Optimal Market Access Pricing*

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Abstract

This paper provides a theoretical explanation for the widespread use of rebate-based access pricing — maker-taker and taker-maker — in present-day securities markets. Given a standard model of trading frictions, we show that exchanges optimally use rebate-based access pricing when dispersion of investor asset valuation is low (and thus potential gains from trade are low), but strictly positive fees for both liquidity makers and liquidity takers with high investor valuation dispersion. In addition, when the trading frequency increases, the incentive to use rebate-based pricing decreases. However, rebate-based pricing is more likely in markets with HFT trading. When rebate-based access pricing is optimal for an exchange, we find that total welfare increases (decreases) when investor valuation dispersion is low (high) without HFTs. However, with HFTs, optimal rebate-based access pricing strictly improves total welfare, although Pareto transfers from exchanges to investors may be needed to improve investor welfare. In addition, we identify an asymmetry in how make fees and take fees affect the trading process. Thus, the effect of maker-taker and taker-maker pricing need not always be symmetric.

JEL classification: G10, G20, G24, D40

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Access fees and rebates in securities markets are at the top of the agenda of financial regulators and market operators around the world. Following Reg NMS (2007) in the US (and related regulation in Europe), market access pricing became a strategic tool for trading platforms and exchanges to attract trading volume. In particular, rebates are used to incentivize investors to submit certain types of orders, while investors using other order types are charged fees. For example, under maker-taker pricing, investors receive rebates when their limit orders (making liquidity) are executed and pay fees on market orders (taking liquidity). However, rebate–based access pricing has been criticized by some practitioners as well as by Angel, Harris, and Spatt (2013) and Spatt (2019).[1] Regulators are now taking actions to study and possibly limit rebate-based pricing. Most notably, on March 14, 2018, the SEC released a proposal for a two–year Transaction Fee Pilot to experiment with reduced access fees and rebates.

The objective of this paper is to study optimal market access pricing by means of a theoretical model of a limit order market with discrete prices and strategic traders. Our approach follows seminal theoretical research by Colliard and Foucault (2012); Foucault, Kadan, and Kandel (2013); and Chao, Yao, and Ye (2018) showing how fees and rebates for taking and making liquidity via market and limit orders can alleviate trading frictions due to price discreteness. Like Chao et al. (2018), we consider the optimal access fees and rebates for a profit-maximizing exchange. Whereas Chao et al. (2018) investigates access pricing and intermarket competition, our paper is the first to consider access pricing in a multiperiod setting and also to allow for flash orders from high-frequency (HFT) traders, who are continuously present in the market. In addition, we provide new insights into the relation between access pricing, the amount of heterogeneity in investor valuations, regulatory constraints on fees, and welfare.

We study optimal access pricing in an equilibrium model of a dynamic limit order market with a discrete

[1] Angel et al. (2013) and Spatt (2019) argue maker-taker pricing reduces the transparency of the true economic spread, negatively impacts agency problems in broker order routing decisions, and puts venues that do not make use of such fees at a competitive disadvantage. Harris (2015) further points out that negative fees allow for intra-tick trading, thus by-passing Reg NMS trade-through rule.

price tick-size in which traders arrive sequentially with heterogeneous private asset valuations. As in Foucault et al. (2013) and Chao et al. (2018), price discreteness creates trading frictions by limiting the prices at which investors can transact. Access fees and rebates enable side-transfers between buyers and sellers to adjust the rewards and costs for liquidity supply and demand. Our paper is most closely related to Chao et al. (2018), which is the first model of optimal access fees for profit-maximizing exchange operators. Our analysis leads to four results that extend previous research on optimal access pricing:

- The optimal access-pricing structure depends on the distribution of gains-from-trade in the population of investors arriving in the market. When potential gains-from-trade are small, optimal access pricing involves a mix of rebates and fees (maker-taker or taker-maker), whereas strictly positive fees are optimal when potential gains-from-trade are large ex ante.

- When the trading frequency increases, the incentive for the exchange to use rebate-based pricing decreases. Furthermore, in a multiperiod market with three rounds of trader-arrival (rather than two), maker-taker and taker-maker pricing are again optimal when trader valuations are not too dispersed, but now they are no longer symmetric. The ability of traders at intermediate times to submit their own limit orders reduces the market power of limit orders posted in the first period.

- The mechanics of liquidity supply and demand change significantly with HFT trading. This is because HFT traders use flash orders to react immediately to orders submitted by slower traders over time. As a result, the range of market parameterizations in which the exchange optimally uses rebate-based access pricing becomes larger relative to the no-HFT market. The increased use of rebate-based pricing is perhaps somewhat surprising because the HFTs simply augment the set of potential counterparties in the market. However, rebate-based access pricing allows liquidity-demanders to reduce the compensation paid to the HFTs for liquidity provision.

- The welfare effect of profit-maximizing rebate-based taker-maker and maker-taker pricing by an exchange are parameter-dependent. When ex ante investor valuations are concentrated, rebate-based access pricing improves welfare. However, when the support of investor valuations is somewhat larger, then without HFTs rebate-based pricing can still maximize exchange profits but can lower overall welfare relative to a zero-fee/zero-rebate pricing. In contrast, with HFT, total welfare increases over the entire parameter region for which rebate-based access pricing is optimal for an exchange.

Taken together, our analysis demonstrates a connection between access pricing and HFT trading. In particular, the presence of HFT traders induces exchanges to use rebate-based access pricing even in active markets (with frequent investor arrival) with large gains-from-trade. In addition, the presence of HFTs induces a substantial redistribution of welfare from slow investors to the exchange.

A sizable empirical literature investigates different aspects of access fees and rebates.[2] Malinova and Park (2015) find evidence following changes in access fees and rebates on the Toronto Stock Exchange (TSX) that appears to support the Colliard and Foucault (2012) irrelevance prediction provided that the TSX price tick-size can be interpreted as being economically small. However, other research finds evidence against Colliard-Foucault irrelevance once there is a discrete tick-size. Panayides, Rindi, and Werner (2017) find that quoted and cum-fee spreads are affected by fees and rebates on the BATS European platforms, CXE and BXE. Using Rule 605 data, O'Donoghue (2015) finds that changes in the split of trading fees between liquidity suppliers and demanders affect order choice and execution quality as predicted by Foucault et al. (2013). Cardella, Hao, and Kalcheva (2015) investigate 108 instances of fee changes for U.S. exchanges in 2008-2010 and find that changes in take fees have a larger impact on trading activity than changes in make fees. Battalio, Corwin, Jennings (2016) find that access fees and rebates appear to affect broker order-routing

[2] In addition to the research discussed here, see also Skjeltorp, Sojli, and Tham (2012), He, Jarnecic, and Liu (2015), Clapham, Gomber, Lausen, and Panz (2017), Anand, Hua, and McCormick (2016), Comerton-Forde, Grégoire, and Zhong (2019) and Lin, Swan, et al. (2017)

decisions.

Empirical research finds that rebate-based access pricing is related to HFTs, but no theory shows the relationship between HFT firms and optimal access pricing chosen by trading platform owners. Menkveld (2013) shows that access rebates are a significant part of HFT profits. We show that in equilibrium the rebate–based fee structure is consistent with HFT market participation. This is also consistent with evidence in Cardella et al. (2015) that Reg NMS was followed by the adoption of rebate–based access pricing by most trading platforms in U.S. markets and by a sharp increase in HFT firm trading. O'Hara (2015) also links HFT trading activity and the increased use of rebate–based access pricing structures around the world.

Angel et al. (2013) and Spatt (2019) emphasize that access fees and rebates have important potential effects in terms of the transparency of economic prices (price + access pricing) vs. quoted prices, the efficacy of regulatory protections based on quoted prices, agency issues when brokers do not pass through fees and rebates to their clients, and impeding intermarket competition. In contrast, our analysis is based on the idea that constraining trade to a discrete price grid creates frictions in the trading process and that access pricing potentially reduces those frictions. Both sets of considerations are likely to be important. Moreover, a complete understanding of access pricing is likely to involve interactions between these various effects.

1 Background information and prior research

U.S. Regulation National Market System (Reg NMS) established the regulatory foundation for the current architecture of US equity markets. This regulation includes an explicit limit on the cost of accessing (i.e., posting and trading on) quotes displayed by U.S. equity trading platforms. Rule 610 caps access fees to no more than \$0.003 per share for stocks priced over \$1, and to no more that 0.3% of the quoted price for stocks priced below \$1. In addition, the Sub-Penny Rule 612 of Reg NMS prohibits exchanges, market makers, and electronic platforms from displaying, ranking or accepting quotes on NMS securities in sub-penny increments unless a stock is priced less than $1 per share. Thus, under Reg NMS, access fees cannot exceed one third of the tick size.[3]

The proposed two-year Transaction Fee Pilot would substantially change the current U.S. equity cap to access fees for 2 groups of NMS stocks with average daily trading volumes $\geq$ 30,000 shares and with a share price $\geq$ $2. The pilot would apply to all equities exchanges but not dark pools and other alternative trading structures. Test Group 1 would lower the access fee to $0.0010 and would still allow rebate-based pricing; whereas Test Group 2 would prohibit all exchange rebates and linked pricing while maintaining the existing $0.003 per share fee cap. Test Group 3, the control group, would maintain the current access fee cap. By lowering the access fee for Test group 1 stocks and banning rebates for Test group 2 stocks, the Transaction Fee Pilot should facilitate an informed, data-driven discussion about the effects of access fees and rebates and their impact on order-routing behaviour, and market quality (SEC Release No. 34-82873).

In Europe, MiFID II (Directive 2014/65/EU) and MiFIR (Regulation 600/2014/EU) mandates a reduction in the tick size for European stocks and thereby implicitly reduced the maximum access fees given that the standard practice on European exchanges is to cap fees relative to the tick size.[4] MiFID II also sharpened the regulation of access fees by requiring new incentives on market making agreements under Stress Market Conditions (RTS 8), a maximum Order-To-Trade ratio for each instrument (RTS 9), and a periodic disclosure by exchanges of the percentage of fees and rebates on total turnover (RTS 27). It also bans "cliff-edge" pricing structures in which customer-specific fees are reduced retroactively for market participants that reach a trading volume threshold (RTS 10).

[3] According to the more recent S.E.C. (2018) Release No.34-82873 on Transaction Fee Pilot for NMS Stocks "For maker-taker exchanges, the amount of the taker fee is bounded by the cap imposed by Rule 610(c) on the fees the exchange can charge to access its best bid/offer for NMS stocks. This cap applies to the fees assessed on an incoming order that executes against a resting order or quote, but does not directly limit rebates paid. The Rule 610(c) cap on fees also typically indirectly limits the amount of the rebates that an exchange offers to less than $0.003 per share in order to maintain net positive transaction revenues. For taker-maker exchanges, the amount of the maker fee charged to the provider of liquidity is not bounded by the Rule 610(c) cap, but such fees typically are no more than $0.003, and the taker of liquidity earns a rebate." If the price of a protected quotation is less than $1.00, the access fee is no more than 0.3% of the quotation price per share SEC (2009).

[4] See Article 49 of MiFID II and the following Regulatory Technical Standard 11 (RTS 11, ESMA 2017). ESMA (2015)

Trading fees have been investigated in a small number of theoretical papers. Colliard and Foucault (2012) show in a competitive market with continuous prices that the breakdown between make and take fees has no effects on the cum-fee-spread (net of fees spread) as traders can neutralize changes in fees by making offsetting changes in the aggressiveness of their orders. However, Foucault et al. (2013) show in a single market with a discrete tick size that the make-take breakdown matters for market quality. Panayides et al. (2017) show how a change in trading fees affects market quality when two trading platforms compete for the provision of liquidity. Chao et al. (2018) models optimal access pricing both in a single-market setting and also with competition between multiple markets.

Our analysis builds on this previous research, and particularly on Chao et al. (2018), in several ways: First, we show that optimal access pricing changes qualitatively with the amount of ex ante dispersion in trader valuations. In the absence of regulation, a two-period market has two equilibria, one with maker-taker and one with symmetric taker-maker pricing. However, when regulation caps the maximum access fee (which thereby limits rebates), this no longer is true. When valuation dispersion is low, maker-taker and symmetric taker-maker pricing is still optimal, but when investor valuation dispersion is sufficiently high, then exchanges optimally charge positive fees to both limit-order submitters and market-order submitters.

Second, we show that the market power of liquidity providers changes with the number of trading rounds, which is a proxy for the rate of trading activity. Longer trading games have more opportunities for arriving investors to trade and, thus, have increased potential liquidity. As a result, the optimal trading strategy of an investor arriving at the market in the first period of the three-period trading game differs from in a two-period game. In a three-period market, the investor at time t_1 is no longer a monopolist in the provision of liquidity (as in a two-period market) and therefore must take into account the fact that, at time t_2, the incoming trader may decide to compete and supply liquidity rather than only taking liquidity.[5]

[5] In a 2-period market, the investor arriving in the second period has no choice either than trading at the offered limit order posted by the liquidity supplier in the first period or leaving the market and not trade.

In addition, if the maximum fee is capped by regulation, as in real markets, then moving from a two-period to a three-period market, the take rebate needs to be larger than the make rebate. Therefore when a regulatory cap reduces access fees, the taker-maker pricing is optimal less of the time than maker-taker pricing. This asymmetry between maker-taker and taker-maker is consistent with the empirical observation that the maker-taker pricing structure is more common in current financial markets.

Third, we are the first to model optimal access pricing in a market in which HFT firms using flash orders are present. In this context, we confirm our earlier intuition about investor valuation heterogeneity and access pricing. The HFTs in our model have no private gains-from-trade. Thus, an exchange with HFT firms has more incentive to use a rebate-based pricing structure. In particular, HFTs in our model use flash market orders to provide liquidity to regular investors. However, given that HFTs do not have private value reasons to want to trade, rebates are needed to induce trading when regular investor gains-from-trade are concentrated.

Fourth, we show that optimal access pricing depends on both the absolute tick size and on the relative magnitude of investor valuation dispersion relative to the tick size. Moreover, when fees are capped relative to the absolute tick size, then a larger tick size is favoured by exchanges because it enlarges the degrees of freedom they have to offer rebates.

Fifth, we show that profit-maximizing rebated-based access pricing by exchanges is Pareto improving when investor valuation dispersion is low. However, without HFTs, once investor valuation dispersion is sufficiently large, rebates may still maximize exchange profits, but they can reduce overall welfare relative to a market with no fees and rebates. In contrast, with HFTs, total welfare improves for the whole parameter region in which an exchange optimally uses rebate-based access pricing.

2 Model

This section describes a model of access fees and rebates in a single limit order market. Traders in the model arrive sequentially over a trading day. In general, there are N periods with arrival times denoted as $t_z \in \{t_1, \ldots, t_N\}$. Section 3 considers a specification with two periods, $t_z \in \{t_1, t_2\}$, and then Section 4 extends the analysis to a trading day of three periods, $t_z \in \{t_1, t_2, t_3\}$. This allows us to investigate the relation between access pricing and investor-arrival frequency. The arriving traders are risk-neutral and are each characterized by a private valuation equal to β_{t_z} for the trader arriving at time t_z, where each β_{t_z} is an i.i.d draw from a uniform distribution, $U[\overline{\beta}, \underline{\beta}]$, where $\overline{\beta}$ and $\underline{\beta}$ are the limits of the trader valuation supports. We denote the mean of the valuation support by v, which is constant over time, and call this the ex ante *asset value*. The *support width* is denoted $\Delta = \bar{\beta} - \underline{\beta}$. Traders with extreme β_{t_z} values are more eager to trade by taking liquidity, whereas traders with β_{t_z} values close to v are more willing to supply liquidity. The wider the support, $[\underline{\beta}, \overline{\beta}]$, the higher is the probability that arriving traders will have strong heterogeneous directional demands to trade, such as, e.g., long–term asset managers. The smaller the support $[\underline{\beta}, \overline{\beta}]$, the higher is the probability that arriving traders will prefer to profit as passive liquidity providers. Later, Section 5 extends the model to allow for high frequency traders who have neutral private values for the asset, but who react to limit order book changes faster than regular arriving investors.

Prices are quoted on a discrete price grid $\{\ldots, P_{-k}, \ldots, P_{-1}, P_1, \ldots, P_k, \ldots\}$ centered around the mean of investor valuation v with a fixed tick size τ. The state of the limit order book at time t_z is a vector:

$$L_{t_z} = [D_{t_z}^{P_k}] \tag{1}$$

where $D_{t_z}^{P_k}$ indicates the total limit order depth at price P_k at time t_z. An investor arriving in the market at time t_z and facing a standing limit order book $L_{t_{z-1}}$ can take one of several different possible actions, x_{t_z}: Post a limit buy or sell order LBP_k or LSP_k at one of the available price levels P_k on the price grid, submit a market buy or sell order $MBP_{k(L_{t_{z-1}},MB)}$ or $MSP_{k(L_{t_{z-1}},MS)}$ that is then executed immediately at the best standing ask price $P_{k(L_{t_{z-1}},MB)}$ or bid price $P_{k(L_{t_{z-1}},MS)}$ on the opposite side of the market where the indices $k(L_{t_{z-1}},MB)$ and $k(L_{t_{z-1}},MS)$ denote the best standing quotes given a market buy or sell given the incoming book $L_{t_{z-1}}$ at time t_z, or not trade by submitting no order (NT).[6] An investor opts not to trade when the payoffs on all available actions are negative. Marketable limit orders that cross with the best available bid/ask on the opposite side of the standing book $L_{t_{z-1}}$ are treated as market orders in terms of both order execution and exchange access pricing. The investor action set at time t_z given a standing book $L_{t_{z-1}}$ is denoted as X_{t_z}. In addition, let $X^L \subset X_{t_z}$ denote the set of possible limit orders at time t_z. For tractability, we assume that limit orders cannot be modified or cancelled after submission and that investors can only send one order of unitary size at a time.[7]

The arrival of new limit and market orders augments or reduces the depth of the limit order book respectively, leading to dynamics:

$$L_{t_z} = L_{t_{z-1}} + Q_{t_z} \quad z = 1, \dots, N \tag{2}$$

where $Q_{t_z} = [Q_{t_z}^{P_k}]$ is a vector of changes in the limit order book due to an arriving investor's action $x_{t_z} \in X_{t_z}$ at t_z. The change $Q_{t_z}^{P_k}$ in depth at price P_k is "$+1$" when an arriving limit order LOP_{k,t_z} adds an additional share and "-1" when a market order executes a limit order when P_k is the best bid or offer (BBO), and otherwise is zero (at other prices unaffected by the arriving order). The changes $Q_{t_z}^{P_k}$ are all zero if no order is submitted.

[6] LO and MO indicate generic limit and market orders, and LB (LS) and MB (MS) indicate the buy (sell) trade direction. For simplicity we will refer to $MBP_{k(L_{t_{z-1}},MB)}$ and $MSP_{k(L_{t_{z-1}},MS)}$ as MBP_k and MSP_k, and we will refer to $P_{k(L_{t_{z-1}},MS)}$ and $P_{k(L_{t_{z-1}},MB)}$ as $P_{k,MS}$ and $P_{k,MB}$.

[7] As noted in Parlour and Seppi (2008), such limit orders are essentially "take it or leave it" offers of liquidity.

Consistent with common practice in today's financial markets, the trading platform may set different access fees $\xi(x)$ for different order types x. An investor offering liquidity by posting a limit order faces a *make fee* (MF). An investor taking liquidity via a market order (or via a marketable limit order) pays a *take fee* (TF). The set of fees is denoted as $\Xi = \{MF, TF\}$. Some fees may be negative (i.e., a rebate), in which case it is a cost for the trading platform and a reward for the investor receiving it. Under a *maker-taker* structure, investors submitting market orders pay a take fee ($TF > 0$) to the trading platform, and investors posting limit orders receive a make rebate equal to $-MF > 0$ whenever their limit order executes. In a *taker-maker* structure, the fees and rebates are reverse so that now limit-order submitters pay make fees ($MF > 0$) and market-order submitters receive take rebates ($-TF > 0$). Consistent with current practice, access fees and rebates in our model are subject to regulation. For notational simplicity, we assume the maximum allowable fee (whether take or make) is one tick. Thus, this regulatory constraint on fees is more binding for smaller tick sizes. Appendix C shows how our results depend on such regulatory restrictions and how this accounts for some of the differences between our model and Chao et al. (2018).

Investor order-submission behaviour depend on both quoted prices and exchange fees and rebates. Given a quoted price P_k, the total amount paid or received by an investor net of fees TF and MF is called the *cum-fee price*. Accordingly, let $P_k^{cum,MS} = P_k - TF$ denote the cum-fee price received net of take fees paid to the exchange when using a market order to sell at the quoted price P_k, and let $P_k^{cum,MB} = P_k + TF$ be the cum-fee price paid including take fees paid to the exchange when using a market order to buy at P_k. Similarly, $P_k^{cum,LS} = P_k - MF$ is the cum-fee price for a limit order to sell and $P_k^{cum,LB} = P_k + MF$ is the cum-fee price for a limit order to buy.

Liquidity supply is endogenous in our model. The limit order book opens empty at the first time period t_1, and so an investor arriving at t_1 can only post limit orders to trade. Similarly, in the final round of order submission t_N, investors can only submit market orders to trade (since new limit orders would be unexecuted). In intermediate periods (e.g., t_2 of a three-period trading game), investors can choose between market and limit orders. As in Chao et al. (2018), the tick size τ and trader valuation support $S = [\overline{\beta}, \underline{\beta}]$ are exogenous input parameters in our analysis.

Consider now the investor order-choice problem. An investor arriving at time t_z chooses his order x_{t_z} to maximize his expected payoff:

$$\max_{x_{t_z} \in X_{t_z}} w(x_{t_z} | S, \tau, \Xi, \beta_{t_z}, L_{t_{z-1}}) = \begin{cases} [\beta_{t_z} - P(x_{t_z}) - \xi(x_{t_z})] \, Pr(\theta_{t_z}^{x_{t_z}} | S, \tau, \Xi, L_{t_{z-1}}) & \text{if } x_{t_z} \text{ is a buy order} \\ [P(x_{t_z}) - \beta_{t_z} - \xi(x_{t_z})] \, Pr(\theta_{t_z}^{x_{t_z}} | S, \tau, \Xi, L_{t_{z-1}}) & \text{if } x_{t_z} \text{ is a sell order} \\ 0 & \text{if } x_{t_z} \text{ is } NT \end{cases} \quad (3)$$

where $\Xi = \{TF, MF\}$ is the set of market access fees $\xi(x_{t_z})$, and $P(x_{t_z})$ is the price at which order x_{t_z} trades if it is executed. The notation $\theta_{t_z}^{x_{t_z}}$ denotes the set of future trading states in which an order x_{t_z} submitted at time t_z is executed, and $Pr(\theta_{t_z}^{x_{t_z}} | S, \tau, \Xi, L_{t_{z-1}})$ is the associated probability of execution. For example, if x_{t_z} is a market order, then $P(x_{t_z})$ is the best standing quote on the other side of the market at time t_z, and $Pr(\theta_{t_z}^{x_{t_z}} | S, \tau, \Xi, L_{t_{z-1}}) = 1$, since market orders are executed immediately at the standing bid or ask (if that side of the book is non-empty). If x_{t_z} is a non-marketable limit order, then the execution price $P(x_{t_z})$ is its limit price, and the execution probability $Pr(\theta_{t_z}^{x_{t_z}} | S, \tau, \Xi, L_{t_{z-1}})$ is between 0 and 1. Table 5 in the Appendix shows explicitly the actions available to traders and their associated payoffs.

The optimal order-submission strategy over time is determined — given market access fees Ξ, an incoming book $L_{t_{z-1}}$, and subsequent order-execution probabilities $Pr(\theta_{t_z}^{x_{t_z}} | S, \tau, \Xi, L_{t_{z-1}})$ at a time t_z — by the upper envelope of the collection of linear functions of the investor valuations β_{t_z} corresponding to the expected investor payoffs for the different possible actions in X_{t_z} in (3). The associated optimal order-submission probabilities $Pr(x_{t_z} | S, \tau, \Xi, L_{t_{z-1}})$ are then the probabilities of investor valuations β_{t_z} in between threshold valuations equating expected payoffs for the different profit-maximizing orders along the support $[\underline{\beta}, \bar{\beta}]$ of investor valuations.

An exchange chooses its fees, Ξ, to maximize its expected payoff from completed transactions:

$$\max_{MF,TF} \pi(MF,TF\,|S,\tau) = \left[\sum_{t_z\in\{t_1,\ldots,t_{N-1}\}} \sum_{x_{t_z}\in X^L} Pr(x_{t_z},\theta_{t_z}^{x_{t_z}}\,|S,\tau,\Xi)\right](MF+TF) \tag{4}$$
$$s.t.:\ -\tau < MF,TF < +\tau$$

given the transaction probabilities $Pr(x_{t_z},\theta_{t_z}^{x_{t_z}}\,|S,\tau,\Xi)$ induced by their fees and the equilibrium investor order–submission strategies that maximizes (3), where the transaction probabilities are the product of the probabilities of different limit orders $x_{t_z}\in X^L$ being submitted and their execution probabilities

$$Pr(x_{t_z},\theta_{t_z}^{x_{t_z}}\,|S,\tau,\Xi) = \sum_{L_{t_{z-1}}} Pr(L_{t_{z-1}}|S,\tau,\Xi)\,Pr(x_{t_z}|S,\tau,\Xi,L_{t_{z-1}})\,Pr(\theta_{t_z}^{x_{t_z}}|S,\tau,\Xi,L_{t_{z-1}}). \tag{5}$$

The formula in (5) reflects the fact that, in a limit order market, transactions only occur when limit orders are submitted and then executed. The regulatory constraint in (4) guarantees traders cannot neutralize the trading fee. In particular, investors cannot adjust the prices at which limit orders are posted on a discrete price grid to exactly offset the impact of small changes in access fees and rebates on their net transaction prices. Note that the exchange has non-negative profits since $TF = MF = 0$ is feasible and gives zero profits. Given the optimization problems solved by investors and the exchange, we can now define an equilibrium:

Definition. A *Subgame Perfect Nash Equilibrium* of the trading game is a collection $\{Pr(x_{t_z}|S,\tau,\Xi^*,\beta_{t_z},L_{t_{z-1}})$, $Pr(\theta_{t_z}^{x_{t_z}}\,|S,\tau,\Xi^*,L_{t_{z-1}})$, $\Xi^*\}$ of order-submission probabilities, order-execution probabilities, and access fees such that:

- The equilibrium order-submission probabilities $Pr(x_{t_z}|S,\tau,\Xi^*,\beta_{t_z},L_{t_{z-1}})$ are the probabilities of optimal orders for investors computed from their optimization problem (3) given the equilibrium execution probabilities $Pr(\theta_{t_z}^{x_{t_z}}|S,\tau,\Xi^*,L_{t_{z-1}})$.

- The order-execution probabilities $Pr(\theta_{t_z}^{x_{t_z}}|S,\tau,\Xi^*,L_{t_{z-1}})$ for an order x_{t_z} submitted at time t_z are consistent with the equilibrium order-submission probabilities $Pr(x_{t_{z'}}|S,\tau,\Xi^*,\beta_{t_{z'}},L_{t_{z'-1}})$ at times $t_{z'} > t_z$.

- The access fees Ξ^* are optimal for the exchange given its optimization problem (4) given the equilibrium order-submission probabilities $Pr(x_{t_z}|S,\tau,\Xi^*,\beta_{t_z},L_{t_{z-1}})$.

Using first principles, we have the following existence result for our model:

Theorem 1. *The equilibrium of a trading game with N periods and a price grid with a fixed number of prices exists and can be constructed analytically via backwards induction.*

Proofs for general N-period models are in Appendix A. Further details for specific versions of the model are in Appendices B through E. However, the functional forms can become complex as the number of periods grows and as the number of possible limit prices increases — i.e., as more limit orders become feasible a priori as larger investor valuation supports encompass more prices or as the price grid becomes finer. As a practical matter, therefore, sometimes (e.g., as in the 3-period model) the first-order conditions that give exchange's equilibrium optimal fees in the final step of the equilibrium derivation are more easily solved numerically. In these cases, rather than explicitly differentiating the analytic exchange expected profit function, we instead evaluated it numerically and use a search algorithm to solve the first-order conditions for Ξ^*.

3 Results for the 2-period trading game

This section examines the 2-period version of the general model ($\{t_1,t_2\}$) centered around a mean asset value normalized to $v = 10$.[8] We consider two possible price grids with different tick sizes. In a large-

[8]Our results are unchanged for other values of v if the price grids and trader-valuation supports are adjusted up or down.

tick market (LTM), the tick size τ is normalized to 1, and the price grid has four possible price levels, $P_k = \{P_{-2}, P_{-1}, P_1, P_2\}$, centered around the mean investor valuation v with $P_{-2} < P_{-1} < v < P_1 < P_2$. The outside quotes are $P_2 = v - \frac{3}{2}\tau$ and $P_{-2} = v + \frac{3}{2}\tau$, and the inside quotes are $P_1 = v - \frac{1}{2}\tau$ and $P_{-1} = v + \frac{1}{2}\tau$. In a small-tick market (STM), the tick size is smaller — which we set here to $\frac{\tau}{3}$ relative to the LTM tick size — and the price grid has ten price levels $p_j = \{p_{-5}, p_{-4}, p_{-3}, p_{-2}, p_{-1}, p_1, p_2, p_3, p_4, p_5\}$, with $p_{-5} < \ldots < p_{-1} < v < p_1 < \ldots < p_5$. The outside quotes of the STM coincide with the outside quotes of the LTM with $p_{-5} = P_{-2} = v - \frac{3}{2}\tau$ and $p_5 = P_2 = v + \frac{3}{2}\tau$.[9]

Our analysis allows for a wide range of trader-valuation supports $S = [\underline{\beta}, \bar{\beta}]$. The smallest support we consider, $[9.8333, 10.1667]$, has a support width Δ of $0.33\,\tau$ and is within the inside quotes of the LTM. This is a market environment in which arriving traders are predisposed to supply liquidity since individual potential gains-from-trade are small. This support is also equal to the inside spread of the STM. The largest support we considered, $[7.50 - 12.50]$, has a width of $5\,\tau$, and corresponds to a market populated by very heterogeneous traders, some of whom have strong trading demands (and prefer to take available liquidity) and others with weaker trading demands (who tend to supply liquidity). The rationale for the specific choice for our largest support is that it is the largest support such that in equilibrium traders never want to post limit orders beyond the outside quotes $\{P_{-2}, P_2\}$ in the LTM or $\{p_{-5}, p_5\}$ in the STM. These two supports let us compare investor and exchange behavior given different ex ante investor valuation dispersion across large and small tick markets. We also consider supports $[\underline{\beta}, \bar{\beta}]$ in between these two extreme cases.

Figure 1 shows the equilibrium optimal fees MF (blue line) and TF (orange line) chosen by the exchange for both the LTM (upper plot) and the STM (lower plot) given the various trader–valuation support widths Δ on the horizontal axes. The gray regions highlight equlibria with taker-maker and maker-taker access pricing involving rebates on market or limit orders. Outside of the gray region, equilibrium access fees are

[9] In real markets, if trading platforms with different tick sizes coexist, then the prices on wider price grids are also on the denser narrow price grids.

non-negative with no rebates. Note that multiple equilibria are possible. For support widths inside the gray region, taker-maker pricing ($TF < 0$ and $MF > 0$) on the left side and maker-taker pricing ($MF < 0$ and $TF > 0$) on the right side are both optimal. Moreover, the taker-maker and maker-taker pricing structures are symmetric here. In contrast, the equilibria with access pricing with strictly positive fees ($MF, TF > 0$) outside the rebate-based (grey) region are unique — i.e., they are identical for support widths $\Delta > 3\,\tau$ on both sides of Figure 1.

Table 1 provides additional details about equilibrium strategies and market properties for the LTM. It reports the equilibrium trading fees and the buyer's trading strategies associated with each support considered here, the cum-fee buy and sell transaction prices $P_k^{cum,LB}$ and $P_k^{cum,MS}$, the equilibrium probabilities of the buyer's order submission ($Pr(x_{t_z}|S,\tau,\Xi,L_{t_{z-1}})$) and execution ($Pr(\theta_{t_z}^{x_{t_z}}|S,\tau,\Xi,l_{t_{z-1}})$), and the equilibrium expected total exchange profit ($\pi(MF,TF|S,\tau)$) associated with each support. When there are two rows for a particular support, they are for the respective maker-taker and taker-maker equilibria.[10] The results are symmetric for limit sells at time t_1. Table 2 in Section 3.2 below provides similar details for the STM.

A general issue explored in our analysis is the relation between the profit-maximizing access pricing for an exchange and, on the other hand, the support S of trader private valuations and the tick size τ. In particular, optimal access pricing is driven by both the relative size of the valuation support width Δ to the tick size τ and also by the absolute tick size τ by itself given that the regulatory cap on fees is tied to the absolute tick size. We explore these issues using two types of comparative statics: First, we hold the tick size τ fixed and vary the trader valuation support width Δ, which changes the amount of potential gains-from-trade. This comparative static describes the effects of the relative valuation-support/tick-size channel alone. Second, we change the tick size τ by comparing LTMs and STMs given the same range of valuation

[10]To economize space, Table 1 does not report the equilibrium strategies of the seller arriving at t_2 as they can be inferred from the buyer's equilibrium strategies at t_1. For example, if a limit buy is posted at t_1, (LBP_1), the equilibrium strategy of the seller taking liquidity at t_2 will be a MSP_1 market sell. In addition, Table 1 does not report the probability of No Trade as it is the complement to 0.5 of the probability of order submission on one side of the market. For example, for the support $[9.8333, 10.1667]$ with the smallest width 0.33τ, the probability of No Trade at t_1 is $0.5 - 0.333 = 0.167$.

supports. This second comparative static depends on both the relative ratio channel and the absolute tick-size channel.

3.1 Large Tick Market

Our 2-period LTM analysis uses equilibria constructed using the analytic recursion in Theorem 1 to demonstrate various equilibrium properties of the the 2-period LTM. Figure 1 and Table 1 provide specific detail.

Proposition 1. *When investor valuation dispersion is low (in that the investor valuation support width is $\Delta < 3\tau$) both maker-taker and taker-maker equilibria exist in the 2-period LTM with fees and rebates that are symmetric. When investor valuation dispersion is higher (in that the valuation support width is $\Delta \in [3\tau, 5\tau]$) the equilibrium fees TF and MF can be jointly positive and unique.*

Proposition 2. *When an exchange optimally uses maker-taker or taker-maker rebate-based access pricing in the 2-period LTM, then rebates are decreasing and fees can be increasing as the trader-valuation support width Δ increases.*

Figure 1 demonstrates these results. To start, consider the LTM with a very narrow trader-valuation support width $0.33\,\tau$. In Figure 1, we see that the LTM has a pair of symmetric equilibria for this support, one with maker-taker pricing and one with taker-maker pricing. Since this valuation support is within the inside LTM quotes, P_{-1} and P_1, there are no prices at which buyers and sellers would transact in the absence of rebates. Thus, a rebate is necessary either on the liquidity-maker or -taker side for investors to be able to trade profitably. Consider a potential buyer with a high personal valuation β_{t_1} who arrives at t_1. (The case of a potential seller with a low valuation at t_1 is symmetric). With maker-taker pricing ($TF = 0.556$ and $MF = -0.444$), the exchange offers a rebate on liquidity-making via limit orders such that the buyer is willing to use an aggressive LBP_1 limit order at t_1 to offer to buy at a quoted price P_1 above his valuation ($\beta_{t_1} \leq \bar{\beta} < P_1$) to earn the make rebate. An investor with a low personal valuation β_{t_2} arriving at t_2 can then sell at P_1 above his valuation ($\beta_{t_2} \leq \bar{\beta} < P_1$) but must also pay a take fee. In this case, maker-taker pricing generates trading by subsidizing liquidity-making via limit orders at aggressive posted prices at t_1 and imposing fees on liquidity-taking via market orders at t_2 (which benefit from the aggressive limit prices). The converse logic applies to the taker-maker equilibrium pricing ($MF = 0.556$ and $TF = -0.444$). Now investors with high personal valuations at t_1 use LBP_{-1} limit orders to try to buy at P_{-1}, and investors at t_2 then either use MSP_{-1} market orders to sell at P_{-1} and receive the take rebate, or they do not trade. In each case, the reason this works is that investor trading decisions depend on the cum-fee prices they pay or received net of market access fees and rebates (rather than on just quoted prices alone), and the exchange can use its access pricing to affect the cum-fee prices.

Figure 1: Make Fees and Take Fees in a 2-Period Market. This figure reports the equilibrium make fees (MF & mf) and take fees (TF & tf) in the Large Tick Market (LTM) (upper panel) and Small Tick Market (STM) (lower panel) corresponding to different investor valuation supports with widths ranging from $0.33\,\tau$ to $5\,\tau$ on the horizontal axes (where $\tau = 1$ is the tick size in the LTM). The figure reports in blue (orange) italics the equilibrium fees MF (TF). The left (right) side of the gray region in the figure gives the equilibrium fees for taker-maker (maker-taker) access pricing. The taker-maker and maker-taker pricing structures are optimal and symmetric.

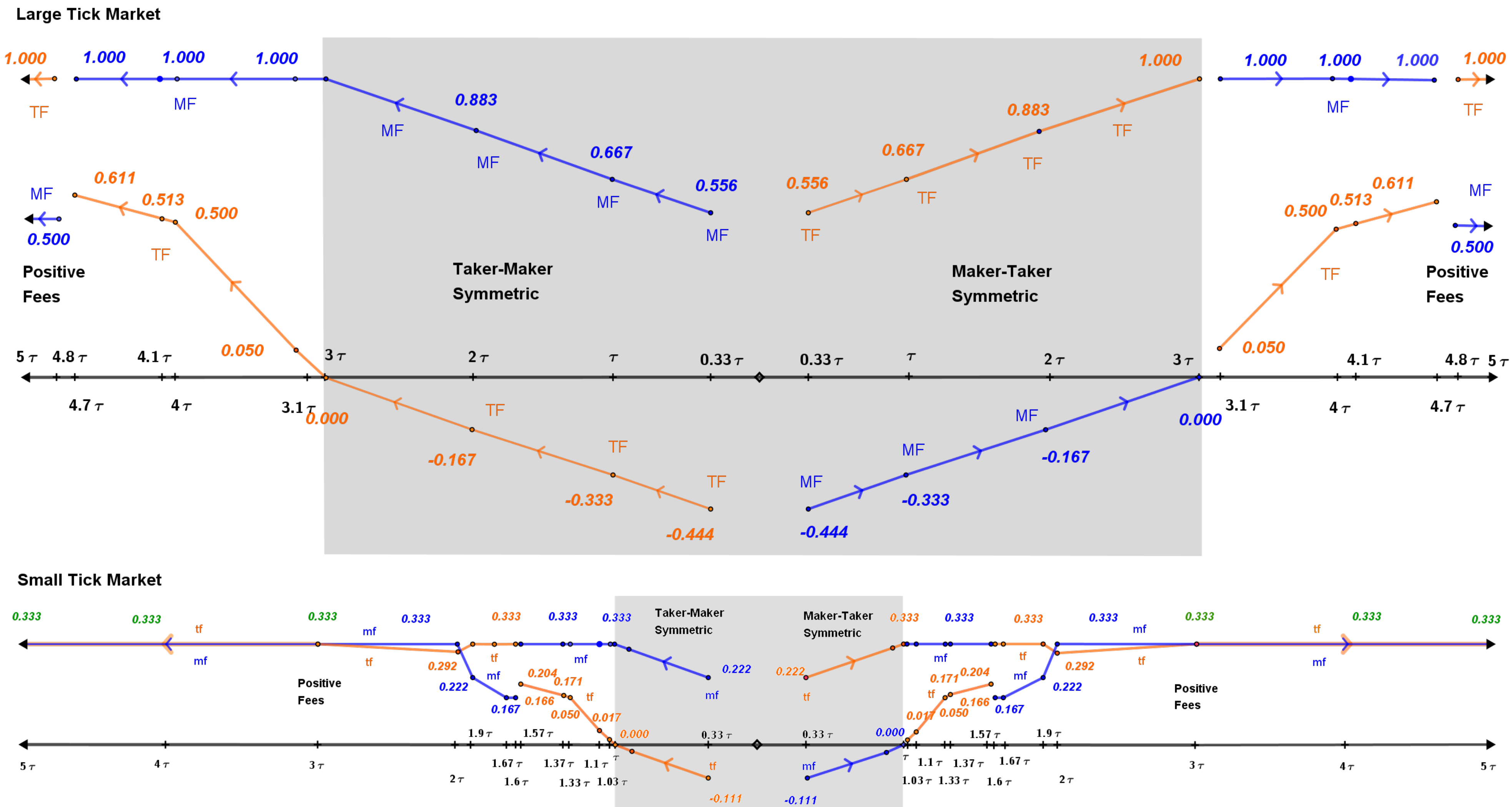
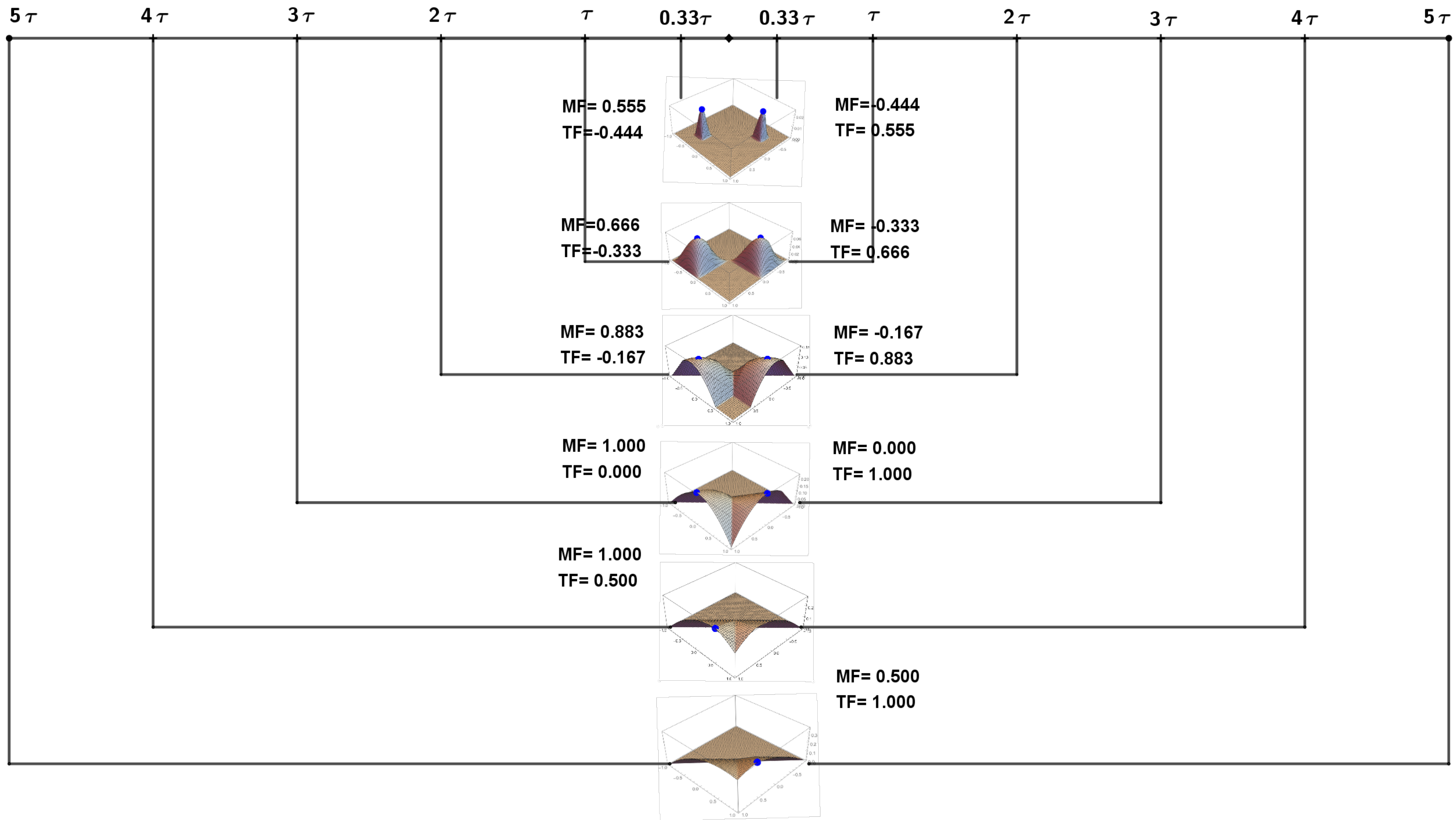


Table 1: 2-Period Large Tick Market (LTM): Equilibrium Fees and Trading Strategies. This table reports for different investor valuation support width, $\Delta = \overline{\beta} - \underline{\beta}$ expressed in terms of the LTM tick size, τ (column 1), the extreme values of the support, $\underline{\beta}$ and $\bar{\beta}$ (column 2), the equilibrium make and take fees, MF and TF (column 3 and 4), the buyer's equilibrium trading strategies at t_1, x_{t_1} other than No Trade (column 5) and the associated probability of submission at t_1, $Pr(x_{t_1}|S,\tau,\Xi,L_{t_0})$ (column 6). The table also shows the cum-fee buy and sell prices ($P_k^{cum,LB}$ and $P_k^{cum,MS}$) (column 7 and 8), the equilibrium probability of execution of the buyer's order posted at t_1, $Pr(\theta_{t_1}^{x_{t_1}}|S,\tau,\Xi,l_{t_0})$, which correspond to the unconditional probability of MS at t_2 (column 9) and the exchange expected profit from both buyers and sellers, $\pi(MF,TF|S,\tau)$ (column 10). When the equilibrium pricing is rebate based, for each support we report first the taker-maker set of fees and then the maker-taker set of equilibrium MF and TF. Results are rounded to the third decimal.

Support width $\Delta = \overline{\beta} - \underline{\beta}$	$\underline{\beta}, \bar{\beta}$	MF	TF	Eq.Strategy x_{t_1} at t_1	Pr. Submission $Pr(x_{t_1}\|S,\tau,\Xi,L_{t_0})$	$P_k^{cum,LB}$	$P_k^{cum,MS}$	Pr. Execution $Pr(\theta_{t_1}^{x_{t_1}}\|S,\tau,\Xi,l_{t_0})$	Exchange E[Profit] $\pi(MF,TF\|S,\tau)$
0.33 τ	9.833, 10.167	0.556	-0.444	LBP_{-1}	0.333	10.056	9.944	0.333	0.025
0.33 τ	9.833, 10.167	-0.444	0.556	LBP_1	0.333	10.056	9.944	0.333	0.025
τ	9.500, 10.500	0.667	-0.333	LBP_{-1}	0.333	10.167	9.833	0.333	0.074
τ	9.500, 10.500	-0.333	0.667	LBP_1	0.333	10.167	9.833	0.333	0.074
2 τ	9.000, 11.000	0.833	-0.167	LBP_{-1}	0.333	10.333	9.667	0.333	0.148
2 τ	9.000, 11.000	-0.167	0.883	LBP_1	0.333	10.333	9.667	0.333	0.148
3 τ	8.500, 11.500	1.000	0.000	LBP_{-1}	0.333	10.500	9.500	0.333	0.222
3 τ	8.500, 11.500	0.000	1.000	LBP_1	0.333	10.500	9.500	0.333	0.222
3.1 τ	8.450, 11.550	1.000	0.050	LBP_{-1}	0.339	10.500	9.450	0.323	0.229
4 τ	8.000, 12.000	1.000	0.500	LBP_{-1}	0.375	10.500	9.000	0.250	0.281
4.1 τ	7.950, 12.050	1.000	0.513	LBP_{-1} , LBP_{-2}	0.369, 0.131	10.500, 9.500	8.987, 7.987	0.253, 0.009	0.286
4.7 τ	7.650, 12.350	1.000	0.611	LBP_{-1} , LBP_{-2}	0.342 , 0.157	10.500 , 9.500	8.889 , 7.889	0.264 , 0.051	0.317
4.8 τ	7.600, 12.400	0.500	1.000	LBP_1 , LBP_{-1}	0.104, 0.396	11.000, 10.000	9.500, 8.500	0.396, 0.188	0.346
5 τ	7.500, 12.500	0.500	1.000	LBP_1 , LBP_{-1}	0.100 , 0.400	11.000 , 10.000	9.500 , 8.500	0.400 , 0.200	0.360

Figure 2: 2-Period Large Tick Market (LTM): Exchange Expected Profit Function and Access Pricing. This figure shows the exchange profit function and the equilibrium make fees and take fees for the LTM corresponding to different investor valuation supports with widths ranging from $0.33\,\tau$ to $5\,\tau$ (where $\tau = 1$ is the tick size in the LTM) as reported on the horizontal axis. The three-dimensional figures indicate (blue dots) the optimal make fee (MF), the optimal take fee (TF), and the associated equilibrium exchange expected profit for each support.



The profit-maximizing access-pricing structure depends on the relationship between the support of traders' evaluation and the tick size. Intuitively, as the support of investor valuations increases, the potential gains-from-trade increase, which increases investor trading demand. As a result, the exchange has less of a need to incentivize trading. Thus, the exchange, in equilibrium, exploits investors' greater ex ante gains-from-trade by increasing fees and reducing rebates. This happens with both taker-maker and maker-taker pricing. Starting from the smallest valuation support, $0.33\,\tau$, Figure 1 and Table 1 show that the exchange monotonically increases both MF and TF as the support width Δ increases up until the point that the regulatory cap on fees binds. For example, when the support width reaches $2\,\tau$, the buyer still buys either at P_{-1} or at P_1 and the exchange sets the symmetric taker-maker and maker-taker fee structure with a positive fee of 0.883 and rebate of -0.167. Taker-maker and maker-taker access pricing persists until, holding the LTM tick size fixed at τ, the investor valuation support width Δ reaches the outside quotes with $\Delta = 3\,\tau$.

Proposition 1 and Figure 1 show that three things happen once $\Delta > 3\,\tau$: First, investor trading demand is sufficiently strong that the exchange ceases giving rebates to incentivize trade and switches instead to a strictly positive-fee access pricing structure. Second, the regulatory cap on fees is reached on one side of the market. Third, the optimal access pricing structure becomes unique. As a result, at this point, the exchange starts charging the highest possible make fee on limit orders given the regulatory cap, $MF = 1.000$, and also charges a positive take fee for market orders. For example, when the support width is $3.1\,\tau$, the optimal take fee is $TF = 0.050$. In these parameterizations, low-valuation investors still profitably sell at the low price P_{-1}. In equilibrium, a high-valuation investor arriving at t_1 knows that, given the wide valuation support and the relatively low TF, there is a sufficiently high probability of a seller arriving in period t_2 willing to demand liquidity at the lower price P_{-1}. Strictly positive fee equilibria are new relative to Chao et al. (2018), who find only rebate-based access pricing. The reasons for the difference between our results and CYY are considered in Appendix C.

The fact that the optimal equilibrium access pricing structure can be unique is also new. For example, numerical calculation (not reported) verifies that using hypothetical symmetric fees $MF = 0.050$ and $TF = 1.000$ leads to lower exchange expected profits than using the equilibrium fees $MF = 1.000$ and $TF = 0.050$ when $\Delta = 3.1\tau$. The reason illustrates a significant asymmetry between make and take fees. In the context of this two-period market, market orders are only used at time t_2 and, thus, take fees simply affect the willingness of the investor at time t_2 to trade with whatever limit orders happen to be in the book. In contrast, limit-order submitters at time t_1 have a decision about the price at which they optimally choose to post a limit order. As a result, make fees potentially affect a more complicated decision between multiple order-submission alternatives for limit-order submitters (i.e., as opposed to the trade/no-trade decision of market-order submitters). In the $\Delta = 3.1\tau$ example, the symmetric fees are suboptimal because, with a hypothetical make fee of 0.050, buy limit orders at P_{-1} and P_1 both have positive expected profits and, given a sufficiently low make fee (i.e., 0.050) and a wide investor valuation support (e.g., $\Delta = 3.1\tau$) — such that there is a sufficient probability of investors at t_2 with very low private valuations who would be willing to sell at a low cum-fee price of $P_{-1} - TF$, — there are some investors at t_1 (with valuations slightly above v) who would post buy limit orders at P_{-1} rather than at P_1. Since such orders have lower execution probabilities than limit orders at P_1, this reduces exchange expected profits relative to the equilibrium fees, thereby making the hypothetical symmetric fees $MF = 0.050$ and $TF = 1.000$ suboptimal. As we will see, our three-period model in Section 4 shows there is a related asymmetry in make and take fees in multi-period markets.

In general, changes in the equilibrium investor strategy at t_1 coincide with changes in the exchange's optimal fee structure. We see this clearly in Table 1 and Figure 1. As the valuation support width Δ increases beyond $4\,\tau$ in the region with strictly positive fees, the buyers start using two possible different limit orders at t_1 — i.e., they now buy at P_{-1} or at P_{-2} — for two different intervals of β_{t_1}. While at $\Delta = 4\,\tau$ the buyer has no incentive to buy at P_{-2} (a seller with the minimum possible valuation 8 would not sell at the cum fee sell price $P_{-2}^{cum,MS} = 8.5 - 0.5 = 8$), at a wider support, e.g., $\Delta = 4.1\tau$, the buyer does have an incentive to post orders at 8.5 as the incoming seller even with the minimum valuation 7.95 would be willing to sell at $P_{-2}^{cum,MS} = 8.5 - 0.513 = 7.987$. The exchange exploits the larger gains-from-trades of the sellers by setting a higher TF, and keeps charging the buyer the maximum $MF = 1.000$ up until the support width reaches $\Delta = 4.8\,\tau$. Once $\Delta \geq 4.8\,\tau$, the buyer switches from using LBP_{-2} to using LBP_1, and the exchange halves the MF to 0.500 and increases the TF to 1.000.

Figure 2 illustrates the exchange's expected profit function for different combinations of fees and rebates given different investor valuation supports. The blue dots denote profit-maximizing combinations of make and take fees. The symmetric pairs of profit-maximizing MF and TF are clearly visible when the investor valuation supports are narrow. However, there is a unique profit-maximizing set of fees once the valuation support is large enough. The intuition for the asymmetry between maker-taker and taker-maker pricing is that additional prices (at P_{-2} and P_2) become a priori possible at time t_1 once the width is $\Delta > 3\,\tau$. The exchange can use make fees MF to directly control the expected profit on these multiple possible limit orders so as to incentivize investors at t_1 to submit orders that maximize the exchanges profits. In contrast, take fees TF only affect the order-submission behavior of investors at t_1 indirectly via its impact on the trading behavior of the investor at t_2.

Proposition 3. *The sum of the make and take fees is one third of the support width, $MF + TF = \Delta/3$ for all support widths $\Delta < 3\,\tau$ in the 2-period LTM.*

This property — which can be verified numerically in Table 1 — is proven analytically in Appendix B. The key part of the proof is that the exchange's expected profit can be expressed as

$$\pi(MF, TF|S, \tau) = 2\max\left\{0, \frac{\bar{\beta} - P_{-1}^{cum,LB}}{\Delta}\right\}(MF + TF)\max\left\{0, \frac{P_{-1}^{cum,MS} - \underline{\beta}}{\Delta}\right\} \tag{6}$$

which is the product of the relevant limit-order submission probability at time t_1 (just one possible buy limit order in equilibrium), the net fee, and the relevant market-order submission probability at time t_2 (i.e., so that the earlier limit order is executed). The specific function form of this expression follows from the uninform valuation distribution assumption and symmetry between the buy and sell sides of the market. Given this representation, we note that the three components $\bar{\beta} - P_{-1}^{cum,LB}$, $MF + TF = P_{-1}^{cum,LB} - P_{-1}^{cum,MS}$, and $P_{-1}^{cum,MS} - \underline{\beta}$ in (6), when they are positive, sum to the valuation support width Δ. It can then be shown that the product in (6) is maximized by the exchange choosing MF and TF to set these three components equal to each other, which implies that $MF + TF = \Delta/3$.

Proposition 1 states that, given a tick size τ, the optimal fee structure depends on the support of traders' evaluations and therefore on the types of traders populating the market. This leads to an empirical prediction:

Empirical Prediction 1: *Markets populated by traders with low valuation dispersion optimally have taker-maker and maker-taker access pricing. Conversely, markets populated by traders with high valuation dispersion optimally have a unique positive-fee access pricing.*

The smaller the support of the traders' evaluation, the more likely the traders will act as liquidity providers, whereas the larger the support of the traders' evaluations the more likely traders will act as hedge funds managers who do not generally trade to speculate on small price increments. Within the logic of our model, high frequency trading firms can be characterized as having asset valuations equal to the fundamental asset value (v), and these results hint at a more general conclusion that we discuss in Section 5 when we will extend the model to include HFT firms.

3.2 Small Tick Market

We next consider the effects of a smaller tick size on the optimal fee structure. The results for the STM with a tick size of $\tau/3$ are in the lower panel of Figure 1 and in Table 2.[11] Given a smaller tick size, as the support of traders' valuations increases, the exchange still has an incentive to increase both its make fee, MF, and take fee, TF. Thus, access pricing changes in the same direction as in the LTM. However, the pricing structure reaches the threshold when both fees are positive earlier since the regulatory cap on fees (which is tied to the tick size) binds sooner. Figure 1 shows that when the support is $[p_{-2}, p_2]$, which corresponds to a width τ in the STM and which is equivalent to the support $[P_{-1}, P_1]$ with width τ in the LTM, the optimal STM access pricing structure has positive fees on both the take and make sides ($MF = 0.333$ and $TF = 0.000$), whereas the optimal LTM access fee structure with the same valuation support $[p_{-2}, p_2]$ is still the symmetric taker-maker and maker-taker pricing.[12] Thus, the exchange's optimal access pricing Ξ^* depends on both the absolute tick size (given the regulatory restriction on fees relative to the tick size) and the relative size of the investor valuation support compared to the tick size.

Figure 1 (lower panel) and Table 2 show that, as for the LTM, when the investors' support widens, the incentive for the STM exchange to offer rebates decreases. All else equal, given a regulation capping fees to be smaller than the tick size (in Appendix C we relax this assumption), when the tick size is smaller the exchange may only set smaller fees (both positive and negative). We show that by starting from the same smallest support as per the LTM, the region of the traders' support consistent with the exchange profitably offering the taker-maker or the maker-taker fee structures (grey regions in Figure 1) is narrower, and the fees themselves are smaller in absolute values.

[11] The exchange profit functions and their maximizers are qualitatively similar in the STM to the Figure 2 for the LTM.

[12] Since the STM tick size is $1/3$ of LTM tick size ($\frac{1}{3}\tau$), the STM equilibrium fees are equal to $\frac{1}{3}$ of the LTM equilibrium fee computed for a support three times larger (e.g., τ). To ease the comparison between the STM and the LTM, we provide finer numerical detail for the STM in the regions of the valuation support where there are discontinuities in optimal access pricing. These correspond to support widths in the LTM where there are discontinuities in access pricing divided by three.

We conclude that our results in the STM are qualitatively the same as in the LTM except that the STM exchange reaches the regulatory fee cap sooner. Hence, in the small tick market the exchange has fewer degrees of freedom to maximize profits by setting a taker-maker or maker-taker fee structure. The results from the STM lead to our next proposition:

Proposition 4. *When the tick size is smaller, the exchange has a smaller incentive to offer rebates in maker-taker and taker-maker fee structures, and the optimal fees can be smaller.*

Proposition 4 for the STM confirm that it is not just the absolute value of the tick size that matters when determining the optimal fee structure but rather the relation between the tick size and the width of the trader valuation support. More precisely, when the tick size is smaller, the exchange has less degree of freedom in setting the trading fees and this leads to our second empirical prediction:

Empirical Prediction 2: *When, holding the trading population constant, the tick size increases (decreases), the exchange has an incentive to offer greater (smaller) rebates.*

Our empirical prediction can be tested by investigating how a change in the tick size alters the incentive for the exchange to offer rebates. Our model predicts that when, all else equal, the tick size increases, the exchange, to attract volume, should increase the rebates offered to the same population of market participants. However, with competition, if the exchange does not adjust the rebates to the new tick size, it runs the risk of seeing orders migrating to other more profitable venues. Comerton-Forde et al. (2019) investigate the effects of an increase in the tick size within the U.S. tick size pilot program started in October 2016 and, interestingly, find that following the increase in the U.S. tick size from 1 penny to 5 pennies a substantial amount of orders migrated from the maker-taker to the taker-maker inverted fees platforms. This finding is consistent with our model's prediction that following an increase in the tick size the exchange should offer greater rebates to ensure that volume is maximized within a trading platform.

Table 2: 2-Period Small Tick Market (SMT): Equilibrium Fees and Trading Strategies. This table reports for different investor valuation support width, $\Delta = \overline{\beta} - \underline{\beta}$ still expressed in terms of the LTM tick size τ (column 1), the extreme values of the support, $\underline{\beta}$ and $\bar{\beta}$ (column 2), the equilibrium make and take fees (MF and TF) (column 3 and 4), the equilibrium trading strategies at t_1, x_{t_1} other than No Trade (column 5) and the associated probability of submission at t_1, $Pr(x_{t_1} | S, \frac{\tau}{3}, \Xi, L_{t_0})$ (column 6). The table also shows the cum-fee buy and sell prices ($P_j^{cum,LB}$ and $P_j^{cum,MS}$) (column 7 and 8), the probability of execution of the order posted at t_1, $Pr(\theta_{t_1}^{x_{t_1}} | S, \frac{\tau}{3}, \Xi, l_{t_0})$, which correspond to the unconditional probability of MS at t_2 (column 9) and the exchange expected profit from both buyers and sellers, $\pi(MF, TF | S, \frac{\tau}{3})$ (column 10). When the equilibrium pricing is rebate based, for each support we report first taker-maker set of fees and then the maker-taker set of equilibrium MF and TF. .

Support $\Delta = \overline{\beta} - \underline{\beta}$	$\underline{\beta}, \bar{\beta}$	MF	TF	Eq. Orders x_{t_1} at t_1	Pr. Submission $Pr(x_{t_1} \| S, \frac{\tau}{3}, \Xi, L_{t_0})$	$P_j^{cum,LB}$	$P_j^{cum,MS}$	Pr. Execution $Pr(\theta_{t_1}^{x_{t_1}} \| S, \frac{\tau}{3}, \Xi, l_{t_0})$	Exchange E[Profit] $\pi(MF, TF \| S, \frac{\tau}{3})$
$0.33\,\tau$	9.833, 10.167	0.222	-0.111	LBp_{-1}	0.333	10.056	9.944	0.333	0.025
$0.33\,\tau$	9.833, 10.167	-0.111	0.222	LBp_1	0.333	10.056	9.944	0.333	0.025
τ	9.500, 10.500	0.333	0.000	LBp_{-1}	0.333	10.167	9.833	0.333	0.074
τ	9.500, 10.500	0.000	0.333	LBp_1	0.333	10.167	9.833	0.333	0.074
$1.03\,\tau$	9.485, 10.515	0.333	0.017	LBp_{-1}	0.338	10.167	9.816	0.322	0.076
$1.1\,\tau$	9.450, 10.550	0.333	0.050	LBp_{-1}	0.348	10.167	9.783	0.303	0.081
$1.33\,\tau$	9.333, 10.667	0.333	0.166	LBp_{-1}, LBp_{-2}	0.375, 0.126	10.167, 9.833	9.667, 9.334	0.251, 0.001	0.085
$1.37\,\tau$	9.315, 10.685	0.333	0.171	LBp_{-1}, LBp_{-2}	0.369, 0.131	10.167, 9.833	9.662, 9.329	0.253, 0.009	0.095
$1.57\,\tau$	9.215, 10.785	0.333	0.204	LBp_{-1}, LBp_{-2}	0.353, 0.147	10.167, 9.833	9.629, 9.296	0.263, 0.051	0.106
$1.6\,\tau$	9.200, 10.800	0.167	0.333	LBp_1, LBp_{-1}	0.104, 0.396	10.334, 10.050	9.834, 9.500	0.396, 0.188	0.115
$1.67\,\tau$	9.165, 10.835	0.167	0.333	LBp_1, LBp_{-1}	0.100, 0.400	10.334, 10.050	9.834, 9.500	0.400, 0.200	0.120
$1.9\,\tau$	9.050, 10.950	0.222	0.333	LBp_1, LBp_{-1}	0.059, 0.351	10.389, 10.055	9.834, 9.500	0.412, 0.237	0.125
				LBp_{-2}	0.091	9.722	9.167	0.061	
$2\,\tau$	9.000, 11.000	0.333	0.292	LBp_{-1}, LBP_{-2}	0.313, 0.187	10.167, 9.833	9.541, 9.208	0.271, 0.104	0.130
$3\,\tau$	8.500, 11.500	0.333	0.333	LBp_{-1}, LBp_{-2}	0.222, 0.222	10.167, 9.833	9.500, 9.167	0.333, 0.222	0.173
				LBp_{-3}	0.056	9.500	8.834	0.111	
$4\,\tau$	8.000, 12.000	0.333	0.333	LBp_{-1}, LBp_{-2}	0.167, 0.167	10.166, 9.833	9.500, 9.167	0.375, 0.292	0.194
				LBp_{-3}	0.167	9.500	8.834	0.208	
$5\,\tau$	7.500, 12.500	0.333	0.333	LBp_{-1}, LBp_{-2}	0.133, 0.133	10.166, 9.833	9.500, 9.167	0.400, 0.333	0.204
				LBp_{-3}, LBp_{-4}	0.133, 0.100	9.500, 9.167	8.834, 8.500	0.267, 0.200	

4 Results for the 3-Period trading game

This section extends our model to three periods ($\{t_1, t_2, t_3\}$) and shows how the equilibrium changes relative to the 2-period equilibrium. Two key intuitions drive these changes. First, in the 2-period model in Section 3, investors in the first period are monopolists in supplying liquidity since there is no opportunity for later traders to compete against the first-period trader's limit orders. In particular, investors at t_2 can only accept or decline liquidity offered by the limit order posted at time t_1 since the game ends after t_2. Once there are more than two periods, the first-period liquidity supply is no longer monopolistic and some amount of intertemporal competition in liquidity supply is possible. Second and relatedly, there is a higher level of trading activity when there are more rounds of investor-arrival. The fact that more traders arrive over time increases the probability of limit order execution.

The equilibrium construction of our 3-period model is entirely analytic, based on Theorem 1, up to the final step of solving the exchange's first-order conditions. However, due to the complexity of the 3-period exchange profit function, we report numerical equilibrium fees and rebates obtained using a combination of a Simulated Annealing (SA) algorithm together with grid search to refine our results once the optimal region has been identified (Appendices C and D).

Figure 3 shows the equilibrium make fees and take fees for the 3-period model for different investor valuation supports.[13] Many of the results for the 3-period model are similar to the 2-period model. There is still a highlighted grey region of investor valuation supports with both taker-maker and maker-taker equilibria and, again, as the investor valuation support width Δ increases, the exchange increases both MF and TF subject to the regulatory cap, and eventually there is a unique equilibrium with strictly positive fees. However, there are also some differences. To help explain these differences, Table 3 shows the equilib-

[13]Once again, the 3-period exchange profit functions look qualitatively similar to those for the 2-period exchange modulo the asymmetry discussed below.

rium strategies for the 3-period LTM market, together with the order-submission probabilities at t_1, $Pr(x_{t_1}$, the execution probabilities, $Pr(\theta_{t_1}^{x_{t_1}}|S, \tau, \Xi, l_{t_0})$, the equilibrium fees, MF and TF, the equilibrium cum-fee prices for buy limit orders ($P_k^{cum,LB}$) and sell market orders ($P_k^{cum,MS}$), and the exchange expected profit, $\pi(MF, TF|S, \tau)$.

Proposition 5. *The set of valuation supports associated with rebates can be smaller in the 3-period model. In addition, fees can be larger and rebates can be smaller in the 3-period model.*

Comparing Figures 1 and 3 shows that the grey region with rebate-based access pricing (maker-taker or taker-maker) is smaller in the 3-period framework. The largest support associated with the rebate-based pricing is $[8.85, 11.15]$ (with a width of $2.3\,\tau$) in the 3-period market as opposed to $[8.50, 11.50]$ (with a width of $3\,\tau$) in the 2-period framework. In addition, because trading volume is higher in the 3-period model, exchange profits are systematically higher. We also note that the levels of MF and TF in the 3-period model are smaller. The intuition for the effect of the number of trading periods on the use of rebates and the level of access pricing is the following: Holding everything fixed, the probability that limit orders are executed increases because there are more opportunities for investors with complementary reasons to trade to arrive and trade with each other. As a result, the exchange has less of an incentive to offer rebates.

Proposition 6. *Maker-taker and taker-maker pricing can be asymmetric in the 3-period model with smaller rebates in the maker-taker equilibrium than in the taker-maker equilibrium.*

This asymmetry is new and in contrast to the symmetry in our 2-period model and also Chao et al. (2018). The equilibrium fees are asymmetric because in the 3-period model the investor at time t_1 is no longer a monopolist in liquidity provision. An incoming investor at time t_2 may try to induce competition with the t_1 limit order by the investor at t_3.

Consider, for example, the equilibrium strategies in Row 1 of Table 3 for a support width $\Delta = 0.3\,\tau$. In the taker-maker equilibrium when the investor in period t_1 tries to limit buy (LBP_{-1}) at the price P_{-1}, an incoming seller in period t_2 has the option of either market selling (MSP_{-1}) at P_{-1} or trying to limit sell (LSP_1) at the higher price P_1 with an investor arriving at time t_3. In contrast, in the maker-taker equilibrium the investor at t_1 tries to limit buy (LBP_1) at P_1 (because of the rebate $MF = -0.428$), which consequently means that the seller arriving at t_2 has no other trading option than market selling (MSP_1) at the high price P_1 — since limit selling at P_{-1} is not allowed given the pre-existing limit buy at P_1 in order to prevent a crossed market — and therefore will be charged a positive fee $TF = 0.557$.[14]

Table 3 shows that in the taker-maker equilibrium the seller at t_2 opts only to market sell at P_{-1}. This choice is driven by the higher TF rebate (-0.443) that encourages transactions at t_2 in the taker-maker equilibrium. It is precisely due to the high TF rebate that the seller does not try to limit sell at t_2 in the taker-maker equilibrium. The two grey rows 3 and 4 in Table 3 show that if, off equilibrium, the exchange used symmetric fees — i.e., the equilibrium taker-maker TF and MF are flipped for the maker-taker MF and TF or if the equilibrium maker-taker TF and MF are flipped and used for the taker-maker MF and TF — the incoming seller would opt for either market selling (MSP_{-1}) or limit selling (LSP_1), and exchange profits would be smaller. This explains why, in equilibrium, the exchange offers a larger TF rebate than the MF rebate.

Observation: *The minimum rebate $|MF|$ in the 3-period taker-maker equilibrium is strictly positive, whereas it is 0 in the maker-taker equilibrium because the regulatory cap on the taker-maker TF binds for smaller support widths than in the maker-taker equilibrium.*

This discontinuity can be seen in Figure 3 where the minimum taker-maker rebate $|TF|$ on the left when Δ is just larger than $2.3\,\tau$ is a little less than $|-1.04|$ whereas the minimum maker-taker rebate $|MF|$ on the right is 0.

[14]The state of the book when the seller arrives at t_2 has a limit order at P_1, hence the seller does not compete for the provision of liquidity as a limit sell order at P_{-1} is dominated by the market sell order at P_1.

The comparison between the 2-period and 3-period frameworks also allows us to study how optimal access fees should differ for stocks with different rates of trading activity. The 3-period framework proxies for a stock with a faster rate of trading activity. This leads to the following empirical prediction:

Empirical Prediction 3: *When stocks have a higher rate of trading activity, the region of valuation supports associated with rebate-based pricing shrinks, and the exchange has an incentive to offer smaller rebates.*

A practical complication here is that exchanges generally have a single set of fees and rebates that are applied across all stocks on an exchange. Thus, access pricing optimization happens for the cross-section of traded stocks. However, actual access pricing typically involves special rules and volume-contingent pricing schedules.[15] We conjecture that this pricing complexity allows exchanges to implement some amount of access pricing customization for different types of stocks.

[15]See, for example, the 2018 LSE access price list at:
https://www.lseg.com/sites/default/files/content/documents/Trading%20Services%20Price%20List_effectiveOct2018.pdf

Figure 3: Make Fees and Take Fees in 3-Period Market. This figure reports the equilibrium make fees (MF) and take fees (TF) in the Large Tick Market (LTM) corresponding to different investor valuation supports ranging from 0.33 τ and 5 τ, (where τ is the tick size in the LTM) on the horizontal black axes. The left (right) part of the figure reports the equilibrium trading fees consistent with the taker-maker (maker-taker) fee structure. The figure reports in blue (orange) italic the equilibrium MF (TF) set by the exchange.

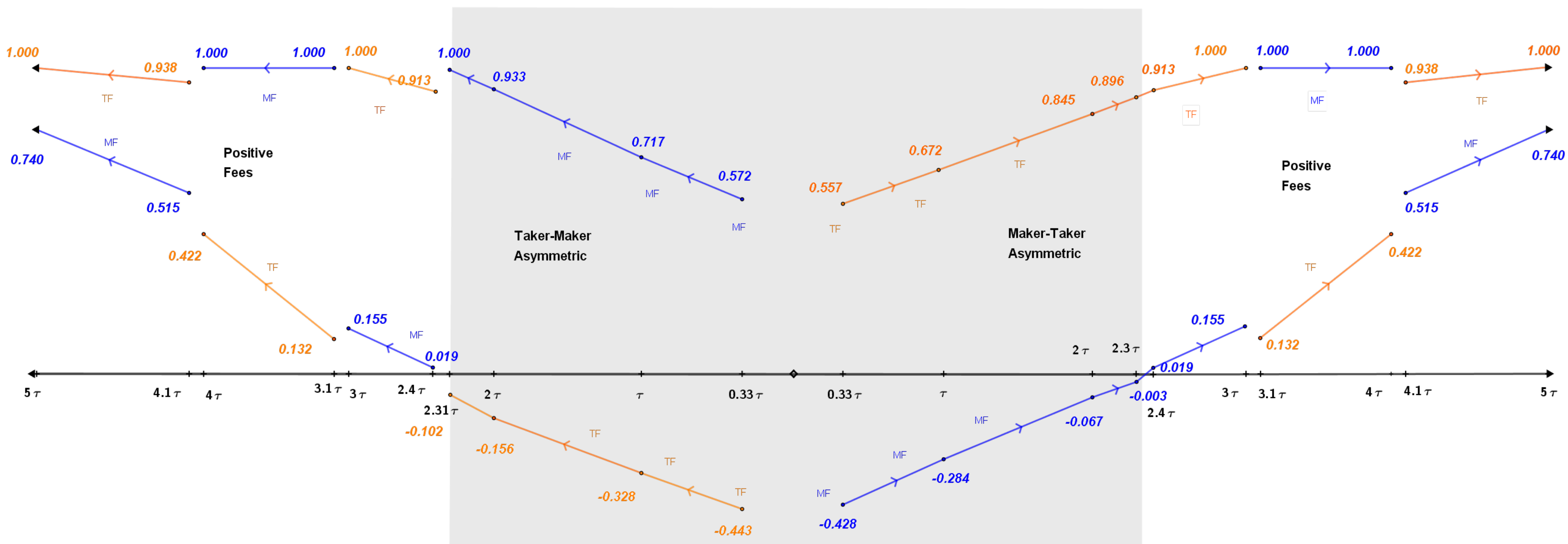


Table 3: 3-Period Large Tick Market (LTM). Equilibrium Fees and Trading Strategies. This table reports for different investor valuation support width, $\Delta = \overline{\beta} - \underline{\beta}$ expressed in terms of the LTM tick size τ (column 1), the extreme values of the support, $\underline{\beta}$ and $\bar{\beta}$ (column 2), the equilibrium make and take fees (MF and TF) (column 3 and 4), the equilibrium orders x_{t_1} at t_1 other than No Trade (column 5) and the equilibrium orders x_{t_2} at t_2, conditional on the trading strategy indicated at t_1 (column 6). The table also shows the associated probability of submission, $Pr(x_{t_1}|S,\tau,\Xi,L_{t_0})$ and $Pr(x_{t_2}|S,\tau,\Xi,L_{t_1})$, (column 7 and 8), as well the cum-fee buy and sell prices ($P_k^{cum,LB}$ and $P_k^{cum,MS}$) (column 8 and 10), the probability of execution of the order posted at t_1, $Pr(\theta_{t_1}^{x_{t_1}}|S,\tau,\Xi,l_{t_0})$, (column 11) and the Exchange Expected Profit, $\pi(MF,TF|S,\tau)$ (column 12). The third and fourth gray rows report results (marked with a *) for off-equilibrium fees that symmetrically flip the corresponding equilibrium fees. When the equilibrium pricing is rebate based, for each support we report first maker-taker set of fees and then the taker-maker set of equilibrium MF and TF.

Support width $\Delta = \overline{\beta} - \underline{\beta}$	$\underline{\beta}, \bar{\beta}$	MF	TF	Eq.Strategies x_{t_z} t_1	t_2	Pr. Submission $Pr(x_{t_z}\|S,\tau,\Xi,L_{t_{z-1}})$ t_1	t_2	$P_k^{cum,LB}$	$P_k^{cum,MS}$	Pr. Execution $Pr(\theta_{t_1}^{x_{t_1}}\|S,\tau,\Xi,l_{t_0})$	Exchange E[Profit] $\pi(MF,TF\|S,\tau)$
0.33 τ	9.833, 10.167	0.572	-0.443	LBP_{-1}	MSP_{-1}	0.284	0.328	10.072	9.943	0.548	0.051
0.33 τ	9.833, 10.167	-0.428	0.557	LBP_1	MSP_1	0.284	0.328	10.072	9.943	0.548	0.051
0.33 τ*	9.833, 10.167	0.557*	-0.428*	LBP_{-1}*	MSP_{-1}* & LSP_1*	0.328*	0.266* & 0.062*	10.057*	9.927*& 9.943	0.475*	0.050*
0.33 τ*	9.833, 10.167	-0.443*	0.572*	LBP_1*	MSP_1*	0.328*	0.284*	10.057*	9.927*	0.487*	0.050*
τ	9.500, 10.500	0.716	-0.328	LBP_{-1}	MSP_{-1}	0.284	0.328	10.216	9.828	0.548	0.152
τ	9.500, 10.500	-0.284	0.672	LBP_1	MSP_1	0.284	0.328	10.216	9.828	0.548	0.152
2 τ	9.000, 11.000	0.933	-0.156	LBP_{-1}	MSP_{-1}	0.284	0.328	10.433	9.656	0.548	0.304
2 τ	9.000, 11.000	-0.067	0.845	LBP_1	MSP_1	0.284	0.328	10.433	9.655	0.548	0.304
2.31 τ	8.850, 11.150	1.000	-0.102	LBP_{-1}	MSP_{-1}	0.284	0.328	10.500	9.602	0.548	0.351
2.31 τ	8.850, 11.150	-0.001	0.898	LBP_1	MSP_1	0.284	0.328	10.499	9.602	0.548	0.351
2.4 τ	8.800, 11.200	0.019	0.913	LBP_1	MSP_1	0.284	0.328	10.519	9.587	0.548	0.365
3 τ	8.500, 11.500	0.155	1.000	LBP_1	MSP_1	0.282	0.333	10.655	9.500	0.556	0.456
3.1 τ	8.450, 11.550	1.000	0.132	LBP_{-1}	LBP_1, MSP_{-1}, LSP_1	0.339	0.016, 0.278, 0.060	10.500, 11.500	9.368, 9.500	0.487	0.468
4 τ	8.000, 12.000	1.000	0.422	LBP_{-1}	LBP_1, MSP_{-1}, LSP_1, LSP_2	0.367	0.125, 0.231, 0.125, 0.270	10.500, 11.500	9.078, 9.500, 10.500	0.404	0.612
				LBP_{-2}	LBP_{-1}, MSP_{-2}, LSP_1, LSP_2	0.133	0.375, 0.000, 0.356, 0.270	9.500, 10.500	8.078, 10.500, 11.500	0.012	
4.1 τ	7.950, 12.050	0.515	0.938	LBP_1	LBP_2, MSP_1	0.149	0.009, 0.393	11.015, 12.015	9.562	0.628	0.627
				LBP_{-1}	LBP_1, MSP_{-1}, LSP_1	0.347	0.252, 0.088, 0,408	10.015, 11.015	8.562, 9.985	0.187	
5 τ	7.500, 12.500	0.740	1.000	LBP_1	LBP_2, MSP_1	0.108	0.052, 0.400	11.240, 12.240	9.500	0.619	0.785
				LBP_{-1}	LBP_1, MSP_{-1}, LSP_1	0.344	0.252, 0.137, 0.315	10.240, 11.240	8.500, 9.760	0.259	

5 High Frequency Trading and Access Pricing

Our previous results show that an exchange's optimal access pricing depend crucially on the type of investors in the market, and that the incentive to offer rebates decreases with traders' ex ante potential gains-from-trade. Thus, the more traders have personal evaluations near the asset's fundamental value, the greater is the exchange's incentive to offer rebates. In real markets, one important type of active traders whose personal valuations typically do not differ from the fundamental value of the asset are high frequency traders (HFT). This section extends our previous our previous analysis to include high frequency trading firms.

HFT firms are profit-maximizing investors that differ from regular investors (INV) in four ways in our model: First, rather than having stochastic private valuations, all HFT firm have the same non-random personal valuation, which we assume is equal to the mean INV valuation v. Second, rather than arriving sequentially, the HFTs are continually present. In particular, unlike Foucault, Kadan, and Kandel (2013), the HFTs bear no monitoring costs. Third, HFT firms can react immediately to take advantage of any profitable trading opportunities in limit orders submitted by arriving regular INVs. For example, if in period t_z an INV posts an aggressive limit buy (sell) order such that the associated cum-fee sell (buy) price for a market order is above (below) the HFT valuation v, the HFT firm can submit a sell (buy) market order within the same period t_z to take the other side of the profitable trade. We call these fast market orders *flash orders*. If more than one HFT submits a flash order, then one is randomly selected for execution, and the rest are cancelled. Fourth, Budish, Cramton, and Shim (2015) show that there is a natural bid-ask spread for HFT limit orders given endogenous picking-off costs for stale orders. Thus, to simplify our analysis, we assume that, while HFTs are willing to provide liquidity ex post to regular INVs using flash orders, they are unwilling to provide ex ante liquidity via limit orders.[16].

[16]Allowing for the possibility that HFTs might sometimes use limit order when they are unwilling to use flash orders given a hypothetical exchange access pricing structure would simply complicate the analysis. In Budish et al. (2015), the break-even condition in a limit order book such that HFT firms supply liquidity is that the payoff from market making is at least equal to the costs of being sniped by other competing HFT firms. Thus, our assumption of no HFT limit orders is simply a convenient

Based on the foregoing, the HFT action set, $X_{t_z}^{HFT} = \{MOP_{k(L_{t_z})}^{HFT}, NT\}$ consists of possible flash market orders $MOP_{k(L_{t_z})}^{HFT}$ given the current book L_{t_z} or no-trade.

In each period t_z, HFT firms choose the order $x_{t_z}^{HFT}$ to maximize their expected payoff depending on whether or not there is an aggressive limit order in the book L_{t_z} that it would be profitable to trade with

$$\max_{x_{t_z}^{HFT} \in X_{t_z}^{HFT}} w(x_{t_z}^{HFT} | \tau, \Xi, v, L_{t_z}) = \begin{cases} [v - P(x_{t_z}^{HFT}) - \xi(x_{t_z}^{HFT})] & \text{if } x_{t_z}^{HFT} \text{ is buy and there is a limit sell in } L_{t_z} \\ [P(x_{t_z}^{HFT}) - v - \xi(x_{t_z}^{HFT})] & \text{if } x_{t_z}^{HFT} \text{ is sell and there is a limit buy in } L_{t_z} \\ 0 & \text{if } x_{t_z}^{HFT} \text{ is } NT. \end{cases} \quad (7)$$

The execution probability for a flash market order is 1 if it is submitted. Note that the current INV order Q_{t_z} is part of the current book L_{t_z} that is the conditioning information of the HFT.

Competition by the HFTs simplifies the structure of equilibrium in a market with HFTs. Since HFTs are always willing to buy and sell at v, the exchange, in equilibrium, can set the fees and rebates Ξ so that in equilibrium the cum-fee prices paid and received by the HFT is their break-even valuation v.

This has two immediate implication: First, limit buys at prices below P_{-1} and limit sells at prices above P_1 are never used in equilibrium. This is because HFTs and INVs know that such limit orders would always be undercut by future HFTs who will be willing to trade via flash market orders at their break-even cum-fee prices. Second, the INVs therefore choose between submitting limit orders at P_{-1} and P_1, market orders (if there are any pre-existing limit orders in the book at P_{-1} and P_1), and NT.

Regular investors (INV) have the same formal action set $X_{t_z}^{INV}$ as before. However, there is a fundamental change in the INV order submission problem when HFTs are present and active. If the HFTs are willing to use flash orders to immediately take the other side of aggressive limit buy (sell) orders at prices above (below) v, then less aggressive standing limit buy (sell) orders at outside prices (P_2 and P_{-2}) below (above)

reduced-form for picking-off risks for a smart trading crowd as first suggested in Seppi (1997).

v are never executed in equilibrium. This is because standing outside limit orders would always be undercut by HFT flash orders. If instead flash orders are not profitable for HFT firms – and therefore HFTs do not use flash orders to execute aggressive limit orders immediately, — then the market looks like the market without HFTs in that execution limit orders depends on the future arrival of later regular investors who are willing to take the other side of the limit order based on their personal gains-from-trade. As a result, the regular investor's objective function with HFTs is as follows. If an INV arrives at time t_z, he chooses his order $x_{t_z}^{INV}$ to maximize his expected payoff:

$$\max_{x_{t_z}^{INV} \in X_{t_z}^{INV}} w(x_{t_z}^{INV} | S, \tau, \Xi, \beta_{t_z}, L_{t_{z-1}})$$
$$= \begin{cases} [\beta_{t_z} - P(x_{t_z}^{INV}) - \xi(x_{t_z}^{INV})] \, Pr(\theta_{t_z}^{x_{t_z}^{INV}} | S, \tau, \Xi, L_{t_{z-1}}) & \text{if } x_{t_z}^{INV} \text{ is a buy order} \\ [P(x_{t_z}^{INV}) - \beta_{t_z} - \xi(x_{t_z}^{INV})] \, Pr(\theta_{t_z}^{x_{t_z}^{INV}} | S, \tau, \Xi, L_{t_{z-1}}) & \text{if } x_{t_z}^{INV} \text{ is a sell order} \\ 0 & \text{if } x_{t_z}^{INV} \text{ is } NT. \end{cases} \quad (8)$$

where now $Pr(\theta_{t_z}^{x_{t_z}^{HT}} | S, \xi, l_{t_{z-1}})$ reflects both the possibility of immediate execution of some limit orders by HFTs and possible future execution by regular INVs for other limit orders. Both HFT firms and INVs maximize their expected terminal payoff conditional on the support of the INVs evaluations, S, a set of fees, Ξ, chosen by the exchange, and the incoming state of the limit order book $L_{t_{z-1}}$. One further difference is that now limit orders are possible in equilibrium for the regular INV in the final trading date t_3 due to the possibility of execution by the HFTs.

Given the behavior of HFTs and regular investors, the exchange sets its access pricing to maximizes its expected payoff. Formally, this problem is the same as in (4). However, the presence of the HFTs potentially affects the behavior of the INV investors and the specific forms of the order-submission and order-execution probabilities.

A Subgame Perfect Nash Equilibrium consists of order-submission strategies $x_{t_z}^{HFT}$ and $x_{t_z}^{INV}$ that maximize expected profits for both the HFT firms and the INVs given the order-execution probabilties they induce and access fees Ξ that maximize the exchange's expected profit.

Theorem 2. *The equilibrium of an N-period model with HFTs exists and can be constructed using an analytic recursion.*

Figure 4 shows the equilibrium fees and rebates in the three-period LTM with HFTs for different INV valuation supports. Comparing these results with the previous 3-period model without HFTs (Figure 3), we see that, all else equal, the gray region characterized by an optimal pricing structure with rebates widens when HFTs are present in the market. With HFT firms present, the INV support consistent with the taker-maker or maker-taker pricing is $4\,\tau$, whereas in the 3-period protocol it was only $2.3\,\tau$. Figure 4 and Table 4 show that the exchange sets either the MF or the TF to attract HFT firms. Starting from the smallest support ($0.33\,\tau$), in the taker-maker regime the exchange offers a rebate on the TF slightly greater than half a tick so that the HFT firms have an incentive take liquidity at P_{-1} ($TF = -0.500^{*}$); in the maker-taker regime the exchange sets the TF just below half a tick ($TF = 0.499^{*}$) so that the HFT firms have an incentive to profitably take the limit order posted at P_1 by the INVs buying at t_1.

As the support of INVs widens and reaches $2\,\tau$, the equilibrium strategies of the liquidity suppliers arriving at t_1 does not change in the taker-maker region (LBP_{-1}) and the equilibrium MF reaches its maximun value (MF=1). The equilibrium strategies of the liquidity suppliers in the maker-taker region (LBP_1) instead changes: at t_1 the liquidity supplier no longer buys at P_1 but rather buys at P_{-1}. The reason why the equilibrium strategy of the buyer is no longer LBP_1 but rather LBP_{-1} is that the exchange exploits the now greater gains from trade and has an incentive to set the maximun MF for all market participants, and INVs anticipate that with a rebate on the TF slightly greater than half a tick ($TF = -0.500^{*}$), at t_2 the HFT firms will be willing to sell at P_{-1}.[17] Note that when the support becomes wider than 2τ, the exchange sets a unique taker-maker symmetric fee structure, thus inducing HFT firms to take liquidity at the inside quotes.

By widening the support even further, the fees only change when the equilibrium trading strategies also change, which is in correspondence of the support 4τ when at t_1 the incoming buyers switch from buying either at P_{-1} or at P_1 to buying both at LBP_{-1} and at LBP_1 depending on their support. As the support reaches 4τ, the exchange finds it optimal to set $TF = 0.499^*$ to induce the HFT firms taking liquidity at P_1 (with a 0.001 profit per execution). The exchange sets $MF = 0.520$ so to make selling at P_{-1} also profitable for INVs, given their now wide support. Table 4 shows that beyond this threshold as the support widens, the exchange holds the TF constant at 0.499^* and gradually increases the MF to take advantage of the larger INVs' gains from trade. These results lead to our fourth empirical prediction:

Empirical Prediction 4: *Markets with HFT traders are more likely to have rebated-based access pricing.*

Our results explain the growing predominance rebate-based access pricing structures in U.S. markets since the advent of Reg NMS. In addition, they are consistent with the empirical findings of Cardella et al. (2015) who use a three year data set (2008-2010) to show that most of the U.S. exchanges adopted after 2008 a rebate based fee structure starting right after the advent of Reg NMS.[18]

Our results are reminiscent of the Foucault et al. (2013) findings that the fee breakdown matters when the tick size is positive. Holding the total fee constant, Foucault et al. (2013) show that when the gains-from-trade to market takers increase relative to the gains-from-trade to the market makers, the optimal trading fees become larger. Independently of the role played by HFT firms, our extension shows that exchange profits sharply increase when HFTs are active in the market and therefore exchanges set their fees to maximize the HFTs activity. As HFT firms can generate a greater amount of volume than INVs, exchanges prioritize HFT

[17] Selling at 9.50 with a rebate larger than half a tick means selling net of TF at a price higher than 10.00, which allows HFTs to make some profits.

[18] Similar results hold also in the small tick STM market. In results available from the Authors upon request, we show that when, all else equal, the tick size is smaller the exchange has a a smaller incentive to offer a rebate-based fee structure.

firms and set the fees to maximize HFTs volume. It is therefore not surprising that the region associated with a rebate on the take fee is larger than in the 3-period model without HFTs.

Figure 4: Pattern of Make Fees and Take Fees: 3-Period Model with High Frequency Traders (HFT) This figure shows the equilibrium make fees (MF) and take fees (TF) in the Large Tick Market (LTM) corresponding to different investor valuation supports ranging from $0.33\,\tau$ to $5\,\tau$ (where τ is the tick size in the LTM) on the horizontal black axes. The left (right) part of the figure reports the equilibrium trading fees consistent with the taker-maker (maker-taker) fee structure. The figure reports in blue (orange) italic the equilibrium MF (TF) set by the exchange. Note that $-0.500^* = -0.5 - 1 \cdot 10^{-7}$ and $0.499^* = 0.5 - 1 \cdot 10^{-7}$.

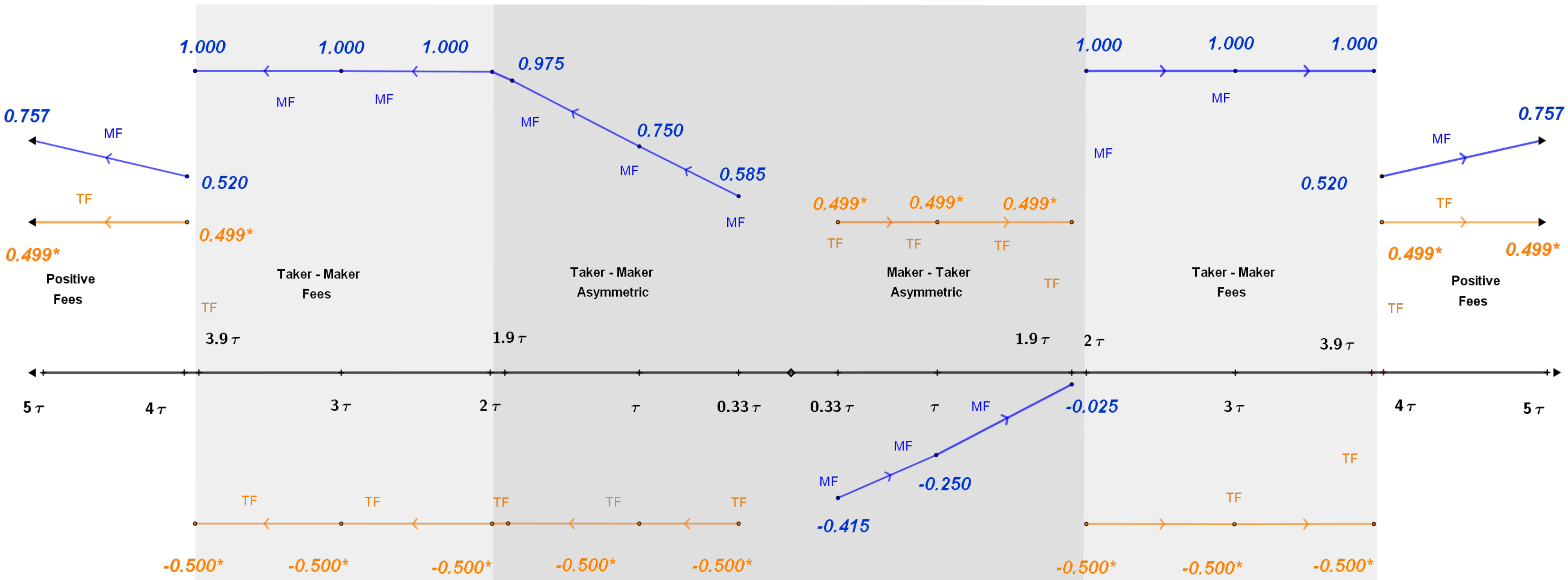


Table 4: **3-Period Large Tick Market with HFTs: Equilibrium Fees and Trading Strategies.** This table reports for different investor valuation support width $\Delta = \overline{\beta} - \underline{\beta}$ are expressed in terms of the LTM tick size τ (column 1), the extreme values of the support, $\underline{\beta}$ and $\bar{\beta}$ (column 2), the equilibrium make and take fee, MF and TF, (column 3 and 4), the equilibrium orders x_{t_1} at t_1, other than No Trade (column 5) and the equilibrium orders x_{t_2} at t_2 conditional on the trading strategy indicated at t_1 (column 6). The table also shows the associated probabilities of submission, $Pr(x_{t_1}|S,\tau,\Xi,L_{t_0})$ and $Pr(x_{t_2}|S,\tau,\Xi,L_{t_1})$ (column 7 and 8), as well the cum-fee buy and sell price, $P_k^{cum,LB}$ and $P_k^{cum,MS}$, (column 9 and 10), the probability of execution of the order posted at t_1, $Pr(\theta_{t_1}^{x_{t_1}}|S,\tau,\Xi,l_{t_0})$, (column 11) and the exchange expected profit $\pi(MF,TF|S,\tau)$ (column 12). When the equilibrium pricing is rebate-based, for each support we report first the taker-maker set of fees and then the maker-taker set of equilibrium MF and TF. We do not report the order-submission probabilities for HFT flash market orders (e.g. $\text{MS}P_{-1}^{HFT}$) after aggressive limit orders because in equilibrium they are always equal to 1. We only report the Pr. Execution of the limit order posted at t_1. Note that $-0.500^* = -0.5 - 1 \cdot 10^{-7}$ and $0.499^* = 0.5 - 1 \cdot 10^{-7}$.

Support width $\Delta = \overline{\beta} - \underline{\beta}$	$\underline{\beta}, \bar{\beta}$	MF	TF	Eq. Orders x_{t_z}		Pr. Submission $Pr(x_{t_z}\|S,\tau,\Xi,L_{t_{z-1}})$		$P_k^{cum,LB}$	$P_k^{cum,MS}$	Pr. Execution $Pr(\theta_{t_1}^{x_{t_1}}\|S,\tau,\Xi,l_{t_0})$	Exchange E[Profit] $\pi(MF,TF\|S,\tau)$
				t_1	t_2	t_1	t_2				
0.33 τ	9.833, 10.167	0.585	-0.500*	LBP_{-1}, MSP_{-1}^{HFT}	LBP_{-1}, MSP_{-1}^{HFT}	0.245	0.245	10.085	10.000*	1	0.127
0.33 τ	9.833, 10.167	-0.415	0.499*	LBP_1, MSP_1^{HFT}	LBP_1, MSP_1^{HFT}	0.245	0.245	10.085	10.000*	1	0.127
τ	9.500, 10.500	0.750	-0.500*	LBP_{-1}, MSP_{-1}^{HFT}	LBP_{-1}, MSP_{-1}^{HFT}	0.250	0.250	10.250	10.000*	1	0.375
τ	9.500, 10.500	-0.250	0.499*	LBP_1, MSP_1^{HFT}	LBP_1, MSP_1^{HFT}	0.250	0.250	10.250	10.000*	1	0.375
1.9 τ	9.050, 10.950	0.975	-0.500*	LBP_{-1},MSP_{-1}^{HFT}	LBP_{-1}, MSP_{-1}^{HFT}	0.250	0.250	10.475	10.000*	1	0.712
1.9 τ	9.050, 10.950	-0.025	0.499*	LBP_1, MSP_1^{HFT}	LBP_1, MSP_1^{HFT}	0.250	0.250	10.475	10.000*	1	0.712
2 τ	9.000, 11.000	1.000	-0.500*	LBP_{-1}, MSP_{-1}^{HFT}	LBP_{-1}, MSP_{-1}^{HFT}	0.250	0.250	10.500	10.000*	1	0.750
3 τ	8.500, 11.500	1.000	-0.500*	LBP_{-1}, MSP_{-1}^{HFT}	LBP_{-1}, MSP_{-1}^{HFT}	0.333	0.333	10.500	10.000*	1	1.000
3.9 τ	8.050, 11.950	1.000	-0.500*	LBP_{-1}, MSP_{-1}^{HFT}	LBP_{-1}, MSP_{-1}^{HFT}	0.372	0.372	10.500	10.000*	1	1.115
4 τ	8.000, 12.000	0.520	0.499*	LBP_1, MSP_1^{HFT}	LBP_{-1}, LBP_1, MSP_1^{HFT}	0.130	0.333, 0.161	11.020, 10.020, 11.020	10.000*, 10.000*	1	1.167
				LBP_{-1}	LBP_1, MSP_{-1}, MSP_1^{HFT}, LSP_1	0.365	0.245, 0.168, 0.327	10.020, 11.020	9.000*, 9.000*, 9.980	0.315	
5 τ	7.500, 12.500	0.757	0.499*	LBP_1, MSP_1^{HFT}	LBP_{-1}, LBP_1, MSP_1^{HFT}	0.120	0.286, 0.162	11.257, 10.257, 11.257	10.000*, 10.000*	1	1.522
				LBP_{-1}	LBP_1, MSP_{-1}, MSP_1^{HFT}, LSP_1	0.328	0.259, 0.236, 0.212	10.257, 11.257	9.000*, 10.000*, 9.743	0.391	

6 Welfare and Market Quality

Access pricing that maximizes exchange profits does not necessarily improve the overall welfare of other market participants. In this section we revisit the markets discussed in the previous sections and investigate how the different profit-maximizing access pricing for exchanges affect the welfare of other market participants. As a reference point for our welfare analysis, we compare equilibria with non-zero access pricing with a benchmark model with no fees or rebates. The solution for the benchmark model is analytic since it follows as a simplification of Theorem 1. Tables 13, 14, 15 and 16, 17, 18, in Appendices D and E provide specifics about how to operationalize the recursion described in Theorem 1. Tables 19 through 23 in Appendix F provide specific numerical details about welfare, market quality and other characteristics.

Figures 5 and 6 show our results about welfare. The figures show total welfare for all agents with and without optimal access pricing and also show the welfare breakdown for the various traders and the exchange. Our findings are consistent for all three model specifications: The exchange's profit-maximizing fees improve total welfare when they are small (for small valuation supports in the PIW regions); when they become larger (for larger valuation supports in the RW regions) they increase total welfare (but investors are worse off unless there are Pareto transfers from the exchange to investors); and when they are very large (in the DL regions), the optimal fees reduce total welfare relative to no access fees. For example, for the 3-period market with HFTs, optimal access pricing is Pareto improving relative to the zero-fee benchmark market with no transfers between the exchange and investors up until an investor valuation support of roughly 1.27τ. For larger valuation supports, total welfare with profit-maximizing access pricing still improves over the zero-fee market but transfers from the exchange to investors are required for investors to be better off. This region extends up until a support of roughly 3.9τ. Lastly, for still larger valuation supports, optimized access pricing actually worsens total welfare.

There are several other features to note here:

- In the 2-period market, the regions with Pareto improvement in welfare (PIW) and the region in which welfare is redistributed (RW) both become smaller in STM relative to LTM. This is intuitive, since the distortions associated with price grid discreteness are decreasing as the tick size shrinks. In particular, the PIW (RW) region extends to supports of $1.27\,\tau$ ($1.88\,\tau$) for the LTM but only to $0.42\,\tau$ ($0.63\,\tau$) for the STM.

- Going from a 2-period to a 3-period market, the Pareto improving and welfare redistribution regions get smaller and the rebates are smaller (i.e., $|MF|$ is smaller). Now the PIW (RW) region only extends to supports of $1.2\,\tau$ ($1.62\,\tau$) for the 3-period LTM down from $1.27\,\tau$ ($1.88\,\tau$) for the 2-period LTM. This is consistent with the positive effect of higher trading activity on trade execution.

- With HFTs, the Pareto improving region increases somewhat and the redistributed welfare region of the parameter space becomes much larger. Now the PIW (RW) region extends to supports of $1.27\,\tau$ ($3.9\,\tau$) for the 3-period LTM with HFTs up from $1.2\,\tau$ ($1.62\,\tau$) without HFTs.

Two key intuitions underling our welfare results — and also the exchange's access pricing behavior more generally through the model analysis — are the roles of two different externalities. On the one hand, total welfare depends on the probability of transaction execution, whereas individual investors care about both the probability of order execution and also on price improvement on their personal payoff conditional on order-execution. Thus, in some (parametric) circumstances individual traders may submit orders with lower execution probabilities (which can reduce overall welfare) if their personal price improvement benefit dominates. However, since exchanges care about transaction execution, their access pricing can offset the individual investor price-improvement externality. This is the reason rebate-based access pricing improves overall welfare when investor valuation dispersion is small relative to the price tick size. On the other hand, there is also an externality in the exchanges behavior. In particular, exchanges care about both the transaction execution probability and also on the net fee they earn conditional on transaction execution. Thus, under other circumstances exchanges may set fees that reduce transaction execution probabilities (which reduces overall welfare) in order to increase the net fees they earn. The shaded areas reported in Figures 5 and 6 shows the DL region due to rebate-based pricing as opposed to positive pricing. The DL due to rebate-based pricing decreases when the tick size decreases and when the trading frequency increases, and it drops to zero when HFT are active in the market.

Welfare is not directly observable, but our model does provide predictions about observable measures of market quality. Tables 19 and 22 in the Appendix report measures of market quality (spread, volume and depth) and also measures of welfare for the 2-period and for the 3-period model with HFTs for a large tick size. For space reasons, the zero-fee benchmark results are reported separately in Table 23. The double vertical lines correspond to points where there are discontinuities in investor strategies and exchange access pricing. Analogous results for the 2-period STM and for the 3-period market without HFTs are in Tables 20 and 21 in the Appendix.

Figure 5: Welfare: 2-period LTM and STM This figure shows how the welfare of the Exchange (EXCH) - dashed line, Investors, (INV - dotted line) and Total Welfare (INV + EXCH - dashed-dotted line) change with the investors' support (S) in the large tick market (LTM) on the left and in the small tick market (STM) on the right. Both figures also report the welfare of investors under the Benchmark regime (solid line) with no trading fees (MF=TF=0). The support is expressed in large tick unit of measure (τ). Both figures show the results for three regions: Pareto Improvement Welfare (PIW), Redistribution Welfare (RW) and Deadweight Loss (DL).

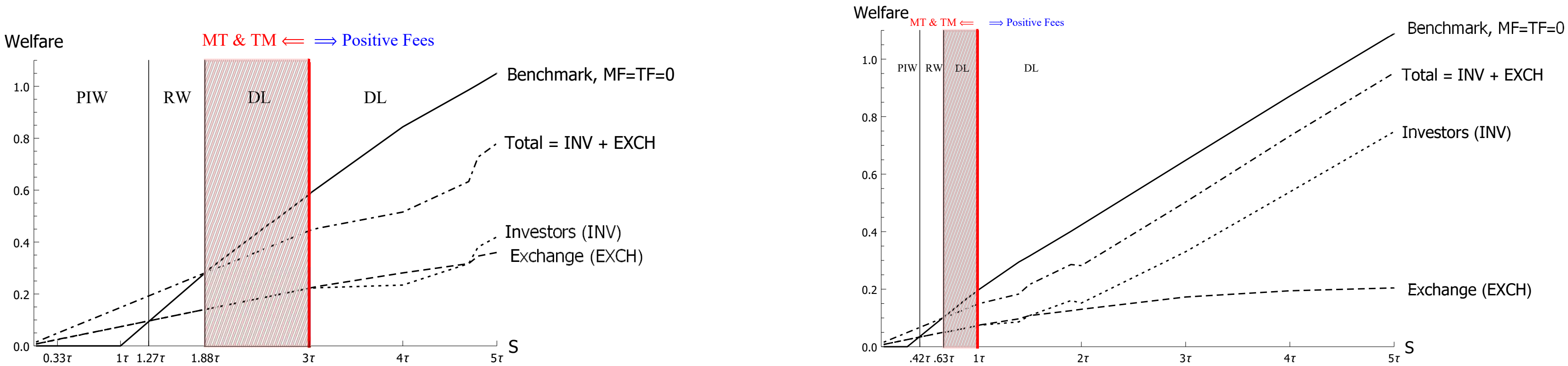
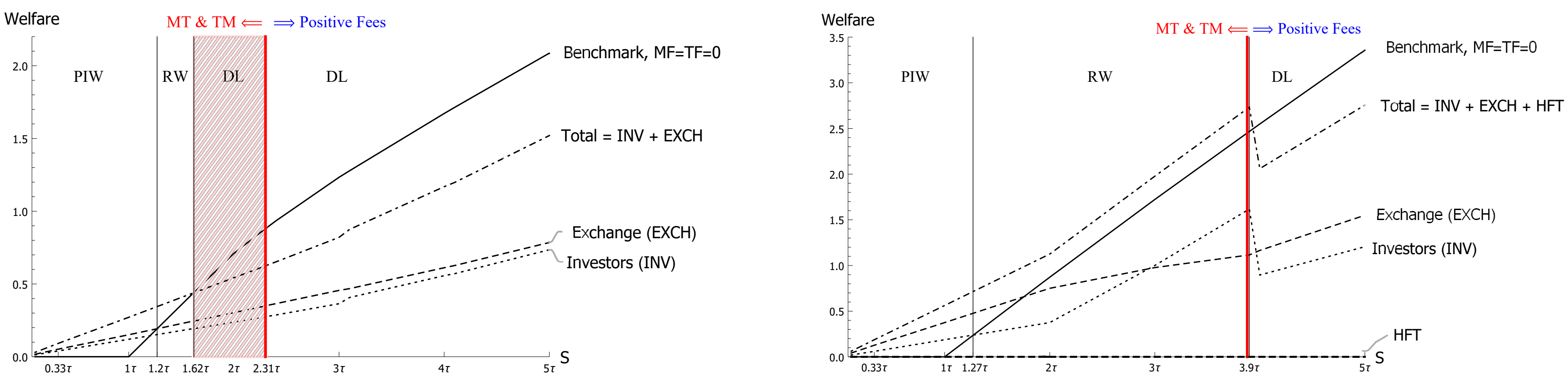


Figure 6: Welfare: 3-period LTM and 3-period LTM with HFT This figure shows on the left how the welfare of the Exchange (EXCH) - dashed line, Investors, (INV - dotted line) and Total Welfare (INV + EXCH - dashed-dotted line) change with the investors' support (S) in the large tick market (LTM); and on the right it shows how the welfare of the Exchange (EXCH) - dashed line, Investors and High Frequency Traders (INV&HFT - dotted line) and Total Welfare of Investors, HFTs and the Exchange (INV&HFT+EXCH - dashed-dotted line) change with the investors' support (S). Both figures also report the welfare of investors under the Benchmark regime (solid line) with no trading fees (MF=TF=0). The support is expressed in large tick unit measure (τ). Both figures show the results for three regions: Pareto Improvement Welfare (PIW), Redistribution Welfare (RW) and Deadweight Loss (DL). The shaded area indicates the DL region with rebate-based pricing.



7 Conclusions

This paper extends the existing theoretical models of optimal access pricing to allow for different populations of market participants, realistic regulatory restrictions, multiple periods, and HFT traders. Our model shows that investor valuation dispersion drives the exchanges' choice of the optimal trading fees. If the market is mainly populated by traders with valuations close to the current asset value, then in equilibrium the exchange chooses a rebate-based pricing structure. If the market is instead populated by traders with disperse valuations, then the exchange chooses jointly positive make and take fees.

Regulatory caps on access fees crucially affect the exchange choice of the optimal pricing. When there is no cap, the exchange chooses a rebate-based fee structure to maximize its total profits (volume times per-trade profit). To achieve the largest possible profits the exchange has to impose fees that induce market participants to trade only at the outside quotes. When exchange access fees are capped relative to the tick size, the exchange chooses a rebate-based pricing only when the support of the traders' evaluation is small and investors need to be subsidized to trade at the inside quotes. When traders instead have dispersed valuations, the exchange imposes positive fees on all market participants. Our model also shows how different tick size regimes affect the equilibrium pricing structure. Thus, the optimal access pricing structure depends on both the absolute tick size and the tick size relative to the dispersion. When the tick size is smaller, the region of the investors' support that is consistent with a rebate-based pricing is smaller and exchanges have less degree of freedom in setting the trading fees.

A natural question here is why the rebate-based fee structure became predominant over the positive fee structures after Reg NMS. Our answer is that technological innovations led to a sharp increase of high frequency trading. Within the context of our model, high frequency traders have valuations concentrated around the asset value. We therefore conclude that the fee structure that governs today's markets is crucially affected by the type of market participants with HFT firms driving the fee structure towards rebates-based pricing. In particular, while the observed increased in the rate of trading activity in market nowadays could be expected to induce exchanges to reduce rebate-based access pricing (based on our results for 2- and 3-period markets), it is precisely the increase in the presence of HFTs (in liquid stocks) that makes the exchange opt for rebate based fee pricings.

Importantly, we show that optimized rebate-based access pricing by exchanges can be Pareto improving, but that there is also a sizeable parameter region where rebates reduce welfare in the absence of transfer payments. In particular, our model shows the effects of different pricing structures on the welfare of different market participants. When the market is populated by investors with small gains-from-trade, the rebate-based pricing Pareto improves welfare, and hence resolves the frictions generated by discrete prices. However, in markets populated by investors with large gains-from-trade optimal access pricing generates deadweight losses. The frictions generated by discrete pricing are less severe when the tick size is smaller and therefore the positive effects of rebate-based pricing decrease when the tick size is smaller. Similarly, when trading activity increases, there is less need for the exchange to subsidise trading via rebate-based pricing and therefore the Pareto improvement in welfare generated by rebate-based pricing decreases. When the gains-from-trade increase beyond the threshold that guarantees a Pareto improvement in welfare rebate-based pricing generate a redistribution of welfare from investors to the exchange. This region decreases when the tick size decreases or the trading activity increases, but becomes overwhelming in presence of HFTs when the DL region generated by trading fees decreases substantially.

Our model is the first that includes more than two trading periods. This extension allows us to show that when the trading period is longer than two period, results may crucially change the reason being that the liquidity suppliers coming to the market in the first period are no longer monopolists of the liquidity provision. While in a 2-period model in the second period traders can only take the liquidity standing on the book or decide not to trade and leave the market, in a 3-period model he/she can also compete for the provision of liquidity. This happens in particular in the taker-maker regime when the liquidity suppliers arriving when the book is empty buy at low prices. The induced greater competition for the provision of liquidity affects the exchange optimal pricing that will try to induce the liquidity taker not to compete for the provision of liquidity, thus leading to optimal asymmetric fee structure.

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Appendices

A General proofs for N-period models

The proof strategies for our general N-period models are relatively standard for finite sequential games.

Proof for Theorem 1: The proof consists of three steps.

The recursion step for deriving analytic investor strategies is the following: Given access pricing fees Ξ, the order-execution probabilities $Pr(\theta_{t_z}^{x_{t_z}}|S,\tau,\Xi,L_{t_{z-1}})$ for computing the investor expected profit for each possible order $x_{t_z} \in X_{t_z}$ at any time t_z in the investor maximization problem (3) are either 1 for market orders at the BBO or are determined recursively for limit orders from the order-submission probabilities $Pr(x_{t_z}|S,\tau,\Xi,L_{t_{z-1}})$ at later dates. The upper envelope of the expected investor payoffs for the different possible actions at a generic time t_z determines the optimal investor actions at t_z and, given the distribution over the investor valuation β_{t_z} the associated order-submission probabilities for the optimal actions in terms of intervals on the investor valuation support S for any incoming book $L_{t_{z-1}}$. Given the assumptions of a discrete number of possible investor actions and discrete tine, the set of possible incoming books is finite.

The initiation step starts the recursion at the terminal period t_N, at which time the order-execution probabilities take a simple form: They are zero for new limit orders (since the game ends after time t_N) and one for market orders (which can only be submitted if the book is non-empty). Thus, investor expected profit for different orders, the upper envelope, the optimal orders, and the order-submission probabilities at time t_N can be derived directly.

The exchange profit optimization step is then as follows: The order-submission and order-execution probabilities from the first two steps can then be used to construct the exchange's expected profit in (4) analytically given arbitrary fees Ξ. Given the analytic exchange expected profit function, the profit-maximizing fees Ξ^* can then be found analytically since the set of possible fees and rebates is compact given the regulatory cap on access fees. QED

Proof of Theorem 2: The proof structure is the same as for Theorem 1 with the addition that INVs and HFTs investors arrive sequentially. First, the recursion step again involves characterizing analytic optimal order submissions and order-submission probabilities in term of intervals of valuations β_{t+z} along the support S associated with the analytic upper envelope of the payoffs of all of the possible investor actions. Again, there are a finite number of possible investor actions with linear payoff and a finite number of periods and, thus, at each point in time t_z, a finite set of possible prior histories L_{tz-1}. Analytic order-execution probabilties can then be computed from the analytic order-submission probabilities. Second, the initiation step at time N involves optimization with only market orders for which the a priori execution probability is one. Third, the exchange profit optimization step is logically similar to the same step in Theorem 1. QED

Comment: The following parts of this Appendix show how to derive the optimal trading strategies and the optimal MF and TF for both the 2-period large tick model (Appendix B), and for the 3-period model (Appendix D) and for the 3-period with HFTs model (Appendix E). Table 5 shows explicitly the orders and payoffs available to investors in the LTM. They are similar in the SMT except for minor notation changes. For our proofs in the following Appendices the following Lemma 1 is relevant:

Lemma 1. *Investors with $\beta_{t_1} > v$ are potential buyers at time t_1 (i.e., they either submit limit buy orders or NT but they never submit limit sell orders). Similarly, investors with $\beta_{t_1} < v$ are potential sellers at time t_1.*

Proof of lemma 1: This result follows immediately from the fact that the investor expected profit functions from limit buy and sell orders are symmetric and increasing in the distance from the posted limit prices.

Table 5: Trading Strategies and Payoffs This table reports the trading strategies and associate payoffs available to investors in the LTM.

Action	Payoff
Market Order to Sell: MSP_{t_z}	$P(x_{t_z}) - \beta_{t_z} - TF$
Limit Order to Sell: LSP_{t_z}	$[P(x_{t_z}) - \beta_{t_z} - MF]\, Pr(\theta_{t_z}^{x_{t_z}} \vert S, \Xi, L_{t_{z-1}})$
No Trade: NT_{t_z}	0
Limit Order to Buy: LBP_{t_z}	$[\beta_{t_z} - P(x_{t_z}) - MF]\, Pr(\theta_{t_z}^{x_{t_z}} \vert S, \Xi, L_{t_{z-1}})$
Market Order to Buy: MBP_{t_z}	$\beta_{t_z} - P(x_{t_z}) - TF$

B Equilibrium of 2-Period Model and Proofs of Propositions 1 and 2

The model is solved by backward induction. Thus, consider first the last round of trading, t_2. Investors arriving at t_2 either choose a market order or do not submit an order (NT) since new limit orders at t_2 have a zero execution probability. An investor at t_2 is willing to submit a market sell order MSP_{k,t_2} to hit a limit buy order at price P_k if his payoff $P_k^{cum,MS}(x_{t_2}) - \beta_{t_2} > 0$ is positive, where $P_k^{cum,MS}(x_{t_2}) = P_{k,MS}(x_{t_2}) - TF$ is the cum-fee market-order sell price for price P_k. Given that the investor's valuation β_{t_2} is drawn from $U[\underline{\beta}, \bar{\beta}]$, the submission probability of a market sell, x_{k,t_2}^{MS}, at t_2 is:[19]

$$Pr(x_{k,t_2}^{MS} \vert S, \Xi, L_{t_1}) = \max\{0, \min\{1, \frac{P_{k,MS}(x_{t_2}) - TF - \underline{\beta}}{\Delta}\}\} = Pr(\theta_{t_1}^{x_k^{LB}} \vert S, \Xi, L_{t_1}) \tag{9}$$

where the submission probability of a market sell order MSP_{k,t_2} at P_k at time t_2 is the execution probability $Pr(\theta_{t_1}^{x_k^{LB}} \vert S, \Xi, L_{t_1})$ of a limit buy order LBP_{k,t_1} posted at P_k at time t_1.[20] By symmetry, the submission probability of a market buy MBP_{-k,t_2} at t_2 given a cum-fee market order buy price $P_{-k}^{cum,MB}(x_{t_2}) = P_{-k,MB}(x_{t_2}) + TF$ is:

$$Pr(x_{-k,t_2}^{MB} \vert S, \Xi, L_{t_1}) = \max\{0, \min\{1, \frac{\bar{\beta} - P_{-k,MB}(x_{t_2}) - TF}{\Delta}\}\} = Pr(\theta_{t_1}^{x_{-k}^{LS}} \vert S, \Xi, L_{t_1}), \tag{10}$$

which is the execution probability $Pr(\theta_{t_1}^{x_{-k}^{LS}} \vert S, \Xi, L_{t_1})$ of a limit sell order, LSP_{-k,t_1}.

Next, consider the initial time t_1 in the 2-period market. The limit order book opens empty, and so the investor arriving at t_1 chooses between submitting limit orders and submitting no order (NT). From Lemma

[19]We extended our previous notation so that, for example, x_{k,t_2}^{MS} and MSP_{k,t_2} are used interchangeably for a market sell order at P_k at t_2. When possible, we simplify the notation to make it consistent with the notation used in the figures.

[20]The book opens empty at t_1 and therefore at t_2 the only possible order a seller can take is the one posted by the buyer at t_1

1, an investor with $\beta_{t_1} > v$ is a potential buyer who only submits limit buy orders or NT. This investor optimally posts a limit buy LBP_{k,t_1} at a price P_k if two conditions hold: First, the expected payoff from LBP_{k,t_1} given a private valuation β_{t_1} is positive

$$(\beta_{t_1} - P_k^{cum.LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_k^{LB}} | S, \Xi, L_{t_1}) > 0 \tag{11}$$

and, second, it is greater than the expected payoff from any other alternative limit order $LBP_{\sim k,t_1}$

$$(\beta_{t_1} - P_k^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_k^{LB}} | S, \Xi, L_{t_1}) > (\beta_{t_1} - P_{\sim k}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{\sim k}^{LB}} | S, \Xi, L_{t_1}) \tag{12}$$

where $\sim k$ indexes other possible limit price $P_{\sim k,t_1}$, and where $P_k^{cum,LB}(x_{t_2}) = P_{k,LB}(x_{t_2}) + MF$ and $P_{\sim k}^{cum,LB}(x_{t_2}) = P_{\sim k,LB}(x_{t_2}) + MF$ are the associated cum-fee limit buy prices. Hence, the probability of submission of LBP_{k,t_1} at t_1 is the probability that conditions (11) and (12) are both satisfied:

$$\begin{aligned} & Pr(x_{k,t_1}^{LB} | S, \Xi, L_{t_0}) = \\ & = Pr\Big[(\beta_{t_1} - P_k^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_k^{LB}} | S, \Xi, L_{t_1}) > 0, \\ & \qquad (\beta_{t_1} - P_k^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_k^{LB}} | S, \Xi, L_{t_1}) > (\beta_{t_1} - P_{\sim k}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{\sim k}^{LB}} | S, \Xi, L_{t_1})\Big] \end{aligned} \tag{13}$$

By symmetry, a potential seller at t_1 with $\beta_{t_1} < v$ submits a limit sell LSP_{-k,t_1} if the analogous conditions hold:

$$(P_{-k}^{cum.LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-k}^{LS}} | S, \Xi, L_{t_1}) > 0 \tag{14}$$

and

$$(P_{-k}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-k}^{LS}} | S, \Xi, L_{t_1}) > (P_{\sim -k}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{\sim -k}^{LS}} | S, \Xi, L_{t_1}) \tag{15}$$

where $P_{-k}^{cum,LS}(x_{t_2}) = P_{-k,LS}(x_{t_2}) + MF$ and $P_{\sim -k}^{cum,LS}(x_{t_2}) = P_{\sim -k,LS}(x_{t_2}) + MF$ are the cum-fee limit sell prices. Thus, the probability of submission of LSP_{-k,t_1} at t_1 is the probability that conditions (14) and (15) both hold:

$$\begin{aligned} & Pr(x_{-k,t_1}^{LS} | S, \Xi, L_{t_0}) = \\ & = Pr\Big[(P_{-k}^{cum.LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-k}^{LS}} | S, \Xi, L_{t_1}) > 0, \\ & \qquad (P_{-k}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-k}^{LS}} | S, \Xi, L_{t_1}) > (P_{\sim -k}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{\sim -k}^{LS}} | S, \Xi, L_{t_1})\Big] \end{aligned} \tag{16}$$

We normalize the tick size to $\tau = 1$, and let the investor valuation support, $[\underline{\beta}, \bar{\beta}]$, vary within the outside LTM quotes so that $P_{-3} \leq \underline{\beta}$ and $\bar{\beta} \leq P_3$. Let $\Delta = \bar{\beta} - \underline{\beta} \leq 5\tau$ denote the support width. The equilibrium MF and TF for $P_{-3} \leq \underline{\beta} < \bar{\beta} \leq P_3$ are then derived in four parametric cases respectively for support widths $0 < \Delta \leq 3\tau$ (case 1), $3\tau < \Delta \leq 4\tau$ (case 2), $4\tau < \Delta \leq 4.7\tau$ (case 3), and $4.7\tau < \Delta \leq 5\tau$ (case 4).

Case 1: $0 < \Delta \leq 3\tau$

The exchange sets its access pricing MF and TF to maximize its expected profit. These fees and rebates can take one of three possible alternative forms: Taker-Maker, $\Xi_{TM} = \{0 \leq MF \leq 1, -1 \leq TF \leq 0\}$; Maker-

Taker, $\Xi_{MT} = \{-1 \leq MF \leq 0, 0 \leq TF \leq 1\}$; and Positive-Fee, $\Xi_{PF} = \{0 \leq MF \leq 1, 0 \leq TF \leq 1\}$. We now show that the exchange optimization problem when $\Delta \leq 3\tau$ results in the following functional forms for the equilibrium MF and TF in the Taker-Maker regime

$$MF^* = \frac{\Delta+3}{6} \qquad TF^* = \frac{\Delta-3}{6} \tag{17}$$

and under the Maker-Taker regime

$$MF^* = \frac{\Delta-3}{6} \qquad TF^* = \frac{\Delta+3}{6} \tag{18}$$

Taker-Maker: $\Xi_{TM} = \{0 \leq MF \leq 1, -1 \leq TF \leq 0\}$

We consider first Taker-Maker pricing Ξ_{TM} with a take rebate and a positive make fee. Given $\Delta < 3$, the lower investor valuation limit in this case is $\underline{\beta} = P_{-2} + \frac{3-\Delta}{2}$, and the upper investor valuation is $\overline{\beta} = P_2 - \frac{3-\Delta}{2}$, as illustrated in Figures 7 and 8. Consider first a potential buyer arriving at t_1 with $\beta_{t_1} > v$. The logic for a potential seller arriving at t_1 is symmetric.

Order-submission probabilities for each possible market order at t_2 can be computed using (9) and (**??**) given the valuation-support restriction $\Delta \leq 3$ and Taker-Maker pricing. Columns 4 and 5 in Table 6 report the market order submission probabilities for the price levels in Column 1:

$$Pr(x^{MS}_{k,t_2}|S,\Xi,L_{t_1}) = \max\{0, \frac{P_{k,MS}(x_{t_2}) - TF - \underline{\beta}}{\Delta}\} = \max\{0, \frac{\Delta}{2} + \frac{P_k - P_{-k}}{2} - TF\} \tag{19}$$

$$Pr(x^{MB}_{-k,t_2}|S,\Xi,L_{t_1}) = \max\{0, \frac{\bar{\beta} - P_{-k,MB}(x_{t_2}) - TF}{\Delta}\} = \max\{0, \frac{\Delta}{2} - \frac{P_k - P_{-k}}{2} - TF\} \tag{20}$$

For example, Rows 3 in Column 4 and Row 4 in Column 3 in Table 6 gives the order-submission probability at t_2 of a market sell at P_{-1}, which is equal to the order-submission probability of a market buy at P_1

$$\begin{aligned} Pr(x^{MS}_{-1,t_2}|S,\Xi,L_{t_1}) = \max\{0, \tfrac{P^{cum,MS}_{-1}(x_{t_2}) - \underline{\beta}}{\Delta}\} = \\ Pr(x^{MB}_{1,t_2}|S,\Xi,L_{t_1}) = \max\{0, \tfrac{\bar{\beta} - P^{cum,MB}_{1}(x_{t_2})}{\Delta}\} = \max\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{1}{2} - TF]\}. \end{aligned} \tag{21}$$

To understand the intuition in the last term in (21), note from Figure 7 that only traders with a β_{t_2} in the interval $[\underline{\beta}, P^{cum,MS}_{-1}(x_{t_2})]$ with width $\frac{\Delta}{2} - \frac{1}{2} - TF$ are willing to use a market order to sell at a posted price P_{-1}. This interval is equal to half of the support minus half the tick size, hence $\frac{1}{2}$, given $\tau = 1$, which is the distance from the fundamental asset value v to P_{-1}, minus TF (negative in the Taker-Maker regime), which increases the interval of the support including βs belonging to sellers. This interval is strictly positive for $\Delta \geq 1$, which means that $Pr(x^{MS}_{-1,t_2}|S,\Xi,L_{t_1}) > 0$ for $\Delta \geq 1$.

The market order submission probabilities at t_2 are, in turn, respectively the corresponding order-execution probabilities of limit orders posted at t_1. Thus, we can now consider the expected profits for all possible limit orders that a potential buyer and symmetrically a potential seller can post at t_1. We verify the conditions under which (11) and (12) hold — and symmetrically (14) and (15) — and finally compute the limit order submission probabilities at t_1 consistent with both (13) and (16) .

To check that conditions (11) and (14) hold, we compute $Pr(\beta_{t_1} > P^{cum,LB}_{k}(x_{t_1}))$ and $Pr(P^{cum,LS}_{-k}(x_{t_1}) > \beta_{t_1})$

for each order in Column 2 of Table 6. For example, for a limit order to buy at P_{-1} and to sell at P_1 we have:

$$
\begin{aligned}
Pr(\beta_{t_1} > P_{-1}^{cum,LB}(x_{t_1})) = \max\{0, \tfrac{\bar{\beta} > P_{-1}^{cum,LB}(x_{t_1})}{\Delta}\} = \\
Pr(P_{1}^{cum,LS}(x_{t_1}) > \beta_{t_1}) = \max\{0, \tfrac{P_{1}^{cum,LS}(x_{t_1}) > \underline{\beta}}{\Delta}\} = \max\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{1}{2} - MF]\}.
\end{aligned}
\quad (22)
$$

To understand the intuition for the final term in (23), notice, for example, from Figure 7 that only traders with a β_{t_1} in the interval $[P_{-1}^{cum,LB}(x_{t_1}), \overline{\beta}_{t_1}]$ with width $\frac{\Delta}{2} + \frac{1}{2} - MF$ will be willing to buy at the quoted price P_{-1}. This interval is equal to half of the investor valuation support (consistent with Lemma 1 only traders with a personal evaluation larger than the fundamental value of the asset will be buying) plus half the tick size (the distance between the mid-point of the support/fundamental asset value v and P_{-1}) and which now increases the interval of the support including β buyers - minus MF, which instead decreases the interval of the support including βs belonging to buyers.

We next need to check whether both conditions (12) and (15) hold for each possible order at t_1:

- First, consider a limit buy at P_2 and symmetrically a limit order to sell at P_{-2}. Given the assumed investor valuation support with width $\Delta < 3$ and given the positive MF with Taker-Maker pricing, the expected payoff associated with limit orders at P_2 (P_{-2}) would be negative since the associated cum-fee buy (sell) price would be above (below) the maximum (minimum) possible trader valuation. Hence, such limit orders would never be submitted.

- Second, consider a limit buy at P_{-2} or limit sell at P_2. For these orders, the expected profit is positive:

$$
\begin{aligned}
(\beta_{t_1} - P_{-2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-2}^{LB}} | S, \Xi, L_{t_1}) = (P_{2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{2}^{LS}} | S, \Xi, L_{t_1}) = \\
\max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{3}{2} - MF]\right\} \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{3}{2} - TF]\right\} > 0.
\end{aligned}
\quad (23)
$$

- Third, the expected profit for a limit buy at P_{-1} or a limit sell at P_1 is:

$$
\begin{aligned}
(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}} | S, \Xi, L_{t_1}) = (P_{1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{1}^{LS}} | S, \Xi, L_{t_1}) \\
\max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{1}{2} - MF]\right\} \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{1}{2} - TF]\right\},
\end{aligned}
\quad (24)
$$

  which is higher than from limit buys at P_{-2} and limit sells at P_2, since the following difference is negative:

$$
\begin{aligned}
(\beta_{t_1} - P_{-2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-2}^{LB}} | S, \Xi, L_{t_1}) - (\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}} | S, \Xi, L_{t_1}) = \\
= (P_{2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{2}^{LS}} | S, \Xi, L_{t_1}) - (P_{1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{1}^{LS}} | S, \Xi, L_{t_1}) = \frac{\mathrm{MF} - \mathrm{TF} - 2}{\Delta^2} < 0.
\end{aligned}
\quad (25)
$$

- Lastly, the expected profit from a limit buy at P_1 and limit sell at P_{-1} is positive:

$$
\begin{aligned}
(\beta_{t_1} - P_{1,LB}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{1}^{LB}} | S, \Xi, L_{t_1}) = (P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}} | S, \Xi, L_{t_1}) \\
\max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{1}{2} - MF]\right\} \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{1}{2} - TF]\right\},
\end{aligned}
\quad (26)
$$

  which is lower than the expected payoff from limit buys at P_{-1} or limit. sells at P_1, since the following difference is negative given Taker-Maker pricing:

$$(\beta_{t_1} - P_1^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_1^{LB}}|S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}}|S,\Xi,L_{t_1}) =$$
$$= (P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}}|S,\Xi,L_{t_1}) - (P_1^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_1^{LS}}|S,\Xi,L_{t_1}) = \frac{TF - MF}{\Delta^2} < 0 \quad (27)$$

Thus, we have shown that limit buys at P_{-1} and limit sells at P_1 are the optimal order submissions at t_1 in the $\Delta \leq 3\tau$ case with Taker-Maker pricing. In particular, we have shown that thelimit orders LBP_{-1} and LBP_1 have positive expected payoffs for the ranges in Table 8 and that they dominate all alternative orders.

To determine its optimal MF and TF, the exchange maximizes its exchange profit given the optimal strategy for potential buyers and sellers posting limit orders LBP_{-1,t_1} and LSP_{1,t_1} at t_1, which we have derived as a function of the trading fees, MF and TF, and of the investors' support, S.[21] In particular, by symmetry, the exchange's expected profit is equal to the submission probability $Pr(x_{-1,t_1}^{LB}|S,\Xi,L_{t_0})$ of LBP_{-1,t_1}, times the associated execution probability $Pr(\theta_{t_1}^{x_{-1}^{LB}}|S,\Xi,L_{t_1})$, times the per share net fee, MF+TF. Table 7 reports the equlibrium order submission probabilities.

$$Pr\Big[(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}}|S,\Xi,L_{t_1}) > (\beta_{t_1} - P_{\sim k}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{\sim k}^{LB}}|S,\Xi,L_{t_1})|S,\Xi,L_{t_1})\Big] = 1.$$

It follows that:

$$Pr(x_{-1,t_1}^{LB}|S,\Xi,L_{t_0}) = Pr[(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1}))] = \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{1}{2} - MF]\right\}$$

and the maximization problem of the exchange is:

$$\begin{aligned}
\max_{MF,TF\in\Xi} \pi^{LTM}(MF,TF\,|S,x_{-1,t_1}^{LB},L_{t_0},x_{-1,t_2}^{MS},L_{t_1}) &= Pr(x_{-1,t_1}^{LB}|S,\Xi,L_{t_0}) \times Pr(\theta_{t_1}^{x_{-1}^{LB}}|S,\Xi,L_{t_1}) \times (MF+TF) \quad (28)\\
&= \max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2} + \frac{1}{2} - MF]\right\}(MF+TF)\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2} - \frac{1}{2} - TF]\right\}\\
s.t.:\ 0 < MF < 1, -1 < TF < 0&\\
s.t.:\ MF + TF > 0& \quad (29)\\
s.t.:\ 0 < \Delta \leq 3&
\end{aligned}$$

From the first-order conditions, we obtain:

$$MF^* = \frac{\Delta + 3}{6} \qquad TF^* = \frac{\Delta - 3}{6} \quad (30)$$

[21]The case of a seller posting LSP_{1,t_1} is symmetric. As in real markets, traders arrive sequentially and, hence, either a buyer or seller may arrive at t_1.

Computing the second and mixed derivatives, as well as the determinant, we obtain

$$\delta_{TF,TF} = -\frac{-2\text{MF}+\Delta+1}{\Delta^2} < 0 \tag{31}$$

$$\delta_{MF,MF} = -\frac{\Delta-2\text{TF}-1}{\Delta^2} < 0 \tag{32}$$

$$\delta_{MF,TF} = \frac{2\text{MF}-\Delta+2\text{TF}}{\Delta^2} \tag{33}$$

$$Det = (-1-4MF^2+2MF(1+\Delta-2TF)+2(-1+\Delta-2TF)TF)/\Delta^4 \tag{34}$$

By substituting the equilibrium fees from (30) into (34) we obtain: $Det(MF^*,TF^*) = \frac{1}{3\Delta^2} > 0$.

By substituting the desired value of Δ into MF^* and TF^* in (30), we obtain the equilibrium Taker-Maker fees presented in Table 1. QED

Maker-Taker: $\Xi_{MT} = \{-1 \leq MF \leq 0, 0 \leq TF \leq 1\}$

Now consider Maker-Taker pricing, Ξ_{MT}, with a make rebate and a positive take fee, as illustrated in Figure 8. Once again, we determine the optimal strategies for arriving investors at times t_1 and t_2 and the associated order-submission probabilities:

- First, given a positive take fee TF and an investor valuation support with a width $\Delta < 3$, the expected profit on limit buys at P_{-2} and limit sells at P_2 at t_1 is zero there will be no sellers (buyers) at t_2 willing to sell (buy) at a cum-fee price smaller (higher) than P_{-2} (P_2). Thus, such limit orders are not used in this case.
- Second, the expected profit for a limit buy at P_2 or limit sells at P_{-2} is positive

$$\begin{gathered}(\beta_{t_1} - P_2^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_2^{LB}}|S,\Xi,L_{t_1}) = (P_{-2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-2}^{LS}}|S,\Xi,L_{t_1}) = \\ \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{3}{2} - MF]\right\} \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{3}{2} - TF]\right\} > 0.\end{gathered} \tag{35}$$

- Third, the expected profit from a limit buy and limit sell at P_{-1} is higher:

$$\begin{gathered}Pr(\beta_{t_1} - P_1^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_1^{LB}}|S,\Xi,L_{t_1}) = Pr(P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}}|S,\Xi,L_{t_1}) = \\ \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{1}{2} - MF]\right\} \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{1}{2} - TF]\right\},\end{gathered} \tag{36}$$

  since the following difference, given $\text{TF} - \text{MF} < 2$ with Maker-Taker pricing, is negative:

$$\begin{gathered}Pr(\beta_{t_1} - P_2^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_2^{LB}}|S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_1^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_1^{LB}}|S,\Xi,L_{t_1}) = \\ = Pr(P_{-2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-2}^{LS}}|S,\Xi,L_{t_1}) - Pr(P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}}|S,\Xi,L_{t_1}) = \frac{\text{TF}-\text{MF}-2}{\Delta^2} < 0.\end{gathered} \tag{37}$$

- Lastly, if the expected profit from a limit buy at P_{-1} or limit sell at P_1 is positive and equal to:

$$\begin{gathered}Pr(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}}|S,\Xi,L_{t_1}) = Pr(P_1^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_1^{LS}}|S,\Xi,L_{t_1}) = \\ \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} - \tfrac{1}{2} - MF]\right\} \max\left\{0, \tfrac{1}{\Delta}[\tfrac{\Delta}{2} + \tfrac{1}{2} - TF]\right\},\end{gathered} \tag{38}$$

which is lower than the expected payoff from a limit buy at P_1 or limit sell at P_{-1}, since the following difference is negative given Maker-Taker pricing with $\Xi_{MT} = \{-1 \leq MF \leq 0, 0 \leq TF \leq 1\}$:

$$Pr(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}} | S, \Xi, L_{t_1}) - Pr(\beta_{t_1} - P_{1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{1}^{LB}} | S, \Xi, L_{t_1}) =$$
$$= Pr(P_{1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{1}^{LS}} | S, \Xi, L_{t_1}) - Pr(P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}} | S, \Xi, L_{t_1}) = \frac{MF - TF}{\Delta^2} < 0 \quad (39)$$

Thus, under the Maker-Taker regime the exchange will set the fees such that an investor arriving at t_1 will optimally choose either LBP_{1,t_1} or LSP_{-1,t_1}.

As for the Taker-Maker regime, the exchange anticipates that the optimal order submission strategy for the buyer (seller) is to buy at P_1 (sell at P_{-1}) and to determine the optimal fees we maximize the exchange profits conditional on the buyer now choosing LBP_{1,t_1}, the case of the seller arriving at t_1 being symmetric:

$$\max_{MF,TF \in \Xi} \pi_{0<\Delta\leq 3\tau}^{LTM}(MF, TF | S, x_{1,t_1}^{LB}, L_{t_0}, x_{1,t_2}^{MS}, L_{t_1}) = Pr(x_{1,t_1}^{LB} | S, \Xi, L_{t_0}) \times Pr(\theta_{t_1}^{x_1^{LB}} | S, \Xi, L_{t_1}) \times (MF + TF) \quad (40)$$
$$= \max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2} - \frac{1}{2} - MF]\right\}(MF + TF) \max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2} + \frac{1}{2} - TF]\right\} \quad (41)$$
$$s.t.: \; -1 < MF < 0, 0 < TF < 1$$
$$s.t.: \; MF + TF > 0$$

From the first-order conditions, we obtain:

$$MF^* = \frac{\Delta - 3}{6} \qquad TF^* = \frac{\Delta + 3}{6} \quad (42)$$

Computing the second and mixed derivatives, as well as the determinant, we obtain

$$\delta_{TF,TF} = -\frac{-2\mathrm{MF} + \Delta - 1}{\Delta^2} < 0 \quad (43)$$
$$\delta_{MF,MF} = -\frac{\Delta - 2\mathrm{TF} + 1}{\Delta^2} < 0 \quad (44)$$
$$\delta_{MF,TF} = \frac{2\mathrm{MF} - \Delta + 2\mathrm{TF}}{\Delta^2} \quad (45)$$
$$Det = (-1 - 4MF^2 + 2MF(-1 + \Delta - 2TF) + 2(1 + \Delta - 2TF)TF)/\Delta^4 \quad (46)$$

By substituting the equilibrium fees from (42) into (46) we obtain: $Det(MF^*, TF^*) = \frac{1}{3\Delta^2} > 0$.

By substituting the value of Δ into MF^* and TF^* in (42), we obtain the equilibrium Maker-Taker fees presented in Table 1. QED

Interestingly, Table 1 shows that when the exchange opts for a Taker-Maker (or Maker-Taker) pricing Proposition (3) holds in equilibrium:

$$Pr(x_{-1,t_1}^{LB} | S, \Xi, L_{t_0}) = Pr(\theta_{t_1}^{x_{-1}^{LB}} | S, \Xi, L_{t_1}) = \frac{(MF + TF)}{\Delta} = \frac{1}{3} \quad (47)$$

and

$$Pr(x^{LB}_{1,t_1}|S,\Xi,L_{t_0}) = Pr(\theta_{t_1}^{x_1^{LB}}|S,\Xi,L_{t_1}) = \frac{(MF+TF)}{\Delta} = \frac{1}{3} \tag{48}$$

As Figure 7 (and 8) shows, to maximize expected profits the exchange has to maximize the product of 3 components, $\bar{\beta} - P^{cum,LB}_{-1}$, (MF+TF), $P^{cum,MS}_{-1} - \underline{\beta}$ (and , $\bar{\beta} - P^{cum,LB}_{1}$,(MF+TF), $P^{cum,MS}_{1} - \underline{\beta}$), and the sum of these three components are constrained to be equal to Δ.

Table 6: Submission and Execution Probability. This table reports the price levels on the LTM price grid (column 1) and the associated probabilities $Pr(\beta_{t_1} > P^{cum,LB}_k(x_{t_1})) = \max\{0, \frac{\bar{\beta}-P^{cum,LB}_k(x_{t_2})}{\Delta}\}$ and $Pr(P^{cum,LS}_k(x_{t_1}) > \beta_{t_1}) = \max\{0, \frac{P^{cum,LS}_{-k}(x_{t_2})-\underline{\beta}}{\Delta}\}$, which, in equilibrium, correspond to the submission probabilities for limit orders posted at P_k at t_1 (columns 2 and 3). In addition, the table reports the associated limit order execution probabilities, $Pr(\theta_{t_1}^{x_k^{LB}}|S,\Xi,L_{t_1}) = Pr(x^{MS}_{k,t_2}|S,\Xi,L_{t_1}) = \max\{0, \frac{P^{cum,MS}_k(x_{t_2})-\underline{\beta}}{\Delta}\}$ and $Pr(\theta_{t_1}^{x_{-k}^{LS}}|S,\Xi,L_{t_1}) = Pr(x^{MB}_{-k,t_2}|S,\Xi,L_{t_1}) = \max\{0, \frac{\bar{\beta}-P^{cum,MB}_{-k}(x_{t_2})}{\Delta}\}$ (columns 4 and 5).

P_k	$Pr(\beta_{t_1} > P^{cum,LB}_k(x_{t_1}))$	$Pr(P^{cum,LS}_{-k}(x_{t_1}) > \beta_{t_1})$	$Pr(\theta_{t_1}^{x_k^{LB}}\|S,\Xi,L_{t_1})$	$Pr(\theta_{t_1}^{x_{-k}^{LS}}\|S,\Xi,L_{t_1})$
P_{-3}	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{5}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{5}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{5}{2}-TF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{5}{2}-TF]\right\}$
P_{-2}	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{3}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{3}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{3}{2}-TF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{3}{2}-TF]\right\}$
P_{-1}	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{1}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{1}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{1}{2}-TF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{1}{2}-TF]\right\}$
P_1	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{1}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{1}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{1}{2}-TF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{1}{2}-TF]\right\}$
P_2	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{3}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{3}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{3}{2}-TF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{3}{2}-TF]\right\}$
P_3	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{5}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{5}{2}-MF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}+\frac{5}{2}-TF]\right\}$	$\max\left\{0, \frac{1}{\Delta}[\frac{\Delta}{2}-\frac{5}{2}-TF]\right\}$

Table 7: Equilibrium Submission Probability This table reports the equilibrium submission probabilities for the buy side, $Pr(x^{LB}_{k,t_1}|S,\Xi,L_{t_0})$, conditional on the size of the support Δ. Equilibrium submission probabilities for the sell side, $Pr(x^{LS}_{-k,t_1}|S,\Xi,L_{t_0})$ are symmetric.

	$0<\Delta\leq 4\tau$		$4<\Delta\leq 4.7\tau$	$4.7<\Delta\leq 5\tau$
	Taker-Maker	Maker-Taker	Positive Fees	Positive Fees
$Pr(x^{LB}_{1,t_1}\|S,\Xi,L_{t_0})$		$\max\left\{0,\frac{1}{\Delta}[\frac{\Delta}{2}-\frac{1}{2}-MF]\right\}$		$\max\left\{0,\frac{1}{\Delta}[TF-MF]\right\}$ for $\beta>\frac{\Delta}{2}+9.5$
$Pr(x^{LB}_{-1,t_1}\|S,\Xi,L_{t_0})$	$\max\left\{0,\frac{1}{\Delta}[\frac{\Delta}{2}+\frac{1}{2}-MF]\right\}$		$\max\left\{0,\frac{1}{\Delta}[TF+1]\right\}$ for $\beta>\mathrm{MF}+\frac{\Delta}{2}-\mathrm{TF}+8$	$\max\left\{0,\frac{1}{\Delta}[\frac{\Delta}{2}+\frac{1}{2}-TF]\right\}$ for $\mathrm{MF}+9.5<\beta<\mathrm{MF}+\frac{\Delta}{2}+9$
$Pr(x^{LB}_{-2,t_1}\|S,\Xi,L_{t_0})$			$\max\left\{0,\frac{1}{\Delta}[\frac{\Delta}{2}+MF-TF-2]\right\}$ for $10<\beta<\mathrm{MF}+\frac{\Delta}{2}-\mathrm{TF}+8$	

Table 8: Difference in expected payoff from different orders. This table reports the difference in the expected payoffs from different orders indicated in column 1. Column 2 reports such differences as a function of Δ, whereas columns 3 to 6 reports the same differences for different values of Δ.

	Δ	$\Delta=1$	$\Delta=2$	$\Delta=3$	$\Delta=4$
$Pr(\beta_{t_1} - P_{2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{2}^{LB}} \mid S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{1}^{LB}} \mid S,\Xi,L_{t_1})$ $Pr(P_{1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{1}^{LS}} \mid S,\Xi,L_{t_1}) - Pr(P_{2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{2}^{LS}} \mid S,\Xi,L_{t_1})$	$\frac{4\text{MF}-3S+2\text{TF}+5}{2S^2}$	$2\text{MF}+\text{TF}+1$	$\frac{1}{8}(4\text{MF}+2\text{TF}-1)$	$\frac{1}{9}(2\text{MF}+\text{TF}-2)$	$\frac{1}{32}(4\text{MF}+2\text{TF}-7)$
$Pr(\beta_{t_1} - P_{2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{2}^{LB}} \mid S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}} \mid S,\Xi,L_{t_1})$ $Pr(P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}} \mid S,\Xi,L_{t_1}) - Pr(P_{2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{2}^{LS}} \mid S,\Xi,L_{t_1})$	$\frac{2\text{MF}-3S+4\text{TF}+5}{2S^2}$	$\text{MF}+2\text{TF}+1$	$\frac{1}{8}(2\text{MF}+4\text{TF}-1)$	$\frac{1}{9}(\text{MF}+2\text{TF}-2)$	$\frac{1}{32}(2\text{MF}+4\text{TF}-7)$
$Pr(\beta_{t_1} - P_{2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{2}^{LB}} \mid S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{-2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-2}^{LB}} \mid S,\Xi,L_{t_1})$ $Pr(P_{-2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-2}^{LS}} \mid S,\Xi,L_{t_1}) - Pr(P_{2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{2}^{LS}} \mid S,\Xi,L_{t_1})$	$\frac{-3S+6\text{TF}+9}{2S^2}$	$3(\text{TF}+1)$	$\frac{3}{8}(2\text{TF}+1)$	$\frac{\text{TF}}{3}$	$\frac{3}{32}(2\text{TF}-1)$
$Pr(\beta_{t_1} - P_{1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{1}^{LB}} \mid S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}} \mid S,\Xi,L_{t_1})$ $Pr(P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}} \mid S,\Xi,L_{t_1}) - Pr(P_{1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{1}^{LS}} \mid S,\Xi,L_{t_1})$	$\frac{\text{TF}-\text{MF}}{S^2}$	$\text{TF}-\text{MF}$	$\frac{\text{TF}-\text{MF}}{4}$	$\frac{\text{TF}-\text{MF}}{9}$	$\frac{\text{TF}-\text{MF}}{16}$
$Pr(\beta_{t_1} - P_{1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{1}^{LB}} \mid S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{-2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-2}^{LB}} \mid S,\Xi,L_{t_1})$ $Pr(P_{-2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-2}^{LS}} \mid S,\Xi,L_{t_1}) - Pr(P_{1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{1}^{LS}} \mid S,\Xi,L_{t_1})$	$\frac{-2\text{MF}+2\text{TF}+2}{S^2}$	$-2\text{MF}+2\text{TF}+2$	$\frac{1}{2}(-\text{MF}+\text{TF}+1)$	$-\frac{2}{9}(\text{MF}-\text{TF}-1)$	$\frac{1}{8}(-\text{MF}+\text{TF}+1)$
$Pr(\beta_{t_1} - P_{-1}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-1}^{LB}} \mid S,\Xi,L_{t_1}) - Pr(\beta_{t_1} - P_{-2}^{cum,LB}(x_{t_1})) \times Pr(\theta_{t_1}^{x_{-2}^{LB}} \mid S,\Xi,L_{t_1})$ $Pr(P_{-2}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-2}^{LS}} \mid S,\Xi,L_{t_1}) - Pr(P_{-1}^{cum,LS}(x_{t_1}) - \beta_{t_1}) \times Pr(\theta_{t_1}^{x_{-1}^{LS}} \mid S,\Xi,L_{t_1})$	$\frac{-\text{MF}+\text{TF}+2}{S^2}$	$-\text{MF}+\text{TF}+2$	$\frac{1}{4}(-\text{MF}+\text{TF}+2)$	$\frac{1}{9}(-\text{MF}+\text{TF}+2)$	$\frac{1}{16}(-\text{MF}+\text{TF}+2)$

Figure 7: Taker-Maker Pricing: $\Xi_{TM} = \{0 \leq MF \leq 1, -1 \leq TF \leq 0\}$ This Figure provides a graphical representation of how to obtain the equilibrium probabilities of order submission and execution for the Taker-Maker pricing structure and the support $\Delta \in [\underline{\beta}, \bar{\beta}]$. P_2 and P_{-2} are the outside quotes of the LTM, whereas P_1 and P_{-1} are the inside quotes of the LTM. $P_{-1}^{cum,LB}$ and $P_{-1}^{cum,MS}$ are the cum-fee buy and sell prices, respectively. LBP_{-1,t_1} is a limit buy order posted at P_{-1} at t_1, and MSP_{-1,t_2} is a market sell order posted at P_{-1} at t_2.

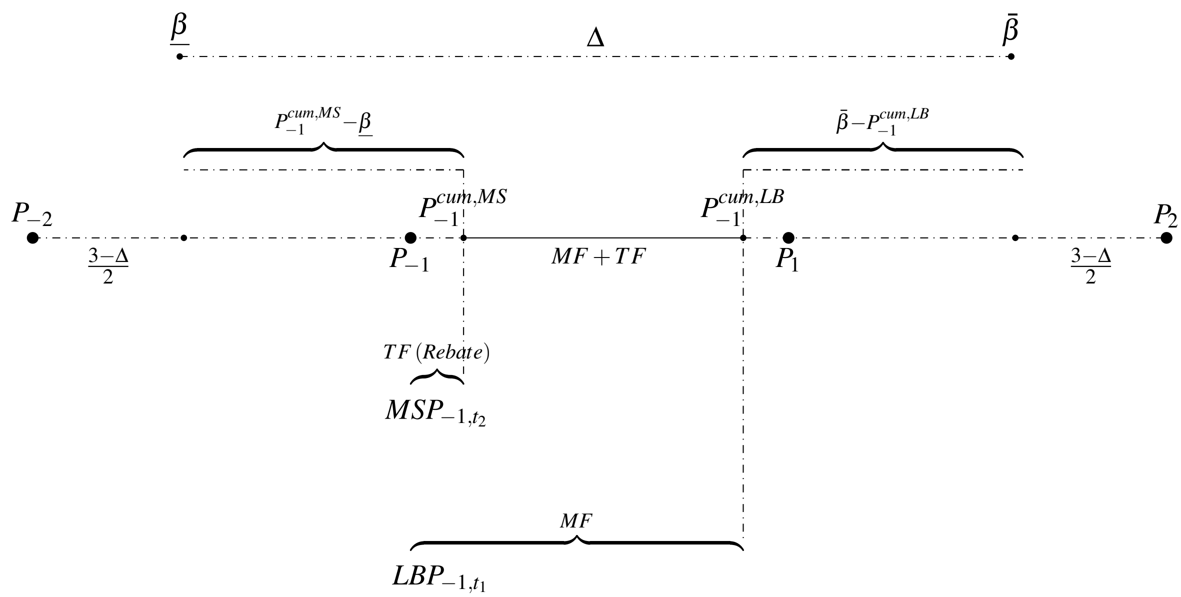
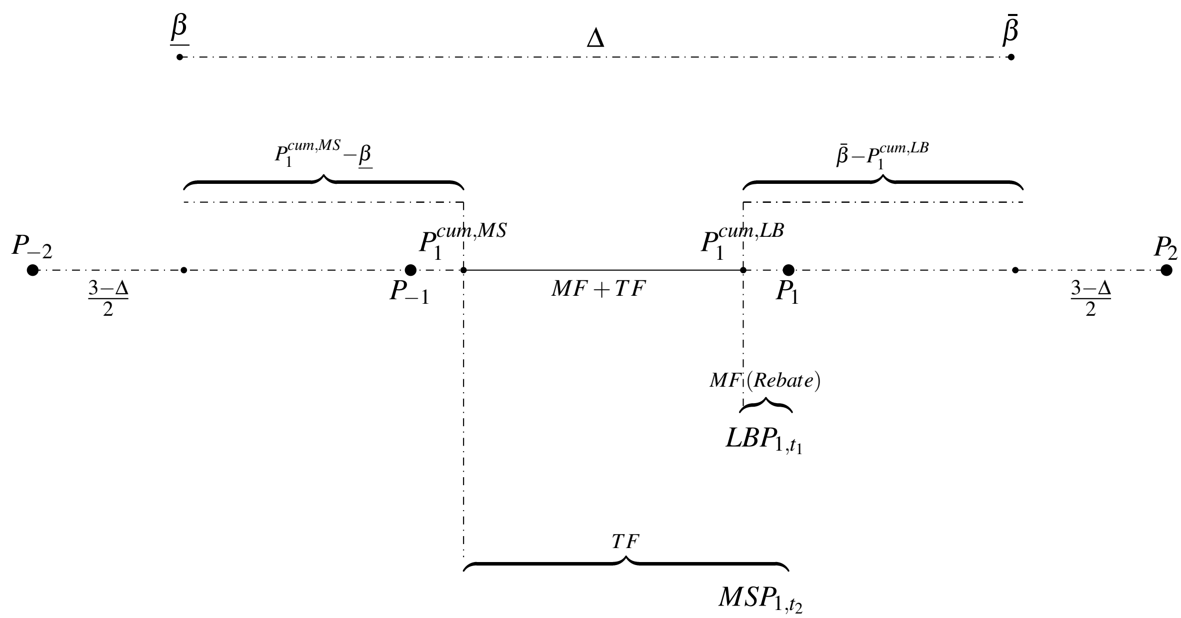


Figure 8: Maker-Taker Pricing: $\Xi_{MT} = \{-1 \leq MF \leq 0, 0 \leq TF \leq 1\}$ This Figure provides a graphical representation of how to obtain the equilibrium probabilities of order submission and execution for the Maker-Taker pricing structure and the support $\Delta \in [\underline{\beta}, \bar{\beta}]$. P_2 and P_{-2} are the outside quotes of the LTM, whereas P_1 and P_{-1} are the inside quotes of the LTM. $P_1^{cum,LB}$ and $P_1^{cum,MS}$ are the cum-fee buy and sell prices, respectively. LBP_{1,t_1} is a limit buy order posted at P_1 at t_1, and MSP_{1,t_2} is a market sell order posted P_1 at t_2.



Positive Fees: $\Xi_{PF} = \{0 \le MF \le 1, 0 \le TF \le 1\}$

Consider now the possibility of access pricing with strictly positive fees, Ξ_{PF}. The expressions in Table 6 (which imply zero probabilities when they are negative) show that under this pricing, the trader's expected profit is zero if he buys at P_2, LBP_{2,t_1}, or sells at P_{-2}, LSP_{-2,t_1}, as no buyers (sellers) would be willing to buy (sell) at a price net of fee higher (lower) than P_2 (P_{-2}).

The trader's expected profit would also be zero if he buys at P_{-2}, LBP_{-2,t_1}, or sells at P_2, LSP_{2,t_1}, as no sellers (buyers) would be willing to market sell (market buy) at t_2 at a price net of fee lower (higher) than P_{-2} (P_2), being $P_{-2} \le \Delta \le P_2$.

Table 6 shows that the trader's expected profit at t_1 would be positive if he buys either at P_{-1} or at P_1 (or sells at either P_1 or P_{-1}); and, considering equations (27) and (39), the difference in the expected profit would depend on the relative size of the MF and TF (Table 8). However, equation (27) shows that the trader would secure higher profits from the equilibrium strategy if the exchange set Ξ_{TM} rather than Ξ_{PF} when either a buyer buys at P_{-1} or a seller sells at P_1 at t_1; similarly equation (39) shows that the trader would get more profits from the equilibrium strategy if the exchange set Ξ_{MF} rather than Ξ_{PF} when either a buyer buys at P_1 or a seller sells at P_{-1} at t_1. We can therefore conclude that Ξ_{PF} is suboptimal when $P_{-2} \le \underline{\beta} < \bar{\beta} \le P_2$. QED

Case 2: $3\tau < \Delta \le 4\tau$

Given that $3\tau < \Delta \le 4\tau$, traders can choose among the same orders considered in Case 1. While the symmetry between buy and sell orders still applies, i.e., $Pr(\theta^{x^{LB}_{i,t_1}}|S,\Xi,L_{t_0}) = Pr(\theta^{x^{MS}_{\sim i,t_1}}|S,\Xi,L_{t_0})$, now $\bar{\beta} > P_2$ and $\underline{\beta} < P_{-2}$, and therefore buying at P_2 as well as selling at P_{-2} can be profitable if $Pr(x^{MS}_{-2,t_2}|S,\Xi,L_{t_1}) > 0$, i.e., if $TF < \frac{S-3}{2}$ (Table 6). However, Table 8 shows that a limit order to buy at P_2 (sell at P_{-2}) are dominated strategies.

Hence, to determine the optimal MF and TF, we maximize the exchange profits conditional on the buyer choosing LBP_{-2,t_1}, or $x^{LB}_{-1,t_1} = LBP_{-1,t_1}$ the case of the seller arriving at t_1 being symmetric:

$$
\begin{aligned}
&\max_{MF,TF \in \Xi} \pi^{LTM}_{3\tau<\Delta\le 4\tau}(MF,TF\,|S,x^{LB}_{-1,t_1},L_{t_0},x^{MS}_{-1,t_2},L_{t_1}) = \\
&= \left(Pr(x^{LB}_{-1,t_1}|S,\Xi,L_{t_0}) \times Pr(\theta^{x^{LB}_{-1}}_{t_1}|S,\Xi,L_{t_1})\right) \times (MF+TF) = \\
&= \frac{0.5(\mathrm{TF}+1.)(\mathrm{MF}+\mathrm{TF})(\Delta-2.\mathrm{TF}-1.)}{\Delta^2} = \\
&s.t.:\ TF < \frac{\Delta-3}{2} \\
&s.t.:\ -1 < MF < 1, -1 < TF < 1 \\
&s.t.:\ MF+TF > 0 \\
&s.t.:\ 3 < \Delta \le 4
\end{aligned}
\tag{49}
$$

The Kuhn-Tucker Lagrangian is:

$$L(MF,TF,\Delta,\lambda_k,v_h) = \\ \pi^{LTM}_{3\tau<\Delta\leq 4\tau}(MF,TF\,|S,x^{LB}_{1,t_1},L_{t_0},x^{MS}_{1,t_2},L_{t_1})- \\ \lambda_1(-TF+\frac{\Delta-3}{2})-\lambda_2(-MF+1)-\lambda_3(-\Delta+3.1)-\lambda_4(-\Delta+4)+v_1MF+v_2TF \quad (50)$$

The Kuhn-Tucker conditions are:

$$\frac{\delta\pi^{LTM}_{3\tau<\Delta\leq 4\tau}}{\delta MF} = \frac{0.5(\mathrm{TF}+1.)(\Delta-2.\mathrm{TF}-1.)}{\Delta^2} \geq 0 \;\&\; MF\times\frac{\delta\pi^{LTM}_{3\tau<\Delta\leq 4\tau}}{\delta MF}=0 \quad (51)$$

$$\frac{\delta\pi^{LTM}_{3\tau<\Delta\leq 4\tau}}{\delta TF} = \frac{\mathrm{MF}(0.5\Delta-2.\mathrm{TF}-1.5)+\Delta(1.\mathrm{TF}+0.5)+(-3.\mathrm{TF}-3.)\mathrm{TF}-0.5}{\Delta^2} \geq 0 \;\&\; TF\times\frac{\delta\pi^{LTM}_{3\tau<\Delta\leq 4\tau}}{\delta TF}=0 \quad (52)$$

$$\frac{\delta\pi^{LTM}_{3\tau<\Delta\leq 4\tau}}{\delta\Delta} = \frac{(\mathrm{MF}+\mathrm{TF})(\Delta(-0.5\mathrm{TF}-0.5)+\mathrm{TF}(2.\mathrm{TF}+3.)+1.)}{\Delta^3} \geq 0 \;\&\; \Delta\times\frac{\delta\pi^{LTM}_{3\tau<\Delta\leq 4\tau}}{\delta\Delta}=0 \quad (53)$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_1} = (-TF+\frac{\Delta-3}{2})\geq 0 \;\&\; \lambda_1\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_1}=0 \quad (54)$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_2} = (-MF+1)\geq 0 \;\&\; \lambda_2\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_2}=0 \quad (55)$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_3} = (-\Delta+3.1)\geq 0 \;\&\; \lambda_3\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_3}=0 \quad (56)$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_4} = (-\Delta+4)\geq 0 \;\&\; \lambda_4\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_4}=0 \quad (57)$$

The equilibrium MF^* and TF^* that satisfy these conditions are:

$$MF^*=1 \qquad TF^*=0.5(\Delta-3) \quad (58)$$

By substituting a given value of Δ into MF^* and TF^* in (58), we obtain the equilibrium fees in Table 1.

Case 3: $4\tau < S \leq 4.7\tau$

We have shown that for investor valuation supports with widths up $\Delta = 4$, there are dominant orders for potential buyers and sellers, and so the optimal order-submission strategy can be obtained by comparing the expected payoff associated with each possible order, as shown in Tables 6 and 8; in the latter we present as an example the differences in expected payoffs conditional on different support sizes. However, for investor valuation supports with widths $\Delta > 4$, there are two possible equilibrium limit orders, and we report the outcome of (13) and (16) in Table 7, which shows that both a limit order at P_{-1} and at P_{-2} are sometimes optimal depending on β_{t_1}. We also report conditions on the value of β such that the equilibrium strategies hold.

To determine the optimal MF and TF, the exchange maximizes its expected profit conditional on the buyer choosing either LBP_{-2,t_1}, or LBP_{-1,t_1} the case of the seller arriving at t_1 being symmetric:

$$\max_{MF,TF\in\Xi} \pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}(MF,TF\,|S,x^{LB}_{-1,t_1},x^{LB}_{-2,t_1},L_{t_0},x^{MS}_{-1,t_2},x^{MS}_{-2,t_2},L_{t_1}) = \tag{59}$$
$$= \Big(Pr(x^{LB}_{-1,t_1}|S,\Xi,L_{t_0})\times Pr(\theta^{x^{LB}_{-1}}_{t_1}|S,\Xi,L_{t_1}) + Pr(x^{LB}_{-2,t_1}|S,\Xi,L_{t_0})\times Pr(\theta^{x^{LB}_{-2}}_{t_1}|S,\Xi,L_{t_1})\Big)\times(MF+TF) =$$
$$= \frac{(MF+TF)(-2.5+\Delta(1.25-0.25\Delta+0.5TF)-2TF+MF(1.5-0.5\Delta+TF))}{\Delta^2}$$
$$s.t.: \; TF < \frac{\Delta-3}{2}$$
$$s.t.: \; -1 < MF < 1, -1 < TF < 1$$
$$s.t.: \; MF+TF > 0$$
$$s.t.: \; 4 < \Delta \leq 4.7$$

The Kuhn-Tucker Lagrangian is:

$$L(MF,TF,\Delta,\lambda_k,v_h) = \tag{60}$$
$$\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}(MF,TF\,|S,x^{B}_{1,t_1},L_{t_0},x^{S}_{1,t_2},L_{t_1}) +$$
$$\lambda_1(-TF+\frac{\Delta-3}{2}) - \lambda_2(-MF+1) - \lambda_3(-\Delta+4) - \lambda_4(-\Delta+4.7) + v_1 MF + v_2 TF$$

The Kuhn-Tucker conditions are:

$$\frac{\delta\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}}{\delta MF} = \frac{\mathrm{TF}(0.5-\mathrm{TF})+\mathrm{MF}(\Delta-2\mathrm{TF}-3)+(0.25\Delta-1.25)\Delta+2.5}{\Delta^2} \geq 0 \;\&\; MF\times\frac{\delta\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}}{\delta MF} = 0 \tag{61}$$

$$\frac{\delta\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}}{\delta TF} = \frac{\mathrm{MF}(0.5-2\mathrm{TF})-\mathrm{MF}^2+\Delta(0.25\Delta-\mathrm{TF}-1.25)+4\mathrm{TF}+2.5}{\Delta^2} \geq 0 \;\&\; TF\times\frac{\delta\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}}{\delta TF} = 0 \tag{62}$$

$$\frac{\delta\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}}{\delta\Delta} = \frac{\mathrm{MF}^2(-0.5\Delta+2\mathrm{TF}+3)+\mathrm{MF}(1.25\Delta+\mathrm{TF}(2\mathrm{TF}-1)-5)+(0.5\Delta-4)\mathrm{TF}^2+(1.25\Delta-5)\mathrm{TF}}{\Delta^3} \geq 0$$
$$\&\; \Delta\times\frac{\delta\pi^{LTM}_{4\tau<\Delta\leq 4.7\tau}}{\delta\Delta} = 0 \tag{63}$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_1} = (-TF+\frac{\Delta-3}{2}) \geq 0 \;\&\; \lambda_1\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_1} = 0 \tag{64}$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_2} = (-MF+1) \geq 0 \;\&\; \lambda_2\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_2} = 0 \tag{65}$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_3} = (-\Delta+4) \geq 0 \;\&\; \lambda_3\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_3} = 0 \tag{66}$$

$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_4} = (-\Delta+4.7) \geq 0 \;\&\; \lambda_4\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_4} = 0 \tag{67}$$

The equilibrium MF^* and TF^* that satisfy these conditions are:

$$MF^* = 1 \qquad TF^* = \frac{0.25\left(\Delta^2-5\Delta+8\right)}{\Delta-2} \tag{68}$$

By substituting a given value of Δ into MF^* and TF^* in (68), we obtain the equilibrium fees in Table 1.

Case 4: $4.7\tau < \Delta \leq 5\tau$

In this case, the investor valuation support width can be as large as 5τ, which is the difference between P_3 and P_{-3}. So we also consider the investor's profit conditional on orders posted at P_3 and P_{-3}. Table 6 shows that the investor's profit is zero if he buys at P_3 or sells at P_{-3}. Table 7 shows that for this interval of the support the equilibrium strategies are either $x^{LB}_{1,t_1} = LBP_{1,t_1}$, or $x^{LB}_{-1,t_1} = LBP_{-1,t_1}$. Therefore, to determine the optimal MF and TF, we maximize the exchange profits conditional on the buyer optimally using these two strategies, the case of the seller arriving at t_1 being symmetric:

$$\max_{MF,TF\in\Xi} \pi^{LTM}_{4.7\tau<\Delta\leq 5\tau}(MF,TF\,|S,x^{LB}_{1,t_1},x^{LB}_{-1,t_1},L_{t_0},x^{MS}_{1,t_2},x^{MS}_{-1,t_2},L_{t_1}) = \tag{69}$$
$$= \Big(Pr(x^{LB}_{1,t_1}|S,\Xi,L_{t_0})\times Pr(\theta_{t_1}^{x_1^{LB}}|S,\Xi,L_{t_1}) + Pr(x^{LB}_{-1,t_1}|S,\Xi,L_{t_0})\times Pr(\theta_{t_1}^{x_{-1}^{LB}}|S,\Xi,L_{t_1})\Big)\times(MF+TF) =$$
$$= \frac{0.25(-2\mathrm{MF}+\Delta-1)(\mathrm{MF}+\mathrm{TF})(\Delta-2\mathrm{TF}+1)}{\Delta^2}$$
$$s.t.:\ TF < \frac{\Delta-3}{2}$$
$$s.t.:\ -1 < MF < 1, -1 < TF < 1$$
$$s.t.:\ MF+TF > 0$$
$$s.t.:\ 4.7 < \Delta \leq 5$$

The Kuhn-Tucker Lagrangian is:

$$L(MF,TF,\Delta,\lambda_k,v_h) = \tag{70}$$
$$\pi^{LTM}_{4.7\tau<\Delta\leq 5\tau}(MF,TF\,|S,x^{B}_{1,t_1},L_{t_0},x^{S}_{1,t_2},L_{t_1})+$$
$$\lambda_1(-TF+\frac{\Delta-3}{2})-\lambda_2(-MF+1)-\lambda_3(-\Delta+4.7)-\lambda_4(-\Delta+5)+v_1MF+v_2TF$$

The Kuhn-Tucker conditions are:

$$\frac{\delta\pi^{LTM}}{\delta MF} = \frac{\mathrm{MF}(-\Delta+2\mathrm{TF}-1)+0.25\Delta^2-\Delta\mathrm{TF}+\mathrm{TF}^2-0.25}{\Delta^2} \geq 0 \ \&\ MF\times\frac{\delta\pi^{LTM}}{\delta MF} = 0 \tag{71}$$
$$\frac{\delta\pi^{LTM}}{\delta TF} = \frac{\mathrm{MF}^2-\mathrm{MF}\Delta+2\mathrm{MFTF}+0.25\Delta^2-\Delta\mathrm{TF}+\mathrm{TF}-0.25}{\Delta^2} \geq 0 \ \&\ TF\times\frac{\delta\pi^{LTM}}{\delta TF} = 0 \tag{72}$$
$$\frac{\delta\pi^{LTM}}{\delta\Delta} = \frac{\mathrm{TF}^2(-2\mathrm{MF}+0.5\Delta-1)+\mathrm{MFTF}(\Delta-2\mathrm{MF})+\mathrm{MF}(\mathrm{MF}(0.5\Delta+1)+0.5)+0.5\mathrm{TF}}{\Delta^3} \geq 0 \ \&\ \Delta\times\frac{\delta\pi^{LTM}}{\delta\Delta} = 0 \tag{73}$$
$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_1} = (-TF+\frac{\Delta-3}{2}) \geq 0 \ \&\ \lambda_1\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_1} = 0 \tag{74}$$
$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_2} = (-MF+1) \geq 0 \ \&\ \lambda_2\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_2} = 0 \tag{75}$$
$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_3} = (-\Delta+4.7) \geq 0 \ \&\ \lambda_3\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_3} = 0 \tag{76}$$
$$\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_4} = (-\Delta+5) \geq 0 \ \&\ \lambda_4\times\frac{\delta L(MF,TF,\Delta,\lambda_k,v_h)}{\delta\lambda_4} = 0 \tag{77}$$

The equilibrium MF^* and TF^* that satisfy these conditions are:

$$MF^* = 0.5 \qquad TF^* = 1 \;\forall\; 4.7 < \Delta \leq 5 \tag{78}$$

which are the equilibrium fees presented in Table 1. QED.

Comment: Proposition 1 follows from the formulas for optimal MF and TF in the parameterizations $\Delta < 4\tau$ for which rebated-based pricing is optimal. Proposition 1 also follows from the optimal fee formula when MF and TF are optionl.

C Regulatory Regimes

Regulatory restrictions can have a major impact on equilibrium access pricing. In this section we disentangle the effects of three alternative regulatory specifications. Our model assumes the trading platform cannot set trading fees that (in absolute value) exceed the tick size, $-\tau \leq MF \leq \tau$ and $-\tau \leq TF \leq \tau$. We call this the *RRS Regulatory Restrictions*.[22] Our results over the $\Delta < 3\,\tau$ region agree qualitatively with Chao et al. (2018) (CYY) regarding the existence of symmetric maker-taker and taker-maker equilibria. However, our results differ in that we find that jointly positive fees occur when the amount of investor valuation dispersion is large ($\Delta > 3\,\tau$), whereas in CYY fees are never jointly positive.

The reason for the difference is that CYY impose different constraints on fees and rebates (footnote 16, Chao et al. (2018)): $0 < MF < \tau$ and $-\tau < TF < 0$. We call this the *CYY Restrictions*. To show the effect of this stronger restriction, we solve our 2-period model with a support equal to $[P_{-1}, P_1] = [p_{-2}, p_2] = [9.5, 10.5]$ (with a support width of τ) under three different tick size specifications (as in CYY) for three different regulatory regimes: The RRS Regulatory Restrictions, the CYY Restrictions, and with no restrictions on access pricing (“No Restrictions").

Another difference between our analysis and CYY is the assumed investor arrival process. We assume investor valuations each period are uniformly distributed on the whole valuation support, whereas CYY assume buyers and sellers alternate each period with sellers' valuations being distributed over the lower half of the support and buyers' valuations being distributed over the upper half of the support.[23] However, we show that the qualitative differences between our results and CYY are due to the different regulatory assumptions and not the mechanical difference in investor arrival.

Table 9 shows that when $\Delta = \tau$ and No Restrictions are imposed on MF and TF, the equilibrium in the CYY model with the CYY investor-arrival assumption delivers the same trading fees at in the equilibrium with the RRS investor-arrival assumption, across each of the three different tick size specifications considered here (τ, $\frac{\tau}{4}$ and $\frac{\tau}{8}$). However, exchange profits in the CYY model are twice as high as in RRS model, because of the alternating buyer and seller assumption in the CYY framework. The results reported in Table 9 show that when No Restrictions are imposed on the trading fees, the taker-maker (shown) and symmetric maker-taker (not shown) pricing structures are both optimal in equilibrium. This holds with both the RRS and CYY investor-arrival assumptions. When, following CYY, we hold the investor valuation support constant and consider different tick sizes, we find that the equilibrium optimal make and take fees do not change. In particular, with No Restrictions on trading fees, the exchange optimally sets a positive fee of

[22] Using the exact fee cap of 0.3 of the tick size from Reg NMS would make our results stronger. In Europe — where there is no formal regulatory fee cap but possibly an informal regulatory understanding — exchanges usually access set access fees smaller than one tick.

[23] The reason we do not assume alternating buyers and sellers is that, when we extend our model to three periods, the assumption that any investor may arrive at each trading period is more suited to modeling liquidity dynamics.

0.667 and a rebate of -0.333 irrespective of the tick size. By doing so, the exchange forces traders to discard the tick size and trade at the outside quotes. Once again, we note the net fee is one third of the valuation support width.

Table 9 also shows the effects of both the RRS Regulatory Restrictions and the CYY Restrictions on the equilibrium trading fees. When we solve for equilibria under both the RRS and CYY trader-arrival models with the RRS Regulatory Restrictions, the taker-maker pricing structure and symmetrically the maker-taker pricing structure prevail only when the tick size is equal to τ. For smaller tick sizes ($\frac{\tau}{4}$ and $\frac{\tau}{8}$) both the optimal MF and TF are positive. Intuitively, under the RRS Regulatory Restrictions the exchange cannot discard the tick size rule and, not being allowed to impose extreme trading fees, maximizes profits by imposing the symmetric taker-maker or maker-taker pricing only when the support is equal to the tick size. When instead the support widens relative to the tick size, the exchange exploits the investors' increased gains from trade and imposes positive fees on both takers and makers.

Notice here that cutting the tick size to $\frac{\tau}{4}$ holding the support width constant at $\Delta = \tau$ (i.e., 1 tick) has the same impact on the support width/tick size ratio as in Table 1 where we hold the tick size equal to 1 and set $\Delta = 4\tau$. There we find that the equilibrium fees are positive, but the results are different because changing the tick size also affects the RRS Regulatory Restriction through which fees are capped relative to the tick size. Changing the support width does not affect the fee cap, but changing the tick size does. When instead the CYY Restrictions constrain the exchange not to impose a positive TF, the taker-maker pricing is the only equilibrium trading fee structure that prevails both under the RRS and under the CYY protocol.

So far, we have shown how the optimal trading fees change when, holding the investor composition constant (i.e., holding the valuation support constant at $\Delta = \tau = 1$), we consider different markets with different tick size regimes. The natural following question is whether we obtain similar results by holding the tick size constant and changing the support of the investors' beliefs. Table 10 shows that under the "No Restrictions" regime, if we hold the tick size constant to 1 and gradually widen the support from one tick ($[9.50, 10.50] = \tau$), to three ticks ($[8.50, 11.50] = 3\tau$), to five ticks ($[7.50, 12.50] = 5\tau$), the taker-maker and symmetrically the maker-taker pricing structure become stronger with the (unconstrained) positive fee increasing from 0.667 to 3.333 and the rebate $|fee|$ increasing from $|-0.333|$ to $|-1.667|$. These results holds for both the RRS and the CYY investor-arrival frameworks, although as before the CYY exchange profits are twice as high in the RRS framework. In addition, notice here yet again that the net fee satifies $MF + TF = \Delta/3$. When instead we impose the RRS Regulatory Restrictions, we are back to Figure 1 and Table 1 that show how, when the support in the LTM reaches three ticks, the taker-maker and maker-taker are no longer equilibrium fee structures. To economize space we do not show the results obtained when running the same extensions with increasing supports for the CYY framework as they lead to the unique taker-maker equilibrium due to the restriction imposed on the TF.

Table 9: Optimal Trading Fees and Restrictions This table reports the equilibrium optimal make (MF) and take fee (TF), Exchange Expected Profit, equilibrium strategies, cum-fee buy and sell prices ($P_k^{LB,cum}$ and $P_k^{MS,cum}$) for a support with width $\Delta = \tau$ for markets with three different tick size specifications (τ, $\frac{\tau}{4}$ and $\frac{\tau}{8}$) and under three different regulatory regimes given both the RRS (our) and the CYY (Chao et al. (2018)) investor-arrival frameworks. The "RRS Regulatory Restrictions" are $-\tau \leq MF, TF \leq \tau$; the "CYY Restrictions" are $0 \leq MF \leq S$ and $-\tau \leq TF \leq 0$; and the "No Restrictions" protocol imposes no restrictions on MF and TF fees.

		τ	$\frac{\tau}{4}$	$\frac{\tau}{8}$
	MF	0.667	0.667	0.667
CYY framework	TF	-0.333	-0.333	-0.333
"No Restrictions"	Exchange E[Profit]	0.148	0.148	0.148
	Eq.Strategies x_{t_1}	$LB_{9.500}$	$LB_{9.500}$	$LB_{9.500}$
$-S \leq MF \leq S$	$P_k^{LB,cum}$	10.167	10.167	10.167
$-S \leq TF \leq S$	$P_k^{MS,cum}$	9.833	9.833	9.833
	MF	0.667	0.667	0.667
RRS framework	TF	-0.333	-0.333	-0.333
"No Restrictions"	Exchange E[Profit]	0.074	0.074	0.074
	Eq.Strategies x_{t_1}	$LB_{9.500}$	$LB_{9.500}$	$LB_{9.500}$
$-S \leq MF \leq S$	$P_k^{LB,cum}$	10.167	10.167	10.167
$-S \leq TF \leq S$	$P_k^{MS,cum}$	9.833	9.833	9.833
CYY framework	MF	0.667	0.206	0.125
"RRS Regulatory	TF	-0.333	0.169	0.125
Restrictions"	Exchange E[Profit]	0.148	0.141	0.125
	Eq.Strategies x_{t_1}	$LB_{9.500}$	$LB_{9.750}$, $LB_{10.000}$	$LB_{9.750}$, $LB_{9.875}$, $LB_{10.000}$
$-\tau \leq MF \leq \tau$	$P_k^{LB,cum}$	10.167	9.956, 10.206	9.875, 10.000, 10.125
$-\tau \leq TF \leq \tau$	$P_k^{MS,cum}$	9.833	9.581, 9.831	9.625, 9.750, 9.875
RRS framework	MF	0.667	0.206	0.125
"RRS Regulatory	TF	-0.333	0.169	0.125
Restrictions"	Exchange E[Profit]	0.074	0.070	0.0625
	Eq.Strategies x_{t_1}	$LB_{9.500}$	$LB_{9.750}$, $LB_{10.000}$	$LB_{9.750}$, $LB_{9.875}$, $LB_{10.000}$
$-\tau \leq MF \leq \tau$	$P_k^{LB,cum}$	10.167	9.956, 10.206	9.875, 10.000, 10.125
$-\tau \leq TF \leq \tau$	$P_k^{MS,cum}$	9.833	9.581, 9.831	9.625, 9.750, 9.875
CYY framework	MF	0.667	0.496	0.387
"CYY Restrictions"	TF	-0.333	-0.121	-0.012
	Exchange E[Profit]	0.148	0.141	0.141
$0 \leq MF \leq S$	Eq.Strategies x_{t_1}	$LB_{9.500}$	$LB_{9.500}$, $LB_{9.750}$	$LB_{9.500}$, $LB_{9.625}$, $LB_{9.750}$
$-\tau \leq TF \leq 0$	$P_k^{LB,cum}$	10.167	9.996, 10.246	9.887, 10.012, 10.137
	$P_k^{MS,cum}$	9.833	9.621, 9.871	9.512, 9.637, 9.762
RRS framework	MF	0.667	0.496	0.387
"CYY Restrictions"	TF	-0.333	-0.121	-0.012
	Exchange E[Profit]	0.074	0.070	0.070
$0 \leq MF \leq S$	Eq.Strategies x_{t_1}	$LB_{9.500}$	$LB_{9.500}$, $LB_{9.750}$	$LB_{9.500}$, $LB_{9.625}$, $LB_{9.750}$
$-\tau \leq TF \leq 0$	$P_k^{LB,cum}$	10.167	9.996, 10.246	9.887, 10.012, 10.137
	$P_k^{MS,cum}$	9.833	9.621, 9.871	9.512, 9.637, 9.762

Table 10: Optimal Trading Fees and No Restrictions. This table reports results on the optimal trading fees (MF and TF), equilibrium trading strategies ($x_{t_1} = LB_{P_k}$), cum-fee prices buy and sell prices ($P_k^{LB,cum}$ and $P_k^{MS,cum}$) and Exchange Expected Profits for the "No Restrictions" protocol on access fees and for both the RRS and CYY investor-arrival frameworks. The tick size τ is equal to 1, and results are reported for three support widths, $\Delta = 1$, $\Delta = 3$ and $\Delta = 5$.

	Support width		
	$\Delta = 1$	$\Delta = 3$	$\Delta = 5$
MF	0.667	2.000	3.333
TF	-0.333	-1.000	-1.667
Eq.Strategies $x_{t_1} = LB_{P_k}$	$LB_{9.500}$	$LB_{8.500}$	$LB_{7.500}$
$P_k^{LB,cum}$	10.167	10.500	10.833
$P_k^{MS,cum}$	9.833	9.500	9.167
Exchange E[Profit] CYY	0.148	0.444	0.741
Exchange E[Profit] RRS	0.074	0.222	0.370

D 3-Period Model (In Progress)

To illustrate how the 3-period model works, we first use the benchmark model and show that in absence of fees, given the investors' support and the tick size, the model has a closed form solution. We present the solution for a support equal to 2τ and the large tick τ. The solution for different supports and for the small tick size can be found in a similar way. The following 3 tables show how to derive analytically the equilibrium order submission probabilities respectively at t_3, t_2 and t_1 for the benchmark model.
To obtain the optimal trading fees set by the exchange we then add the profit function of the exchange to the benchmark model without fees and we maximize the exchange profits π by using both the Simulated Annealing (SA) algorithm and the optimizing algorithm, the Fee Optimizing (FO) algorithm, that we created to refine the solutions provided by the SA algorithm. Results are shown in Table 3.
Table 21 shows market quality and welfare results for the 3-Period large tick market.

Here below we explain how we integrate the SA algorithm with the FO algorithm to maximize the exchange profits, π.

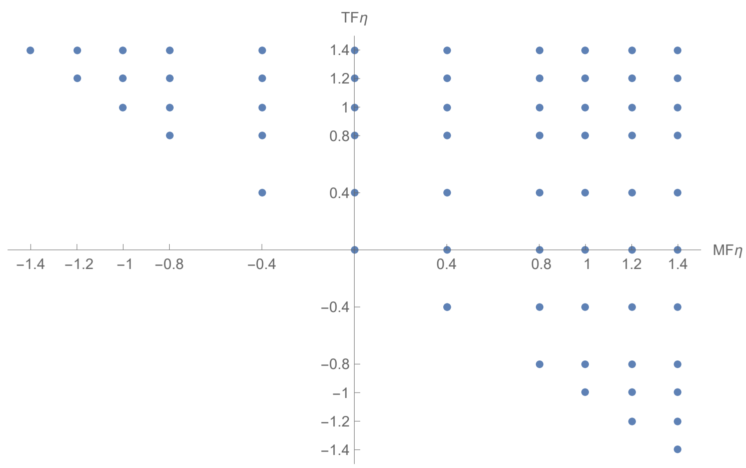
Simulated Annealing (SA) and Fee Optimizing (FO) Algorithms

We use the model described in Tables 13, 14 and 15 to initialize the variables that we need to compute π, i.e., the investors' support, the tick size, and the probability of order submission at each node of the trading game. We then use both the SA and the FO algorithms to determine the equilibrium MF^* and TF^* set by the exchange conditional on the support and the large tick size.[24]

[24]Results for the STM can be obtained in a similar way and are available from the authors upon request.

The simulated annealing (SA) algorithm is an iterative procedure that starts at time η with an initial set of combinations of MF_η and TF_η, Ξ_η, with $-1.5\tau < MF_\eta, TF_\eta < 1.5\tau$, and search for the maximum profit of the exchange, π, conditional on the tick size and the support of traders' evaluations, S. Figure 9 reports the initial combinations of fees that we chose for the large tick market. The SA will then search

Figure 9: Simulated Annealing (SA) Algorithm: Large Tick Market (LTM) initial Sets of MF_η and TF_η. This Figure reports the initial combinations of MF_η and TF_η, from which the SA algorithm starts to numerically maximize the exchange profits π.



for the maximum π within a neighborhood of amplitude $2 \times \varepsilon$ of each initial combination of fees. We set $\varepsilon = 0.25$ so that the amplitude of the region explored around each fee is equal to half a tick. For example, given the initial set of fees, $\Xi_\eta = \{MF_\eta = 0, TF_\eta = 0.4\}$, the SA algorithm will select a value for $MF_{\eta+1}$ within the interval $\{MF_\eta = 0 - \varepsilon, MF_\eta = 0 + \varepsilon\}$ and a value for $TF_{\eta+1}$ within the interval $\{TF_\eta = 0.4 - \varepsilon, TF_\eta = 0.4 + \varepsilon\}$ with Uniform probability. Assume for example that the randomly selected set of fees is $\Xi_{\eta+1} = \{MF_{\eta+1} = 0.1, TF_{\eta+1} = 0.5\}$. If $\Xi_{\eta+1}$ is associated with an exchange profit that is higher than the exchange profit associated with the initial set of fees Ξ_η, then the SA algorithm will select the next combination of fees, $\Xi_{\eta+2}$, starting from $\Xi_{\eta+1}$, within the interval $\{MF_{\eta+2} = 0.1 - \varepsilon, MF_{\eta+2} = 0.1 + \varepsilon\}$ for $MF_{\eta+2}$ and $\{TF_{\eta+2} = 0.5 - \varepsilon, TF_{\eta+2} = 0.5 + \varepsilon\}$ for $TF_{\eta+2}$. If instead $\Xi_{\eta+1}$ is associated with an exchange profit which is lower or equal than the exchange profit associated with Ξ_η, then the algorithm will choose the new combination of fees, $\Xi_{\eta+2}$ starting from $\Xi_{\eta+1}$, within the interval $2 \times \varepsilon$ with probability $\zeta_\eta = e^{\frac{\pi_\eta - \pi_{\eta-1}}{\chi_\eta}}$, whereas it will choose the new combination of fees starting from Ξ_η with probability $1 - \zeta_\eta$, where χ_η is a parameter that starts with value $\chi_\eta = 0.8$ and decreases by $0.9\chi_\eta$ at each η iteration until it reaches its minimum that we set at 0.066667. This means that as the number of iterations increases, the probability ζ_η with which the SA algorithm will explore the neighborhood of the out-of-equilibrium sets of fees will also tend to increase.

Starting from the initial 66 combinations of fees, the SA algorithm explores approximately 10700 sets of fees for each support and produces a number (approximately 8) of possible equilibrium set of fees ($\Xi^\dagger = \{MF^\dagger, TF^\dagger\}$) for each support that differ approximately by 10^{-4} in terms of the associated π. The objective of the FO algorithm is to refine the equilibrium sets of fees generated by the SA algorithm.

We consider the 8 combinations of fees [with the highest associated π and with $-\tau < MF^\dagger, TF^\dagger < \tau$], of which 4 combinations of fees such that $MF^\dagger > TF^\dagger$ and 4 combinations of fees such that $MF^\dagger < TF^\dagger$. To illustrate how the FO algorithm works, assume that one the 8 optimal sets of SA fees chosen is $\Xi^{\dagger'} =$

$\{MF^{\dagger'} = -0.270, TF^{\dagger'} = 0.494\}$. The FO algorithm will generate the first grid (Grid #1) with the combinations of fees that differ by 6 steps of $\Delta = 0.02$ $(-0.06, -0.04, -0.02, 0.0, 0.02, 0.04, 0.06)$ from $\Xi^{\dagger'}$. The FO algorithm will then evaluate and compare the 49 combinations of fees reported in Table 11 and select the set of fees with the highest associated π. Assume that the optimal set of fees generated by Grid #1 is

Table 11: Grid#1 This Table reports the combinations of MF and TF that differ by 6 steps of $\Delta = 0.02$ from $\Xi^{\dagger'}$.

		-0.06	-0.04	-0.02	0	0.02	0.04	0.06
		-0.33	-0.31	-0.29	-0.27	-0.25	-0.23	-0.21
-0.06	0.434	-0.33,0.434	-0.31,0.434	-0.29,0.434	-0.27,0.434	-0.25,0.434	-0.23,0.434	-0.21,0.434
-0.04	0.454	-0.33,0.454	-0.31,0.454	-0.29,0.454	-0.27,0.454	-0.25,0.454	-0.23,0.454	-0.21,0.454
-0.02	0.474	-0.33,0.474	-0.31,0.474	-0.29,0.474	-0.27,0.474	-0.25,0.474	-0.23,0.474	-0.21,0.474
0	0.494	-0.33,0.494	-0.31,0.494	-0.29,0.494	-0.27,0.494	-0.25,0.494	-0.23,0.494	-0.21,0.494
0.02	0.514	-0.33,0.514	**-0.31,0.514**	-0.29,0.514	-0.27,0.514	-0.25,0.514	-0.23,0.514	-0.21,0.514
0.04	0.534	-0.33,0.534	-0.31,0.534	-0.29,0.534	-0.27,0.534	-0.25,0.534	-0.23,0.534	-0.21,0.534
0.06	0.554	-0.33,0.554	-0.31,0.554	-0.29,0.554	-0.27,0.554	-0.25,0.554	-0.23,0.554	-0.21,0.554

$\Xi^{\dagger''} = \{MF^{\dagger''} = -0.310, TF^{\dagger''} = 0.514\}$, the FO algorithm will now generate a second grid (*Grid*#2) that differ by 6 steps of $\Delta = 0.01$ $(-0.03, -0.02, -0.01, 0.0, 0.01, 0.02, 0.03)$ from $\Xi^{\dagger''}$. The FO algorithm will then evaluate and compare the new 49 combinations of fees presented in Table 12, and will repeat this procedure 6 times starting from the new possible equilibrium set of fees, each time reducing Δ according to the following vector: $\Delta \in \{0.02, 0.01, 0.005, 0.0025, 0.000125, 0.000065\}$. The set of fees associated with the highest π derived from the last grid will be finally compared with the optimal sets of fees obtained by starting from the other 7 best combinations of fees generated by the SA. The resulting set of fees associated with the highest π will be the optimal set of fees, $\xi^* = MF^*, TF^*$.

Table 12: Grid#2 This Table reports the combinations of MF and TF that differ by 6 steps of $\Delta = 0.01$ from $\Xi^{\dagger''}$.

		-0.03	-0.02	-0.01	0	0.01	0.02	0.03
		-0.34	-0.33	-0.32	-0.31	-0.3	-0.29	-0.28
-0.03	0.484	-0.33,0.484	-0.31,0.484	-0.29,0.484	-0.27,0.484	-0.25,0.484	-0.23,0.484	-0.21,0.484
-0.02	0.494	-0.33,0.494	-0.31,0.494	-0.29,0.494	-0.27,0.494	-0.25,0.494	-0.23,0.494	-0.21,0.494
-0.01	0.504	-0.33,0.504	-0.31,0.504	-0.29,0.504	-0.27,0.504	-0.25,0.504	-0.23,0.504	-0.21,0.504
0	0.514	-0.33,0.514	-0.31,0.514	-0.29,0.514	-0.27,0.514	-0.25,0.514	-0.23,0.514	-0.21,0.514
0.01	0.524	-0.33,0.524	-0.31,0.524	-0.29,0.524	-0.27,0.524	-0.25,0.524	-0.23,0.524	-0.21,0.524
0.02	0.534	-0.33,0.534	-0.31,0.534	-0.29,0.534	-0.27,0.534	-0.25,0.534	-0.23,0.534	-0.21,0.534
0.03	0.544	-0.33,0.544	-0.31,0.544	-0.29,0.544	-0.27,0.544	-0.25,0.544	-0.23,0.544	-0.21,0.544

E 3-Period Model With HFT (In Progress)

As for the 3-period model, we now show how to obtain the closed form solution for the benchmark model (this time with HFTs) without fees, a support equal to 2τ and the large tick size, τ.

Table 13: 3-Period Large Tick Market (LTM). Equilibrium Strategies at t_3. This table shows how to derive the equilibrium order submission strategies at t_3 for the benchmark model which has no trading fees ($MF = TF = 0.00$) and for an investors' support equal to 2τ. At t_1 the market opens with an empty book, [0000], where each element in the square bracket, $L_{t_z} = D_{t_z}^{P_i}$, corresponds to the depth of the book at each price level at time t_z, $[L_{t_z}^{P_2}, L_{t_z}^{P_1}, L_{t_z}^{P_{-1}}, L_{t_z}^{P_{-2}}]$. Given the chosen set of fees, four are the equilibrium strategies at t_1, LBP_1 and LBP_{-1} on the buy side and LSP_1 and LSP_{-1} on the sell side. At t_1 Table **??** presents both the buy and the sell equilibrium strategies. However, as the equilibrium strategies consistent with the states of the book derived from the buy side are symmetric to those derived from the sell side, to economize space at t_2 we only present the equilibrium strategies that are consistent with the states of the book derived from the sell equilibrium strategies at t_1. Given the equilibrium limit buy orders at t_1, the possible states of the books at the beginning of t_2 are: $[00B0]$ following a LBP_{-1} and $[0B00]$ following a LBP_1. Given the equilibrium strategies at t_2 and therefore the possible states of the books at the beginning of t_3, this table shows the equilibrium strategies at t_3 (column 1), their payoffs (column 2), the β thresholds (column 3) and the order submission probabilities (column 4).

Equilibrium Strategy	Payoff	β Threshold	Order Submission Probability
at t_1 the book opens empty [0000]: equilibrium strategy LBP_{-1}			
at t_2 the book opens [00B0]			
	t_2 equilibrium strategy: MSP_{-1,t_2}		
at t_3 the book opens empty [0000]			
NT_{t_3}	0	{9.000,11.000}	1
	t_2 equilibrium strategy: LSP_1		
at t_3 the book opens [0SB0]			
MSP_{-1,t_3}	$P_{-1} - \beta_{t_3} - TF = 9.500 + \beta_{t_3}$	{9.000,9.500}	0.250
NT_{t_3}	0	{9.500,10.500}	0.500
MBP_{1,t_3}	$\beta_{t_3} - P_1 - TF = -10.500 + \beta_{t_3}$	{10.500,11.000}	0.250
	t_2 equilibrium strategy: LBP_1		
at t_3 the book opens [0BB0]			
MSP_{1,t_3}	$P_1 - \beta_{t_3} - TF = 10.500 - \beta_{t_3}$	{9.000,10.500}	0.750
NT_{t_3}	0	{10.500,11.000}	0.250
at t_1 the book opens empty [0000]: equilibrium strategy LBP_1			
at t_2 the book opens [0B00]			
	t_2 equilibrium strategy: MSP_{1,t_2}		
at t_3 the book opens empty [0000]			
NT_{t_3}	0	{9.000, 11.000}	1
	t_2 equilibrium strategy: NT_{t_2}		
at t_3 the book opens [00B0]			
MSP_{1,t_3}	$P_1 - \beta_{t_3} - TF = 10.500 - \beta_{t_3}$	{9.000, 10.500}	0.750
NT_{t_3}	0	{10.500,11.000}	0.250

To obtain the optimal trading fees, we use - as for the 3-period model - both the SA and the FO algorithms and we run them for different supports of the market participants. Results are shown in Table 4.

Table 14: 3-Period Large Tick Market (LTM). Equilibrium Strategies at t_2. This table shows how to derive the equilibrium order submission strategies at t_2 for the benchmark model which has no trading fees ($MF = TF = 0.00$) and for a support equal to 2τ. At t_1 the market opens with an empty book, [0000], where each element in the square bracket, $L_{t_z} = D_{t_z}^{P_i}$, corresponds to the depth of the book at each price level at time t_z, $[L_{t_z}^{P_2}, L_{t_z}^{P_1}, L_{t_z}^{P_{-1}}, L_{t_z}^{P_{-2}}]$. Given the chosen set of fees, four are the equilibrium strategies at t_1: LBP_1 and LBP_{-1} on the buy side and LSP_1 and LSP_{-1} on the sell side. At t_1 we present both buy and sell the equilibrium strategies; to economize space, at t_2 we present the equilibrium strategies that are consistent with the states of the book derived from the sell equilibrium strategies at t_1, as the equilibrium strategies consistent with the states of the book derived from the buy side are perfectly symmetric. Given the equilibrium limit buy orders at t_1, the possible states of the books at the beginning of t_2 are: [00B0] following a LBP_{-1} and [0B00] following a LBP_1. Column 1 shows the Equilibrium Strategies at t_2, column 2 shows the corresponding payoffs, and columns 3 and 4 show the β thresholds and the order submission probabilities respectively.

Equilibrium Strategy	Payoff	β Threshold	Order Submission Probability
at t_1 the book opens empty [0000]: equilibrium strategy LBP_{-1}			
at t_2 the book opens [00B0]			
MSP_{-1,t_2}	$P_{-1} - \beta_{t_2} - TF = 9.500 - \beta_{t_2}$	{9.000,9.167}	0.083
LSP_1	$(P_1 - \beta_{t_2} - MF)Pr(\theta_{t_2}^{LSP_1} \| S, \Xi, L_{t_2}) = 2.625 - 0.250\beta_{t_2}$	{9.167,10.500}	0.667
LBP_1	$(\beta_{t_2} - P_1 - MF)Pr(\theta_{t_2}^{LBP_1} \| S, \Xi, L_{t_2}) = -7.875 + 0.750\beta_{t_2}$	{10.500,11.000}	0.250
at t_1 the book opens empty [0000]: equilibrium strategy LBP_1			
at t_2 the book opens [0B00]			
MSP_{1,t_2}	$P_1 - \beta_{t_2} - TF = 10.500 - \beta_{t_2}$	{9.000,10.500}	0.750
NT_{t_2}	0	{10.500,11.000}	0.250

Table 15: 3-Period Large Tick Market (LTM). Equilibrium Strategies at t_1. This table shows how to derive the equilibrium order submission strategies at t_1 for the benchmark model which has no trading fees ($MF = TF = 0.00$) and for a support equal to 2τ. At t_1 the market opens with an empty book, [0000], where each element in the square bracket, $L_{t_z} = D_{t_z}^{P_i}$, corresponds to the depth of the book at each price level at time t_z, $[L_{t_z}^{P_2}, L_{t_z}^{P_1}, L_{t_z}^{P_{-1}}, L_{t_z}^{P_{-2}}]$. Given the chosen set of fees, four are the equilibrium strategies at t_1: LBP_1 and LBP_{-1} on the buy side and LSP_1 and LSP_{-1} on the sell side (column 1). Column 2 shows their payoffs, and columns 3 and 4 shows the β thresholds and the order submission probabilities respectively.

Equilibrium Strategy	Payoff	β Threshold	Order Submission Probability
at t_1 the book opens empty [0000]			
LSP_{-1}	$(P_{-1} - \beta_{t_1} - MF)Pr(\theta_{t_1}^{LSP_{-1}} \| S, \Xi, L_{t_1}) = 8.906 - 0.938\beta_{t_1}$	{9.000,9.136}	0.068
LSP_1	$(P_1 - \beta_{t_1} - MF)Pr(\theta_{t_1}^{LSP_1} \| S, \Xi, L_{t_1}) = 2.625 - 0.250\beta_{t_1}$	{9.136,10.000}	0.432
LBP_{-1}	$(\beta_{t_1} - P_{-1} - MF)Pr(\theta_{t_1}^{LBP_{-1}} \| S, \Xi, L_{t_1}) = -2.375 + 0.250\beta_{t_1}$	{10.000,10.863}	0.432
LBP_1	$(\beta_{t_1} - P_1 - MF)Pr(\theta_{t_1}^{LBP_1} \| S, \Xi, L_{t_1}) = -9.844 + 0.938\beta_{t_1}$	{10.863,11.000}	0.068

Table 16: 3-Period Large Tick Market (LTM) with HFTs. Equilibrium Strategies at t_3. This table shows how to derive the equilibrium order submission strategies at t_3 for the benchmark model with HFTs which has no trading fees ($MF = TF = 0.00$) and for the Investors' support equal to 2τ. At t_1 the market opens with an empty book, [0000], where each element in the square bracket, $L_{t_z} = D_{t_z}^{P_i}$, corresponds to the depth of the book at each price level at time t_z, $[L_{t_z}^{P_2}, L_{t_z}^{P_1}, L_{t_z}^{P_{-1}}, L_{t_z}^{P_{-2}}]$. Given the chosen set of fees, four are the equilibrium strategies at t_1: LBP_1 (followed by a MSP_1 from an HFT firm) and LBP_{-1} on the buy side, and LSP_1 and LSP_{-1} (followed by a MBP_{-1} from an HFT firm) on the sell side. At t_1 Table 18 presents both the buy and the sell equilibrium strategies. However, as the equilibrium strategies consistent with the states of the book derived from the buy side are symmetric to those derived from the sell side, to economize space at t_2 we only present the equilibrium strategies that are consistent with the states of the book derived from the sell equilibrium strategies at t_1. Given the equilibrium limit buy orders at t_1, the possible states of the books at the beginning of t_2 are: [00B0] following a LBP_{-1} and [0000] following a LBP_1 and a MSP_1 from an HFT firm. Given the equilibrium strategies at t_2 and therefore the possible states of the books at the beginning of t_3, this table shows the equilibrium strategies at t_3 (column 1), their payoffs (column 2), the β thresholds (column 3) and the order submission probabilities (column 4).

Equilibrium Strategy	Payoff	β Threshold	Order Submission Probability
at t_1 the book opens empty [0000]: equilibrium strategy LBP_{-1}			
at t_2 the book opens [00B0]			
	t_2 equilibrium strategy: MSP_{-1,t_2}		
at t_3 the book opens empty [0000]			
NT_{t_3}	0	{9.000,11.000}	1
	t_2 equilibrium strategy: LSP_1		
at t_3 the book opens [0SB0]			
MSP_{-1,t_3}	$P_{-1} - \beta_{t_3} - TF = 9.500 - \beta_{t_3}$	{9.000,9.500}	0.250
NT_{t_3}	0	{9.500,10.500}	0.500
MBP_{1,t_3}	$\beta_{t_3} - P_1 - TF = -10.500 + \beta_{t_3}$	{10.500,11.000}	0.250
	t_2 equilibrium strategy: $LBP_1 \longrightarrow$ HFT: $MSP_{1_{t_2}}$		
at t_3 the book opens [00B0]			
NT_{t_3}	0	{9.000,9.500}	0.750
MSP_{-1,t_3}	$P_{-1} - \beta_{t_3} - TF = 9.500 - \beta_{t_3}$	{9.500,11.000}	0.250
at t_1 the book opens empty [0000]: equilibrium strategy $LSP_{-1} \longrightarrow$ HFT: $MBP_{-1_{t_2}}$			
at t_2 the book opens empty [0000]			
	t_2 equilibrium strategy: $LBP_1 \longrightarrow$ HFT: $MSP_{1_{t_2}}$		
at t_3 the book opens empty [0000]			
NT_{t_3}	0	{9.000, 11.000}	1
	t_2 equilibrium strategy: LBP_{-1}		
at t_3 the book opens [00B0]			
MSP_{-1,t_3}	$P_{-1} - \beta_{t_3} - TF = 9.500 - \beta_{t_3}$	{9.000, 9.500}	0.250
NT_{t_3}	0	{9.500, 11.000}	0.750
	t_2 equilibrium strategy: LSP_1		
at t_3 the book opens [0S00]			
NT_{t_3}	0	{9.000, 10.500}	0.750
MBP_{1,t_3}	$\beta_{t_3} - P_1 - TF = -10.500 + \beta_{t_3}$	{10.500,11.000}	0.250
	t_2 equilibrium strategy: $LSP_{-1} \longrightarrow$ HFT: $MBP_{-1_{t_2}}$		
at t_3 the book opens empty [0000]			
NT_{t_3}	0	{9.000, 11.000}	1

Table 17: 3-Period Large Tick Market (LTM) with HFTs. Equilibrium Strategies at t_2. This table shows how to derive the equilibrium order submission strategies at t_2 for the benchmark model with HFT which has no trading fees ($MF = TF = 0.00$) and for an investors' support equal to 2τ. At t_1 the market opens with an empty book, [0000], where each element in the square bracket, $L_{t_z} = D_{t_z}^{P_i}$, corresponds to the depth of the book at each price level at time t_z, $[L_{t_z}^{P_2}, L_{t_z}^{P_1}, L_{t_z}^{P_{-1}}, L_{t_z}^{P_{-2}}]$. Given the chosen set of fees, four are the equilibrium strategies at t_1: LBP_1 (followed by a MSP_1 from an HFT firm) and LBP_{-1} on the buy side, and LSP_1 and LSP_{-1} (followed by a MBP_{-1} from an HFT firm) on the sell side. At t_1 Table 18 presents both the buy and the sell equilibrium strategies. However, as the equilibrium strategies consistent with the states of the book derived from the buy side are symmetric to those derived from the sell side, to economize space at t_2 we only present the equilibrium strategies that are consistent with the states of the book derived from the sell equilibrium strategies at t_1. Given the equilibrium limit buy orders at t_1, the possible states of the books at the beginning of t_2 are: [00B0] following a LBP_{-1} and [0000] following a LBP_1 and a MSP_1 from an HFT firm. Column 1 shows the Equilibrium Strategies at t_2, column 2 shows the corresponding payoffs, and columns 3 and 4 show the β thresholds and the order submission probabilities respectively. We present the β Thresholds and the Order Submission Probabilities only for the regular investors; HFT firms have $\beta = 1$ and take profitable liquidity offered by aggressive orders with probability 1.

Equilibrium Strategy	**Payoff**	**β Threshold**	**Order Submission Probability**
at t_1 the book opens empty [0000]: equilibrium strategy LBP_{-1}			
at t_2 the book opens [00B0]			
MSP_{-1,t_2}	$P_{-1} - \beta_{t_2} - MF = 9.500 - \beta_{t_2}$	{9.000,9.167}	0.083
LSP_1	$(P_1 - \beta_{t_2} - MF)Pr(\theta_{t_2}^{LSP_1} \vert S, \Xi, L_{t_2}) = 2.625 - 0.250\beta_{t_2}$	{9.167,10.500}	0.667
$LBP_1 \longrightarrow HFT: MSP_{1,t_2}$	$(\beta_{t_2} - P_1 - TF) \times 1 = -10.500 + \beta_{t_2}$	{10.500,11.000}	0.250
at t_1 the book opens empty [0000]: equilibrium strategy $LBP_1 \longrightarrow$ HFT: MSP_{1,t_2}			
at t_2 the book opens empty [0000]			
$LSP_{-1} \longrightarrow HFT: MBP_{-1,t_2}$	$(P_{-1} - \beta_{t_1} - MF) \times 1 = 9.500 - \beta_{t_1}$	{9.000,9.167}	0.083
LSP_1	$(P_1 - \beta_{t_1} - MF)Pr(\theta_{t_1}^{LSP_1} \vert S, \Xi, L_{t_1}) = 2.625 - 0.250\beta_{t_1}$	{9.167,10.000}	0.417
LBP_{-1}	$(\beta_{t_1} - P_{-1} - MF)Pr(\theta_{t_1}^{LBP_{-1}} \vert S, \Xi, L_{t_1}) = -2.375 + 0.250\beta_{t_1}$	{10.000,10.833}	0.417
$LBP_1 \longrightarrow HFT: MSP_{1,t_2}$	$(\beta_{t_1} - P_1 - MF) \times 1 = -10.500 + \beta_{t_1}$	{10.833,11.000}	0.083

Table 18: 3-Period Large Tick Market (LTM) with HFTs. Equilibrium Strategies at t_1. This table shows how to derive the equilibrium order submission strategies at t_1 for the benchmark model with HFTs which has no trading fees ($MF = TF = 0.00$) and for an investors' support equal to 2τ. At t_1 the market opens with an empty book, [0000], where each element in the square bracket, $L_{t_z} = D_{t_z}^{P_i}$, corresponds to the depth of the book at each price level at time t_z, $[L_{t_z}^{P_2}, L_{t_z}^{P_1}, L_{t_z}^{P_{-1}}, L_{t_z}^{P_{-2}}]$. Given the chosen set of fees, four are the equilibrium strategies at t_1: LBP_1 (followed by a MSP_1 from an HFT firm) and LBP_{-1} on the buy side, and LSP_1 and LSP_{-1} (followed by a MBP_{-1} from an HFT firm) on the sell side (column 1). Column 2 shows their payoffs, and columns 3 and 4 shows the β thresholds and the order submission probabilities respectively. We present the β Thresholds and the Order Submission Probabilities only for the regular investors; HFT firms have $\beta = 1$ and take profitable liquidity offered by aggressive orders with probability 1.

Equilibrium Strategy	**Payoff**	**β Threshold**	**Order Submission Probability**
at t_1 the book opens empty [0000]			
$LSP_{-1} \longrightarrow HFT: MBP_{-1,t_1}$	$(P_{-1} - \beta_{t_1} - MF) \times 1 = 9.500 - \beta_{t_1}$	{9.000,9.167}	0.083
LSP_1	$(P_1 - \beta_{t_1} - MF)Pr(\theta_{t_1}^{LSP_1} \vert S, \Xi, L_{t_1}) = 2.625 - 0.250\beta_{t_1}$	{9.167,10.000}	0.417
LBP_{-1}	$(\beta_{t_1} - P_{-1} - MF)Pr(\theta_{t_1}^{LBP_{-1}} \vert S, \Xi, L_{t_1}) = -2.375 + 0.250\beta_{t_1}$	{10.000,10.833}	0.417
$LBP_1 \longrightarrow HFT: MSP_{1,t_1}$	$(\beta_{t_1} - P_1 - MF) \times 1 = -10.500 + \beta_{t_1}$	{10.833,11.000}	0.083

F Market Quality and Welfare

Tables 19, 20,, 22, 23 show the market quality and welfare results for the 2-period model, both large and small tick, and for the 3-period model with HFTs.

Table 19: 2-Period Large Tick Market: Market Quality and Welfare with profit-maximizing access pricing. This table reports for each support of the investors' personal evaluation considered (row 1), our metrics of market quality and welfare. The equilibrium make and take fee (MF and TF) are reported in rows 2 and 3, and the extremes of the investors' support ($\underline{\beta}$ and $\bar{\beta}$ max) are reported in rows 4 and 5. The investors' supports are expressed in terms of the tick size of the large tick, τ. The shaded area indicates the DL region with rebate-based pricing.

Δ	**0.333 τ**		**1.000 τ**		**1.270 τ**		**1.880 τ**		**2.000 τ**		**3.000 τ**		**4.000 τ**	**5.000 τ**
MF	-0.444	0.556	-0.333	0.667	-0.288	0.712	-0.187	0.813	-0.167	0.833	0.000	1.000	1.000	0.500
TF	0.556	-0.444	0.667	-0.333	0.712	-0.288	0.813	-0.187	0.833	-0.167	1.000	0.000	0.500	1.000
$\bar{\beta}$	10.167	10.167	10.500	10.500	10.635	10.635	10.940	10.940	11.000	11.000	11.500	11.500	12.000	12.500
$\underline{\beta}$	9.833	9.833	9.500	9.500	9.365	9.365	9.060	9.060	9.000	9.000	8.500	8.500	8.000	7.500
Depth P_2 (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Depth P_1 (t_1)	0.333	0.033	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.375	0.500
Depth P_{-1} (t_1)	0.333	0.033	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.375	0.500
Depth P_{-2} (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Depth (t_1)	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.750	1.000
Volume (t_2)	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.187	0.240
Eff Spread (t_2)	-1.000	1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000	-1.000	1.000	1.000	1.000	1.000	0.333
Welfare LO INV	0.012	0.012	0.037	0.037	0.047	0.047	0.070	0.070	0.074	0.074	0.111	0.111	0.141	0.260
Welfare MO INV	0.012	0.012	0.037	0.037	0.047	0.047	0.070	0.070	0.074	0.074	0.111	0.111	0.094	0.160
Welfare INV	0.025	0.025	0.074	0.074	0.094	0.094	0.139	0.139	0.148	0.148	0.222	0.222	0.234	0.420
Welfare Exchange	0.025	0.025	0.074	0.074	0.094	0.094	0.139	0.139	0.148	0.148	0.222	0.222	0.281	0.360
Welfare Tot	0.049	0.049	0.148	0.148	0.188	0.188	0.279	0.279	0.296	0.296	0.444	0.444	0.516	0.780
Depth P_2 Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000		0.000		0.000	0.100
Depth P_1 Benchmark (t_1)	0.000		0.000		0.500		0.500		0.500		0.500		0.500	0.400
Depth P_{-1} Benchmark (t_1)	0.000		0.000		0.500		0.500		0.500		0.500		0.500	0.400
Depth P_{-2} Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000		0.000		0.000	0.100
Depth Benchmark (t_1)	0.000		0.000		1.000		1.000		1.000		1.000		1.000	1.000
Volume Benchmark (t_2)	0.000		0.000		0.106		0.234		0.250		0.333		0.375	0.360
Eff Spread Benchmark (t_2)	0.000		1.000		1.000		1.000		1.000		1.000		1.000	1.222
W. Benchmark LO INV	0.000		0.000		0.087		0.227		0.250		0.417		0.563	0.710
W. Benchmark MO INV	0.000		0.000		0.007		0.051		0.063		0.167		0.281	0.340
W. Benchmark Tot	0.000		0.000		0.094		0.279		0.313		0.583		0.844	1.050

Table 20: **2-Period Small Tick Market: Market Quality and Welfare.** This Table reports for each support of the investors' personal evaluation considered (row 1), our metrics of market quality and welfare. The equilibrium make and take fee (MF and TF) are reported in rows 2 and 3, and the extremes of the investors' support (β min and β max) are reported in rows 4 and 5. The investors' supports are expressed in terms of the tick size of the small tick, 1/3 τ.

Δ	**0.333 τ**		**0.420 τ**		**0.630 τ**		**1.000 τ**		**2.000 τ**	**3.000 τ**	**4.000 τ**	**5.000 τ**
MF	-0.111	0.222	-0.097	0.237	-0.062	0.272	0.333	0.000	0.333	0.333	0.333	0.333
TF	0.222	-0.111	0.237	-0.097	0.272	-0.062	0.000	0.333	0.292	0.333	0.333	0.333
β **min**	9.833	9.833	9.790	9.790	9.685	9.685	9.500	9.500	9.000	8.500	8.000	7.500
β **max**	10.167	10.167	10.210	10.210	10.315	10.315	10.500	10.500	11.000	11.500	12.000	12.500
Ave Depth p_5 (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ave Depth p_4 (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.100
Ave Depth p_3 (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.167	0.133
Ave Depth p_2 (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.187	0.222	0.167	0.133
Ave Depth p_1 (t_1)	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.313	0.222	0.167	0.133
Ave Depth p_{-1} (t_1)	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.313	0.222	0.167	0.133
Ave Depth p_{-2} (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.187	0.222	0.167	0.133
Ave Depth p_{-3} (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.167	0.133
Ave Depth p_{-4} (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.100
Ave Depth p_{-5} (t_1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Depth (t_1)	0.666	0.668	0.667	0.667	0.667	0.667	0.667	0.667	1.000	1.000	1.000	1.000
Volume (t_2)	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.222	0.208	0.259	0.292	0.307
Ave Eff Spread (t_2)	-0.333	0.333	-0.333	0.333	-0.333	0.333	-0.333	0.333	0.458	0.651	0.873	1.097
Welfare INV MO	0.012	0.012	0.016	0.016	0.023	0.023	0.037	0.037	0.050	0.109	0.179	0.248
Welfare INV LO	0.012	0.012	0.016	0.016	0.023	0.023	0.037	0.037	0.102	0.221	0.359	0.500
Welfare INV	0.025	0.025	0.031	0.031	0.047	0.047	0.074	0.074	0.152	0.330	0.538	0.748
Welfare Exchange	0.025	0.025	0.031	0.031	0.047	0.047	0.074	0.074	0.130	0.173	0.194	0.204
Wefare Tot	0.049	0.049	0.062	0.062	0.093	0.093	0.148	0.148	0.282	0.503	0.733	0.952
Ave Depth p_5 Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000	0.000	0.000	0.000
Ave Depth p_4 Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000	0.000	0.000	0.100
Ave Depth p_3 Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000	0.056	0.167	0.133
Ave Depth p_2 Benchmark (t_1)	0.000		0.000		0.000		0.000		0.167	0.222	0.167	0.133
Ave Depth p_1 Benchmark (t_1)	0.000		0.500		0.500		0.500		0.333	0.222	0.167	0.133
Ave Depth p_{-1} Benchmark (t_1)	0.000		0.500		0.500		0.500		0.333	0.222	0.167	0.133
Ave Depth p_{-2} Benchmark (t_1)	0.000		0.000		0.000		0.000		0.167	0.222	0.167	0.133
Ave Depth p_{-3} Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000	0.056	0.167	0.133
Ave Depth p_{-4} Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000	0.000	0.000	0.100
Ave Depth p_{-5} Benchmark (t_1)	0.000		0.000		0.000		0.000		0.000	0.000	0.000	0.000
Depth Benchmark (t_1)	0.000		1.000		1.000		1.000		1.000	1.000	1.000	1.000
Volume Benchmark (t_2)	0.000		0.103		0.235		0.333		0.361	0.370	0.375	0.373
Ave Eff Spread Benchmark (t_2)	0.000		0.333		0.333		0.333		0.487	0.689	0.901	1.127
Welfare Benchmark INV LO	0.000		0.028		0.076		0.139		0.287	0.434	0.581	0.727
Welfare Benchmark INV MO	0.000		0.002		0.017		0.056		0.137	0.214	0.291	0.361
Welfare Benchmark Tot	0.000		0.030		0.094		0.194		0.424	0.648	0.872	1.089

Table 21: 3-Period Large Tick Market: Market Quality and Welfare. This Table reports for each support of the investors' personal evaluation considered (row 1), our metrics of market quality and welfare. The equilibrium make and take fee (MF and TF) are reported in rows 2 and 3, and the extremes of the investors' support (β min and β max) are reported in rows 4 and 5. The investors' supports are expressed in terms of the tick size of the large tick, τ.

Δ	**0.333** τ		**1.000** τ		**1.200** τ		**1.620** τ		**1.667** τ		**2.000** τ		**3.000** τ	**4.000** τ	**5.000** τ
MF	0.572	-0.428	0.717	-0.284	0.760	-0.241	0.850	-0.150	0.860	-0.139	0.933	-0.068	0.983	1.000	0.740
TF	-0.443	0.557	-0.328	0.672	-0.294	0.707	-0.221	0.779	-0.213	0.786	-0.156	0.845	0.115	0.422	1.000
β **min**	9.833	9.833	9.500	9.500	9.400	9.400	9.190	9.190	9.167	9.167	9.000	9.000	8.500	8.000	7.500
β **max**	10.167	10.167	10.500	10.500	10.600	10.600	10.810	10.810	10.833	10.833	11.000	11.000	11.500	12.000	12.500
Ave Depth P_2 (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.101
Ave Depth P_1 (t_1, t_2)	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.329	0.419	0.526
Ave Depth P_{-1} (t_1, t_2)	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.298	0.329	0.419	0.526
Ave Depth P_{-2} (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.101
Depth (t_1, t_2)	0.597	0.597	0.597	0.597	0.597	0.597	0.597	0.597	0.597	0.597	0.597	0.597	0.659	1.239	1.254
Volume (t_2, t_3)	0.391	0.393	0.392	0.392	0.391	0.391	0.392	0.392	0.392	0.392	0.392	0.391	0.395	0.430	0.451
Ave Eff Spread (t_2, t_3)	0.392	-0.392	0.392	-0.392	0.392	-0.392	0.392	-0.392	0.392	-0.392	0.392	-0.392	0.109	0.475	0.045
Welfare INV MO	0.021	0.021	0.064	0.064	0.077	0.077	0.104	0.104	0.107	0.107	0.128	0.128	0.197	0.264	0.351
Welfare INV LO	0.019	0.019	0.056	0.056	0.067	0.067	0.090	0.090	0.093	0.093	0.111	0.111	0.167	0.293	0.384
Welfare INV	0.040	0.040	0.120	0.120	0.144	0.144	0.194	0.194	0.200	0.200	0.239	0.239	0.364	0.557	0.735
Welfare Exchange	0.051	0.051	0.152	0.152	0.183	0.183	0.246	0.246	0.253	0.253	0.304	0.304	0.451	0.612	0.785
Welfare Tot	0.091	0.091	0.272	0.272	0.326	0.326	0.440	0.440	0.453	0.453	0.544	0.544	0.815	1.169	1.520

Δ	**0.333** τ	**1.000** τ	**1.200** τ	**1.620** τ	**1.667** τ	**2.000** τ	**3.000** τ	**4.000** τ	**5.000** τ
Ave Depth P_2 Benchmark (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.073
Ave Depth P_1 Benchmark (t_1, t_2)	0.000	0.000	0.740	0.694	0.689	0.689	0.578	0.539	0.516
Ave Depth P_{-1} Benchmark (t_1, t_2)	0.000	0.000	0.740	0.694	0.689	0.689	0.578	0.539	0.516
Ave Depth P_{-2} Benchmark (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.073
Depth Benchmark (t_1, t_2)	0.000	0.000	1.479	1.389	1.378	1.378	1.156	1.190	1.177
Volume Benchmark (t_2, t_3)	0.000	0.000	0.246	0.528	0.548	0.548	0.767	0.767	0.769
Ave Eff Spread Benchmark (t_2, t_3)	0.000	0.000	0.136	0.070	0.064	0.064	-0.076	0.014	0.070
Welfare Benchmark INV LO	0.000	0.000	0.111	0.281	0.296	0.393	0.619	0.852	1.072
Welfare Benchmark INV MO	0.000	0.000	0.057	0.195	0.0.211	0.319	0.614	0.819	1.015
Welfare Benchmark Tot	0.000	0.000	0.168	0.476	0.507	0.712	1.233	1.672	2.087

Table 22: 3-Period Large Tick Market with HFTs: Market Quality and Welfare with access fees or rebates. This Table reports for each support of the investors' personal evaluation considered (row 1) our metrics of market quality and welfare. The equilibrium make and take fee (MF and TF) are reported in rows 2 and 3, and the extremes of the investors support ($\underline{\beta}$ and $\bar{\beta}$) are reported in rows 4 and 5. The investors' supports are expressed in terms of the tick size of the large tick, τ. Note that $0.000^* = 1 \cdot 10^{-7}$ and $0.185^* = 0.185 + 1 \cdot 10^{-7}$.

Δ	**0.333 τ**		**1.000 τ**		**1.270 τ**	**2.000 τ**	**3.000 τ**	**3.900 τ**	**4.000 τ**	**5.000 τ**
MF	-0.415	0.585	-0.250	0.750	0.817	1.000	1.000	1.000	0.520	0.757
TF	0.500	-0.500	0.500	-0.500	-0.500	-0.500	-0.500	-0.500	0.500	0.500
$\bar{\beta}$	10.167	10.167	10.500	10.500	10.635	11.000	11.500	11.950	12.000	12.500
$\underline{\beta}$	9.833	9.833	9.500	9.500	9.365	9.000	8.500	8.050	8.000	7.500
Ave Depth P_2 (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ave Depth P_1 (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.439	0.373
Ave Depth P_{-1} (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.439	0.373
Ave Depth P_{-2} (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Depth (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.878	0.747
Volume (t_1, t_2)	0.980	0.980	1.000	1.000	1.001	1.000	1.333	1.487	0.649	0.671
Volume (t_2, t_3)	0.980	0.980	1.000	1.000	1.001	1.000	1.333	1.487	0.884	0.970
Volume (t_1, t_2, t_3)	1.470	1.470	1.500	1.500	1.502	1.500	2.000	2.231	1.144	1.211
Ave Quoted Spread (t_1, t_2)	5.000	5.000	5.000	5.000	5.000	5.000	5.000	5.000	3.245	3.506
Ave Eff Spread (t_1, t_2)	0.000*	1.000*	0.000*	1.000*	1.000*	1.000*	1.000*	1.000*	0.000*	0.000*
Ave Eff Spread (t_2, t_3)	0.623*	0.377*	0.625*	0.375*	0.375*	0.375*	0.333*	0.314*	0.928*	0.911*
Ave Eff Spread (t_1, t_2, t_3)	0.082*	0.585*	0.083*	0.583*	0.583*	0.583*	0.556*	0.543*	0.285*	0.274*
Ave Eff SpreadMid (t_1, t_2)	2.000*	3.000*	2.000*	3.000*	1.125*	3.000*	3.000*	3.000*	2.149*	2.160*
Ave Eff SpreadMid (t_2, t_3)	0.755*	1.132*	0.750*	1.125*	1.125*	1.125*	1.000*	0.942*	1.729*	1.645*
Ave Eff SpreadMid (t_1, t_2, t_3)	1.170*	1.755*	1.167*	1.750*	1.750*	1.750*	1.667*	1.628*	1.819*	1.764*
Welfare INV MO	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.210	0.329
Welfare INV LO	0.060	0.060	0.188	0.188	0.239	0.375	1.000	1.617	0.686	0.876
Welfare INV	0.060	0.060	0.188	0.188	0.239	0.375	1.000	1.617	0.896	1.206
Welfare HFTs	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
Welfare Exchange	0.125	0.125	0.375	0.375	0.476	0.750	1.000	1.115	1.167	1.522
Welfare Tot	0.185*	0.185*	0.562*	0.562*	0.715*	1.125*	2.000*	2.732*	2.063*	2.728*

Table 23: 3-Period Large Tick Market with HFTs: Market Quality and Welfare, with no access fees or rebates. This Table reports for each support of the investors' personal evaluation considered (row 1) our metrics of market quality and welfare. The extremes of the investors support ($\underline{\beta}$ and $\bar{\beta}$) are reported in rows 4 and 5. The investors' supports are expressed in terms of the tick size of the large tick, τ.

Δ	**0.333 τ**	**1.000 τ**	**1.270 τ**	**2.000 τ**	**3.000 τ**	**3.900 τ**	**4.000 τ**	**5.000 τ**
$\bar{\beta}$	10.167	10.500	10.635	11.000	11.500	11.950	12.000	12.500
$\underline{\beta}$	9.833	9.500	9.365	9.000	8.500	8.050	8.000	7.500
Ave Depth P_2 (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Ave Depth P_1 (t_1, t_2)	0.000	0.000	0.705	0.573	0.444	0.368	0.361	0.303
Ave Depth P_{-1} (t_1, t_2)	0.000	0.000	0.705	0.573	0.444	0.368	0.361	0.303
Ave Depth P_{-2} (t_1, t_2)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Depth (t_1, t_2)	0.000	0.000	1.410	1.146	0.888	0.736	0.722	0.607
Volume (t_1, t_2)	0.000	0.000	0.142	0.236	0.389	0.965	0.491	0.564
Volume (t_2, t_3)	0.000	0.000	0.329	0.403	0.556	1.269	0.641	0.698
Volume (t_1, t_2, t_3)	0.000	0.000	0.354	0.324	0.481	1.709	0.577	0.643
Ave Quoted Spread (t_1, t_2)	5.000	5.000	2.179	2.708	3.222	3.528	3.556	3.787
Ave Eff Spread (t_1, t_2)	0.500	0.500	0.000	0.000	0.000	0.000	0.000	0.000
Ave Eff Spread (t_2, t_3)	0.500	0.500	0.851	0.941	0.944	0.927	0.925	0.906
Ave Eff Spread (t_1, t_2, t_3)	0.333	0.333	0.234	0.294	0.296	0.285	0.284	0.270
Ave Eff SpreadMid (t_1, t_2)	0.000	0.000	2.052	2.104	2.111	2.104	2.103	2.093
Ave Eff SpreadMid (t_2, t_3)	0.000	0.000	1.690	1.793	1.681	1.553	1.540	1.418
Ave Eff SpreadMid (t_1, t_2, t_3)	0.000	0.000	1.793	1.862	1.787	1.702	1.693	1.612
Welfare INV LO	0.000	0.000	0.148	0.482	0.972	1.472	1.531	2.140
Welfare INV MO	0.000	0.000	0.015	0.120	0.269	0.372	0.381	0.465
Welfare HFT	0.000	0.000	0.073	0.269	0.481	0.623	0.637	0.753
Welfare Exchange	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Welfare Tot	0.000	0.000	0.236	0.872	1.722	2.467	2.549	3.358