M (q_3) = \frac{1}{2} (100 - q_3)$$

Firm 3's maximization program is the same as in the previous point (substituting $\sum_{j \neq i} q_j$ with q_M), so its best response function is

$$q_3 (q_M) \equiv R_3 (q_M) = \frac{1}{2} (100 - q_M)$$

To solve this problem, we could exploit symmetry, because firm M and firm 3 show the same marginal costs (which are the only costs that affect their maximization programs). Anyway, since they differ in their fixed costs, we solve the system of their best response functions and prove that even in this case symmetry holds.

Therefore, we have to solve the following system

$$\begin{cases} q_M = \frac{1}{2} (100 - q_3) \\ q_3 = \frac{1}{2} (100 - q_M) \end{cases}$$

Substituting the second equation into the first one yields

$$q_M = \frac{1}{2} \left[100 - \frac{1}{2} (100 - q_M) \right] = \frac{1}{4} (100 + q_M)$$

Hence, the quantity produced by the new entity is

$$q_M^{C'} = \frac{100}{3} = 33.\overline{3}$$

The quantity produced by firm 3 is

$$q_3^{C'} = \frac{1}{2}\left[100 - \frac{100}{3}\right] = \frac{200}{6} = \frac{100}{3}$$

Notice that actually the two firms produce the same output in equilibrium since they show the same marginal cost, even if their profit will be different because they bear different fixed costs.

Total output is given by

$$Q^{C'} = q_M^{C'} + q_3^{C'} = \frac{100}{3} + \frac{100}{3} = \frac{200}{3} = 66.\bar{6} < Q^C = 75$$

Market price is

$$P^{C'} = 150 - Q^{C'} = 150 - \frac{200}{3} = \frac{250}{3} = 83.\bar{3} > P^C = 75$$

(c) Firm M's profits are given by

$$\pi_M^{C'} = P^{C'} \cdot q_M^{C'} - 50q_M^{C'} - 200 = \frac{250}{3} \cdot \frac{100}{3} - 50 \cdot \frac{100}{3} - 200 =$$

$$= \frac{1}{9}\left[25,000 - 15,000 - 1,800\right] = \frac{8,200}{9} = 911.\bar{1} < \pi_1^C + \pi_2^C = 2 \cdot 525 = 1050$$

It appears that the merger is not profitable for the firms involved. In absence of efficiency gains, firm M's strategy to reduce its output in order to enjoy higher profit margins leads to the paradoxical result that the only firm which is better off is the one not involved in the merger, i.e. firm 3, which can expand its production and whose profit increases from $\pi_3^C = 525$ to

$$\pi_3^{C'} = P^{C'} \cdot q_3^{C'} - 50 \cdot q_3^{C'} - 100 = \frac{1}{9}\left(25,000 - 15,000 - 900\right) = \frac{9100}{9} = 1,011.\bar{1}$$

(d) Synergies created by the merger imply that now firm M's marginal cost is given by

$$c_M = 50 - 0.4 \cdot 50 = 30$$

Firm M's maximization program is now given by

$$\max_{q_M} \left(150 - q_M - q_3\right) \cdot q_M - 30q_M - 200$$

The first-order condition for q_M is

$$-q_M + 150 - q_M - q_3 - 30 = 0$$

from which we get

$$q_M\left(q_3\right) \equiv R_M^{'}\left(q_3\right) = \frac{1}{2}\left(120 - q_3\right)$$

Since the best response function of firm 3 is unchanged, we have to solve the following system

$$\begin{cases} q_M = \frac{1}{2}(120 - q_3) \\ q_3 = \frac{1}{2}(100 - q_M) \end{cases}$$

Substituting the second equation into the first one yields

$$q_M = \frac{1}{2}\left[120 - \frac{1}{2}(100 - q_M)\right] = \frac{1}{4}(140 + q_M)$$

Hence, the output produced by the new entity is

$$q_M^{C''} = \frac{140}{3} = 46.\overline{6}$$

while the quantity supplied by firm 3 amounts to

$$q_3^{C''} = \frac{1}{2}\left[100 - \frac{140}{3}\right] = \frac{160}{6} = \frac{80}{3} = 26.\overline{6}$$

Total output is

$$Q^{C''} = q_M^{C''} + q_3^{C''} = \frac{140}{3} + \frac{80}{3} = \frac{220}{3} = 73.\overline{3} < Q^C = 75$$

Market price is

$$P^{C''} = 150 - Q^{C''} = 150 - \frac{220}{3} = \frac{230}{3} = 76.\overline{6} > P^C = 75$$

We can conclude that the merging firms' claim is not correct since the merger yields higher prices and then a lower consumers' surplus.

(e) Firm 3's profits are

$$\pi_3^{C''} = P^{C''} \cdot q_3^{C''} - 50 \cdot q_3^{C''} - 100 = \frac{230}{3} \cdot \frac{80}{3} - 50 \cdot \frac{80}{3} - 100 =$$

$$= \frac{1}{9}(18,400 - 12,000 - 900) = \frac{5500}{9} = 611.\overline{1}$$

It appears that the merger benefits firm 3, whose profit increases from $\pi_3^C = 525$ to $\pi_3^{C'} = 1.011.\overline{1}$ in absence of effiency gains and $\pi_3^{C''} = 611.\overline{1}$ when there are synergies. As stressed above, the rationale is that the firm can expand its production, given the two merging firms' strategy of reducing their output in order to have higher profit margins. Notice that, when there are efficiency gains, the entity resulting from the merger limits less its production (since it has become more efficient) and this leads firm 3 to expand less its production. Consequently, firm 3's profits increase less than in absence of synergies.

(f) Efficiency gains now imply

$$c_M^{'} = 50 - 0.80 \cdot 50 = 10$$

Firm M's maximization program is

$$\max_{q_M} (150 - q_M - q_3) \cdot q_M - 10q_M - 200$$

The first-order condition for q_M is

$$-q_M + 150 - q_M - q_3 - 10 = 0$$

from which we get

$$q_M (q_3) \equiv R_M^{'''} (q_3) = \frac{1}{2} (140 - q_3)$$

Since firm 3's best response function is unchanged, we have to solve the following system

$$\begin{cases} q_M = \frac{1}{2} (140 - q_3) \\ q_3 = \frac{1}{2} (100 - q_M) \end{cases}$$

Substituting the second equation into the first one yields

$$q_M = \frac{1}{2} \left[140 - \frac{1}{2} (100 - q_M) \right] = \frac{1}{4} [180 + q_M]$$

Hence, the output produced by the new entity is

$$q_M^{C^{'''}} = \frac{180}{3} = 60$$

The quantity supplied by firm 3 is equal to

$$q_3^{C^{'''}} = \frac{1}{2} (100 - 60) = 20$$

Total output is

$$Q^{C^{'''}} = q_M^{C^{'''}} + q_3^{C^{'''}} = 60 + 20 = 80 > Q^C = 75$$

Market price is given by

$$P^{C^{'''}} = 150 - Q^{C^{'''}} = 150 - 80 = 70 < P^C = 75$$

Now, the merging firms' claim is correct and the anti-trust agency should approve the merger, since market price is lower. Hence, only efficiency gains from the merger which are large enough benefit consumers.

Firm 3's profits are

$$\pi_3^{C^{'''}} = P^{C^{'''}} \cdot q_3^{C^{'''}} - 50 \cdot q_3^{C^{'''}} - 100 = 70 \cdot 20 - 50 \cdot 20 - 100 =$$

$$= 1,400 - 1,000 - 100 = 300 < \pi_3^C = 525$$

In this case, firm 3's profit is lower after merger, since the firm is much more inefficient than the firm resulting from the merger.

(g) In this case, efficiency gains imply

$$f_M = 0.6 \cdot 2f = 120$$

Since the fixed cost does not affect each firm's maximization program, equilibrium quantities and price will be unchanged, so consumers will not benefit from this kind of efficiency gains. Only the two merging firms will be better off since they bear lower costs.

Exercise 2

At the second stage, firm i's $(i = 1, ...n)$ maximization problem is

$$\max_{q_i} \pi_i = q_i \cdot p = q_i \left[a - q_i - \sum_{j \neq i} q_j \right]$$

The first order condition for q_i is

$$a - q_i - \sum_{j \neq i} q_j - q_i = 0$$

from which we get

$$q_i (q_j) \equiv R_i (q_j) = \frac{1}{2} \left[a - \sum_{j \neq i} q_j \right]$$

Symmetry implies

$$q_i = \frac{1}{2} \left[a - (n-1) q_i \right]$$

Hence firm i produces

$$q_i^C (n) = \frac{a}{n+1}$$

and total quantity is

$$q^C (n) \equiv n q_i^C = \frac{an}{n+1}$$

The equilibrium price is

$$p^C (n) = a - q^C (n) = a - \frac{an}{n+1} = \frac{a}{n+1}$$

Finally, firm i's profit (gross of fixed costs) is

$$\pi_i^C(n) = p^C(n) \cdot q_i^C(n) = \frac{a}{n+1} \cdot \frac{a}{n+1} = \frac{a^2}{(n+1)^2}$$

(II) Imposing the zero-profit condition (which must hold in the long run free entry equilibrium) entails

$$\pi_i^C - F = 0$$

which becomes

$$\frac{a^2}{(n+1)^2} - F = 0 \Leftrightarrow \sqrt{F}(n+1) = a$$

The number of firms under free entry equilibrium is

$$n^* = \frac{a}{\sqrt{F}} - 1$$

It is immediate to see that the higher the market size a the higher n^*; an increase in the fixed cost F reduces n^*.

Exercise 3

(I) Let us move by backward induction. At stage 2, firm E decides whether to enter or not. If it does not enter, $\pi_E^{NE} = 0$. If it enters, firm E's maximization program is

$$\max_{q_E} \pi_E = q_E [1 - q_I - q_E] - F^2$$

The first order condition for q_E is

$$1 - q_I - q_E - q_E = 0$$

from which we get

$$q_E(q_I) \equiv R_E(q_I) = \frac{1 - q_I}{2}$$

At stage 1, firm I must decide whether to accomodate entry or deter it.

If accomodation occurs, firm I competes à la Stackelberg with firm E and solves the following maximization problem

$$\max_{q_I} \pi_I = q_I [1 - q_I - q_E(q_I)] = q_I \left[1 - q_I - \frac{1 - q_I}{2}\right]$$

The first order condition for q_I is

$$1 - q_I - \frac{1 - q_I}{2} + q_I \left(-1 + \frac{1}{2}\right) = 0$$

from which we get

$$q_I^A = \frac{1}{2}$$

and

$$q_E^E = \frac{1}{2}\left[1 - q_I^A\right] = \frac{1}{4}$$

Firm I's profits are

$$\pi_I^A = q_I^A\left[1 - q_I^A - q_E^E\right] = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$$

while firm E's profits are

$$\pi_E^E = q_E^E\left[1 - q_I^A - q_E^E\right] = \frac{1}{4}\cdot\frac{1}{4} - F^2 = \frac{1}{16} - F^2.$$

(II) If firm I deters entry, it must produce a quantity q_I^C such that the rival makes zero profits. In other terms, $q_E(q_I) = \frac{1-q_I}{2}$ must be such that

$$q_E(q_I)\left[1 - q_I - q_E(q_I)\right] - F^2 = 0$$

which becomes

$$\frac{1-q_I}{2}\left[1 - q_I - \frac{1-q_I}{2}\right] - F^2 = 0$$

Finally, we get

$$(1-q_I)^2 - 4F^2 = 0$$

which yields

$$q_I^D(F) = 1 - 2F.$$

(III) Firm I prefers to accomodate if and only if $\pi_I^A \geq \pi_I^D$. In other words,

$$q_I^A\left[1 - q_I^A - q_E^E\right] \geq q_I^D\left[1 - q_I^D - q_E^D\right]$$

where $q_E^D = 0$, since firm E cannot produce when entry is deterred.
Hence, we get

$$\frac{1}{8} \geq (1-2F)\left[1 - (1-2F) - 0\right]$$

which can be rewritten as

$$4F^2 - 2F + \frac{1}{8} \geq 0$$

The roots of the characteristic equation are $F_1 = \frac{2-\sqrt{2}}{8} \approx 0.07$ and $F_2 = \frac{2+\sqrt{2}}{8} \approx 0.43$. Since $F \in \left[0, \frac{1}{4}\right]$, we conclude that if $F \in \left[0, \frac{2-\sqrt{2}}{8}\right]$, entry is accomodated. When $F \in \left(\frac{2-\sqrt{2}}{8}, \frac{1}{4}\right]$ entry is deterred.

This vindicates Bain's result that the equilibrium for F high enough is one of deterred entry.

(IV) Let us move backward. At the second stage, firm $i = 1, 2$ competing à la Cournot has the following maximization program

$$\max_{q_i} q_i \left[1 - q_i - q_j\right] - F^2, \text{ for } q_j \text{ given}$$

The first order condition is

$$1 - q_i - q_j - q_i = 0$$

which implies

$$q_i\left(q_j\right) \equiv R_i\left(q_j\right) = \frac{1 - q_j}{2}$$

Simmetry implies

$$q_i\left(q_j\right) = \frac{1 - q_i}{2}$$

which yields

$$q_i^C = \frac{1}{3}$$

Firm i's profits are

$$\pi_i^C = q_i^C \left[1 - 2q_i^C\right] - F^2 = \frac{1}{3} \cdot \frac{1}{3} - F^2 = \frac{1}{9} - F^2.$$

At the first stage each firm will enter if and only if

$$\frac{1}{9} - F^2 \geq 0$$

which implies

$$F^2 \leq \frac{1}{9}$$

Hence, if $F^2 < \frac{1}{16} < \frac{1}{9}$ both firms will participate in the market.