Political Economics
Problem set 1

Due (before the class start) on October 6, 2008

Answer both questions

1 Capital Taxation

Assume the following set up in the context of chapter 12.1–12.2 of PT(2000) . Preferences are given by:

$$w = c_1 + U(c_2) + V(1 - l)$$

The budget constraints are:

$$\begin{aligned} c_1 &\leq 1 - k \\ c_2 &\leq (1 - \tau) k + (1 - \tau_L) l \end{aligned}$$

where $V(.)$ is a strictly concave and increasing utility function. Agents decisions are taken sequentially: (c_1, k) in period 1 and (c_2, l) in period 2. All variables are constrained to be non-negative.

The government faces a budget constraint in period 2:

$$g = \tau k + \tau_L L$$

1. Solve the representative agent problem for a given g.
2. For a given amount of public spending g, compute the optimal tax structure $\boldsymbol{\tau} = (\tau_L, \tau_K)$ under commitment (i.e., assuming that the tax structure is chosen once and for all at the beginning of period 1), and compute the full equilibrium. (Hint: pay attention to how the equilibrium depends on the value of the spending parameter g).
3. Solve the same problem under discretion (i.e. assuming that the tax structure is chosen at the beginning of period 2, once investment decisions have been made) and compute the equilibrium assuming rational expectations. Carachterize all the equilibria illustrating how they depend on the value of g.

Solution

1.1 The representative agent problem

Given τ, g :

$$\underset{k,x}{Max}\ w = (1 - k) + U((1 - \tau) k + (1 - \tau_L)(1 - x)) + V(x) \tag{1}$$

The FOC with respect to saving k is:

$$\frac{\partial w}{\partial k} = -1 + U_c(c_2)\,(1-\tau) \leq 0 \ , k \geqslant 0 \tag{2}$$

So there exists $\overline{\tau}$, such that:

$$U_c(c_2)\,(1-\overline{\tau}) = 1, k > 0 \tag{3}$$

Thus if:
$\tau \leqslant \overline{\tau} \Longrightarrow k^* \in (0,1]$;
$\tau > \overline{\tau} \Longrightarrow k^* = 0$;
The optimal labor supply can be derived from the FOC with respect to x:

$$\frac{\partial w}{\partial x} = V_x\,(x) = U_c(c_2)\,(1-\tau_l) \tag{4}$$

And plugging in the first FOC, it becomes:

$$V_x(x) = \frac{(1-\tau_l)}{(1-\tau)}$$

Since the utility function is quasi linear but not separable, each tax base depends on its own tax but also on the other tax rate;
$l^* = l(\tau, \tau_l)$;
$k^* = k(\tau, \tau_l)$;

1.2 Ex-ante optimal policy

Commitment implies that the government sets the tax rates, $\boldsymbol{\tau} = (\tau_l, \tau)$, ex-ante, i.e. before agents take their savings and labor supply decisions, and then sticks to its announcement. The government maximizes:

$$\underset{t,t_l}{Max}\, w = (1 - k^*(\tau,\tau_l)) + U[(1 - x^*(\tau,\tau_l))(1-\tau_l) + (1-\tau)\,k^*(\tau,\tau_l)] + V\,(x^*(\tau,\tau_l)) + \mu(\tau k + \tau_l l - g)$$

Applying the Envelope theorem we can write down the FOCs:
$\tau: \quad -U_c(c_2)k^* + \mu(k + \tau_l l_\tau + \tau k_t) = 0$
$\tau_l: \quad -U_c(c_2)l^* + \mu(l + \tau_l l_{\tau_l} + \tau k_{t_l}) = 0$

$$\Longrightarrow \quad \frac{\tau_l l_{\tau_l}}{l} + \frac{\tau k_{t_l}}{k}\frac{k}{l} = \frac{\tau_l l_\tau}{l}\frac{l}{k} + \frac{\tau k_t}{k}$$

Where the first term in the left hand side and the last in the right hand side represent elasticities of each tax base with respect to its own tax rate;while the other terms represent cross elasticities of each tax base with respect to the other tax rate.

From point one we know that capital is completely inelastic to τ up to $\overline{\tau}$ and, if taxes are any higher, $k = 0$ and all government expenditure must be financed through a distortive labor tax.

Ex-ante it is optimal for the government to finance G with a capital tax alone if it possible. Assume that:
$\overline{g}$ is such that : $\overline{g} = \overline{\tau} k^*$.
Therefore:
If $g \leq \overline{g}$, that is a tax on capital is enough to finance $g \Longrightarrow \tau^* < \overline{\tau}$ and $\tau_l = 0$.
If $g > \overline{g}$, $\tau^* = \overline{\tau}$ and the labor tax must finance the rest of the government expenditure g: $\tau_l = \dfrac{g - \tau^* k^*}{l^*}$.

1.3 Without commitment

"Without commitment" means that governments can set capital taxes after consumers have taken their savings decision. In period 2, k is given and subject to taxation $(\varepsilon_k(\tau) = 0)$. Ex-post it is optimal to tax capital as much as possible instead of taxing labor which is more distortive. We have to distinguish two basic cases: the case in which the government can cover all of its expenses from the capital tax, and the case in which it cannot.

If $g > \overline{g}$, a tax on capital smaller than $\overline{\tau}$ is not enough to finance g. Thus the government will always want to impose a tax rate $\tau > \overline{\tau}$ once consumers have made their savings decision. Since agents anticipate the government's behavior, they will always set savings to zero, so that the government needs to raise all revenues with labor taxation. Therefore, we have a unique equilibrium with zero savings and high labor taxation.

If $g \leqslant \overline{g}$, there are three possible equilibria: in the good equilibrium, agents believe that in the second period,the government will impose the tax rate $\tau < \overline{\tau}$ thus saving all their money, so that no labor taxation is necessary in the second period. In the bad equilibrium, agents (who do not trust the government) expect some tax rate $\widehat{\tau} > \overline{\tau}$ and set savings to zero, so that the government (which is now indifferent with respect to capital tax rates) will have to finance all of its expenditure by labor taxation in the second period. The third equilibrium is more subtle: if agents believe that the government will impose exactly $\tau = \overline{\tau}$ they are indifferent with respect to the saving rate. As a consequence, it is possible that agents choose a savings rate $k^* \epsilon (0, 1)$ such that $k^*\overline{\tau} = g$; given k^*,it is optimal for the government to impose $\overline{\tau}$, so that $(k^*, \overline{\tau}, 0)$ constitutes another possible equilibrium for the case when $g < \overline{\tau}R$.

2 Risk Sharing and Targeted Redistribution

Consider an economy where population is spread over three regions, $J = A, B, C$. Individuals are divided in employed and unemployed. Employed individuals consume $c = y(1 - \tau)$, where y is income and τ a non-distorting tax. Unemployed individuals receive an unemployment subsidy, s. Individuals are risk averse and evaluate private consumption with a concave utility function $U(\cdot)$. Moreover individuals differ in the probability of being employed; in particular let e^i denote the probability that an individual of type i is employed. The average value of e^i in the population is e, which also denotes the fraction of employed individuals (while $1 - e$ is the unemployment rate). There are I different types: $i = 1, 2...I$ and each type forms a continuum. Assume that the expected utility function of an individual of type i residing in region J is the following:

$$w^{iJ} = e^i U(c) + (1 - e^i)U(s)$$

Summing over risk types i, the government budget constraint can be written as:

$$ey\tau = (1 - e)s$$

There is a single electoral districts that includes all regions, and there are two office-seeking candidates, 1 and 2, running for the election. The policy platforms they announce are respectively: $\mathbf{q}_1 = [\tau_1, s_1]$ and $\mathbf{q}_2 = [\tau_2, s_2]$.

1. Suppose that citizens behave as Downsians voters (i.e. they only care about the policy implemented). Which policy platforms will be announced by the candidates? Does an equilibrium exist?

2. Assume now that voters care also about candidates' ideology. Let $W^{iJ}(q)$ be the indirect utility function of voter of type i in region J, as a function of the policy vector $\mathbf{q} = [\tau, s]$. Then the swing voter of type i in region J is defined by:

$$\sigma^{iJ} = W^{iJ}(\mathbf{q}_1) - W^{iJ}(\mathbf{q}_2) - \delta$$

Finally, suppose that the distribution of individual ideological preferences is uniform and specific to each region *cum* risk type, with density ϕ^{iJ} for type i in region J. To simplify make the assumption that in each region all types have the same distribution of ideological preference: $\phi^{iJ} = \phi^J$ for all i. Notice that σ^i has a region specific uniform distribution on $[-\frac{1}{2\phi^J}, \frac{1}{2\phi^J}]$ while δ is uniformly distributed on $[-\frac{1}{2\psi}, \frac{1}{2\psi}]$. Determine the equilibrium in this probabilistic voting framework and give the intuition.

3. Suppose now that citizens also draw utility from *local* public consumption, and f^J denotes local public consumption per capita in region J. Assume that the utility from local public consumption is linear. Thus, an individual of type i residing in region J has preferences:

$$w^{iJ} = e^i U(c) + (1 - e^i)U(s) + f^J$$

The government budget becomes:

$$ey\tau = (1 - e)s + \frac{1}{3}\sum_J f^J$$

   (a) As before, assume Downsian voters. Determine the political equilibrium (if it exists), namely the policies announced by the candidates. Contrast this answer with the one you provided at point 1.

   (b) Now suppose that voters behave as in the probabilistic voting model (as in point 2). Pin down the equilibrium in this case.Compare your results with those given in point 2 and explain intuitively what you found.

Solution

2.1 Median Voter Equilibrium (I)

After substituting with the government budget constraint, the policy preferences of individual i in region j can be written as the following:

$$w^{ij} = e^i U(y - \frac{1-e}{e}s) + (1 - e^i)U(s)$$

It is easy to verify that these policy preferences satisfy the Intermediate Preference Property, since they can be rewritten in this form:
$W(q, e^i) = e^i[U(y - \frac{1-e}{e}s) - U(s)] + U(s)$

This property guarantees the existence of a Condorcet winner.
To determine the bliss point of the median voter we have to solve:

$$\underset{s}{Max} W^{ij} = e^i U(y - \frac{1-e}{e}s) + (1 - e^i)U(s)$$

$$s: \ \frac{U_c(c)}{U_s(s)} = \frac{(1-e^i)}{(1-e)}\frac{e}{e^i}$$
$$\Longrightarrow s^m \ \text{ is such that: } \frac{U_c(c)}{U_s(s^m)} = \frac{(1-e^m)}{(1-e)}\frac{e}{e^m}$$

The average voter (e) bliss point ($\overline{s}$) implies full insurance with $\overline{s} = c$.
Assuming that $e^m > e$ (i.e. the probability of employment for the median voter is greater than the one of the average voter), then the median voter would prefer a smaller unemployment insurance with respect to the average voter, such that: $s^m < \overline{s}$.

2.2 Probabilistic Voting Equilibrium(I)

Assume for simplicity that the share of each type in group j is $\frac{1}{I}$. Then party 1 vote share can be written as:

$$\pi_1 = \sum_i \sum_j \frac{1}{3I} \phi^j (\sigma^j + \frac{1}{2\phi^j})$$

and as a consequence party 1 probability of winning will be:

$$p_1 = \Pr ob(\pi_1 \geqslant \frac{1}{2}) = \Pr ob\left(\sum_i \sum_j \frac{1}{3I}\phi^j(\sigma^j + \frac{1}{2\phi^j}) \geqslant \frac{1}{2}\right) = \Pr ob\left(\sum_i \sum_j \frac{1}{3I}\phi^j \sigma^j \geqslant 0\right) =$$
$$= \Pr ob\left(\sum_i \sum_j \frac{1}{3I}\phi^j [W^{ij}(s_1) - W^{ij}(s_2) - \delta] \geqslant 0\right) = \left[\frac{\psi}{\phi}\sum_i \sum_j \frac{1}{3I}\phi^j [W^{ij}(s_1) - W^{ij}(s_2)] + \frac{1}{2}\right]$$

Thus party 1 solves:

$$\underset{s_1}{Max}\left\{\frac{\psi}{\phi}\sum_i \sum_j \frac{1}{3I}\phi^j \left[\left(e^i U(y - \frac{1-e}{e}s_1) + (1-e^i)U(s_1)\right) - W^{ij}(s_2)\right] + \frac{1}{2}\right\}$$

FOC:
$s_1: \; \frac{U_c(c)(1-e)}{e} e = U_s(s)(1-e)$

simplifying: $U_c(c) = U_s(s)$
which implies full insurance: $c = s$

2.3 Median Voter Equilibrium (II)

In the presence of a local public good (f^j), individual policy preferences do not satisfy anymore the intermediate preference property. This implies that the M.V. equilibrium does not exist; in this case voters preferences for a multidimensional policy cannot be projected on a unidimensional space in which voters can be ordered by their type, since type e^i bliss point is different in different regions.

2.4 Probabilistic Voting Equilibrium(II)

The computation is exactly as before. The only thing that changes is the expression for the utility function (and the government budget constraint) which now entails also the local public good.

Party one will solve:

$$\underset{s_1, f^j}{Max}\left\{\frac{\psi}{\phi}\sum_i \sum_j \frac{1}{3I}\phi^j \left[\left(e^i U(y - \frac{1-e}{e}s_1 - \frac{1}{3e}\sum_j f^j) + (1-e^i)U(s_1) + f^j\right) - W^{ij}(s_2)\right] + \frac{1}{2}\right\}$$

The FOCs are:

s_1: $\frac{U_c(c)(1-e)}{e} e = U_s(s)(1-e)$ which as before leads to full insurance: $c = s$

$f^j : \sum_j \frac{\phi^j}{3} U_c(c) = \phi^j$

Substituting with the optimality condition for the unemployment insurance we obtain:

$$\sum_j \frac{\phi^j}{3} U_s(s) = \phi^j \tag{5}$$

This condition will only holds for region B, the one with a higher number of swing voters.

(5) tells us that in the optimum, the political gain(gain in votes equal to ϕ^B)of an additional unit of local public good for region B must be equal to the political cost of less unemployment benefits in all the regions (left hand side).