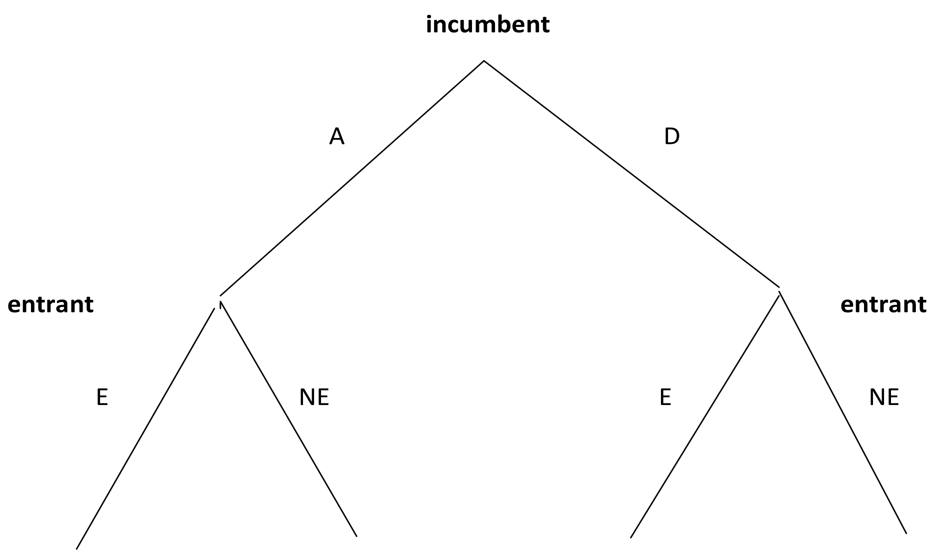
CLASS VI: Foreclosure
SOLUTIONS

November 16, 2009

Exercise 1

The extensive form of a game relies on the conceptual apparatus known as game tree. In our exercise, we have the following (the payoffs are derived from the calculations below)



$\pi_I^{A,E} = \frac{1}{8}$ $\quad$ $\pi_I^{A,NE} = \frac{1}{4}$ $\quad$ $\pi_I^{D,E} = \frac{3}{16}$ $\quad$ $\pi_I^{D,NE} = \frac{3}{16}$

$\pi_E^{A,E} = \frac{3}{64}$ $\quad$ $\pi_E^{A,NE} = 0$ $\quad$ $\pi_E^{D,E} = 0$ $\quad$ $\pi_E^{D,NE} = 0$

Notice that $\pi_I^{A,NE}$ is the monopoly profit. Moreover, $\pi_I^{D,E} = \pi_I^{D,NE}$ since if the incumbent decides to deter entry, it will produce a quantity such that entry will not occur, independently of the entrant's decision.

(I.b) At the second stage, if the entrant does not enter, it produces zero, pays no entry costs and makes zero profits ($\pi_E^{NE} = 0$), independently of the incumbent's strategy (A, D).

If the entrant does enter, its maximization program is

$$\max_{q_E} q_E \left[1 - q_I - q_E\right] - F$$

The first-order condition for q_E is

$$1 - q_I - q_E - q_E = 0$$

from which we get

$$q_E\left(q_I\right) \equiv R_E\left(q_I\right) = \frac{1 - q_I}{2}$$

The entrant's profit (as a function of q_I) is

$$\pi_E^E\left(q_I\right) = \frac{1 - q_I}{2} \cdot \left[1 - \frac{1 - q_I}{2} - q_I\right] - F = \frac{\left(1 - q_I\right)^2}{4} - F$$

(I.c) At the first stage, if the incumbent accomodates, it acts as a Stackelberg leader and solves the following

$$\max_{q_I} q_I \left[1 - q_I - \frac{1 - q_I}{2}\right]$$

The first-order condition for q_I is

$$1 - q_I - \frac{1 - q_I}{2} - \frac{1}{2} q_I = 0$$

from which we get

$$q_I^{A,E} = \frac{1}{2}$$

and

$$q_E^{A,E} = \frac{1}{4}$$

The incumbent's profit under accomodation is

$$\pi_I^{A,E} = \frac{1}{2} \cdot \left[1 - \frac{1}{2} - \frac{1}{4}\right] = \frac{1}{8}$$

while the entrant's profit is given by

$$\pi_E^{A,E} = \frac{1}{4} \cdot \frac{1}{4} - \frac{1}{64} = \frac{3}{64}$$

(I.d) At the first stage, if the incumbent deters entry, it sets a quantity such that the entrant's profit is zero, i.e.

$$\frac{(1-q_I)^2}{4} - \frac{1}{64} = 0$$

Taking the square root yields

$$\frac{1-q_I}{2} = \frac{1}{8}$$

from which we get

$$q_I^{D,E} = \frac{3}{4}$$

and

$$q_E^{D,E} = 0$$

since the entry is deterred.

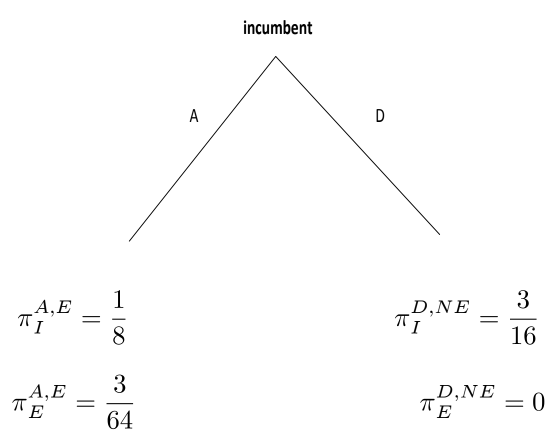
Moreover, the incumbent's profit under deterrence is

$$\pi_I^{D,E} = \frac{3}{4} \cdot \left[1 - \frac{3}{4} - 0\right] = \frac{3}{16}$$

while the entrant's profit is of course

$$\pi_E^{D,E} = 0$$

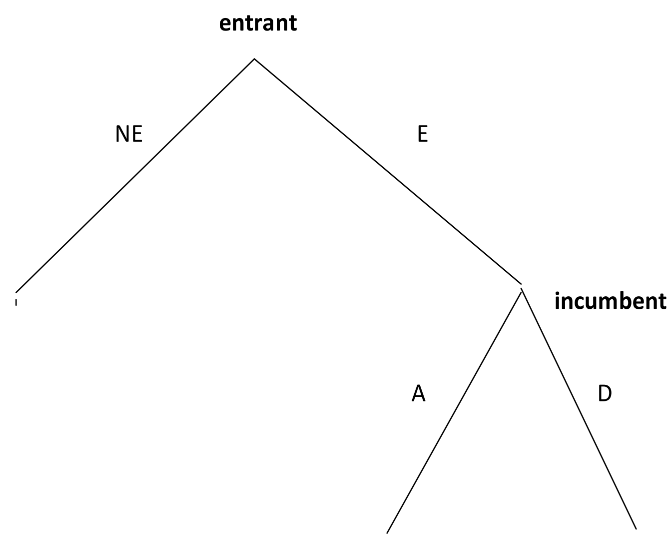
(I.e) To find the subgame perfect Nash equilibrium, we move backwards. At the second stage, the entrant will prefer not to enter if the incumbent deters - really, it is indifferent between enter and not to enter ($\pi_E^{D,E} = 0 = \pi_E^{D,NE}$) and thus there is no incentive to enter -, while it will find it convenient to enter if the incumbent accomodates ($\pi_E^{A,E} = \frac{3}{64} > \pi_E^{A,NE} = 0$). Hence, our game tree boils down to



Since at the first stage the incumbent will prefer to deter ($\pi_I^{D,NE} = \frac{3}{16} > \pi_I^{A,E} = \frac{1}{8}$), we get $SPNE \equiv \{\sigma_I; \sigma_E\} = \{\text{deter; do not enter if firm } I \text{ deters}\} = \{D, NE \text{ if } D\}$.

Remember that the SPNE is a Nash equilibrium in every subgame of the game as a whole and isolates only the Nash equilibria which are *reasonable*, that is the Nash equilibria which satisfy the principle of sequential rationality.

(II.a) The extensive form of the game is as follows (the payoffs are derived from the calculations below)



$\pi_E^{NE} = 0$ $\qquad$ $\pi_E^{E,A} = \dfrac{7}{64}$ $\qquad$ $\pi_E^{E,D} = 0 - F$

$\pi_I^{NE} = \pi_I^M = \dfrac{1}{4}$ $\qquad$ $\pi_I^{E,A} = \dfrac{1}{16}$ $\qquad$ $\pi_I^{E,D} = 0$

(II.b) Moving backwards, at the second stage, the incumbent decides whether to accomodate or deter.

If the incumbent accomodates, its maximization problem is the following

$$\max_{q_I} q_I \left[1 - q_I - q_E\right]$$

The first-order condition for q_I is

$$1 - q_I - q_E - q_I = 0$$

from which we get

$$q_I(q_E) \equiv R_I(q_E) = \frac{1 - q_E}{2}$$

The incumbent's profit (as a function of q_E) is given by

$$\pi_I^A(q_E) = \frac{1-q_E}{2} \cdot \left[1 - \frac{1-q_E}{2} - q_E\right] = \frac{(1-q_E)^2}{4}$$

If the incumbent deters, it is assumed to start a price war and so it will choose $p = MC = 0$, which implies $\pi_I^D = 0$.

At the first stage, the entrant decides whether to enter or not.

If it does enter, its maximization problem (given that the incumbent accomodates) is the following

$$\max_{q_E} q_E \left[1 - \frac{1-q_E}{2} - q_E\right] - F$$

The first-order condition for q_E is

$$1 - \frac{1-q_E}{2} - q_E - \frac{1}{2}q_E = 0$$

from which we get

$$q_E^{E,A} = \frac{1}{2}$$

and

$$q_I^{E,A} = \frac{1}{4}$$

Finally, the entrant's profit is

$$\pi_E^{E,A} = \frac{1}{2}\left[1 - \frac{1}{2} - \frac{1}{4}\right] - \frac{1}{64} = \frac{1}{8} - \frac{1}{64} = \frac{7}{64}$$

while the incumbent's profit is given by

$$\pi_I^{E,A} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

(II.c) The incumbent would prefer that the entrant does not enter, because this way it can enjoy monopoly profits ($\pi_I^{NE} = \frac{1}{4}$).

(II.d) Before answering this question, let us compute the strategic (or normal) form of the game

		FIRM I	
		A	D
FIRM E	E	$\frac{7}{64}; \frac{1}{16}$	$0 - F; 0$
	NE	$0; \frac{1}{4}$	$0; \frac{1}{4}$

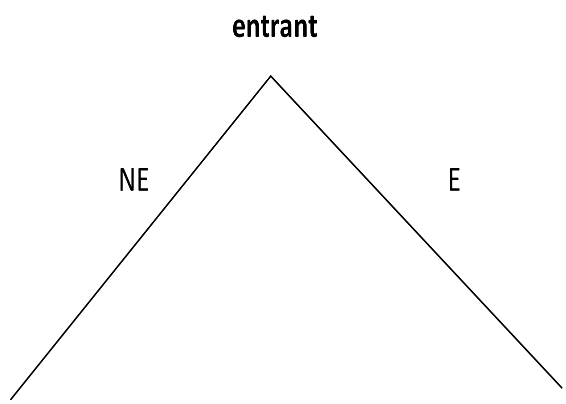
The Nash equilibria of this game are the following

$$NE_1 = \{\sigma_E; \sigma_I\} = \{NE; D \text{ if firm E plays } E\}$$

and

$$NE_2 = \{\sigma_E; \sigma_I\} = \{E; A \text{ if firm E plays } E\}$$

Now, the incumbent's threat to deter entry is not credible. Once the entrant has entered, the incumbent finds it optimal to accomodate rather than fight ($\pi_I^{E,A} = \frac{1}{16} > \pi_I^{E,D} = 0$). Hence, the Nash equilibrium NE_1 is not a reasonable prediction of the game. Through backward induction we find that the only Nash equilibrium which is also subgame perfect is $NE_2 = SPNE = \{\sigma_E; \sigma_I\} = \{E; A \text{ if firm E plays } E\}$. As a matter of fact, given that at the second stage the incumbent will prefer to accomodate if the entrant enters ($\pi_I^{E,A} = \frac{1}{16} > \pi_I^{E,D} = 0$), our game tree boils down to



$$\pi_E^{NE} = 0 \qquad\qquad \pi_E^{E,A} = \frac{7}{64}$$

$$\pi_I^{NE} = \pi_I^M = \frac{1}{4} \qquad\qquad \pi_I^{E;A} = \frac{1}{16}$$

It is immediate to see that at the first stage the entrant will prefer to enter ($\pi_E^{E,A} = \frac{7}{64} > \pi_E^{NE} = 0$) and thus the only reasonable Nash equilibrium is $\{\sigma_E; \sigma_I\} = \{E; A \text{ if firm E plays } E\}$.

Exercise 2

(I) A consumer who is indifferent between the two firms is located at t_{12}, where t_{12} is given by equating utilities from buying variety $x_1 = 0$ and $x_2 = 1$. Put in another way,

$$u^* - p_1 - (0 - t_{12})^2 = u^* - p_2 - (1 - t_{12})^2$$

from which we get

$$t_{12} = D_1(p_1, p_2) = \frac{p_2 - p_1 + 1}{2}$$

Notice that $t_{12} = D_1(p_1, p_2)$ since all consumers who are uniformely distributed and buy each a unit of good - to the left of t_{12} will purchase variety x_1.

Firm 2's demand is given by

$$D_2(p_1, p_2) = 1 - t_{12} = 1 - \frac{p_2 - p_1 + 1}{2} = \frac{p_1 - p_2 + 1}{2}$$

(II) The monopolist will choose the highest possible price compatible with the participation constraint of the most distant consumer, that is the one located at $\frac{1}{2}$. Hence,

$$u^* - p - \left(0 - \frac{1}{2}\right)^2 = 0$$

from which we get

$$p^M = u^* - \frac{1}{4}$$

If it could choose, the monopolist would select varieties $\frac{1}{4}$ and $\frac{3}{4}$, since these locations minimize average transportation costs. As a matter of fact, symmetry of the problem implies that each variety will be bought by half of consumers. Then, for each market segment (of length $\frac{1}{2}$) the location that minimizes average transportation costs is at the middle of this market segment (given uniform distribution), which implies $\frac{1}{4}$ and $\frac{3}{4}$. Not surprisingly, these are also the locations that maximize social welfare.

As long as average transportation costs are minimized, the monopolist can charge the price as high as possible, i.e.

$$u^* - p - \left(\frac{1}{4} - \frac{1}{2}\right)^2 = 0$$

whic implies

$$p^M_{MAX} = u^* - \frac{1}{16}$$

(III) The consumer who is indifferent between varieties $x_1 = 0$ and $x_3 = \frac{1}{2}$ is located at t_{13} defined as

$$u^* - p_1 - (0 - t_{13})^2 = u^* - p_3 - \left(\frac{1}{2} - t_{13}\right)^2$$

from which we get

$$t_{13} = p_3 - p_1 + \frac{1}{4}$$

The consumer who is indifferent between varieties $x_2 = 1$ and $x_3 = \frac{1}{2}$ is located at t_{23} such that

$$u^* - p_2 - (1 - t_{23})^2 = u^* - p_3 - \left(\frac{1}{2} - t_{23}\right)^2$$

from which we get

$$t_{23} = \frac{3}{4} + p_2 - p_3$$

Hence, the demand for variety x_1 will be

$$D_1(p_1, p_3) = t_{13} = p_3 - p_1 + \frac{1}{4}$$

The demand for x_2 is

$$D_2(p_2, p_3) = 1 - t_{23} = 1 - \left(\frac{3}{4} + p_2 - p_3\right) = \frac{1}{4} - p_2 + p_3$$

Finally, the demand for x_3 is given by

$$D_3(p_1, p_2, p_3) = t_{23} - t_{13} = \frac{3}{4} + p_2 - p_3 - \left(p_3 - p_1 + \frac{1}{4}\right) = \frac{1}{2} + p_2 - 2p_3 + p_1$$

In the market for variety x_1, the monopolist's problem is

$$\max_{p_1} \pi_1 = p_1 D_1 = p_1 \left(p_3 - p_1 + \frac{1}{4}\right)$$

The first order condition for p_1 is

$$p_3 - p_1 + \frac{1}{4} - p_1 = 0$$

from which we get

$$p_1(p_3) \equiv R_1(p_3) = \frac{1}{8} + \frac{1}{2} p_3$$

In the market for variety x_2, the monopolist's maximization program is

$$\max_{p2} \pi_2 = p_2 D_2 = p_2 \left(\frac{1}{4} - p_2 + p_3\right)$$

The first order condition for p_2 is

$$\frac{1}{4} - p_2 + p_3 - p_2 = 0$$

from which we get

$$p_2(p_3) \equiv R_2(p_3) = \frac{1}{8} + \frac{1}{2} p_3$$

Finally, in the market for variety x_3, the entrant's maximization problem is

$$\max_{p_3} \pi_3 = p_3 D_3 = p_3 \left(\frac{1}{2} + p_2 - 2p_3 + p_1 \right)$$

The first order condition for p_1 is

$$\frac{1}{2} + p_2 - 2p_3 + p_1 - 2p_3 = 0$$

from which we get

$$p_3 (p_1, p_2) \equiv R_3 (p_1, p_2) = \frac{1}{8} + \frac{1}{4} p_1 + \frac{1}{4} p_2$$

Equilibrium prices come from the following system

$$\begin{cases} p_1 = \frac{1}{8} + \frac{1}{2} p_3 \\ p_2 = \frac{1}{8} + \frac{1}{2} p_3 \\ p_3 (p_1, p_2) = \frac{1}{8} + \frac{1}{4} p_1 + \frac{1}{4} p_2 \end{cases}$$

First and second equations imply $p_1 = p_2 = \frac{1}{8} + \frac{1}{2} p_3$. Substituting into the third one yields

$$p_3^* = \frac{1}{8} + \frac{1}{4} \left(\frac{1}{8} + \frac{1}{2} p_3 \right) + \frac{1}{4} \left(\frac{1}{8} + \frac{1}{2} p_3 \right) = \frac{1}{8} + \frac{2}{4} \left(\frac{1}{8} + \frac{1}{2} p_3 \right) = \frac{1}{4}$$

and

$$p_1^* = p_2^* = \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{4}$$

(IV) The entrant's profit is

$$\pi_3^* = p_3^* D_3^* - F = \frac{1}{4} \left(\frac{1}{2} + p_2^* - 2p_3^* + p_1^* \right) - F = \frac{1}{4} \left(\frac{1}{2} + \frac{1}{4} - \frac{2}{4} + \frac{1}{4} \right) - \frac{1}{6} = \frac{1}{4} \cdot \frac{1}{2} - \frac{1}{6} = -\frac{1}{24} < 0$$

The entrant's profit is negative with $F = \frac{1}{6}$, so it does not find it profitable to enter the market.

(V) If the entrant locates at $x_3 = 0$, Betrand competition occurs. This implies

$$p_1^{**} = p_3^{**} = 0$$

and

$$\pi_1^{**} = \pi_3^{**} = 0$$

The consumer who is indifferent between varieties $x_1 = x_3 = 0$ and $x_2 = 1$ is lacated at t_{12} such that

$$u^* - 0 - (0 - t_{12})^2 = u^* - p_2 - (1 - t_{12})^2$$

which implies

$$t_{12} = \frac{1 + p_2}{2}$$

Hence, t_{12} captures the aggregate demand for the entrant and the incumbent (former monopolist) at $x_1 = 0$.

The demand for the incumbent in $x_2 = 0$ is the residual one, i.e.

$$D_2(p_2) = 1 - t_{12} = \frac{1 - p_2}{2}.$$

In the market for variety x_2, the incumbent's maximization problem is

$$\max_{p_2} \pi_2 = p_2 D_2 = p_2 \cdot \frac{1 - p_2}{2}$$

The first order condition for p_2 is

$$\frac{1 - p_2}{2} - \frac{1}{2} p_2 = 0$$

from which we get

$$p_2^{**} = \frac{1}{2}$$

and

$$\pi_2^{**} = p_2^{**} \cdot D_2(p_2^{**}) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

Summing up, the incumbent's profits are

$$\pi_I^{**} = \pi_1^{**} + \pi_2^{**} = 0 + \frac{1}{8} = \frac{1}{8}$$

while the entrant's profits are

$$\pi_E^{**} = \pi_3^{**} = 0$$

If the incumbent withdraws the variety $x_1 = 0$, the demand for variety $x_3 = 0$ is given by the "location" t_{23} of the consumer who is indifferent between $x_3 = 0$ and $x_2 = 1$, i.e.

$$u^* - p_3 - (0 - t_{23})^2 = u^* - p_2 - (1 - t_{23})^2$$

from which we get

$$t_{23} = D_3(p_2, p_3) = \frac{p_2 - p_3 + 1}{2}$$

and

$$D_2(p_2, p_3) = 1 - t_{23} = 1 - \frac{p_2 - p_3 + 1}{2} = \frac{p_3 - p_2 + 1}{2}$$

The entrant's maximization problem is

$$\max_{p_3} \pi_3 = p_3 D_3 = p_3 \cdot \frac{p_2 - p_3 + 1}{2}$$

The first order condition for p_3 is

$$\frac{p_2 - p_3 + 1}{2} - \frac{1}{2} p_3 = 0$$

from which we get

$$p_3(p_2) \equiv R_3(p_2) = \frac{1 + p_2}{2}$$

The incumbent's maximization problem is

$$\max_{p_2} \pi_2 = p_2 D_2 = p_2 \cdot \frac{p_3 - p_2 + 1}{2}$$

The first order condition for p_2 is

$$\frac{p_3 - p_2 + 1}{2} - \frac{1}{2} p_2 = 0$$

from which we get

$$p_2(p_3) \equiv R_2(p_3) = \frac{1 + p_3}{2}$$

We know that in equilibrium we must have

$$p_3(p_2) = R_3[R_2(p_3)]$$

which means

$$p_3 = \frac{1}{2}[1 + p_2] = \frac{1}{2}\left[1 + \frac{1 + p_3}{2}\right]$$

Finally we get

$$p_3^{***} = 1$$

$$p_2^{***} = \frac{1}{2}[1 + p_3^{***}] = 1$$

Moreover,

$$\pi_3^{***} = p_3^{***} \cdot D_3^{***} = p_3^{***} \cdot \frac{p_2^{***} - p_3^{***} + 1}{2} = \frac{1}{2}$$

and

$$\pi_2^{***} = p_2^{***} \cdot D_2^{***} = p_2^{***} \cdot \frac{p_3^{***} - p_2^{***} + 1}{2} = \frac{1}{2}$$

Since $\pi_3^{***} = \frac{1}{2} > \pi_3^{**} = 0$ and $\pi_2^{***} = \frac{1}{2} > \pi_I^{**} = \pi_1^{**} + \pi_2^{**} = 0 + \frac{1}{8} = \frac{1}{8}$, withdrawing variety $x_1 = 0$ is Pareto improving, so not only the entrant finds it profitable to enter but also the incumbent prefers to accomodate.

Exercise 3

(I) Firm M represents a natural monopoly, as its average costs are everywhere decreasing

$$\frac{d}{dq}\left[\frac{C(q)}{q}\right] = \frac{d}{dq}\left[c + \frac{F}{q}\right] = -\frac{F}{q^2} < 0$$

According to Baumol *et al.* (1982), an industry is a natural monopoly if, over the relevant range of outputs, the cost function is subadditive, i.e.

$$\sum_{i=1}^{n} C(q_i) > C\left(\sum_{i=1}^{n} q_i\right), \text{ for any } n\text{-tuple of outputs } q_1, ..., q_n$$

This means that it costs less to produce the various outputs together than to produce them separately. It can be easily shown that everywhere decreasing average costs imply subadditivity.

(II.a) The benevolent regulator maximizes social welfare and thus its maximization problem is

$$\max_{p,S} CS + \pi$$

$$s.t.$$

$$\pi \geq 0 \qquad \text{(PC)}$$

where (PC) is the firm's participation constraint. Notice that, as funds are transferable (with zero costs of public funds), the regulator has two instruments: the price p to be charged to consumers and the subsidy S to the firm. Hence, we get

$$\max_{p,S} \frac{1}{2}(1-p)^2 - S + (p-c)\cdot(1-p) - F + S$$

$$s.t.$$

$$\pi \geq 0 \qquad \text{(PC)}$$

Since for a given p there exists a bijective correspondence between T and π, we can replace the choice variable T with π and get

$$\max_{p,\pi} \frac{1}{2}(1-p)^2 - [\pi - (p-c)(1-p) + F] + \pi$$

$$s.t.$$

$$\pi \geq 0 \tag{PC}$$

Since π disappears in the objective function, in principle any value of $\pi \geq 0$ will maximize social welfare. Hence, ignoring for the moment the participation constraint, we have

$$\max_{p} \frac{1}{2}(1-p)^2 + (p-c)(1-p) - F$$

The first-order condition for p is

$$-(1-p) + 1 - p - (p-c) = 0$$

which immediately yields

$$p = c$$

that is marginal cost pricing.

The lowest subsidy compatible with the firm's participation constraint is $S = F$ as setting a price equal to marginal (constant) costs only covers variable costs.

(II.b) When public funds are not available, the regulator's maximization problem is

$$\max_{p} \frac{1}{2}(1-p)^2 + (p-c)(1-p) - F$$

$$s.t.$$

$$\pi \geq 0 \tag{PC}$$

We know that maximizing social welfare means minimizing the deadweight welfare loss. To do that, price must be set as close as possible to marginal costs, consistently with the nonnegative profit condition. In other terms, p must be such that

$$(p-c)(1-p) - F = 0$$

Since $q = 1 - p$, we get

$$p^* = c + \frac{F}{q}$$

that is average cost pricing.

(II.c) If public funds are socially costly ($\lambda > 0$), the regulator's maximization problem becomes

$$\max_{p,S} \frac{1}{2}(1-p)^2 - (1+\lambda) S + (p-c)\cdot(1-p) - F + S$$

$$s.t.$$

$$\pi \geq 0 \qquad \text{(PC)}$$

Substituting as before the choice variable S with π yields

$$\max_{p,\pi} \frac{1}{2}(1-p)^2 - (1+\lambda)[\pi - (p-c)(1-p) + F] + \pi$$

$$s.t.$$

$$\pi \geq 0 \qquad \text{(PC)}$$

Since the objective function is decreasing in π $[-(1+\lambda)\pi + \pi = -\lambda\pi]$, the regulator finds it optimal to set $\pi = 0$. The maximization problem becomes

$$\max_{p} \frac{1}{2}(1-p)^2 + (1+\lambda)(p-c)(1-p) - (1+\lambda) F$$

The first-order condition for p is

$$-(1-p) + (1+\lambda)[1-p-(p-c)] = 0$$

which can be rewritten as follows

$$-p + c + \lambda[-2p + 1 + c] = 0$$

If we sum and subtract by λp, we find

$$-p(1+\lambda) + c(1+\lambda) + \lambda(1-p) = 0$$

Finally, we get

$$\frac{p-c}{p} = \frac{\lambda}{1+\lambda}\frac{1-p}{p}$$

where the left-hand side is the Lerner index (LI). The second term on right-hand side captures the inverse of the demand elasticity $\eta(p)$ (in absolute value), since $|\eta(p)| = \left|\frac{\Delta q/q}{\Delta p/p}\right| = \left|\frac{p}{1-p}(-1)\right| = \frac{p}{1-p}$.

Notice that

a) $\frac{\partial LI}{\partial \lambda} = \frac{1-p}{p}\frac{1}{(1+\lambda)^2} > 0$, which means that the higher λ, the higher the price-cost mark up. As public funds are more costly, the fixed costs will be less financed with subsidy and more with price;

b) $\frac{\partial LI}{\partial |\eta|} < 0$, which means that the more elastic is the demand, the lower will be the price-cost markup, since the welfare loss from $p > c$ is higher. So, F will be more financed with S and less with the price.

(I.c) If the regulator still applies the complete-information pricing policy (with zero costs of public funds), the profit of a firm with costs c_i, $i = L, H$ which declares $\widehat{c}_i$ is

$$\pi_i(\widehat{c}_i, c_i) = \widehat{c}_i(1 - \widehat{c}_i) - c_i(1 - \widehat{c}_i) + S - K = \pi_i(\widehat{c}_i, \widehat{c}_i) + (\widehat{c}_i - c_i)(1 - \widehat{c}_i)$$

where $S = K$. Notice that $\pi_i(\widehat{c}_i, \widehat{c}_i) = 0$ since in complete information any firm has zero profits. Hence we get

$$\pi_i(\widehat{c}_i, c_i) = (\widehat{c}_i - c_i)(1 - \widehat{c}_i)$$

This implies that the firm can get a positive profit as long as it reports $\widehat{c}_i > c_i$. When $c_i \in \{c_L, c_H\}$ the efficient firm has an incentive to overstate its costs, i.e. to declare $\widehat{c}_i = c_H$. It can be shown that after taking into account the firm's participation constraint and the incentive compatibility constraints we find

a) $p_L^{**} = c_L$ (no-distortion at the top)

b) $p_H^{**} = c_H + \frac{\phi}{1-\phi}(1 - \alpha)(c_H - c_L) > c_H$, where α is the weight on the profit in the social welfare function (trade-off between allocative efficiency and informational rent).