CLASS VI: Stackelberg, Bundling, Exclusive Dealing
SOLUTIONS

November 16, 2009

Exercise 1

(I) We solve the Stackelberg game by backward induction. At stage 2, the follower's maximization problem is

$$\max_{q_F} q_F \left[1 - q_L - q_F\right]$$

The first-order condition for q_F is

$$1 - q_L - q_F - q_F = 0$$

from which we get

$$q_F\left(q_L\right) = \frac{1}{2}\left[1 - q_L\right]$$

At the first stage, the leader's maximization problem is

$$\max_{q_L} q_L \left[1 - q_L - \frac{1}{2}\left(1 - q_L\right)\right]$$

The first-order condition for q_L is

$$1 - q_L - \frac{1}{2}\left(1 - q_L\right) - \frac{1}{2} q_L = 0$$

from which we get

$$q_L^* = \frac{1}{2}$$

and

$$q_F^* = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Substituting into the demand function yields the equilibrium price

$$p^* = 1 - q_L^* - q_F^* = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

and the profits of the two firms

$$\pi_L^* = p^* q_L^* = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$\pi_F^* = p^* q_F^* = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

(II) If firms move simultaneously, we solve a Cournot game between firm F's owner and firm L's manager (on behalf of the owner). Each player will maximize their objective function, namely π_F for firm F's owner and s_L for firm L's manager. In other terms, firm F owner's maximization problem is the same as before, which implies

$$q_F(q_L) \equiv R_F(q_L) = \frac{1}{2}(1 - q_L)$$

Now, firm L manager's maximization problem is

$$\max_{q_L} s_L = q_L [1 - q_L - q_F - \beta] - k$$

The first-order condition for q_L is

$$1 - q_L - q_F - \beta - q_L = 0$$

from which we get

$$q_L(q_F) \equiv R_L(q_F) = \frac{1}{2}[1 - q_F - \beta]$$

We have to solve the following system

$$\begin{cases} q_F = \frac{1}{2}[1 - q_L] \\ q_L = \frac{1}{2}[1 - q_F - \beta] \end{cases}$$

Substituting the second equation into the first one implies

$$q_F = \frac{1}{2}\left[1 - \frac{1}{2}(1 - q_F - \beta)\right] = \frac{1}{4}[1 + q_F + \beta]$$

from which we get

$$q_F^{**}(\beta) = \frac{1}{3}(1 + \beta)$$

Substituting yields

$$q_L^{**}(\beta) = \frac{1}{2}\left[1 - \frac{1}{3}(1 + \beta) - \beta\right] = \frac{1}{3}(1 - 2\beta)$$

If we replace into the demand function we get

$$p^{**}(\beta) = 1 - q_L^{**} - q_F^{**} = 1 - \frac{1}{3}(1 - 2\beta) - \frac{1}{3}(1 + \beta) = \frac{1}{3}(1 + \beta)$$

Firm L's owner chooses β and k in order to

$$\max_{\beta,k} \pi_L^{**}(\beta, k) = p^{**}(\beta) \cdot q_L^{**}(\beta) - s_L(\beta) = \frac{1}{3}(1 + \beta) \cdot \frac{1}{3}(1 - 2\beta) - \frac{1}{3}(1 - 2\beta)\left[\frac{1}{3}(1 + \beta) - \beta\right] + k$$

which becomes

$$\max_{\beta,k} \frac{1}{9}(1 + \beta) \cdot (1 - 2\beta) - \frac{1}{9}(1 - 2\beta)^2 + k$$

The first-order condition for β is given by

$$1 - 2\beta - 2(1 + \beta) + 2 \cdot (1 - 2\beta) \cdot 2 = 0$$

from which we get

$$\beta^{**} = +\frac{1}{4}$$

Now notice that the objective function is increasing in k. Thereføeore, firm L' owner will find it optimal to set the highest possible k compatible with the manager's participation constraint. This observation implies

$$s_L(k) = q_L^{**}[p^{**} - \beta^{**}] - k = 0$$

which becomes

$$s_L(k) = \frac{1}{3}\left[1 - \frac{1}{2}\right] \cdot \left[\frac{1}{3}\left(1 + \frac{1}{4}\right) - \frac{1}{4}\right] - k = 0$$

$$\Leftrightarrow \left(\frac{1}{3} \cdot \frac{1}{2}\right) \cdot \left(\frac{5}{12} - \frac{1}{4}\right) - k = 0$$

Hence, we get

$$k^{**} = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

Finally, the equilibrium quantities are

$$q_L^{**} = \frac{1}{3}(1 - 2\beta^{**}) = \frac{1}{3} \cdot \left(1 - \frac{1}{2}\right) = \frac{1}{6}$$

$$q_F^{**} = \frac{1}{3}(1 + \beta^{**}) = \frac{1}{3} \cdot \left(1 + \frac{1}{4}\right) = \frac{5}{12}$$

while the firms' profits are equal to

$$\pi_L^{**} = \frac{1}{9}(1+\beta^{**})\cdot(1-2\beta^{**}) - \frac{1}{9}(1-2\beta^{**})^2 + k^{**} = \frac{1}{9}\cdot\frac{5}{4}\cdot\frac{1}{2} - \frac{1}{9}\left(\frac{1}{2}\right)^2 + \frac{1}{36} = \frac{5}{72}$$

$$\pi_F^{**} = q_F^{**}\left[1 - q_L^{**} - q_F^{**}\right] = \frac{5}{12}\cdot\left[1 - \frac{1}{6} - \frac{5}{12}\right] = \frac{25}{144}$$

Notice that firm F gets a higher profit than firm L, even though they show the same production costs. This is exactly the opposite of what we found in point (I). The rationale is that firm L's owner delegates its power to a manager who does not internalize profit maximization and only cares about his salary.

Exercise 2

Let us solve the game by backward induction, analyzing the price decisions at the last stage and the entry decision, taking as given the buyers' decisions about exclusivity at the first stage.

(i) If both buyers have accepted exclusivity and entry has occurred, the entrant will not be able to sell its good and will obtain a payoff $\pi_E^{ED,E} = -20$ (since it has paid the fixed cost $F = 20$). If entry has not occurred, the entrant's payoff is $\pi_E^{NE} = 0$ (since it has not paid the fixed cost). Therefore, given that both buyers have accepted exclusivity, the entrant will not enter the market.

The equilibrium price paid by each buyer is the monopoly price, which comes from

$$\max_p 2\cdot(p-6)\cdot(10-p)$$

The first order condition for p is

$$10 - p - p + 6 = 0$$

from which we get

$$p^m = 8$$

The aggregate quantity purchased by the two buyers is equal to

$$q^m = 2\cdot(10-8) = 4$$

while the quantity bought by each of them is

$$q_{B_i}^m = 2$$

The total profit of the incumbent is

$$\pi_I^m = (8-6)\cdot 4 - x_1 - x_2 = 8 - x_1 - x_2$$

where x_1 and x_2 are the compensations for buyer 1 and 2 respectively.

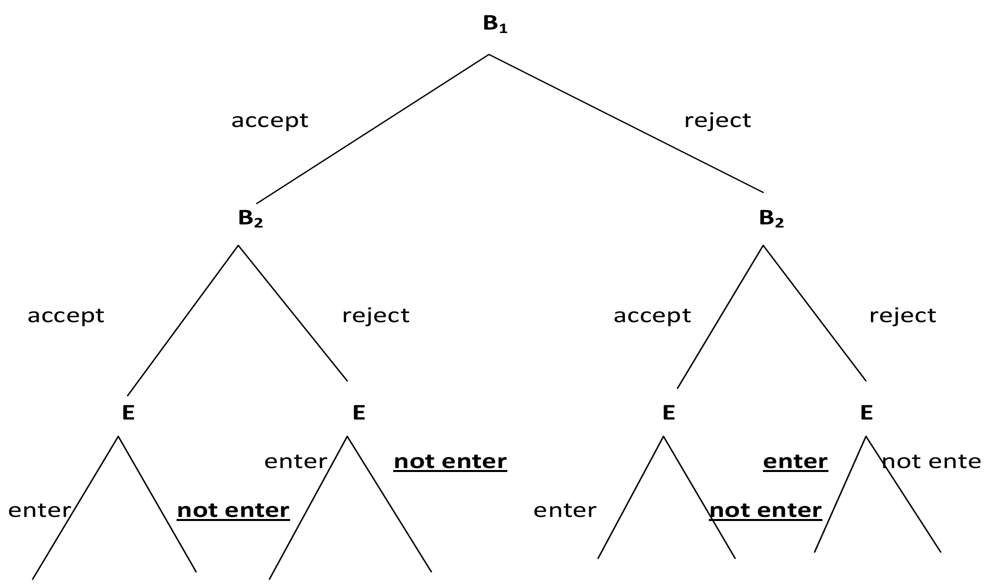
The surplus for each buyer is equal to

$$CS_{B_i} = \frac{1}{2}(10-8)\cdot 2 + x_i = 2 + x_i$$

(ii) If just one buyer has accepted exclusivity and entry has occurred, the entrant competes à la Bertrand with the incumbent for the free buyer. In equilibrium, the entrant will sell the good to the free buyer by charging a price $p = c_I = 6$ (Bertrand-Nash equilibrium price with asymmetric costs) and will earn $\pi_E^{ED_i,E} = (6-2)\cdot(10-6) - 20 = -4$. We know that if the entrant does not enter it gets $\pi_E^{NE} = 0$. Hence, if just one buyer has accepted exclusivity, the entrant will prefer to stay out of the market $(-4 < 0)$. Even in this case both buyers will pay the monopoly price $p^m = 8$. However, buyer i which has accepted the exclusivity will get $CS_{B_i} = 2 + x_i$ as before, while the other will receive $CS_{B_j} = 2$. The incumbent extracts the monopoly profit from each buyer and earns $\pi_I^m = 8 - x_i$.

(iii) If both buyers have rejected exclusivity and entry has occurred, Betrand competition takes place for each buyer and the entrant gets $\pi_E^{NED,E} = 2\cdot(6-2)\cdot(10-6) - 20 = 12 > 0$. In this case, the entrant finds it convenient to enter the market, each buyer's payoff is $CS_{B_i} = \frac{1}{2}(10-6)\cdot(10-6) = 8$ and the incumbent's payoff is $\pi_I = 0$.

The extensive form of the game can be represented as follows



The payoffs of the entrant are

$$\pi_E^{ED,E} = -20 \quad \pi_E^{ED,NE} = 0 \quad \pi_E^{ED_1,E} = -4 \quad \pi_E^{ED_1,NE} = 0$$

$$\pi_E^{ED_2,E} = -4 \quad \pi_E^{ED_2,NE} = 0 \quad \pi_E^{NED,E} = 12 \quad \pi_E^{NED,NE} = 0$$

The payoffs of the incumbent are

$$\pi_I^{ED,E} = 8 - x_1 - x_2 \quad \pi_I^{ED,NE} = 8 - x_1 - x_2 \quad \pi_I^{ED_1,E} = 4 + 0 - x_1 \quad \pi_I^{ED_1,NE} = 8 - x_1$$

$$\pi_I^{ED_2,E} = 0 + 4 - x_2 \quad \pi_I^{ED_2,NE} = 8 - x_2 \quad \pi_I^{NED,E} = 0 \quad \pi_E^{NED,NE} = 8$$

The payoffs of buyer 1 are

$$CS_{B_1}^{ED,E} = 2 + x_1 \quad CS_{B_1}^{ED,NE} = 2 + x_1 \quad CS_{B_1}^{ED_1,E} = 2 + x_1 \quad CS_{B_1}^{ED_1,NE} = 2 + x_1$$

$$CS_{B_1}^{ED_2,E} = \frac{1}{2}(10-6)^2 = 8 \quad CS_{B_1}^{ED_2,NE} = 2 \quad CS_{B_1}^{NED,E} = 8 \quad CS_{B_1}^{NED,NE} = 2$$

The payoffs of buyer 2 are

$$CS_{B_2}^{ED,E} = 2 + x_2 \quad CS_{B_2}^{ED,NE} = 2 + x_2 \quad CS_{B_2}^{ED_1,E} = 8 \quad CS_{B_2}^{ED_1,NE} = 2$$

$$CS_{B_2}^{ED_2,E} = 2 + x_2 \quad CS_{B_2}^{ED_2,NE} = 2 + x_2 \quad CS_{B_2}^{NED,E} = 8 \quad CS_{B_2}^{NED,NE} = 2$$

Let us analyze now the decision of buyer 2 for any possible decision of buyer 1. If buyer 1 has accepted exclusivity, buyer 2 knows that the entrant will stay out of the market and the monopoly price will be charge, irrespectively of whether it accepts exclusivity or not. Hence, buyer 2 is willing to accept exclusivity with a compensation equal to x_2 such that $CS_{B_2}^{ED,NE} = 2 + x_2 \geq CS_{B_2}^{ED_1,NE} = 2$, which implies $x_2 \geq 0$. Hence, if buyer 1 has accepted exclusivity, the incumbent will offer a compensation $x_2 = 0$ to buyer 2 and the latter will accept.

If buyer 1 has rejected exclusivity, buyer 2 anticipates that, if he also rejects, entry will occur and both of them will pay the competitive price $p^B = 6$, while, if it accepts exclusivity, entry will not take place and both buyers will pay the monopoly price $p^m = 8$. Hence, buyer 2 requires a compensation x_2 such that $CS_{B_2}^{ED_2,NE} = 2 + x_2 \geq CS_{B_2}^{NED,E} = 8$, which implies $x_2 \geq 6$ to accept exclusivity. Since the incumbent will extract the monopoly profit from both buyers ($\pi_I^{ED_2,NE} = 8 - x_2$) if buyer 2 accepts, it is willing to offer $x_2 = 6$ to elicit buyer 2's acceptance ($\pi_I^{ED_2,NE} = 8 - 6 > \pi_I^{NED,E} = 0$). Hence, if buyer 1 has rejected exclusivity, the incumbent will offer a compensation $x_2 = 6$ to buyer 2 and the latter will accept.

Let us turn now to buyer 1's decision. This buyer anticipates that buyer 2 will accept exclusivity independently of its decision. Hence, buyer 1 anticipates that entry will never occur and it will pay the monopoly price anyway. Buyer 1 is then willing to accept exclusivity if x_1 is such that $CS_{B_1}^{ED,NE} = 2 + x_1 \geq CS_{B_1}^{ED_2,NE} = 2$, which implies $x_1 \geq 0$. In equilibrium, the incumbent will offer $x_1 = 0$ to buyer 1, who accepts, and $x_2 = 0$ to buyer 2, who accepts as well. Therefore, entry is deterred even if the incumbent does not pay any compensation to buyers.

(II) If the buyer's loss from accepting exclusivity is 8 while the monopoly profit that the incumbent can extract from a buyer amounts to 3, in the subgame which follows buyer 1's rejection the equilibrium changes. Indeed, buyer 2, which anticipates that its rejection will imply Bertrand competition, will require $x_2 \geq 8$ to accept exclusivity. However, the incumbent's monopoly profits (gross of compensations) from both buyers $\pi_I^m = 2 \cdot 3 = 6$ are not large enough to make it profitable to offer $x_2 = 8$ to buyer 2. It follows that buyer 2 will not accept exclusivity if buyer 1 has rejected.

The equilibrium in the subgame which follows buyer 1's acceptance of exclusivity is the same as the one described before, since buyer 2 anticipates that entry will not occur.

Therefore, buyer 1 anticipates that if it accepts exclusivity buyer 2 will also accept, while, if it rejects, buyer 2 will also reject and entry will occur. Buyer 1 will then require a compensation $x_1 \geq 8$ to accept exclusivity, but the incumbent is not willing to offer it (since $\pi_I^m = 6 < 8 = x_1$). In equilibrium, both buyers will reject exclusivity and the entry will take place.

When monopoly profits are large enough ($2 \cdot \pi_I^m \geq x^*$), sequential offers allow the incumbent to fully exploit the externality that a buyer exerts on the other by accepting exclusivity and to exclude the entrant at zero costs. When the monopoly profits are lower ($2 \cdot \pi_I^m < x^*$), sequential offers allow the buyers to "coordinate" (in a non-cooperative way) their decisions and the unique equilibrium is such that both buyers reject and entry will occur.

Exercise 3

(I) Entrant E_i anticipates that, if its R&D process is successful and it enters the market, it will obtain a profit (gross of investment costs) equal to $\pi_{E_i} = (4-2) \cdot 1 = 2 > 0$, where 4 is the Bertrand-Nash equilibrium price with asymmetric costs selected through the Pareto-dominance criterion. Hence, if the R&D process is successful firm E_i will enter market for good i.

Firm E_i will decide its investment x_i in R&D by solving

$$\max_{x_i} 2 \cdot \frac{x_i}{1+x_i} + 0 \cdot \left[1 - \frac{x_i}{1+x_i}\right] - \frac{1}{2}x_i^2$$

The first-order condition for x_i is given by

$$\frac{2}{(1+x_i)^2} - x_i = 0$$

where the first addend is the (expected) marginal benefit from the R&D investment and the second one the marginal cost.

This condition may be rewritten as follows

$$2 - x_i (1 + x_i)^2 = 2 - x_i - x_i^3 - 2x_i^2 = 0$$

Numerical investigations show that

$$x_i^{*NB} \approx 0.69562$$

Hence, when the incumbent commits to sell the two products separately, the optimal level of investment is $x_i^{*NB} \approx 0.69562$. The probability of entry (which is equal to the probability of investment) will be $p\left(x_i^{*NB}\right) = \frac{0.69562}{1+0.69562} \approx 0.41$.

(II) Firm E_i anticipates that it is convenient to enter the market (and make profits equal to 2) only if its R&D process is successful and firm E_j's R&D process is successful as well. As a matter of fact, as long as the incumbent has decided to bundle the two products, if firm E_j's R&D process is not successful, consumers would not buy firm E_i's product because using it without the complementary good j does not generate any utility.

Hence, firm E_i will decide its investment x_i in R&D by solving

$$\max_{x_i} 2 \cdot \frac{x_i}{1 + x_i} \cdot \frac{x_j}{1 + x_j} + 0 \cdot \left[1 - \frac{x_i}{1 + x_i} \cdot \frac{x_j}{1 + x_j}\right] - \frac{1}{2} x_i^2$$

where $\frac{x_i}{1+x_i} \cdot \frac{x_j}{1+x_j}$ is the joint probability that the R&D processes of both firms are successful, computed as the product of the probabilities that each of them is successful (since they are thought of as independent events).

The first-order condition for x_i is given by

$$2 \cdot \frac{x_j}{1 + x_j} \cdot \frac{1}{(1 + x_i)^2} - x_i = 0$$

where the first addend is the (expected) marginal benefit from the R&D investment and the second one the marginal cost.

Since now the marginal benefit if the R&D investment is lower ($2 \cdot \frac{x_j}{1+x_j} \cdot \frac{1}{(1+x_i)^2} < \frac{2}{(1+x_i)^2}$) while the marginal cost is unchanged, the investment will be smaller in equilibrium.

Since firms are symmetric, it seems sensible to impose $x_i = x_j \equiv x$ in equilibrium. Therefore, we get

$$2 \cdot \frac{x}{(1 + x)^3} - x = 0$$

from which we get

$$(1 + x)^3 = 2$$

Finally we have

$$x^{*B} = \sqrt[3]{2} - 1 \approx 0.259992 < x^{*NB} \approx 0.69562$$

The associated probability of entry is

$$p\left(x^{*B}\right) = \left(\frac{x^{*B}}{1 + x^{*B}}\right)^2 = \left(\frac{0.259992}{1.259992}\right)^2 \approx 0.04258 < 0.41 \approx p\left(x_i^{*B}\right)$$

Hence, bundling reduces the probility of entry (anti-competitive effect).