FINANCIAL MATHEMATICS (6008 BIEM)
Detailed Syllabus for Academic Year 2008/2009

1. Multivariable differential calculus and static optimization

General notions. Vector functions and real functions of *n* real variables. Real functions of two real variables: domain, 3-dimensional graph, level curves. The space $\mathbb{R}^n$: norm, distance, neighbourhoods, interior points.

Differential calculus. Generalities on limits and continuity. First partial derivatives, gradient. Second partial derivatives, Hessian matrix. Schwarz's Theorem. Differentiability, total differential. Taylor's formula of the I order. Jacobian matrix.

Optimization. Maxima/minima: global and local, strict (or strong) and weak. Unconstrained optimization problems. Necessary condition of the first order for unconstrained local maxima/minima. Stationary points. Saddle points. NW principal minors of the Hessian matrix. Sufficient condition for unconstrained strict local maxima/minima. Constrained optimization problems, with equality constraints (*n* variables and *m* constraints, where $m < n$). Lagrangean function; Lagrange multipliers. Intuitive justification for our interest in the saddle points of the Lagrangean. Necessary condition of the first order for constrained local maxima/minima. Sufficient condition for constrained strict local maxima/minima. Interpretation of Lagrange multipliers as shadow prices of the resources; sensitivity analysis. Applications: method of least squares, beta coefficients of stocks; optimal choices by consumers and firms (*).

2. Probability

Introduction to probability. Random experiment. Classical, empirical and subjective approach. Axiomatic approach. Sample space. Random events, algebra/σ-algebra of events. Probability of an event, probability space. Probability axioms. Additivity/σ-additivity; conditional probability. Stochastically independent events, positively/negatively correlated events. Bayes's Theorem and its applications.

Random numbers. Borel-measurable functions. Random numbers, distribution function. Discrete random numbers, probability function. Notable discrete distributions: uniform, Poisson. Continuous random numbers, probability density function. Notable continuous distributions: uniform, exponential, normal. Expected value of a random number. Variance and standard deviation of a random number. Expected value of a function of a random number. Moments of a random number, reinterpretation of expected value and variance in terms of moments. Expected value, variance and standard deviation of a linear affine function of a random number.

Discrete random vectors. Random vectors, distribution function. Discrete random vectors, probability function. Discrete random vectors with two components; probability functions with double entry tables. Stochastically independent random numbers. Vector of the expected values of a random vector. Expected value of a linear affine function of a random vector. Covariance of two components of a random vector. Linear correlation between components. Variance-covariance matrix of a random vector. Variance of a linear affine function of a random vector. Bravais coefficient of linear correlation. Application: mean-variance principle and portfolio selection model (layout only).

3. Financial calculus

Financial laws. Accumulation, discount. Principal, accumulated amount (final value), interest, accumulation factor; nominal value, present value (discounted value), discount, discount factor. Interest rate, discount rate. Conjugate factors. One-variable financial laws. Usual laws: simple interests, compound interests, anticipated simple interests, simple discount (or rational discount), compound discount, bank discount. Equivalent rates with simple interests. Equivalent rates with compound interests; convertible nominal annual rate, effective

annual rate. Generality levels for one-variable laws. Continuation accumulation factor. Two-variable financial laws; examples. Generality levels for two-variable laws. Force of interest for one-variable laws; the case of compound interest. Decomposability for one-variable laws. Force of interest for two-variable laws. Decomposability for two-variable laws. Decomposability by product, Cantelli's Theorem. Application: the financial-demographic accumulation factor.

Cash flows. Financial operations, cash flows. Present value and final value of a cash flow. Annuities, investments, loans. Simple annuities with constant instalments, in compound interests (temporary annuities/perpetuities, ordinary/due); the symbols $a_{\overline{n}|i}$, $s_{\overline{n}|i}$, $\ddot{a}_{\overline{n}|i}$, $\ddot{s}_{\overline{n}|i}$, $a_{\overline{\infty}|i}$, $\ddot{a}_{\overline{\infty}|i}$. *DCF*, *NPV*, internal rates of a financial operation. Graph of the *DCF* for investments and loans. *IRR* of an investment, effective cost of a loan.

Amortizations. Amortization of a loan. Maturities, instalments, capital shares, interest shares, residual debt, debt repaid, global interests amount. Amortization plan. Closure conditions: elementary, initial, final. Elementary approach, financial approach. The case of compound interest. Contractual rate. Formulae for interest shares, recursive formulae for residual debts. Change of conditions. Italian amortization, French amortization. Applications: consumer credit, leasing. Problems on instalments; the time profile of payments. Problems on rates; legal indicators of cost.

Term structure. The term structure of prices. Spot prices of unitary zero-coupon bonds, forward prices of unitary zero-coupon bonds. Determination of the forward prices in terms of spot prices, relations with decomposability. The no-arbitrage assumption. Prices of bonds with coupons. The term structure of interest rates. Spot rates, forward rates. The case of a flat term structure.

Fixed-income bonds. Zero-coupon bonds: gross compound yield; gross simple yield to maturity, yield from trading before the maturity, relations with non-decomposability; net simple yield to maturity, with tax deductions at issuance or repayment. Bullet bonds. Issuing conditions: face value, current price, reimbursement value; issuing premium, reimbursement premium; coupons, annual coupon rate. Purchasing conditions: dirty price, current price. Dependence of the price on the term structure. Yield to maturity, immediate yield. Duration of a bond. Price volatility of a bond, with respect to the variations of the market rate. Modified duration. Estimation of price variations via the duration.

Financial decisions. General notions. The opportunity cost of capital. The *NPV* criterion; underlying assumptions and equivalence with the criterion of final wealth. Meaning of the *NPV* as a threshold selling price. The case of an investment: the *IRR* criterion. The case of a loan: the effective cost criterion. Critique of the internal rate criterion. Limitations in the use of the *NPV*, generalizations of it: *GNPV*, *APV*, *GAPV*. Financial leverage. Critique of other evaluation criteria: *WACC*, *ROE*. Outstanding capitals coherent with the internal rate, exogenous outstanding capitals. Decomposition of the *GNPV* into period components. Decomposition of the *GAPV* into period components.

Adopted textbooks

1. E. Castagnoli, L. Peccati, *Mathematical Analysis for Business (Complements)*, EGEA, Milano (2004).
2. E. Castagnoli, M. Cigola, L. Peccati, *Probability. A Brief Introduction*, EGEA, Milano (2009, second edition).

3a. E. Castagnoli, L. Peccati, *Financial Calculus with Applications*, EGEA, Milano (2002).

3b. L. Peccati, *Complementary Notes for the Course of Financial Mathematics,* EGEA, Milano (2004).

(*) For this topic some additional pages are available online.