Separate or Joint Financing?
Coinsurance, Risk Contamination, and
Optimal Conglomeration with Bankruptcy Costs*

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Abstract

This paper analyzes the determinants of the optimal scope of incorporation in the presence of bankruptcy costs. Bankruptcy costs alone generate a non-trivial tradeoff between the benefit of coinsurance and the cost of risk contamination associated with financing projects jointly through debt. This tradeoff is characterized for projects with binary returns, depending on the distributional characteristics of returns (mean, variability, skewness, heterogeneity, correlation, and number of projects), the structure of the bankruptcy cost, and the tax advantage of debt relative to equity. Our predictions are broadly consistent with existing empirical evidence on conglomerate mergers, spin-offs, project finance, and securitization.

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1 Introduction

Consider a firm that needs to finance a number of risky projects through a competitive credit market. The firm has the choice of financing the projects either separately with a number of independent loans or jointly with a single loan. With either financing regime, part of the returns are lost to bankruptcy costs when creditors do not obtain full repayment. When does joint financing lead to lower costs than separate financing? Answering this question allows us to shed light on the profitability of various corporate financial arrangements, such as:

- mergers that combine cash flows and the financing of otherwise separate corporations;
- holding companies, which protect the assets of individual subsidiaries from creditors' claims against other subsidiaries;
- spin-offs in which divisions are set up as independent corporations;
- project finance and securitization, in which projects or loans are financed through separate special-purpose vehicles.

At least since Lewellen (1971), conventional wisdom in corporate finance has largely settled on the view that bankruptcy costs always generate positive financial synergies, so that joint financing is more profitable than separate financing. According to this view, conglomeration brings about a reduction in the probability of bankruptcy by allowing a firm to use the proceeds of a successful project to save an unsuccessful one, which would otherwise have failed. By aggregating imperfectly correlated cash flows, the argument goes, joint financing should reduce expected bankruptcy costs and increase borrowing capacity. As aptly summarized by Brealey, Myers, and Allen's (2006, page 880) textbook, "merging decreases the probability of financial distress, other things equal. If it allows increased borrowing, and increased value from the interest tax shields, there can be a net gain to the merger."

In this paper, we amend this conventional view by revisiting the *purely financial* effects of conglomeration. We argue that bankruptcy costs alone create a non-trivial tradeoff for conglomeration, even abstracting from tax considerations and changes in borrowing capacity. While the literature has mostly focused on the *coinsurance* benefits of conglomeration, we show that *risk-contamination* losses can turn the logic of the conventional argument on its head. In risk contamination, the failure of one project drags down another, successful project that is financed jointly, thus increasing the probability of bankruptcy and its expected costs. This increase in the probability of financial distress and the associated losses can be substantial.

To illustrate the value of breaking up a conglomerate to avoid risk-contamination losses, consider the spin-off of R. J. Reynolds Tobacco Company from Nabisco's food business in 1999. As Steven F. Goldstone, chairman and chief executive officer of RJR Nabisco, commented in the official news release, this sale "paves the way for us to separate the domestic tobacco business from the rest of our organization on a sound and prudent financial basis." Similar considerations led many commentators to favor a split of UBS during the recent financial crisis, as the troubled investment-banking unit was dragging down the highly profitable private-banking business. As suggested by the Financial Times, UBS benefited ex ante from perceived coinsurance gains ("the main reason its investment bank had access to such cheap funding during the boom that led to such huge losses was because UBS had a high credit rating, supported by its private banking business") but ended up suffering the effects of risk contamination ("the losses have prompted clients to withdraw cash from UBS's core wealth management business").[1]

To best understand the determinants of the tradeoff between coinsurance and risk contamination, we focus on a simple setting in which each project has two possible realizations of returns, either low or high. In the baseline model we constrain financing to be obtained only through standard debt. The low-return realization is insufficient to cover the initial investment outlay, thus generating the possibility of bankruptcy. Separate financing involves a number of nonrecourse loans, so that when the repayment obligation on one loan is not met, creditors do not have access to the returns of other

[1] See "UBS does not have luxury of time before it splits up," Financial Times, March 17, 2008, and "Integration loses its attraction," Financial Times, August 13, 2008.

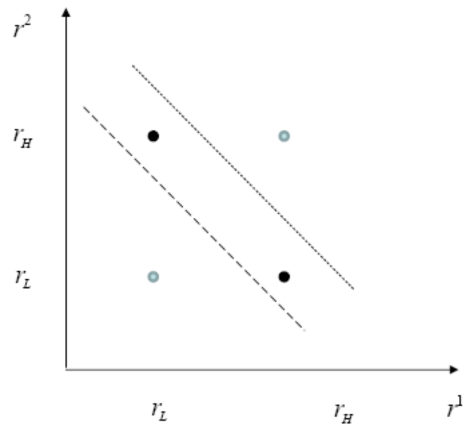

Figure 1: **Joint distribution of returns.** Each project $i = 1, 2$ yields an independent random return r^i with a binary distribution. The return is either low, $r^i = r_L > 0$, with probability $1 - p_i$, or high, $r^i = r_H > r_L$, with probability p_i.

projects. By contrast, joint financing aggregates the returns of multiple projects, so that bankruptcy costs are only incurred when the sum of the returns of the projects falls below the overall repayment obligation required by the creditors.

To develop an initial intuition, consider a setting with two ex ante identical and independent projects, as in the baseline specification of our model. The repayment obligation is endogenously determined and depends on the financing regime (separate or joint). In either regime, competition forces creditors to set the repayment obligation at a level that allows the firm to obtain the projects' present value net of the expected bankruptcy costs. If the projects are financed separately, each loan defaults when the corresponding project yields a low return. If, instead, the projects are financed jointly, default occurs if the *per-project* repayment obligation is higher than the average realized return of the two projects. Similar to the case of separate financing, default occurs if the returns of both projects are low (bottom-left realization of the joint distribution of returns in Figure 1) and does not occur if the returns of both projects are high (top-right realization). The key to the comparison with separate financing is whether or not the required repayment obligation can be met when one project yields a low return and the other project yields a high return, as illustrated by the top-left and bottom-right realizations in Figure 1.

There are two scenarios. First, suppose that the repayment obligation is *below* the average of the high and the low return, as illustrated by the dashed diagonal line in the figure. In this case, the probability of bankruptcy is reduced with joint financing. Ex post, a low-return project, which would have defaulted if it had been financed separately, is saved if the other project yields a high return. Ex ante, the two projects coinsure each other and the expected inefficiency associated with bankruptcy is reduced. A higher probability of full repayment forces creditors to reduce the interest rate below the level required under separate financing. This coinsurance effect drives the classic logic of "good" conglomeration stressed by Lewellen (1971).

This result is reversed if the per-project repayment obligation is above the average of the high and the low return, as illustrated by the dotted diagonal line in the figure. In this second scenario, the probability of bankruptcy is actually higher under joint financing. Ex post, a high-return project, which would have stayed afloat had it been financed separately, is now dragged into bankruptcy when the other project has a low return. Ex ante, projects risk-contaminate each other and joint financing increases the expected inefficiency associated with bankruptcy. If the bankruptcy recovery rate is low, competing creditors are forced to increase the required interest rate above the level that results under separate financing because the loan will be repaid in full less often with joint financing. In this case, conglomeration is "bad" due to risk contamination.

Our key observation is that a meaningful tradeoff regarding corporate structure arises in the presence of bankruptcy costs alone, without need of handicapping joint financing by introducing an additional friction as done by Diamond (1984). As we explain in Section 6.2, Diamond (1984) disregards the possibility of bad conglomeration by implicitly assuming that the (endogenous) per-project repayment obligation with joint financing is always less than the one obtained with separate financing. This paper characterizes conditions on the primitives that overturns this condition. In those situations, conglomeration is bad even when joint financing is not subject to additional frictions compared to separate financing.

The thrust of our analysis consists in characterizing the conditions on the primitives of the model such that good and bad conglomeration arise. To this end, we first solve for the equilibrium repayment obligations that result in the two financing regimes, and then determine the region of parameters for which the borrower finds separate financing more profitable than joint financing. In the context of the baseline model with two identical and independent projects, we illustrate that separate financing can be optimal for empirically plausible parameter values and derive a number of testable comparative statics predictions, such as the following:

- A reduction in the bankruptcy recovery rate decreases the profitability of joint financing. Given that the amount available to creditors following bankruptcy is lower when bankruptcy costs are higher, the repayment obligation associated with joint financing increases with the level of bankruptcy costs. It is then more difficult for the repayment obligation to be below the average of the high and the low return. Thus, the profitability of joint financing is reduced. Consistent with this theoretical prediction, Rossi and Volpin (2004) show that improvements in judicial efficiency and creditor rights significantly increase M&A activity, while Subramanian, Tung, and Wang (2009) find that project finance is more prevalent than corporate finance in countries with less-efficient bankruptcy procedures and weaker creditor rights.

- For projects where good returns are more likely than bad ones, joint financing is also less profitable when the projects are riskier. This is consistent with project finance being more widespread in riskier countries, as shown empirically by Kleimeier and Megginson (2000) among others.

- A mean-preserving increase in the negative skewness of the distribution of returns reduces the attractiveness of joint financing. This result is consistent with the finding that projects with negatively skewed returns, due, for example, to expropriation risk, are likely to be financed on a project basis (see Esty, 2003). Also, since debt returns are negatively skewed, this suggests a motive for the use of separate subsidiaries and securitization structures by banks and other lenders.

In the discussion so far we have compared the profitability of separate and joint financing when both financing regimes are feasible. In the paper, we also characterize situations in which it is feasible to finance projects with positive net present value either only separately or only jointly. When the coinsurance effect prevails, joint financing increases the borrowing capacity, resulting in projects that can be financed jointly but cannot be financed separately. When risk contamination prevails, instead, joint financing decreases the borrowing capacity, so that there are projects that can be financed separately but not jointly.

We also show that a rule of thumb that prescribes adopting the financing regime associated with the lowest interest rate can be suboptimal. We illustrate situations in which it is more profitable for a firm to finance projects separately, even though joint financing at a *lower* interest rate is feasible. Indeed, when risk contamination prevails, joint financing can result in a lower interest rate despite being associated with a higher probability of bankruptcy. When the bankruptcy recovery rate is sufficiently high (or, equivalently, bankruptcy costs are sufficiently low), at any given exogenous promised repayment rate, creditors expect to obtain more with joint financing than with separate financing because bankruptcy occurs more frequently. As a result, competition forces creditors to offer a lower rate to firms that finance projects jointly. This theoretical finding can explain the widespread use of project finance despite the fact that "project debt is often more expensive than corporate debt," solving one of the "apparently counterintuitive features [of project finance]" (Esty, 2003).

Next, we turn to the case of projects with *heterogeneous* distributions of returns. Coinsurance and risk contamination may then be present *simultaneously* when two heterogeneous projects are financed jointly. We characterize situations in which a first project either saves or drags down a second project, depending on whether the first project succeeds or fails. This situation arises when projects differ in their riskiness, measured by second-order stochastic dominance. We show that the relative profitability of separate financing increases in the difference of the riskiness of two projects. This theoretical prediction is in line with empirical findings by Gorton and Souleles (2005) that riskier originator banks are more likely to securitize.

We then examine the impact of correlation across projects' returns. Intuitively, when returns are perfectly negatively correlated, the risk-contamination effect is absent and the coinsurance effect is so strong that it eliminates bankruptcy altogether when projects are financed jointly. As the correlation between project returns increases, separate financing is more likely to dominate.

In an extension of the model to allow for more than two projects, we characterize situations in which partial conglomeration of projects into subgroups of intermediate size is optimal. By grouping subsets of projects into small conglomerates, some of the benefits of coinsurance can be obtained while also containing the costs of risk contamination. Exploiting the logic of the law of large numbers, we also show that full conglomeration results when the number of independent projects becomes arbitrarily large; in the limit, the risk-contamination effect vanishes and it becomes optimal to finance all the projects jointly.

In our baseline model, bankruptcy costs are proportional to the value of the assets under bankruptcy, as is often assumed in the theoretical and empirical literature. In a more general model with variable returns to scale in bankruptcy costs, we show that economies of scale (according to which per-project bankruptcy costs are lower when projects are financed jointly) favor joint financing, while diseconomies of scale favor separate financing. Nevertheless, our main results on the optimality of separate financing are robust to the introduction of mild (dis)economies of scale in bankruptcy costs. We also show that the logic of risk contamination still applies when bankruptcy costs depend on the number of projects that go bankrupt rather than on the value of assets under bankruptcy. In fact, separate financing is now optimal for a larger set of parameters because it becomes easier to obtain joint financing, but only at a rate for which intermediate bankruptcy occurs.

Finally, in the context of a version of the model with normal returns, we identify a simple sufficient condition for the optimality of separate financing. Our comparative statics predictions on the optimal scope of conglomeration are thus robust to a continuous specification of returns. Nevertheless, our baseline specification with binary returns allows us to investigate the role played by asymmetries in the distribution of returns as well as to reach a more thorough understanding and characterization of the determinants of the optimal scope of conglomeration.

In the baseline model, we restrict financing to be obtained through debt.[2] In a supplementary appendix we extend the analysis to allow financing through equity in addition to debt. As in the tradeoff theory of capital structure, equity saves on bankruptcy costs but is subject to higher taxation. We show that if the incremental tax advantage of debt is sufficiently low, joint financing is inconsequential because bankruptcy can be avoided altogether under either joint or separate financing. If the tax advantage is somewhat higher, joint financing becomes more profitable than in the baseline model, because equity financing makes it possible to obtain a repayment rate that avoids intermediate default when one project yields a high return and the other yields a low return. Finally, if the tax advantage is sufficiently high, separate and joint financing are profitable in the same situations as in the baseline model, because then no equity is used in either financing regime. In our simple model with binary project returns, whenever separate financing is more profitable than joint financing, only debt financing is used. Equity is more expensive and is only used if it helps to obtain a repayment rate that decreases the probability of default, in which case joint financing is optimal. This dominance of debt in separate financing is consistent with the many empirical studies that find that a large proportion of funding in project finance is in the form of debt (see, e.g., Kleimeier and Megginson, 2000).

By clarifying the conditions for the value of conglomeration in the presence of bankruptcy costs, this paper contributes to a voluminous literature on the analysis of purely financial motives for mergers. In his discussion to Lewellen (1971), Higgins (1971) notes that joint financing also affects the riskiness of the lender's returns; hence, we abstract from risk concerns by assuming risk neutrality.

[2] As we discuss in the next section, the costly state verification literature shows that debt is the optimal contractual arrangement if returns are privately observed by the borrower and can be verified by creditors only by inducing bankruptcy and incurring the bankruptcy costs.

Scott (1977) suggests that, by separating liabilities and selling secured debt, firms can increase the value of their equity by expropriating wealth from their existing unsecured creditors, such as suppliers and/or unsatisfied customers who are then unable to obtain compensation from the firm.[3] Similarly, Sarig (1985) shows that if cash flows can be negative, as "part of any production process (e.g., when customer or employee liabilities exceed future income)", a firm can exploit the limited liability shelter of the shareholders and creditors by financing projects through separate corporations, imposing again a loss on third-party holders of unsecured claims, such as customers, employees or government. Our baseline model explicitly abstracts from these limited liability effects by assuming positive cash flows. Creditors always break even and third parties are not affected. The financing regime affects the firm's payoffs because the creditors zero-profit condition creates an *endogenous* limited liability constraint.[4] The tradeoff in our model can be viewed as a borrowing firm's choice of replacing a single endogenously determined limited liability constraint by two separate constraints. As a result, in our model separate financing does not always dominate joint financing, contrary to the setting of Scott (1977) and Sarig (1985) with exogenous limited liability constraints.

In a precursor of this paper couched in the context of bank lending, Winton (1999) is the first to uncover the possibility of bad conglomeration. Our Proposition 4 develops Winton's (1999) third case of Proposition 3.1 in which a bank prefers to specialize even though the repayment rate for pooled projects is lower. Our systematic analysis of the tradeoff between coinsurance and risk contamination delivers a rich set of comparative statics predictions depending on the distributional characteristics of returns, the structure of bankruptcy costs, and the tax advantage of debt relative to equity.[5]

[3] However, this "judgement proofness" effect is inconsistent with the notion of rationality on the part of customers and suppliers. Once the lower willingness to pay of customers and suppliers is taken into account, Smith and Warner (1979) argue that the firm's earnings should not be affected by the capital structure.

[4] A number of papers (e.g., Higgins and Schall, 1975, and Kim and McConnell, 1977) have analyzed the effect of the current capital structure on merger incentives. These papers noted that, while mergers may increase total firm value, bondholders may gain at the expense of shareholders. We abstract from such a distributional conflict among (cashless) stakeholders, by considering the ex ante choice of corporate structure by shareholders and forcing bondholders to compete and therefore obtain no surplus.

[5] The literature on financial intermediation under costly state verification is also somewhat related, insofar as this focuses on how diversification across borrowrs can reduce the verification costs of bank depositors when the bank

In a closely related paper, Leland (2007) compares the profitability of separate and joint financing for a borrower who trades off bankruptcy costs with tax shields by adjusting the mix of debt and equity. His work assumes that returns are normally distributed, allows for only two projects, and is largely numerical, though he does present some analytical results which rely on the assumption that the firm's value is convex in the volatility of its returns. In both the baseline binary-return version of our model and later with normal returns, we consider fixed-investment projects that must be financed only with debt and thus we explicitly rule out the possibility of increasing leverage and re-optimizing the capital structure. As a result, unlike Leland (2007), our analysis uncovers situations in which separate financing is optimal even when the amount borrowed through debt does not depend on whether projects are financed jointly or separately. In addition, we obtain a more comprehensive set of analytical predictions, including the general impact of heterogeneous projects, the effect of skewness and other features linked to a nonsymmetric return distribution, and the role of different types of bankruptcy costs, as well as the case with multiple projects. See Section 8.4 for a more detailed comparison.[6]

John (1993), Hege and Ambrus-Lakatos (2002), and Inderst and Müller (2003) analyze the optimal corporate structure in models with agency costs due to debt overhang rather than bankruptcy costs. For example, in Inderst and Müller's (2003) two-project version of Bolton and Scharfstein (1990), financing two projects within the same corporation can reduce the firm's ability to borrow when the firm is able to finance follow-up investments internally without returning to the external capital market.[7] Our predictions for the case with bankruptcy costs are different (see, for example, the

defaults. Most of this work examines diversification across large numbers of independent borrowers, which parallels our analysis in Section 6.2 below. The main exception is Bond (2004), who contrasts conglomerate financing with bank financing in the case of two independent projects. His work relies on the assumption that each project's scale requires large numbers of individual investors who cannot coordinate on costly state verification.

[6] Our results are also very different from those of Shaffer (1994), who studies the effect of joint financing on the probability of *joint* failure. Instead, we compare the firm's expected payoff when the interest rate is endogenously determined by competition among creditors.

[7] See also Faure-Grimaud and Inderst (2005), who focus on the trade-off between coinsurance and winner-picking incentives in this setting.

discussion following Prediction 2).

The paper proceeds as follows. Section 2 formulates the model. Focusing on the baseline version of the model with two identically and independently distributed projects, financed with debt only, and with bankruptcy costs proportional to returns, Section 3 analyzes the conditions setting apart good from bad conglomeration and performs comparative statics with respect to the distribution of returns (mean, variance, and skewness) and the bankruptcy recovery rate. Section 4 turns to the case of projects with heterogeneous returns. Section 5 shows that an increase in the correlation of returns favors separate financing. Generalizing the optimal conglomeration conditions to a setting with multiple projects, Section 6 characterizes situations in which partial conglomeration is profitable and demonstrates that joint financing is optimal when the number of independent projects is sufficiently large. Section 7 shows that our results are robust to different specifications of bankruptcy costs including economies of scale. Section 8 characterizes conditions for bad conglomeration to result when projects' returns are normally distributed. Section 9 concludes with a summary of the main predictions of our theory and a discussion of avenues for future research. The Appendix collects the proofs. A supplementary appendix extends the analysis to a setting in which equity financing is available, albeit with a tax disadvantage relative to debt financing.

2 Baseline Model

This section formulates the simplest possible model to analyze how multiple projects should be optimally financed in the presence of bankruptcy costs. In the rest of the paper we derive results for special cases or extensions of this baseline scenario.

A risk-neutral firm has access to n projects. Project i requires at $t = 1$ an investment outlay normalized to $I = 1$ and yields at $t = 2$ a random payoff or *return* r^i with a binary distribution: the return is either low, $r^i = r^i_L > 0$, with probability $1 - p_i$, or high, $r^i = r^i_H > r^i_L$, with probability p_i. Each project has a positive net present value, $(1 - p_i)\, r^i_L + p_i r^i_H - 1 > 0$. The low return is insufficient to cover the initial investment outlay, $r^i_L < 1$. Returns may be correlated across projects.

Before raising external finance, the firm chooses how to group projects into corporations, or equivalently into separate nonrecourse loans. This means that investors in each corporation have access to the returns of all projects in that corporation, but they do not have access to the returns of the projects in the other corporations set up by the firm. Financing for each corporation can be obtained in a competitive credit market. For notational simplicity, we stipulate that the firm seeks financing only when expecting to obtain a *strictly* positive expected payoff.

Creditors are risk neutral and lend money through standard debt contracts. Without loss of generality, we normalize the risk-free interest rate to $r_f = 0$. Therefore, creditors expect to make zero expected profits. This is equivalent to assuming that each corporation makes a take-it-or-leave-it repayment offer to a single creditor for each loan j, promising to repay r_j^* at $t = 2$ for each unit borrowed at $t = 1$.[8] Thus r_j^* denotes the promised repayment per project. According to our accounting convention, this repayment rate comprises the amount borrowed as well as net interest.[9]

Creditors are repaid in full when the total realized return of the projects pledged is sufficient to cover the promised repayment. If instead the total realized return falls short of the repayment obligation, the corporation defaults and the ownership of the projects' realized returns is transferred to the creditor. Following default, the creditor is only able to recover a fraction $\gamma \in [0, 1]$ of the realized returns r, so that the bankruptcy costs following default are equal to $B(r) = (1 - \gamma)\, r$.[10] In Section 7, we show that our results hold robustly with a more general structure of bankruptcy costs, provided that economies or diseconomies of scale in bankruptcy are not too extreme.

For the baseline specification of the model we restrict external financing to be obtained through debt. Note that debt is the optimal contractual arrangement if we assume that returns are privately observed by the borrower and can be verified by creditors only at a cost. In the context of the

[8] Thus, for the case in which each loan (or corporation) is financed by multiple creditors, we implicitly assume that there are no coordination failures across the creditors who syndicate the same loan.

[9] The net interest rate i satisfies $1 + i = r_j^*$ and therefore the repayment obligation can be interpreted as the gross interest rate.

[10] For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2007).

classic analyses of the costly state verification model (see Townsend, 1979, Diamond, 1984, and Gale and Hellwig, 1985), the verification of returns can be interpreted as a costly bankruptcy process. In this context, they show that the optimal contract turns out to be the standard debt contract under which returns are observed if and only if the borrower cannot repay the loan in full. Once bankruptcy costs are re-interpreted as CSV verification costs, the optimal contractual agreement between the entrepreneur and the creditor is thus a debt contract. That is, if two projects are available, the optimal contracting strategy is either two separate debt contracts, each of which is backed by the returns of one project, or one debt contract, which is backed by the returns of the two projects. A supplementary appendix extends the model to also allow for financing through tax-disadvantaged equity.

3 Two Identical and Independent Projects

This section analyzes the simplest possible specification of the model to develop our main insight. The firm has access to two identically and independently distributed projects. Each project i yields a low return $r_L^i \equiv r_L$ with probability $1-p_i \equiv 1-p$ and a high return $r_H^i \equiv r_H > r_L$ with probability $p_i \equiv p$.

In Section 3.1 we proceed to examine the conditions for when the borrower is able to finance the two projects separately and jointly. In Section 3.2 we compare the profitability of separate and joint financing, when they are both feasible. In Section 3.3 we illustrate that separate financing can be optimal for empirically plausible parameter values. In Section 3.4 we characterize the effect of conglomeration on the firm's borrowing capacity. In Section 3.5 we derive a set of comparative statics predictions for the occurrence of joint and separate financing. Finally, in Section 3.6 we show that the financing option with the lowest repayment rate *is not* necessarily optimal.

3.1 Financing Conditions

Consider first the possibility of financing the two projects through two separate nonrecourse loans or, equivalently, through two different limited liability corporations. Given that the two projects are ex ante identical, financing of each project, if possible, takes place at the same rate. In order for the creditor to break even, the rate r_i^* must satisfy $r_i^* > 1 > r_L$, so that there is a positive probability that the loan is not repaid in full.

Given that the credit market is competitive, creditors must make zero expected profits. Thus the repayment requested by the creditor, r_i^*, is such that the gross expected proceeds, $pr_i^* + \gamma(1-p)r_L$, are equal to the initial investment outlay 1. As a result, each project can be financed through a separate loan if and only if

$$r_i^* := \frac{1-\gamma(1-p)r_L}{p} \leq r_H. \tag{1}$$

The repayment obligation, which is fully paid only in the case of a high return, is equal to the investment outlay, 1, less the expected proceeds from bankruptcy, $\gamma(1-p)r_L$, divided by the probability of staying afloat, p. Intuitively, the creditor needs to recover the expected shortfall in the event of bankruptcy from the event in which the project yields a high return.

Next, consider joint financing of the two projects through a single loan or, equivalently, within the same corporation. Denote by r_m^* the equilibrium repayment obligation *per unit of investment*, so that $2r_m^*$ is the total repayment promised to the creditor in return for the initial financing of the two projects, $2I = 2$. Two cases need to be distinguished, depending on whether or not the required repayment rate induces bankruptcy in the case when one project yields a high return while the other project yields a low return ("intermediate returns").

Suppose first that the equilibrium repayment rate r_m^* is such that $r_L \leq r_m^* \leq \frac{r_H+r_L}{2}$, so that there is no default with intermediate returns. As a result, the probability of default is reduced to $(1-p)^2$. Substituting again in the expected creditor profits, the borrower would only be able to obtain this rate in a competitive market if and only if

$$r_m^* := \frac{1-\gamma\,(1-p)^2\,r_L}{1-(1-p)^2} \leq \frac{r_H+r_L}{2}. \tag{2}$$

Suppose now that the equilibrium rate r_m^{**} is such that $\frac{r_H+r_L}{2} \leq r_m^{**} \leq r_H$ and therefore the borrower defaults in the event of a high and a low return. Hence, default occurs with probability $1-p^2$. In a competitive credit market, this rate can be obtained if and only if

$$r_m^{**} := \frac{1-\gamma\,(1-p)\,(pr_H+r_L)}{p^2} \leq r_H. \tag{3}$$

Since the borrower's expected profits for a given distribution are decreasing in the equilibrium rate, if both conditions (2) and (3) are satisfied, the borrower prefers rate r_m^* to rate r_m^{**}.[11] Summarizing the results so far, we have the following proposition.

Proposition 1 (Financing conditions) *Two independent and identical projects can be financed separately if and only if condition (1) is satisfied, in which case the equilibrium rate is r_i^*. Projects can be financed jointly if and only if conditions (2) or (3) are satisfied. If condition (2) is satisfied, the equilibrium rate is r_m^*, and if it is not satisfied, the rate is r_m^{**}.*

Figure 2 depicts how *per-project* expected returns (equal to the area above the distribution function up to 1) are divided between the borrower and the creditor in the three scenarios described by Proposition 1.[12] In all panels, the net expected return for the borrower corresponds to the light gray area. The gross expected return of the creditor is the sum of (i) the medium gray area, corresponding to the profits when the project stays afloat, and (ii) a fraction γ of the dark gray and black areas, corresponding to the expected proceeds in case of bankruptcy. The remaining fraction $1-\gamma$ of the dark gray and black areas is equal to the expected bankruptcy costs.

The equilibrium rate r^* in the three panels is such that the gross expected return of the creditor

[11] It is straightforward to show that if $r_m^* > (r_H+r_L)/2$, then $r_m^{**} > (r_H+r_L)/2$. Therefore, if it is not possible to obtain r_m^*, then we can disregard the $r_m^{**} > (r_H+r_L)/2$ constraint.

[12] Joint financing steepens the average return distribution around the center, $(r_H+r_L)/2$ and as a result the distribution of (per-project) returns with separate financing is a mean-preserving spread of the distribution with joint financing.

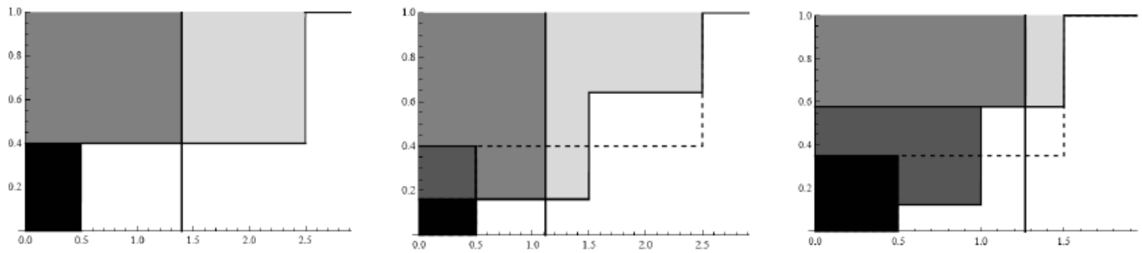

Figure 2: **Financing Conditions and Optimal Conglomeration.** Panel (a) represents the outcome with separate financing, while panels (b) and (c) represent the outcome of joint financing depending on whether coinsurance or risk contamination results. The parameters used in panels (a) and (b) are $p = 0.6,\ r_L = 0.5$, $r_H = 2.5$ and $\gamma = 0.8$ and in (c) $p = 0.65,\ r_L = 0.5$, $r_H = 1.5$ and $\gamma = 0.9$.

is equal to 1. In panel (a), projects can be financed separately because the creditor's per-project expected returns at a rate equal to r_H are greater than 1. In panel (b), projects can be financed jointly at a rate that avoids intermediate bankruptcy because the creditor's expected returns at the crossing point, $(r_H + r_L)/2$, are greater than 1. In panel (c), projects can be financed jointly only at a rate that does not avoid intermediate bankruptcy, because the creditor's expected returns at $(r_H + r_L)/2$ are lower than 1 and at r_H they are greater than 1.

3.2 Good and Bad Conglomeration

When both separate and joint financing are feasible, which regime is more profitable and thus optimal for the borrower? Obviously, in the absence of bankruptcy costs (i.e., when $\gamma = 1$) the borrower is indifferent between financing the projects separately or jointly. The next proposition states the gains and losses when $\gamma < 1$.

Proposition 2 (Separate v. joint financing) *When the borrower can finance two independent and identical projects separately as well as jointly:*

(a) If condition (2) is satisfied, it is optimal to finance the projects jointly to enjoy the coinsurance gains: $p\,(1-p)\,(1-\gamma)r_L$.

(b) If condition (2) is not satisfied, it is optimal to finance the projects separately to avoid the risk-contamination losses: $p\,(1-p)\,(1-\gamma)r_H$.

Intuitively, when the borrower obtains a rate that avoids intermediate bankruptcy, the probability of default under joint financing is lower than under separate financing. The low-return project is saved from default when the other project yields a high return, thereby reducing the inefficiency associated with bankruptcy. Per-project expected savings when the projects are financed jointly rather than separately—the "coinsurance effect"—are equal to the probability that the first project yields a low return while the second project yields a high return, $p(1-p)$, multiplied by the losses avoided due to bankruptcy costs, $(1-\gamma)r_L$. Graphically, per-project savings due to the coinsurance effect associated with joint financing are represented by a fraction $(1-\gamma)$ of the dark gray area in Panel (b) of Figure 2.

If, instead, the borrower obtains a joint rate that does not avoid intermediate bankruptcy, a project with low return drags down the other project, increasing the probability of default. Per-project expected losses when projects are financed jointly rather than separately—the "risk-contamination effect"—are equal to the probability that the first project yields a high return while the second project yields a low return, $p(1-p)$, multiplied by the additional losses in bankruptcy costs incurred, $(1-\gamma)r_H$. Graphically, the per-project costs due to the risk-contamination effect associated with joint financing are represented by a fraction $(1-\gamma)$ of the darker gray area in Panel (c) of Figure 2.

The key is whether the equilibrium repayment rate for joint financing is below or above the crossing point, $(r_H + r_L)/2$. Notice that the crossing point is not necessarily at the mean. In particular, if $p > 1/2$, so that the distribution is skewed to the left (i.e., returns are negatively skewed), the crossing point is below the mean. As a result, equilibrium rates above the crossing point are consistent with a probability of default below 50%. The resulting default probabilities are then $1-p$ for separate financing and $1-p^2$ for joint financing, which for a high enough p may be very low, as illustrated in the following numerical example.

3.3 Illustration

We now illustrate how conglomeration can result in an increase in expected bankruptcy costs for empirically plausible parameter values under the maintained assumption that returns are binary. To this end, we perform a calibration of the four parameters (r_H, r_L, p, and γ) of the baseline version of the model with separate financing. As representative values, we set:

(i) the probability of bankruptcy at 2.09% (parametrized by $1-p^5 = 0.1$) by using Longstaff, Mittal, and Neis (2005) estimate of 10% for the default probability on bonds for BBB rated firms with a five-year horizon;

(ii) the mean return at 5% (so that $[pr_H + (1-p)\gamma r_L - 1]/1 = 0.05$), as in Parrino et al. (2005), who use a mean return of 10.63% given a risk-free rate of 5.22%;

(iii) the bankruptcy recovery rate at $\gamma = 65\%$ (based on 35% liquidation losses as percentage of going concern value) from Alderson and Betker (1995); and

(iv) bankruptcy costs as a fraction of a firm's value at 11% (so that $(1-\gamma)r_L/[pr_H+(1-p)\gamma r_L] = 0.11$), at the mid point of Bris et al.'s (2006) range of estimates of 2% to 20%, at the low end of Altman's (1984) estimate of 11–17% for bankruptcy costs as a fraction of firm value up to three years before default and more conservative than Korteweg's (2010) estimate of 15–30% of firm value at the point of bankruptcy.

The calibrated values are then $r_H = 1.07$; $r_L = 0.33$; $p = 0.98$; $\gamma = 0.65$, for which it is feasible to finance the projects separately, since $r_i^* = 1.02 < 1.07 = r_H$, as well as jointly, since $r_m^{**} = 1.02 < 1.07 = r_H$, but not at the rate below the crossing point, because $r_m^* = 1.01 > 0.70 = (r_H + r_L)/2$. Thus, separate financing is more profitable than joint financing. In this illustration, the risk-contamination effect identified in Proposition 2 is $p(1-p)(1-\gamma)r_H = 0.04$, 4% of the investment outlay $I = 1$, corresponding to 15% of the project's net present value.

3.4 Borrowing Capacity

So far we have compared the profitability of separate and joint financing when both financing regimes are feasible. As we have seen in Section 3.1, there are situations in which it is feasible to finance projects with positive net present value either only separately or only jointly. Thus, conglomeration does not necessarily increase the firm's ability to finance projects.

Proposition 3 (Borrowing capacity) *Consider two identical and independent projects:*

(a) If condition (2) is satisfied, there are projects that can be financed jointly but not separately.

(b) If condition (2) is not satisfied, any project that can be financed jointly can be financed separately and there are projects that can only be financed separately.

When the coinsurance effect prevails, there are projects that can be financed jointly but cannot be financed separately. In this first case, conglomeration increases the firm's borrowing capacity, as in Lewellen (1971). However, when risk contamination prevails, joint financing decreases the firm's borrowing capacity, so that there are projects that can be financed separately but not jointly.

3.5 Testable Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects: the recovery rates and the distribution of returns (mean, variability, and skewness). For each attribute, we study whether separate or joint financing is optimal for a larger range of the remaining parameters. At the same time, we contrast our predictions with those from existing theories and discuss how our predictions on joint and separate financing match existing empirical evidence. Note that joint financing corresponds to mergers, especially conglomerate mergers, whereas separate financing corresponds to spin-offs of divisions. Also, as argued by Leland (2007) asset securitization and project finance are also methods for separately finance activities from originating or sponsoring organizations by placing them in bankruptcy-remote special-purpose vehicles (SPVs). From an analytical perspective, these entities have the key features of separate corporations.

Prediction 1 (Bankruptcy costs) *For higher bankruptcy costs (lower γ) then (a) both joint and separate financing can be obtained for a smaller region of parameters and (b) joint financing is optimal for a smaller region of the remaining parameters.*

Higher bankruptcy costs decrease pledgeable returns, since the recovered returns in case of default are lower (higher discount in the black area). Since bankruptcy costs do not affect the crossing point, $(r_H + r_L)/2$, financing at a rate that avoids intermediate bankruptcy is more difficult and thus joint finance is less likely. To the best of our knowledge, this prediction has not been formulated before.

Still, this prediction is consistent with empirical evidence indicating that merger activity is less likely and project finance is more likely in countries with weaker investor protection. Rossi and Volpin (2004) show that improvements in judicial efficiency and creditor rights significantly increase M&A activity. Comparing the incidence of bank loans for project finance with regular corporate loans for large investments, Subramanian, Tung, and Wang (2009) show that project financing is more frequent in countries with less efficient bankruptcy procedures and weaker creditor rights. Increases in these two measures of investor protection decrease bankruptcy costs and should favor, according to our model, joint financing (mergers or direct investment) over separate financing (project finance).

Prediction 2 (Mean) *For higher probability of a high return (higher p) then (a) both joint and separate financing can be obtained for a larger region of parameters and (b) joint financing is optimal for a larger region of the remaining parameters.*

If the probability of a high return increases, the expected return pledgeable to creditors also increases. It becomes easier to finance projects, and to finance them jointly at a rate that avoids intermediate bankruptcy. Graphically, all the horizontal lines in Figure 2 are then lowered, thereby increasing the expected value (equal to the area above the distribution function) without affecting the crossing point.

This prediction contrasts with that of Inderst and Müller (2003). In their model, it is optimal to keep better projects separate to avoid self-financing and thus commit to return to the capital market.

These two contrasting effects might explain the conflicting empirical evidence on the productivity of conglomerate firms. While Maksimovic and Phillips (2002) find that conglomerate firms, for all but the smallest firms in their sample, are less productive than single-segment firms, Schoar (2002) finds that the productivity of plants in conglomerate firms is higher than in stand-alone firms.[13]

During booms, projects might have a higher expectation across-the-board. Our prediction would then be consistent with a large body of empirical evidence that shows that merger activity usually heats up during economic booms and slows down in recessions (see, for example, Maksimovic and Phillips, 2001). Similarly, Cantor and Demsetz (1993) show that off-balance sheet activity (separate financing) grows following a recession.

Prediction 3 (Mean-preserving spread) *Consider the effect of a mean-preserving spread in the project's return consisting of an increase in the high return r_H and a reduction in the low return r_L so as to maintain the mean return constant. Then, there exists $\overline{p} < 1/2$ such that the region of parameters for which joint financing is optimal decreases if and only if $p > \overline{p}$.*

That is, a mean preserving spread in the distribution of returns favors separate financing as long as the distribution of returns is not too positively skewed. If the distribution is symmetric ($p = 1/2$), a mean preserving spread increases r_H by as much as it reduces r_L. While the crossing point is unaffected, the joint financing rate that avoids intermediate bankruptcy becomes more difficult to obtain because the low return is even lower and the pledgeable returns before the crossing point are lower. In the graph, the black area shrinks. If the distribution of returns is negatively skewed ($p > 1/2$), the crossing point is decreased and it becomes even more difficult to obtain joint financing below the crossing point.[14]

[13] Still, Shoar (2002) finds that conglomerates are less valued than focused firms (the so-called market diversification discount), and argues that the discrepancy can be attributed to conglomerates leaving more rents to workers. A number of papers have also argued that the diversification discount could also be spurious, because of measurement problems and selection biases. For example, Graham, Lemmon, and Wolf (2002) show that acquirers' excess values decline because the business units acquired are already discounted, thus explaining the diversification discount with a self-selection argument. See also Campa and Kedia (2002), Villalonga (2004), and Custodio (2009).

[14] To maintain the mean constant, a given increase in r_H must be combined with a larger decrease in r_L, resulting in a reduction in the crossing point. Formally, from $r'_H = r_H + \varepsilon$ and $r'_L = r_L - \varepsilon p/(1-p)$, we have $(r'_H + r'_L)/2 =$

This prediction is consistent with a similar prediction obtained by Leland (2007). Empirical support can be found in the project finance literature. Kleimeier and Megginson (1999), for example, find that project finance loans are far more likely to be extended to borrowers in riskier countries, particularly countries with higher political and economic risks. They claim that: "As a whole, these geographic lending patterns are consistent with the widely held belief that project finance is a particularly appropriate method of funding projects in relatively risky (non-OECD) countries."

It is also worth noting that loans and other forms of debt typically have default rates well under 50%. Thus, according to our prediction, increases in loan risk should make it more likely that the loans are securitized. On the other hand, the relative risk of the loan originator and the loans will also play a role. We return to this issue in Section 4 below.

Prediction 4 (Skewness) *Consider the effect of a mean-preserving increase in negative skewness in the project's return consisting of a reduction in the low return level* r_L *and an increase in the probability of high return* p *so as to maintain the mean return constant. Then, it becomes optimal to finance the projects jointly for a smaller region of parameters if and only if the high return level* r_H *is sufficiently large.*

An increase in the negative skewness has two conflicting effects. On the one hand, as r_L decreases, the crossing point is reduced and the returns in case of bankruptcy are lower, so that joint financing at the rate that avoids intermediate bankruptcy becomes more difficult. On the other hand, as p increases so as to keep the mean constant, the probability that both projects' returns are low is reduced, so that it becomes easier to finance the projects at the rate avoiding intermediate bankruptcy. Graphically, the black area (creditor's expected returns in case of default) becomes less wide and less high and the gray area (creditor's expected returns if staying afloat) becomes less wide but also higher at the crossing point. If r_H is sufficiently high, the first effect dominates and separation becomes optimal for a larger set of parameters. Indeed, for a given increase in p, one needs a higher

$(r_H + r_L)/2 - \varepsilon(2p-1)/2(1-p)$.

reduction in r_L to ensure a constant mean.

We can find support for this prediction in the literature on project finance. For example, Esty (2003) shows that project finance is widespread when it is possible to lose the entire value due to expropriation. This type of risk generates returns with large negative skewness, as opposed to more symmetric risks such those affecting exchange rates, prices, and quantities. Moreover, project finance is typically used for projects with high potential upside, satisfying the requirement that r_H be sufficiently high.

3.6 Managerial Implications

We now show that the financing regime with the lowest repayment rate does not necessarily entail the lowest likelihood of bankruptcy and is thus not necessarily optimal. Thus borrowers would be misguided by choosing the scope of conglomeration by choosing the option with lowest interest rate. The following proposition characterizes when it is more profitable to finance projects separately, even though joint financing is available at a lower rate.

Proposition 4 (Separate financing at higher rate) *Separate financing is optimal even though it results in a higher interest rate if and only if (i) condition (3) is satisfied but condition (2) is not satisfied and (ii)* $\gamma [pr_H + (1-p)r_L] > 1$.

To see what is going on, first suppose there were no bankruptcy costs. Because the creditor's payoff is a concave function of firm cash flows, it is immediate that, for any fixed repayment rate r, the expected return to the creditor would be higher for joint financing than for separate financing, because joint financing has per unit return that are less risky in the sense of second order stochastic dominance (see Rothschild and Stiglitz, 1970). As a result, the breakeven rate for the creditor would be lower for joint financing than for separate financing—regardless of whether bankruptcy occurred more often or not under joint financing. Nevertheless, the firm's expected cash flows would be the same under either financing method, so repayment rate is not a good indicator of which financing method to use.

Since there are in fact bankruptcy costs, the breakeven repayment rate must increase to offset the reduced cash flows in bankruptcy states. If joint financing does not involve intermediate bankruptcy (condition (2) holds), then expected bankruptcy costs are lower under joint financing, the breakeven rate is lower, and the firm prefers joint financing to separate financing. But if joint financing involves intermediate bankruptcy (condition (2) does not hold but condition (3) holds), then expected bankruptcy costs are higher under joint financing: default occurs more often, and costs once in bankruptcy are at least as high as under separate financing. In this case, bankruptcy costs make the repayment rate increase more under joint financing than under separate, and the firm's net expected cash flow is lower under joint financing. Still, since without bankruptcy costs the repayment rate under conglomerate financing would definitely be lower than that for separate financing, the repayment rate with such costs may still be lower. Condition (ii) of the proposition guarantees that this is the case.[15]

4 Heterogeneous Projects

So far, we have assumed that projects are ex ante symmetric. In this section, we extend the baseline setup to allow for heterogeneity across projects. Project i, $i = 1, 2$, yields (independent) returns

[15] The logic can be further illustrated by Panel (c) of Figure 2. For an (exogenous) repayment rate above the crossing point, $r > (r_H + r_L)/2$, as the one depicted, the creditor's expected returns might be higher if projects are financed jointly in spite of the increased occurrence of bankruptcy. Indeed, with joint financing, the creditor obtains the part of the gray area above the dashed line as well as a fraction γ of the dark gray and black areas. With separate financing, the creditor obtains the gray area, the upper part of the dark gray area and a fraction γ of the black area. Subtracting, the creditor's returns are higher if proceeds from the fraction γ of the dark gray area, $p(1-p)\gamma r_H$, are greater than the sum of the upper part of the dark gray area and the part of the gray area below the dashed line, $p(1-p)r$. That is, if and only if $\gamma r_H > r$. If this condition is satisfied by the equilibrium rate with separate financing, $\gamma r_H > r_i^*$ (as in the statement of the proposition), the equilibrium rate with joint financing must be lower, $r_m^{**} < r_i^*$, despite a higher probability of bankruptcy. Intuitively, creditors obtain higher proceeds from a bankrupt high value project than what they can charge for separate loans, so they are forced by competition to offer a lower interest rate. Thus, the borrower obtains a higher expected payoff with separate financing at a higher interest rate.

Note if the distribution of returns was continuous (as in the extension considered in Section 8), the extra losses from higher probability of bankruptcy if the equilibrium rate with joint financing were marginally above the crossing point would *always* be compensated by the increased proceeds from bankruptcy. Therefore, interest-rate reducing but profit-reducing conglomeration always appears when the project's returns are continuously distributed, because then there would be no discrete jump in the probability of bankruptcy at the crossing point (as there is with binary returns).

r_H^i with probability p_i and r_L^i with probability $1 - p_i$. Without loss of generality, we assume that $r_H^1 + r_L^2 > r_L^1 + r_H^2$, interchanging the indices if necessary. Note that this is equivalent to $r_H^1 - r_L^1 > r_H^2 - r_L^2$, so that project 1 has a greater spread of possible outcomes than project 2.

With heterogeneous projects, four (rather than three) levels of combined returns are possible, adding an extra case to the conditions for joint financing. Now, the possibility arises that default is avoided if project 1 yields a high return and project 2 a low return, whereas default is not avoided if the reverse occurs.

4.1 Financing Conditions

In the case of joint financing, there are now three possible rates and therefore three financing conditions. As in the baseline setup, there exists r_m' such that bankruptcy can be avoided if one project's return is high and the other is low, $r_m' \leq (r_L^1 + r_H^2)/2$.[16] If projects are heterogeneous, there exists r_m'' such that bankruptcy can be avoided if project 1's return is high and project 2's is low but not viceversa, $r_m'' \leq (r_H^1 + r_L^2)/2$. Finally, as in the baseline case, there exists r_m''' such that bankruptcy cannot be avoided if any of the two projects' return is low, which can be obtained if and only if $r_m''' \leq (r_H^1 + r_H^2)/2$. The dotted and dashed lines in Figure 3 depict the cumulative distribution of returns of two heterogeneous projects, whereas the thick line depicts the distribution of the average returns of the two projects. The three possible types of rates correspond to the three flat parts of the average distribution.

4.2 Good and Bad Conglomeration

We now turn to the question of whether the borrower should finance the projects jointly or separately when both financing regimes are feasible. As in the symmetric case, if a rate that avoids bankruptcy in both intermediate situations can be obtained, projects coinsure each other and should be financed jointly. If the firm can only obtain a rate that does not avoid bankruptcy in any of the intermediate situations, projects should be financed separately because they drag each other down. If bankruptcy

[16] The precise expression is included in the Appendix, in the proof of the forthcoming Proposition 5.

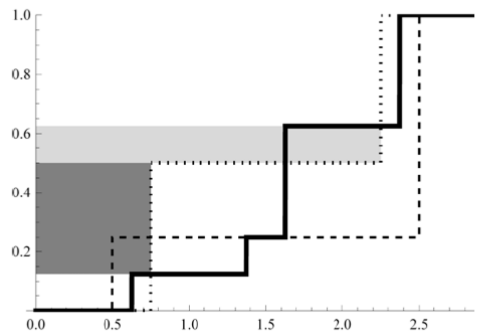

Figure 3: **Heterogeneous Projects**: The dotted and dashed lines depict the cumulative distribution of returns of two heterogeneous projects, whereas the thick line depicts the distribution of the average returns of these two projects. The three possible types of rates in joint financing correspond to the three flat parts of the thick line. If a rate r''_m is obtained (case b of Proposition 5), project 1 (dashed lines) coinsures project 2 (dotted lines) if it has a high return but risk-contaminates it if it has a low return. In this case, the reduction in expected bankruptcy costs obtained with joint rather than separate financing (co-insurance effect) is equal to the darker gray area whereas the increase in expected bankruptcy costs obtained with joint rather than separate financing (risk contamination effect) is equal to the darker gray area. The parameters used in the graph are $p_1 = 0.75$, $p_2 = 0.5$, $r^1_L = 0.5$, $r^2_L = 0.75$, $r^1_H = 2.5$, $r^2_H = 2.25$.

can only be avoided for the more favorable intermediate situation, then both coinsurance and contamination effects are present at the same time. On the one hand, project 1, when it yields a high return, saves project 2 when project 2 yields a low return; on the other hand, project 1, when it yields a low return, contaminates project 2 when project 2 yields a high return. The optimality of separate or joint financing depends on whether the gains from coinsurance dominate the losses from risk contamination.

Proposition 5 (Separate v. joint financing with heterogeneous projects) *When the borrower can finance two heterogeneous projects separately as well as jointly, there exist r'_m, r''_m and r'''_m such that*

(a) If $r'_m \leq (r^1_L + r^2_H)/2$, it is optimal to finance the projects jointly to enjoy the coinsurance gains: $(1 - p_1)\, p_2(1 - \gamma)r^1_L + p_1\,(1 - p_2)\,(1 - \gamma)r^2_L$.

(b) If $r''_m \leq (r^1_H + r^2_L)/2$ but $r'_m > (r^1_L + r^2_H)/2$, it is optimal to finance the projects separately if and only if the risk-contamination losses dominate the coinsurance gains: $(1 - p_1)p_2(1 - \gamma)r^2_H >$

$p_1(1-p_2)(1-\gamma)r_L^2$.

(c) If $r_m''' \leq (r_H^1 + r_H^2)/2$ only is satisfied, it is optimal to finance the projects separately to avoid the risk-contamination losses: $p_1(1-p_2)(1-\gamma)r_H^1 + (1-p_1)p_2(1-\gamma)r_H^2$.

In the new case (b), the probability of default with joint financing is (i) increased by $(1-p_1)p_2$, because a successful project 2 is dragged down by a failing project 1, but (ii) decreased by $p_1(1-p_2)$, because a failing project 2 is saved by a successful project 1. Project 2, however, is saved when it yields a low return but it is dragged down following a high return. Thus, if project 1 has a chance of success that is no greater than that for project 2 ($p_1 \leq p_2$), the risk-contamination effect always dominates the coinsurance effect.

The tradeoff between coinsurance and risk contamination in the new case (b) is depicted in Figure 3. The risk-contamination losses, equal to $(1-p_1)p_2(1-\gamma)r_L^2$, are represented by the light gray area and correspond to the added bankruptcy costs on the high-return project 2 that is dragged down when project 1 has a low return. The coinsurance gains, equal to $p_1(1-p_2)(1-\gamma)r_H^2$, are represented by the gray area and correspond to reduced bankruptcy costs on the low-return project 2 that is saved when project 1 has a high return. For the numerical value used in the figure, it is more profitable to finance the projects separately because the risk-contamination losses are larger than the coinsurance gains.

4.3 Testable Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects (bankruptcy costs, mean, and variability). As in the homogeneous-project case, we show first that an increase in bankruptcy costs increases the desirability of separate financing.

Prediction 5 (Bankruptcy costs) *For higher bankruptcy costs (lower γ) then (a) both joint and separate financing can be obtained for a smaller region of parameters and (b) joint financing is optimal for a smaller region of the remaining parameters.*

In the homogeneous-project case, higher means induce less separation. The next result establishes that this is true even if the two projects have different probabilities of success.

Prediction 6 (Mean) *If project 1 first-order stochastically dominates project 2, and in particular,* $r_H^1 = r_H^2$ *and* $r_L^1 = r_L^2$ *and* $p_1 > p_2$*, for a higher mean of any of the two projects (higher* p_1 *or* p_2*), the region of parameters for which joint financing is optimal increases.*

For the case in which one project is a mean preserving spread of the other, the next result establishes that more risk typically induces more separation.

Prediction 7 (Mean-preserving spread) *If project 2 second-order stochastically dominates project 1 so that* $p_1 = p_2$ *and* $r_H^1 = r_H^2 + \varepsilon$ *and* $r_L^1 = r_L^2 - \frac{p_1}{1-p_1}\varepsilon$ *for* $\varepsilon > 0$*, a higher spread of the risky project (higher* ε*) leads to a decrease in the region of parameters for which joint financing is optimal.*

As explained after Proposition 5, if the probabilities of success are the same joint financing is optimal only if r_m' can be obtained. This condition becomes more stringent as the spread of the risky project increases. Indeed, the less favorable intermediate returns $(r_L^1 + r_H^2)$ decrease in the spread of project 1 and the repayment rate (r_m') increases, as the creditor recovers less in the event of bankruptcy (when both projects yield low returns). In addition, it is easier to finance the projects separately as the increase in the high realization of the return is not fully compensated by the increase in the repayment rate (r_i).

Gorton and Souleles (2005) and Bannier and Hansel (2008) provide evidence that riskier originator banks are more likely to securitize their loans, consistent with our prediction that separate financing is more attractive when the risky project (the bank) is riskier.[17] Similarly, Mills and Newberry (2005) find that nonfinancial firms with greater credit risks are more prone to use off-balance sheet debt.

[17] Of course, these findings are consistent with other explanations. For example, riskier banks may have a higher shadow cost of equity capital, which may make securitization more attractive as a means of conserving costly capital. Also, riskier banks may be more prone to risk-shifting behavior, making it more attractive to shield assets from this through securitization (Kahn and Winton, 2004).

4.4 Managerial Implications

In Section 3.6 we showed that the option with the lowest repayment rate does not need to result in the lowest likelihood of bankruptcy and it is therefore not necessarily optimal. Here we characterize situations in which the financing option with the lowest probability of bankruptcy is not optimal either.

Proposition 6 (Separate financing with higher bankruptcy probability) *Separate financing is optimal even though it results in a higher probability of bankruptcy if and only if (i) the risk-contamination losses dominate the coinsurance gains in case (b) of Proposition 5, i.e.,* $(1-p_1)p_2r_H^2 > p_1(1-p_2)r_L^2$*, but (ii) the probability of dragging down the second project is lower than the probability of saving it, i.e.,* $(1-p_1)p_2 < p_1(1-p_2)$*.*

Notice first that if the levels of bankruptcy costs are "small", so that the borrower can finance the two projects jointly at a rate r_m' (case (a) of Proposition 5), then joint financing results in lower probability of bankruptcy than with separate financing ($(1-p_1)(1-p_2)$ as compared to $1-p_1$ and $1-p_2$) and in lower inefficiency losses. If the levels of bankruptcy costs are, instead, "large", so that the borrower can finance the two projects jointly only at a rate r_m''' (case (c) of Proposition 5), then joint financing results in higher probability of bankruptcy ($1-p_1p_2$ as compared to $1-p_1$ and $1-p_2$) and in higher inefficiency losses. In both cases, it is optimal to finance the option (joint or separate) with the lowest probability of bankruptcy.

Suppose now that the levels of bankruptcy costs are "intermediate" so that the borrower can finance the two projects jointly at a rate r_m'' but not at a rate r_m' (i.e. we are in case (b) of Proposition 5). In this case, (i) if project 1 yields a low return, it drags down project 2 if project 2 has a high return (whereas project 2 would have stayed afloat with separate financing) and, at the same time, (ii) if project 1 yields a high return, it saves project 2 if project 2 has a low return (whereas project 2 would have defaulted with separate financing). As shown in Proposition 5, projects should be financed separately if the expected benefits from coinsuring project 2 are dominated by the expected losses from risk-contaminating it. Proposition 6 highlights that the risk-contamination losses can be greater even if the probability of saving project 2 is higher than the probability of contaminating it ($p_1(1-p_2) > p_2(1-p_1)$), given that the losses from dragging down the second project are greater than the gains from saving it ($r_H^2 > r_L^2$). This situation is likely to occur if (i) the probability of success of the first project is slightly higher than that of the second project ($p_1 > p_2$), and (ii) the difference in realized returns of the second project is large ($r_H^2 >> r_L^2$).

Figure 3 is an example in point. Provided that the joint financing rate is r_m'', the risk-contamination losses, represented by the light gray area, dominate the coinsurance gains, represented by the gray area, and therefore it is more profitable to finance the projects separately, even if the probability of risk-contamination (height of the light gray area) is smaller than the probability of coinsurance (height of the gray area). The borrower might then feel tempted to finance the projects jointly, but this is suboptimal. In this case, a lower probability of bankruptcy associated with joint financing is deceptively attractive.

5 Correlated Projects

To allow for correlation, we now modify the distribution of joint returns for the baseline case with two identical projects. Suppose that the probability of two high returns result is equal to $p\,[1-(1-p)\,(1-\rho)]$, the probability of two low returns is equal to $(1-p)\,[1-p\,(1-\rho)]$, and the probability that one of the projects yields a high return whereas the other yields a low one is equal to $p\,(1-p)\,(1-\rho)$. Thus ρ is the correlation coefficient between the two projects. For the joint probability distribution to be well defined, it is necessary to assume that $\rho \geq \max \langle -(1-p)/p, -p/(1-p)\rangle$. Clearly, if $\rho = 0$ we are back to the baseline scenario with independent returns.

Prediction 8 (Correlation) *If the correlation between the projects increases (ρ is larger), then separate financing is optimal for a larger set of parameters.*

This prediction is similar to the one obtained by Inderst and Müller (2003), but it is driven by a different logic. The probability of having two high returns and the probability of having two low returns increase simultaneously with ρ. As a result, the repayment rate when intermediate bankruptcy is avoided is higher because the probability of two low returns is higher. When intermediate bankruptcy cannot be avoided, the repayment rate is lower because the probability of two high returns also increases. As a consequence, the financing conditions avoiding intermediate bankruptcy are tighter and those not avoiding it looser.

The effects of correlation on the optimality conditions are also intuitive. In the extreme case with perfect negatively correlation (i.e., if $\rho = -1$ and $p = 1/2$), when one project has a high return the other necessarily has a low one, so that projects can always be jointly financed at a rate that avoids intermediate bankruptcy.[18] Thus, it is clearly optimal to always finance projects jointly when the negative correlation is perfect. As correlation increases above $\rho = -1$, conglomeration is optimal for a smaller region of parameters. However, the probability of having intermediate returns decreases, so the difference in expected bankruptcy costs between joint and separate financing shrinks. If projects have perfect positive correlation ($\rho = 1$), the conditions for joint and separate financing are identical and the firm is clearly indifferent between them.

6 Multiple Projects

In this section, we consider a borrower with access to a general number of identical projects with independent returns. In Section 6.1, we characterize the size (and the number) of the groups that it is optimal to finance jointly, thereby identifying conditions for partial conglomeration. In Section 6.2, we show that if the number of independent projects is sufficiently large, it becomes possible and optimal for the borrower to finance all of them jointly, so that full conglomeration results.

Consider a group with k projects. Generalizing our baseline analysis for a group with two projects,

[18] This is not true for $p \neq 1/2$ because either the probability of two high realizations or the probability of two low realizations is greater than 0, even when the correlation is at the lowest possible level.

the per-project repayment rates depend on the number of projects with high return m $(1 \leq m \leq k)$ that are necessary to avoid bankruptcy,

$$r_k(m) := \frac{1-\gamma\left[\sum_{s=0}^{m-1} h(s)\frac{sr_H+(k-s)r_L}{k}\right]}{1-H(m-1)}, \tag{4}$$

where $h(s)$ is the probability that s out of the k projects yield a high return,

$$h(s) := \binom{k}{s} p^s (1-p)^{k-s} \text{ for } s = 0, 1, .., k, \tag{5}$$

and $H(s)$ is the corresponding probability distribution, $H(s) := \sum_{t=0}^{s} h(t)$.[19] As before, the equilibrium repayment rate is the one which requires the minimum number of high returns, i.e. $r_k^* = r_k(m')$ where m' is the lowest m that satisfies $r_k(m) < [(m-1)r_L + r_H]/m$.

6.1 Partial Conglomeration

To simplify the comparison in this section, assume that the firm can only form groups of symmetric sizes. Then, the number of available projects n is such that $n = 2^z$ for some $z \in \mathbb{N}$. In this context, the firm should choose the size of the group k, where $k = 2^w$ for $w = 0, 1, .., z$. If k is the size of the groups then n/k is the number of groups. The following result generalizes Proposition 2 to the case with n projects.

Proposition 7 (Partial conglomeration) *Suppose that there are n projects that can be financed in symmetric groups. If the probability of high return is sufficiently small, $p \leq p^*$, then it is optimal to finance the projects in groups of size k^*, where k^* is the largest k that satisfies $r_k^* < [(k-1)r_L + r_H]/k$.*

First, if a rate that satisfies $r_{k^*}^* < [(k^*-1)r_L + r_H]/k^*$ can be obtained by financing the projects in groups of size k^*, then it is better to finance the projects in groups of size k^* rather than in smaller groups. In this case, a single high return and k^*-1 low returns allow all the projects in the group to stay afloat, so that a single project coinsures the rest of the group. Groups of smaller size cannot be better because one high-return project would save, at most, only the low-return projects of the

[19] For notational convenience we define here the density and distribution functions at the number of projects with high return s rather than at the corresponding return, $\widehat{r}(s) := [sr_H + (k-s)r_L]/k$.

smaller group. If all the projects in the other group(s) yield a low return, they will go bankrupt and the bankruptcy losses would be higher.

Second, if p is small, forming a group of size k^* also dominates forming groups of larger size k' in which $r^*_{k'} > ((k'-1)r_L + r_H)/k'$. In this case, if k' projects are financed jointly and $k'-1$ low returns are realized, risk-contamination would result. Instead, if the projects had been financed in smaller groups of size k^*, the group with a single high-return realization would have been saved. To illustrate, consider the case with four projects ($n = 4$), with $(3r_L + r_H)/4 < r^*_4 < (r_L + r_H)/2$ and $r^*_2 < (r_L + r_H)/2$. In this case, if all four projects are financed jointly, three low returns risk-contaminate the fourth, while two high returns coinsure the other two projects. If the projects are financed in two groups composed of two projects each, a high return in one project coinsures the other project in the same group. The advantage of financing projects in two partial conglomerates with two projects each is that in the event of three low returns, one of the partial conglomerates is saved through coinsurance, while risk contamination is contained. The disadvantage is that if one group yields two low returns and the other group yields two high returns, it would have been possible to save the two projects with low returns through coinsurance if all projects had been financed jointly in a full conglomerate. If p is small (below $p^* = 2/3$ if $n = 4$), the first effect dominates and it is optimal to finance projects in groups of two.

Overall, this proposition generalizes the intuition obtained from the baseline model to the case of multiple projects for p small ($p < p^*$). As in the two-project case, projects should be financed in small groups if, when financing in groups of larger sizes, we cannot obtain rates that would make a successful project save the rest. Higher bankruptcy costs, for example, makes funding of groups of smaller size more likely to be optimal because it is more difficult to get rates that make one project save the rest in larger groups, extending the logic of Prediction 1. Following the same reasoning, higher probability of high return (as long as $p < p^*$) makes funding of groups of larger size more likely to be optimal, as in Prediction 2.

The specific circumstances that this result requires—probability of success not too high, high return large enough to "rescue" the other projects if their returns are low—bears some resemblance to the case of venture capital funds. These funds are limited partnerships that typically target firms with a small chance of very high returns and a large probability of failure, and are funded with convertible preferred equity from limited partners (see Sahlman, 1990, and Fenn, Liang, and Prowse, 1995). Although failure to pay dividends on the preferred equity does not cause bankruptcy per se, it does hurt the reputation of the fund manager. (Note that the manager may run several funds at any one time.) Taking this as a generalized cost of "default," Proposition 7 suggests that each venture capital fund should be limited enough that one success can balance out failures in the rest of its portfolio.

If p is large ($p > p^*$), it might be optimal to form groups of larger sizes even if, in such groups, one cannot obtain a rate that makes a successful project save the rest.[20] That is again consistent with Prediction 2: an increase in the probability of high return favors larger groups. But that makes it difficult to state a necessary and sufficient condition on group formation for a given p. Still, for the case in which projects are symmetric ($p = 1/2$), we can state a sufficient condition on full separation and full conglomeration, thus expanding the results of Proposition 7 for the two extreme group sizes. These sufficient conditions become necessary and sufficient if $n = 2$, thus also generalizing Proposition 2.

Proposition 8 (Joint v. separate financing of multiple projects) *If there are n symmetric projects which have to be financed in symmetric groups, then they should all be financed jointly if $r_n^* < [(n-1)r_L + r_H]/n$ and all separately if $r_k^* > (r_H + r_L)/2$ for any $k \geq 2$.*

Proposition 7 shows that all projects should be financed jointly if, with a full conglomerate, it is possible to obtain a rate such that a single high-return project coinsures all the other projects. At the other extreme, it shows that if, in any group of projects, it is not possible to obtain a rate such that

[20] In the case of four projects, if $p > p^* = 2/3$, the second effect described above dominates the first and it is optimal to finance all four projects jointly.

a high-return project coinsures the rest of the group, it is better to finance all projects separately. Proposition 8 expands the set of cases for which full separation is optimal for $p = 1/2$. It exploits the fact the distribution functions of the average returns of any group of symmetric projects cross the distribution of the single return at the mean and, as a result, the probability of bankruptcy of groups with repayment rates above the mean is higher than the probability of bankruptcy of stand-alone projects. Hence, full separation is optimal if, for any group of projects, it is not possible to obtain a rate below the mean return, or equivalently, a rate such that half of the projects save the other half. For example, this occurs for a larger set of parameters when bankruptcy costs are high or when the mean return is low.

6.2 Large Number of Projects

As the analysis of our baseline model shows, the set of parameters for which joint financing is optimal does not necessarily increase with the number of projects. This result stands in contrast with claims often made in the literature; for example, see the discussion on page 400 and footnote 3 in Diamond (1984). Compared to our model, Diamond (1984) adds an intermediary who contracts with several entrepreneurs to achieve joint financing; in his model this intermediary can observe the entrepreneurs' returns only by paying a cost. Joint financing in our model can be seen as a special case of Diamond's (1984) model with an intermediary who can costlessly observe the entrepreneurs' returns. In the last paragraph on page 400, Diamond (1984) claims informally that the per-entrepreneur delegation costs associated to intermediary financing (which correspond to the expected bankruptcy costs in our model) decrease monotonically with the number of entrepreneurs. When illustrating this result as the number of entrepreneurs increases from one to two, in footnote 3 Diamond (1984) implicitly assumes away bad conglomeration by focusing on the case in which the repayment obligation when using the intermediary (i.e., with joint financing) is less than twice the one obtained without the intermediary (with separate financing). His analysis, however, is incomplete because it disregards the possibility of bad conglomeration. As we show in this paper, if the repayment rate with joint financing rate is above the crossing point, conglomeration is bad even when the intermediary can observe freely the entrepreneurs' returns.

Thus, our paper shows that there is a meaningful tradeoff between joint and separate financing without need of handicapping joint financing through the monitoring cost associated to intermediation. Nevertheless, using an argument based on the law of large numbers we can show that joint financing dominates separate financing when the number of independent and identical projects increases to infinity. This result is the analogue in our setting of Diamond's (1984) Proposition 2.

Proposition 9 (Many projects) *There exists n' and $q \in (0, p)$ such that when the number of projects satisfies $n > n'$, joint financing of all projects can be obtained at a repayment rate that avoids bankruptcy when nq projects have high returns. The resulting per-project return approaches the net present expected value of each project as n grows.*

If the number of independent projects is sufficiently large, it always becomes possible for the borrower to finance all the projects jointly. This result exploits the law of large numbers. Namely, as the number of projects n increases, the probability that the average number of projects with high returns differs from p, the probability of a high return, by more than a small amount ε tends to zero. We can then construct a rate offer to finance all projects jointly that is acceptable to the creditors. The borrower's returns when financing all projects jointly is then arbitrarily close to the first-best as the number of projects increases. Therefore, when the number of projects is large, financing all the projects jointly is approximately optimal for the borrower because the resulting payoff is close to the highest possible level.

Prediction 9 (Full conglomeration) *If there is a large number of independent projects, it is optimal to finance all of them jointly.*

In practice, however, there is an important caveat to this result: for any given firm, projects are likely to be generally positively correlated, due to common shocks to the firm's industry or the general economy. As we have seen, such correlation can reverse the optimality of full joint financing.

7 Structure of Bankruptcy Costs

In line with most of the theoretical and empirical literature, in our baseline specification bankruptcy costs are proportional to realized returns, with $B(r) = (1-\gamma)r$. Note that this baseline specification entails constant returns to scale: $B(2r) = (1-\gamma)2r = 2B(r)$. To investigate the robustness of our results to the structure of the bankruptcy costs, this section consider a general specification that allows for economies or diseconomies of scale in bankruptcy. We retain the feature that bankruptcy costs are larger for higher levels of realized returns, so that $B(r)$ is increasing in r.

As is intuitive, economies of scale in bankruptcy favor joint financing whereas diseconomies of scale favor separation. As demonstrated in the next result, if economies of scale are sufficiently strong, so that $B(2r_L) - 2B(r_L)$ is negative enough, then joint financing is optimal. Separate financing is optimal if, instead, there are sufficiently strong diseconomies of scale, so that $B(2r_L) - 2B(r_L)$ is positive enough. In the intermediate case, which includes constant returns to scale as well as weak economies and diseconomies of scale, separation is optimal if the rate that avoids intermediate bankruptcy cannot be obtained.

Proposition 10 (Scale economies in bankruptcy costs) *With a general structure of bankruptcy costs, there exist thresholds $\underline{S} < 0$ and $\overline{S} > 0$ such that*

(i) If $B(2r_L) - 2B(r_L) < \underline{S}$, joint financing is always optimal;

(ii) If $\underline{S} < B(2r_L) - 2B(r_L) < \overline{S}$, separate financing is optimal if and only if

$$\frac{1-(1-p)^2\left[r_L - B(2r_L)/2\right]}{1-(1-p)^2} > \frac{r_H + r_L}{2};$$

(iii) If $B(2r_L) - 2B(r_L) > \overline{S}$, separate financing is always optimal.

To further characterize the thresholds independently of the level of returns, consider bankruptcy costs given by $B(r) \equiv (1-\gamma)r + \alpha(r - r_L)r$. This specification allows for economies ($\alpha < 0$) and diseconomies of scale ($\alpha > 0$) and includes our baseline case with constant returns to scale as a special case ($\alpha = 0$). Following the procedure set out in the previous proposition, if $\alpha < \underline{\alpha} :=$

$-(1-\gamma)pr_H/\left[(1-p)r_L^2+pr_H^2+pr_Lr_H\right]$, joint financing is optimal (case (i)); if $\underline{\alpha}<\alpha<\overline{\alpha}$, separate financing is optimal if and only if the rate that avoids intermediate bankruptcy cannot be obtained (case (ii)); and if $\alpha>\overline{\alpha}:=p(1-\gamma)/[(1-p)r_L]$, separate financing is optimal (case (iii)).

An alternative specification with constant returns to scale consists in assuming a fixed per-project bankruptcy cost b $(<r_L)$, so that $B(r)=b$ for $r=r_H,r_L$ and $B(r)=2b$ for $r=2r_H,r_H+r_L,2r_L$. Thus, we have $B(2r_L)-2B(r_L)=0$, so that case (ii) always results. In addition, it can be shown that with per project bankruptcy costs separate financing is optimal for a relatively larger set of parameters than in our baseline case with proportional bankruptcy costs.[21] Next, if there is a fixed recovery rate per project w $(<r_L)$, case (ii) also results.

In sum, joint financing is optimal if there are significant economies of scale in bankruptcy, while separate financing is optimal if there are sufficiently strong diseconomies of scale. For weaker economies or diseconomies of scale, as well as for several specifications with constant returns of scale in bankruptcy, separate financing is optimal as long as intermediate bankruptcy cannot be avoided. Higher bankruptcy costs then favor separate financing more generally, as in our baseline specification.

8 Normal Returns

This section analyzes the model when returns are normally distributed rather than binary. The purpose of this extension is twofold. First, we show that our results on bad conglomeration and the main comparative statics predictions are robust to continuous distributions. Second, this specification of returns allows us to make a precise comparison with Leland's (2007) results; our analytical characterization clarifies that the optimality of separate financing holds even when the capital structure mix

[21] To show this, set per-project bankruptcy costs at the same level as the proportional losses of the low-return project, $b=(1-\gamma)r_L$. Then, the proceeds from a bankrupt high-return project are relatively higher in the per-project case compared to the proportional case. Rates r_i^* and r_m^* are the same as those resulting from proportional bankruptcy costs, so that conditions (1) and (2) do not change. However, the rate r_m^{**} is now lower and it becomes easier to satisfy condition (3). As a result, it becomes easier to obtain joint financing, but only at the rate for which intermediate bankruptcy occurs. Therefore, when both separate and joint financing are feasible, separate financing is optimal for a relatively larger set of parameters.

(in terms of debt and equity) is not adjusted to the scope of incorporation. As part of the analysis, we also provide an easy-to-verify sufficient condition for the optimality of separate financing.

8.1 Model Extension

A firm has access to n symmetric, normally distributed projects, $r_i \sim N\left(\mu, \sigma^2\right)$ for $i = 1, ..., n$, with symmetric correlation coefficient ρ. As in the binary case, the distribution function of the average returns lies below the distribution of a single return until a unique crossing point (here equal to the mean because of symmetry), after which the ordering is reserved. Indeed, the average of two normal random variables is also normal with a density that is more peaked around the mean than the original normal density. To retain analytical tractability, we assume (i) that there is a fixed per-project recovery rate w ($w < 1 < \mu$);[22] (ii) that the firm can only form symmetric groups of projects (and therefore $n = 2^z$ for some $z \in \mathbb{N}$), as in Section 6.1; and (iii) that projects need to be financed exclusively with debt.

8.2 Financing Conditions

As in Section 6.1, the firm should choose the size of the groups k, where $k = 2^w$ for $w = 0, 1, .., z$. The *per project* repayment requested by a creditor in a competitive market to finance a group of size k, r_k^*, is defined by

$$kr_k^* \left[1 - G\left(kr_k^*\right)\right] + wkG\left(kr_k^*\right) = k, \tag{6}$$

where G is the distribution function of the sum of k normal random variables. Noting that the distribution of the sum computed at kr is

$$G\left(kr\right) = \Pr\left(r_1 + ... + r_k \leq kr\right) = \Pr\left(\frac{r_1 + ... + r_k}{k} \leq r\right) =: H(r), \tag{7}$$

[22] Given that returns are normally distributed, with positive probability there realized return is lower than the recovery rate. For simplicity, we disregard this problem, given that the probability of these realizations can be made arbitrarily small with an appropriate choice of parameters. Alternatively, the proof of Proposition 11 holds for the general class of log-concave symmetric distributions, which allow for positive support and recovery rates below the support. The key property driving the result is that the density of the average of n random variables is more peaked around the mean compared to the original density. As shown by Proschan (1965), this property holds generally for log-concave symmetric distributions.

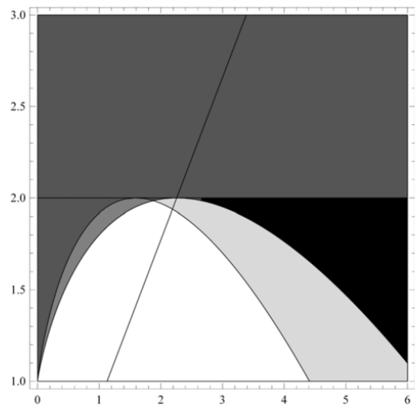

Figure 4: **Financing and Optimality Regions with Normally Distributed Returns.** In the (σ, μ) combinations of the light gray area financing is only possible separately. In the medium gray area financing is only possible jointly. In the dark gray and black areas both separate and joint financing is possible. In the black area (delimited by the two straight lines depicting the two conditions in Proposition 12) separate financing is optimal. In this picture, we take $w = 0$ and $\rho = 0$.

where H is the distribution of the average of $r_1, ..., r_k$, this condition is equivalent to

$$r_k^* \left[1 - H\left(r_k^*\right)\right] + wH\left(r_k^*\right) = 1. \tag{8}$$

The firm's per-project payoff is then

$$\int_{r_k^*}^{+\infty} \frac{r_1 + ... + r_k}{k} dH - r_k^* \left[1 - H(r_k^*)\right] = \int_{r_k^*}^{+\infty} \left(\frac{r_1 + ... + r_k}{k} - r_k^*\right) dH. \tag{9}$$

Given that this payoff is a decreasing function of r_k^*, it is optimal for the firm to select the lowest r_k^* at which condition (8) is satisfied, if such a r_k^* exists. Financing is obtained in such a case. Figure 4 represents the mean-variance parameters allowing projects to be financed separately ($k = 1$) and in groups of two ($k = 2$).

8.3 Good and Bad Conglomeration

We now turn to the question of when is it optimal to finance the projects separately when there are multiple options available.

Proposition 11 (Optimality of separate financing) *If it is feasible to finance separately n nor-*

mally distributed projects with mean μ and standard deviation σ, separate financing is optimal if

$$\mu + w < 2 \text{ and } \mu - w < \sigma\sqrt{[1+\rho(n-1)]\,\pi/2n}. \tag{10}$$

These conditions identify the region of parameters for which separate financing is optimal in Figure 4. We obtain the same comparative statics as in the baseline model. Separation holds for a larger region of parameters if the mean returns are low (Prediction 2) and if the variance is high (Prediction 3). Indeed, it is more difficult to satisfy both conditions in (10) if μ increases, and it is easier to satisfy the second condition if σ increases. As in Prediction 8, when the coefficient of correlation increases the region for which separate financing is optimal increases. Similar to the binary case in Section 6.2, when the number of projects increases the region for which separation is optimal shrinks. In the limit, if there is a large number of independent projects the second condition is never satisfied, in accordance with Prediction 9.

Similarly, an increase in the recovery rate favors the optimality of joint financing (Prediction 1). To see this, consider the mean-variance parameter combinations for which joint and separate financing are both feasible for two levels of recovery rates, $w = w_1$ and $w = w_2$ where $w_1 < w_2$. Then, the region for which separate financing is optimal is smaller for $w = w_2$ than for $w = w_1$. Indeed, an increase from w_1 to w_2 makes it more difficult for the first condition in (10) to be satisfied, thereby shrinking the region in which separate financing is optimal. Even though it becomes easier to satisfy the second condition, the new parameter values for which separate financing is optimal belongs to a region in which it is not feasible to finance the projects.

8.4 Comparison with Leland (2007)

Leland (2007) argues that separate financing becomes optimal because of the endogenous choice between debt and equity. Our baseline model, both in the version with binary returns as well as in the version with continuous returns, considers fixed-investment projects that must be financed with debt. Thus, we explicitly rule out the possibility of increasing leverage and re-optimizing the capital structure, as Leland (2007) does. A key contribution of our analysis is that separate financing can dominate joint financing, even when the capital structure (in terms of the mix of debt and equity) is not optimized as the corporate structure (in terms of separate rather than joint financing) is changed. This case was overlooked by Leland's numerical analysis.

To illustrate this point, consider Leland's (2007) decomposition of financial synergies into three components (see his equation 21, Section IV.B, page 778),

$$\Delta = \Delta V0 + \Delta TS - \Delta DC. \tag{11}$$

(a) $\Delta V0$ is equal to the limited liability effect (equation 22), which is always negative. "As noted by Scott (1977) and Sarig (1985), the LL effect is never positive, and is strictly negative if operational cash flows have a positive probability of being negative and are less than perfectly correlated" (pages 778–779).

(b) "The second component of financial synergies from mergers, ΔTS, is the gain (or loss) in tax savings solely related to the effects of optimal merged leverage versus optimal separate leverage. The examples in Section V show that ΔTS can have either sign" (page 779).

(c) "The final component of financial synergies is the change in the value of default costs at the optimal leverage levels, ΔDC. This term is negative in all examples considered, indicating that although leverage may increase after a merger, the expected losses from default are nonetheless reduced by the lower operational risk of the merged firm" (page 779).

By abstracting from the limited liability effect, from taxes, and the possibility to reoptimize leverage, our baseline model assumes away terms (a) and (b) and focuses uniquely on term (c). This term can have a positive or a negative sign, contrary to Leland's (2007) conjecture that "this term is negative in all examples considered" (page 779). When this term is positive, expected bankruptcy costs are higher with joint financing and therefore financial synergies are negative.[23] Our analysis uncovers that bad conglomeration arises even when the capital structure mix in terms of debt and

[23] This term being negative translates into positive financial synergies, because the difference appears with a negative sign in equation (11).

equity is not reoptimized with the scope of conglomeration. This result and the simple logic that underlies it were not envisioned by Leland's (2007) analysis.[24]

9 Conclusion

This paper analyzes the simple economics of conglomeration with bankruptcy costs. Our results qualify the long-standing claim that joint financing generates financial benefits by economizing on bankruptcy costs. By turning on its head the classic logic that generates coinsurance savings from conglomeration, we characterize instances in which expected bankruptcy costs increase because of risk contamination. For projects with binary returns we provide a complete characterization of the tradeoff between coinsurance and risk contamination. Broadly consistent with empirical evidence, the theory predicts that:

- An increase in the bankruptcy recovery rate and an increase in the probability of a high return favor joint financing;
- An increase in the riskiness of (sufficiently negatively skewed) projects favors separate financing;
- An increase in the negative skewness of projects (with a sufficiently high return) favors separate financing;
- An increase in the differences in terms of risk profiles of two heterogeneous projects favors separate financing;
- An increase in the correlation of projects favors separate financing;
- Joint financing of a sufficiently large number of independent projects is optimal;
- Economies of scale in bankruptcy costs favor joint financing.

In addition, we show that *separate* financing can be optimal even when joint financing involves paying a *lower* repayment rate or results in a *lower* probability of bankruptcy.

[24] Our earlier results on skewness (see Prediction 4) suggest that, with continuous distributions that are negatively skewed, separate financing can be optimal for repayment rates that are below the median return and thus incur default with realistic probabilities.

Our modeling framework is tractable and can be extended in many further directions. In our setup, either investors in each of the two projects have recourse to the returns of the other project (with joint financing) or none of them have access to the returns of the other project (with separate financing). In reality, an asymmetric, intermediate situation could also arise whereby investors in one (recourse) project have access to the returns of the other (nonrecourse) project, but not conversely. In this case, one of the diagonal entries in Figure 1 would be akin to separate financing. That is, if the project without recourse yielded a low return while the project with recourse yielded a high return, the former project would go bankrupt while the latter project would stay afloat. In the other diagonal entry, however, both projects would stay afloat provided that the recourse project is saved by the nonrecourse project. If this is the case, this intermediate solution would dominate separate financing, but the reverse would hold when the recourse project is dragged down by the nonrecourse project. A complete analysis for the resulting tradeoff is left to future research; see Nicodano and Luciano (2009) for an investigation in this direction in a setting with both bankruptcy costs and taxes.

Saving an unsuccessful project might sometimes be optimal for reputational reasons, even if it has been financed with (nonrecourse) debt and the firm is under no legal obligation to save it. Gorton (2008), for example, points out that securitization issuers retain substantial *implicit* exposure even after mortgages are securitized. In the credit card asset-based securities (ABS) market, for example, Higgins and Mason (2004) document instances in which issuers of credit card ABS have taken back non-performing loans despite not being contractually required to do so. Similarly, Gorton and Souleles (2006) show that prices paid by investors in credit card ABS take into account issuers' ability to bail out their ABS. To capture this tradeoff, one could extend our static model to a dynamic framework. It is also natural to extend the model to allow for multiple (and possibly risk-averse) investors, as in Bond's (2004) analysis of conglomeration versus bank intermediation in the costly state verification model.

Finally, our model can also be extended to analyze the public policy problem of optimal conglomeration in the presence of systemic spillovers, a topic that has recently attracted attention (see, for example, Acharya, 2009, and Ibragimov, Jaffee, and Walden, 2011). In this case, bankruptcies create significant negative externalities and the borrower should minimize the probability of bankruptcy instead of maximizing net returns. For the case with normally distributed returns, it can be shown that the two conditions identified in Proposition 11 become necessary and sufficient. On the one hand, if the equilibrium repayment rate lies above the crossing point (the two conditions in (10) are satisfied), the equilibrium rates and the probability of bankruptcy are lower with separate financing. On the other hand, if the repayment rate is below the crossing point (either or both conditions in (10) are not satisfied), joint financing reduces the equilibrium rates and the probability of bankruptcy. We leave the development of this extension to future research.

Appendix: Proofs

Proof of Proposition 1: The proof follows from the analysis reported in the text. Q.E.D.

Proof of Proposition 2: If projects can be financed separately, i.e. condition (1) is satisfied, the entrepreneur obtains a per-project return of $p(r_H - r_i^*)$, which is equal to the ex post net present value

$$pr_H + \gamma(1-p)r_L - 1. \tag{12}$$

Similarly, if condition (2) is satisfied, the entrepreneur obtains a per-project return of $p^2(r_H - r_m^*) + 2p(1-p)\left[(r_H + r_L)/2 - r_m^*\right]$, or

$$p^2 r_H + 2p(1-p)(r_H + r_L)/2 + \gamma\left(1-p\right)^2 r_L - 1, \tag{13}$$

and, if condition (3) but (2) is not satisfied, she obtains $p^2(r_H - r_m^{**})$, or

$$p^2 r_H + \gamma 2p(1-p)(r_H + r_L)/2 + \gamma\left(1-p\right)^2 r_L - 1. \tag{14}$$

Subtracting (13) from (12), we obtain $(1-\gamma)\,p(1-p)r_L$ and therefore joint financing is more profitable than separate financing. Instead, subtracting (12) from (13), we obtain $(1-\gamma)(1-p)pr_H$ and therefore separate financing is more profitable than joint financing. Q.E.D.

Proof of Prediction 1: The statements follow from the fact that the derivatives of the left-hand of (1), (2), and (3) with respect to γ are negative. Q.E.D.

Proof of Prediction 2: The statements follow from the fact that the derivatives of the left-hand of (1), (2), and (3) with respect to p are negative. Q.E.D.

Proof of Prediction 3: Letting ε be such that $\widehat{r}_H = r_H + \varepsilon$, we have that, in order to have a mean preserving spread, $\widehat{r}_L = r_L - \frac{p}{1-p}\varepsilon$. Substituting into condition (2), the derivative of the left-hand side less the derivative of the right-hand side is equal to

$$\frac{1-p}{2-p}\gamma + \frac{1}{2(1-p)} - 1,$$

which is positive if and only if $p > \overline{p}$, where $\overline{p} \equiv \left[1 + 4(1-\gamma) - \sqrt{1 + 8(1-\gamma)}\right]/2(1-\gamma)$. Therefore, condition (2) is less likely to be satisfied following an increase in ε if and only if $p > \overline{p}$. It can be easily checked that $\overline{p} < 1/2$ for any γ. Q.E.D.

Proof of Prediction 4: Letting ε be such that $\widehat{r}_L = r_L - \varepsilon$, we have that, in order to have a mean preserving spread, $\widehat{p} = p - \frac{(1-p)\varepsilon}{r_H - r_L + \varepsilon}$. Following the same procedure as in the proof of the previous prediction, there exists $\overline{r}_H$, such that condition (2) is less likely to be satisfied following an increase in ε if and only if $r > \overline{r}_H$. Q.E.D.

Proof of Proposition 3: (i) Suppose that γ and r_L are arbitrarily close to 1, condition (2) is arbitrarily close to $\frac{r_H + r_L}{2} > 1$ whereas condition (1) simplifies to $r_H > 1$. Clearly there are situations in which condition (2) is satisfied, and therefore projects can be financed jointly, but condition (1) is not satisfied, and therefore projects cannot be financed separately.

(ii) If condition (2) is not satisfied, projects can only be financed jointly if condition (3) is satisfied.

Condition (3) can be rewritten as

$$pr_H - p(1-p)r_H(1-\gamma) + (1-p)\,\gamma r_L > 1.$$

This implies that $pr_H + (1-p)\,\gamma r_L > 1$, which implies that projects can be financed separately. Of course, the opposite is not true, if the parameters are such that $pr_H + (1-p)\,\gamma r_L$ is arbitrarily close to 1, then condition (3) is not satisfied. Q.E.D.

Proof of Proposition 4: Suppose first that a rate below the crossing point can be obtained. We have that

$$r_m^* = \frac{1-(1-p)^2\,\gamma r_L}{1-(1-p)^2} < \frac{1-(1-p)\gamma r_L}{p} = r_i^*,$$

because $1 > \gamma r_L$. Next, suppose that only a rate r_m^{**} above the crossing point can be obtained and therefore the probability of bankruptcy is higher with joint financing. Nevertheless, the rate r_m^* associated with joint financing is lower than r_i^* associated with separate financing whenever

$$r_m^{**} = \frac{1-(1-p)\,\gamma\,(pr_H + r_L)}{p^2} < \frac{1-(1-p)\,\gamma r_L}{p} = r_i^*,$$

or equivalently when

$$\gamma r_H > \frac{1-(1-p)\gamma r_L}{p} = r_i^*,$$

as claimed. Q.E.D.

Proof of Proposition 5: We first derive the financing conditions. Following the same procedure as in the symmetric case, the repayment rate should satisfy $1 < r_i' < r_H^i$. The creditor's zero profit condition is now

$$pr_i' + (1-p_i)\gamma r_L^i - 1 = 0, \tag{15}$$

and project i can be financed (at r_i') if and only if

$$r_i' := \frac{1-(1-p_i)\gamma r_L^i}{p_i} < r_H^i. \tag{16}$$

There are three cases in which joint financing is feasible depending on whether bankruptcy can be avoided in both cases with intermediate returns, or only when project 1 yields a high return and project 2 yields a low return, or in neither case. If bankruptcy can be avoided in both cases with intermediate returns, competition in the credit market results in

$$[1-(1-p_1)(1-p_2)]\,2r'_m+(1-p_1)(1-p_2)\,\gamma\left(r_L^1+r_L^2\right)-2=0, \tag{17}$$

so that this case is possible if and only if

$$r'_m:=\frac{1-(1-p_1)(1-p_2)\,\gamma\frac{r_L^1+r_L^2}{2}}{1-(1-p_1)(1-p_2)}<\frac{r_L^1+r_H^2}{2}. \tag{18}$$

If bankruptcy can be avoided with high intermediate returns but not with low intermediate returns, then

$$p_1p_2 2r''_m+p_1(1-p_2)2r''_m+(1-p_1)p_2\gamma\left(r_L^1+r_H^2\right)+(1-p_1)(1-p_2)\,\gamma\left(r_L^1+r_L^2\right)-2=0, \tag{19}$$

and therefore this case is possible if and only if

$$\frac{r_L^1+r_H^2}{2}<r''_m:=\frac{1-(1-p_1)p_2\gamma\frac{r_L^1+r_H^2}{2}-(1-p_1)(1-p_2)\,\gamma\frac{r_L^1+r_L^2}{2}}{p_1}<\frac{r_H^1+r_L^2}{2}. \tag{20}$$

If bankruptcy cannot be avoided with either intermediate returns, then

$$p_1p_2 2r'''_m+p_1(1-p_2)\gamma\left(r_H^1+r_L^2\right)+(1-p_1)p_2\gamma\left(r_L^1+r_H^2\right)+(1-p_1)(1-p_2)\,\gamma\left(r_L^1+r_L^2\right)-2=0, \tag{21}$$

and therefore this is possible if and only if

$$\frac{r_H^1+r_L^2}{2}<r'''_m<\frac{r_H^1+r_H^2}{2}, \tag{22}$$

where

$$r'''_m:=\frac{1-p_1(1-p_2)\gamma\frac{r_H^1+r_L^2}{2}-p_2(1-p_1)\gamma\frac{r_L^1+r_H^2}{2}-(1-p_1)(1-p_2)\,\gamma\frac{r_L^1+r_L^2}{2}}{p_1p_2}.$$

Again, since the borrower obtains all the ex post net present value, rate r'_m is preferred to r''_m and r''_m is preferred to r'''_m. To complete the proof we only need to show that the lower bound conditions for r''_m and r'''_m are irrelevant. From (17) and (19), and rearranging, we have

$$p_1(r'_m-r''_m)=p_2(1-p_1)\left[\gamma\left(\frac{r_L^1+r_H^2}{2}\right)-r'_m\right],$$

and therefore if $r'_m>\frac{r_L^1+r_H^2}{2}$ then the right-hand side is negative. As a consequence, we have

$r''_m > r'_m > \frac{r_L^1 + r_H^2}{2}$. Similarly, from (19) and (21) and rearranging, we have

$$p_2 \left(r''_m - r'''_m \right) = (1 - p_2) \left[\gamma \left(\frac{r_H^1 + r_L^2}{2} \right) - r''_m \right]$$

and therefore if $r''_m > \frac{r_H^1 + r_L^2}{2}$ then the right-hand side is negative. As a consequence, we have $r'''_m > r''_m > \frac{r_H^1 + r_L^2}{2}$.

We now turn to the choice between joint and separate financing. Substituting r'_m in the right-hand side of (17) and r'_i in the right-hand side of (15) and subtracting the latter from the former, we have

$$p_2 \left(1 - p_1\right) (1 - \gamma) r_L^1 + p_1 \left(1 - p_2\right) (1 - \gamma) r_L^2 \ (> 0).$$

Similarly, substituting r''_m in the right-hand side of (19) and subtracting again the ex post net present value of financing the two projects separately from this, we obtain

$$-(1 - p_1) p_2 (1 - \gamma) r_H^2 + p_1 (1 - p_2)(1 - \gamma) r_L^2,$$

which can be positive or negative. Lastly, substituting r'''_m in the right-hand side of (21) and subtracting the ex post net present value of financing the two projects separately from this, we have

$$-p_1 (1 - p_2)(1 - \gamma) r_H^1 - p_2 (1 - p_1)(1 - \gamma) r_H^2 \ (< 0),$$

as desired. Q.E.D.

Proof of Prediction 5: The statements follow from the fact that the derivatives with respect to γ of r'_m, r''_m and r'''_m, defined in (18), (20), and (22), are negative. Q.E.D.

Proof of Prediction 6: From the proof of Proposition *5*, if $r_L^1 = r_L^2$ and $r_L^1 = r_L^2$, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r'_m can be obtained. The statement follows from the fact that the derivatives of the left-hand of (18) with respect to p_1 and p_2 are negative. Q.E.D.

Proof of Prediction 7: Given that one project is obtained from an elementary increase in risk from the other and returns should still be binary, we must have that $p_1 = p_2 \equiv p$. Letting ε be such that $r_H^1 = r_H^2 + \varepsilon$, we have $r_L^1 = r_L^2 - \frac{p}{1-p}\varepsilon$. Indeed, $p(r_H^2 + \varepsilon) + (1-p)r_L^1 = pr_H^2 + (1-p)r_L^2$. We can also check that $r_L^1 + r_H^2 = r_L^2 - \frac{p}{1-p}\varepsilon + r_H^2 < r_L^2 + \varepsilon + r_H^2 = r_H^1 + r_L^2$.

As shown in the previous proposition, given that the probabilities of success are equal, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r_m' can be obtained. Moreover, the region for which joint financing is optimal shrinks as the repayment rate r_m' is more difficult to obtain if ε increases. Indeed, the left-hand side of condition (18) decreases in ε and the repayment rate (the right-hand side) increases in ε.

On the other hand, the region for which separate financing is possible expands if ε increases. Indeed, the derivative of the left-hand side of condition (16) is equal to γ whereas the right-hand side is equal to 1. Hence, this condition is more easily satisfied as ε increases. Q.E.D.

Proof of Proposition 6: Clearly, from Proposition 5, if statements (i) and (ii) are satisfied, separation is optimal. The probability of default of project 1 is the same in both financing regimes. With separate financing, the probability of default of project 2 is (i) reduced by $(1-p_1)p_2$, as a successful project 2 would not be dragged down if project 1 fails, but (ii) increased by $p_1(1-p_2)$, as a failing project 2 would not be saved if project 1 is successful. Given that, according to (iii), $p_1 > p_2$, we have that $p_1(1-p_2) > (1-p_1)p_2$. As a result, the probability of default with separate financing is higher. Q.E.D.

Proof of Prediction 8: Clearly, separate financing is not affected by correlation. The joint financing repayment rates, r_m^* and r_m^{**} in Proposition 1, and the corresponding financing conditions, are now replaced by $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$, respectively, where

$$r_{m,\rho}^* := \frac{1-(1-p)\left[1-p\left(1-\rho\right)\right]\gamma r_L}{1-(1-p)\left[1-p\left(1-\rho\right)\right]} < \frac{r_H + r_L}{2},$$

and

$$r_{m,\rho}^{**} := \frac{1-(1-p)\,\gamma r_L}{p\left[1-(1-p)\left(1-\rho\right)\left(1-\gamma\right)\right]} < r_H.$$

Note that $r^*_{m,\rho}$ and $r^{**}_{m,\rho}$ are respectively increasing and decreasing in ρ. Q.E.D.

Proof of Proposition 7: We first show that if $r^*_k < [(k-1)r_L + r_H]/k$ then it is better to form groups of k projects rather than smaller groups. If this condition is satisfied, the per-project expected bankruptcy losses are given by $(1-p)^k \gamma r_L$. In groups of smaller size, $m < k$, the minimum loss is given by $(1-p)^m \gamma r_L$, which is larger.

We now show that if $r^*_k < [(k-1)r_L + r_H]/k$ and $r^*_m > [(m-1)r_L + r_H]/m$ for $m > k$ then it is better to form groups of size k rather than groups of larger size. Indeed, the bankruptcy losses of a group of size $m > k$, are greater or equal to $(1-p)^m \gamma r_L + m(1-p)^{m-1} p\gamma[(m-1)r_L + r_H)]/m$ which is in turn greater than $\left[(1-p)^m + m(1-p)^{m-1}p\right] \gamma r_L$. Then, subtracting the per-project bankruptcy costs of a group of size k, the difference is given by $(1-p)^k w(p)$ where $w(p) := (1-p)^{m-k-1}(1+mp)-1$. But, $w(0) = 0$, $w(1) < 0$ and $w'(p) := (1-p)^{m-k-2}\left[-(m-k)mp + k + 1\right]$. Given that $w'(0) > 0$ and $w'(1) \leq 0$ there exists a unique $0 < p' \leq 1$ such that $w'(p) > 0$ if $p < p'$ and $w'(p) < 0$ if $p > p'$. Therefore, there exists a unique $0 < p^* < 1$, such that $w(p) > 0$ if $p < p^*$ and $w(p) < 0$ if $p > p^*$. As a result, the per-project bankruptcy costs for a group of m projects are larger than for a group of k projects if $p < p^*$. Q.E.D.

Proof of Proposition 8: The first statement follows from the same argument as in the first part of the proof of Proposition 7.

With respect to the second statement, note that for any discrete symmetric distribution $H((r_H + r_L)/2 - \varepsilon) = 1 - H((r_H + r_L)/2 + \varepsilon)$ for any $\varepsilon > 0$, where the density and distribution functions are defined at the average returns (rather than at the number of projects with high return s as in the text). Given that $H(\cdot)$ is increasing and $h((r_H + r_L)/2) > 0$, we have that $H((r_H + r_L)/2 - \varepsilon) < H((r_H + r_L)/2 + \varepsilon)$ and substituting $H((r_H + r_L)/2 - \varepsilon) < 1/2$ and therefore $H((r_H + r_L)/2 + \varepsilon) > 1/2$. Hence, we have that $H(r) > 1/2$ for any $(r_H + r_L)/2 < r \leq r_H$ and therefore the distribution of the average return of any group of projects for $(r_H + r_L)/2 < r \leq r_H$ is above that of the returns of the projects financed separately, which is equal to $1/2$ for $r_L \leq r < r_H$.

Now, if $r_k^* > (r_H+r_L)/2$ for any group of k projects, then we have that the per project bankruptcy losses is greater than $H((r_H+r_L)/2+\varepsilon)\gamma r_L$, which is in turn greater than the losses in the case of separate projects, $(1/2)\gamma r_L$.   Q.E.D.

Proof of Proposition 9: First statement. Define $g(\delta) := \delta r_H + (1-\delta)\, r_L$. We have that $g(p) > 1$ because of the positive net present value condition, and trivially $g(0) = r_L < 1$ and $g'(\delta) > 0$. Then there exists a unique $\delta^* \in (0,p)$ such that $g(\delta^*) = 1$. For a fixed rational number ε (small) define $q := \delta^* + \varepsilon$. Clearly, $qr_H + (1-q)\, r_L > 1$

Take any number of projects n such that nq is an integer number. Suppose that we were to finance all these n projects jointly at an interest rate that avoids bankruptcy when at least nq of them have high returns. This is possible if and only if the per-project repayment satisfies

$$r_n^* \leq qr_H + (1-q)\, r_L.$$

Given that the returns recovered in the event of bankruptcy are positive, we have that the equilibrium repayment rate in (4) satisfies

$$r_n^* \leq \frac{1}{1-H(nq-1)} < \frac{1}{1-H(nq)}.$$

From the law of large numbers we have that $H(nq)$ tends to 0 as n grows large (remembering that $q < p$). Therefore r_n^* is bounded above by a number that is arbitrarily close to 1. Given that $qr_H + (1-q)\, r_L > 1$, there exists n' such that for all $n > n'$ then r_n^* is such that

$$r_n^* \leq qr_H + (1-q)\, r_L,$$

as was to be shown.

Second statement: From the loan described above, the borrower obtains a per-project gross profit

$$\pi_n = \gamma \sum\nolimits_{k=0}^{nq-1} h(k) \left[\frac{k}{n} r_H + \left(1-\frac{k}{n}\right) r_L\right] + \sum\nolimits_{k=nq}^{n} h(k) \left[\frac{k}{n} r_H + \left(1-\frac{k}{n}\right) r_L\right].$$

Fix a small rational number ε and an integer n such that $n(p-\varepsilon)$ and $n(p+\varepsilon)$ are integer numbers. Then, given that $q < p-\varepsilon$, and that all terms in the first and in the second sum are positive, we have that

$$\pi_n \geq \sum\nolimits_{k=n(p-\varepsilon)}^{n(p+\varepsilon)} h(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right].$$

Given that the terms in the second factor in the sum are larger for larger k, the sum is reduced by replacing the summand of a given k by that of $n(p-\varepsilon)$, the smallest term. Then, rearranging, we obtain

$$\pi_n \geq \left[(p-\varepsilon) r_H + \left[1-(p-\varepsilon)\right] r_L\right] \left[H\left(n(p+\varepsilon)\right) - H\left(n(p-\varepsilon)\right)\right].$$

From the law of large numbers, $H\left[n(p+\varepsilon)\right] - H\left[n(p-\varepsilon)\right]$ tends to 1 as n grows. Indeed from Chebyshev's inequality we know that

$$H\left(n(p+\varepsilon)\right) - H\left(n(p-\varepsilon)\right) \geq 1 - \frac{(p+\varepsilon)(1-p)}{n\varepsilon^2} - \frac{(1-p+\varepsilon)p}{n\varepsilon^2} = 1 - \frac{2p(1-p)+\varepsilon}{n\varepsilon^2}$$

and therefore

$$\pi_n \geq \left[p r_H + (1-p) r_L - \varepsilon(r_H - r_L)\right] \left(1 - \frac{2p(1-p)+\varepsilon}{n\varepsilon^2}\right).$$

That is for n large, the gross per-project profit differs from the (gross) present value of each project by an amount that is arbitrarily small, $\varepsilon(r_H - r_L)$. Similarly,

$$\frac{\pi_n}{\pi^*} \geq \left(1 - \frac{\varepsilon(r_H - r_L)}{p r_H + (1-p) r_L}\right) \left(1 - \frac{2p(1-p)+\varepsilon}{n\varepsilon^2}\right)$$

where π^* is equal to first-best gross profits, $\pi^* = p r_H + (1-p) r_L$. Q.E.D.

Proof of Prediction 9: The proof follows directly from Proposition 9. Q.E.D.

Proof of Proposition 10: With separate financing the rate satisfies $p r_i^* + (1-p)\left[r_L - B(r_L)\right] = 1$ and therefore the condition is

$$r_i^* := \frac{1 - (1-p)\left[r_L - B(r_L)\right]}{1-(1-p)} < r_H,$$

and the per-project net present value is $p r_H + (1-p) r_L - (1-p) B(r_L) - 1$. Similarly, the condition for obtaining a rate for joint financing that saves both projects when one has low return is given by

$$r_m^* := \frac{1 - (1-p)^2 \left[r_L - B(2r_L)/2\right]}{1-(1-p)^2} < \frac{r_H + r_L}{2}, \tag{23}$$

and the net present value is $pr_H + (1-p)r_L - (1-p)^2 B(2r_L)/2 - 1$. Finally, the rate for joint financing that saves both projects only when both give high returns is given by

$$r_m^{**} = \frac{1-(1-p)\left(p\left[r_L + r_H - B(r_L + r_H)\right] - (1-p)\left[r_L - B(2r_L)/2\right]\right)}{p^2} < r_H.$$

and the per-project net present value is $pr_H + (1-p)r_L - (1-p)[pB(r_L + r_H) + (1-p)B(2r_L)/2] - 1$.

As in the baseline case, if it is possible to choose, the second rate is better than the third, as the net present value is larger. Separate financing is therefore optimal if the first rate (and therefore also the second) is better than the second, that is if $(1-p)B(2r_L)/2 > B(r_L)$ or equivalently

$$B(2r_L) - 2B(r_L) > \frac{2p}{1-p}B(r_L) \equiv \overline{S}.$$

Similarly, joint financing is optimal if the third rate (and therefore also the second) is better than the first, that is if $pB(r_L + r_H) + (1-p)B(2r_L)/2 < B(r_L)$, or equivalently

$$B(2r_L) - 2B(r_L) < -\frac{2p}{1-p}\left[B(r_L + r_H) - B(r_L)\right] \equiv \underline{S}.$$

Finally, if the first rate is better than the third but worse than the second we have, as in our baseline case, that joint financing is optimal if and only if the second rate can be obtained in joint financing, i.e. if condition (23) is satisfied. Q.E.D.

Proof of Proposition 11: Consider two symmetric groups of $n/2$ normally distributed projects with mean μ and variance σ^2. The average distribution of returns of each group of $n/2$ projects, denoted as $F(r)$, is a normal distribution with mean μ and variance $[1+\rho(n/2-1)]\sigma^2/(n/2)$. The average distribution of the total set of n projects, denoted as $H(r)$, is a normal distribution with mean μ and variance $[1+\rho(n-1)]\sigma^2/n$. The two distributions cross at $r = \mu$ and the second distribution is more peaked around $r = \mu$ than the first. Thus we have

$$F(r) \gtreqless H(r) \Leftrightarrow r \lesseqgtr \mu \tag{24}$$

and as a result, for $r > w$,

$$(r-w)\left[1-F(r)\right] + w \lesseqgtr (r-w)\left[1-H(r)\right] + w \Leftrightarrow r \lesseqgtr \mu, \tag{25}$$

and rearranging

$$r\left[1-F(r)\right]+wF(r)\lesseqgtr r\left[1-H\left(r\right)\right]+wH\left(r\right)\Leftrightarrow r\lesseqgtr\mu. \tag{26}$$

Note first that the equilibrium repayment rates for each of the two groups separately (r_i^*) and jointly (r_n^*) satisfy r_i^*,$r_n^*>w$. Indeed, if $r_n^*<w$, the creditor's profits would be $r_n^*\left[1-H\left(r_n^*\right)\right]+wH\left(r_n^*\right)<w$. Given that by assumption $w<1$, the creditor would not be able to recover the initial investment. Applying the same reasoning, we conclude also that $r_i^*<w$ cannot hold. From now on, we thus restrict to repayment rates r_i^*,$r_n^*>w$.

If r_n^*, the lowest r such that $r\left[1-H\left(r\right)\right]+wH\left(r\right)=1$, is such that $r_n^*<\mu$, then $r_n^*<r_i^*$. Indeed, even though r_i^* exists by assumption, it is not possible that $r_i^*<r_n^*$ because, by (26) and single-peakedness of the profit function, we have that for $r<r_n^*$, $r\left[1-F(r)\right]+wF(r)<r\left[1-H\left(r\right)\right]+wH\left(r\right)<r_n^*\left[1-H\left(r_n^*\right)\right]+wH\left(r_n^*\right)=1$. As a result, from (24) and monotonicity of F, we conclude that the probability of bankruptcy is lower with joint financing, $H\left(r_n^*\right)<F(r_n^*)<F(r_i^*)$.

On the other hand, if r_n^* is such that $r_n^*>\mu$, then $r_n^*>r_i^*$. Indeed, given that the creditor's proceeds at $r=w$ are equal to $w<1$ and they are higher than 1 at $r=r_n^*$, as $r_n^*\left[1-F(r_n^*)\right]+wF\left(r_n^*\right)>r_n^*\left[1-H\left(r_n^*\right)\right]+wH\left(r_n^*\right)=1$, by the intermediate value theorem there exists some $r_i^*<r_n^*$ at which $r_i^*\left[1-F\left(r_i^*\right)\right]+wH\left(r_i^*\right)=1$. As a result, from (24) and monotonicity of H, we have that the probability of bankruptcy is lower with separate financing, $F(r_i^*)<H\left(r_i^*\right)<H(r_n^*)$. Since $F(r)<H(r)$ for $r>\mu$, the net surplus of the borrower is then

$$\int_0^{r_n^*}\left[1-H(x)\right]dx<\int_0^{r_n^*}\left[1-F(x)\right]dx<\int_0^{r_i^*}\left[1-F(x)\right]dx.$$

Therefore financing the two groups separately is optimal.

By single-peakedness, r_n^* is such that $r_n^*>\mu$ if and only if the following two conditions hold

$$r\left[1-H\left(r\right)\right]+wH\left(r\right)\big|_{r=\mu}<1 \text{ and } \left.\frac{\partial}{\partial r}\left(r\left[1-H\left(r\right)\right]+wH\left(r\right)\right)\right|_{r=\mu}>0,$$

which are equivalent to

$$\mu+w<2 \text{ and } (\mu-w)h(\mu)<\frac{1}{2}.$$

We then obtain conditions (10) by substituting for the density h of a normal distribution with mean μ and variance $[1 + \rho(n-1)]\,\sigma^2/n$.

Note that if this condition is satisfied for n then it is better to finance each half of the n available projects separately rather than all the n projects jointly. Now, if this condition is satisfied for n then it is also be satisfied for $n/2$. As a result, it is optimal to finance each half of the $n/2$ projects separately rather than jointly. Iterating this reasoning, we conclude that it is optimal to finance all projects separately, as claimed. Q.E.D.

References

Acharya, Viral V., 2009, A theory of systemic risk and the design of prudential bank regulation, *Journal of Financial Stability* 5, 224–255.

Alderson, Michael J., and Brian L. Betker, 1995, Liquidation costs and capital structure, *Journal of Financial Economics* 39, 45–69.

Altman, Edward, 1984, A further empirical investigation of the bankruptcy cost question, *Journal of Finance* 39, 1067–1089.

Bannier, Christina E., and Dennis Hänsel, 2008, Determinants of European banks' engagement in loan securitization, Discussion Paper Series 2: Banking and Financial Studies No. 10/2008, Deutsche Bundesbank, Frankfurt/Main.

Bolton, Patrick, and David S. Scharfstein, 1990, A theory of predation based on agency problems in financial contracting, *American Economic Review* 80, 93–106.

Bond, Philip, 2004, Bank and nonbank financial intermediation, *Journal of Finance* 59, 2489–2529.

Brealey, Richard A., Stewart C. Myers, and Franklin Allen, 2006, *Principles of Corporate Finance* (McGraw-Hill Irwin, New York).

Bris, Arturo, Ivo Welch, and Ning Zhu, 2006, The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization, *Journal of Finance* 61, 1253–1303.

Campa, Jose Manuel, and Simi Kedia, 2002, Explaining the diversification discount, *Journal of Finance* 57, 1731–1762.

Cantor, Richard, and Rebecca Demsetz, 2003, Securitization, loan sales, and the credit slowdown, *Federal Reserve Bank of New York Quarterly Review*, 18, 27–38.

Custódio, Cláudia, 2009, Mergers and acquisitions accounting can explain the diversification discount, Working Paper, London School of Economics.

Diamond, Douglas W., 1984, Financial intermediation and delegated monitoring, *Review of Economic Studies* 51, 393–414.

Esty, Benjamin C., 2003, The economic motivations for using project finance, Working Paper, Harvard Business School.

Faure-Grimaud, Antoine, and Roman Inderst, 2005, Conglomerate entrenchment under optimal financial contracting, *American Economic Review*, 95, 850–861.

Fenn, George, Nellie Liang, and Stephen Prowse, 1995, The economics of the private equity market, Staff Study, Board of Governors of the Federal Reserve System, Washington, DC.

Gale, Douglas, and Martin Hellwig, 1985, Incentive-compatible debt contracts: The one-period problem, *Review of Economic Studies* 52, 647–663.

Gorton, Gary, 2008, The panic of 2007, Yale ICF Working Paper No. 08–24.

Gorton, Gary, and Nicholas S. Souleles, 2006, Special purpose vehicles and securitization, in Mark Carey and René M. Stulz, eds.: *The Risks of Financial Institutions* (University of Chicago Press, Chicago).

Graham, John R., Michael L. Lemmon, and Jack G. Wolf, 2002, Does corporate diversification destroy value?, *Journal of Finance* 57, 695–720.

Hege, Ulrich, and Lorand Ambrus-Lakatos, 2002, Internal Capital Markets: The Insurance-Contagion Trade-off, SSRN Working Paper.

Higgins, Eric, and Joseph Mason, 2004, What is the value of recourse to asset-backed securities? A clinical study of credit card banks, *Journal of Banking and Finance* 28, 875–899.

Higgins, Robert C, 1971, A pure financial rationale for the conglomerate merger: Discussion, *Journal of Finance* 26, 543–545.

Higgins, Robert C., and Lawrence D. Schall, 1975, Corporate bankruptcy and conglomerate merger, *Journal of Finance* 30, 93–113.

Ibragimov, Rustam, Dwight Jaffee, and Johan Walden, 2011, Diversification disasters, *Journal of Financial Economics* 99, 333–348.

Inderst, Roman, and Holger Müller, 2003, Internal versus external financing: An optimal contracting approach, *Journal of Finance* 58, 1033–1062.

John, Teresa A., 1993, Optimality of Spin-Offs and Allocation of Debt, Journal of Financial and Quantitative Analysis, 28, 139–160.

Kahn, Charles, and Andrew Winton, 2004, Moral hazard and optimal subsidiary structure for financial institutions, *Journal of Finance* 59, 2531–2575.

Kale, Jayant, Thomas Noe, and Gabriel Ramirez, 1991, The evidence of business risk on corporate capital structure: Theory and evidence, *Journal of Finance* 46, 1693–1715.

Kim, E. Han, 1978, A mean-variance theory of optimal capital structure and corporate debt capacity, *Journal of Finance* 33, 45–63.

Kim, E. Han, and John McConnell, 1977, Corporate mergers and the co-insurance of corporate debt. *Journal of Finance* 32, 349–365.

Kleimeier, Stefanie, and William L. Megginson, 2000, Are project finance loans different from other syndicated credits?, *Journal of Applied Corporate Finance* 13, 75–87.

Korteweg, Arthur, 2007, The costs of financial distress across industries, Working Paper, Stanford Graduate School of Business.

Korteweg, Arthur, 2010, The net benefits to leverage, *Journal of Finance* 65, 2137–2170.

Leland, Hayne, 2007, Purely financial synergies and the optimal scope of the firm: Implications for mergers, spin-offs and structured finance, *Journal of Finance* 62, 765–807.

Lewellen, Wilbur G., 1971, A pure financial rationale for the conglomerate merger, *Journal of Finance* 26, 521–537.

Longstaff, Francis, Sanjay Mittal, and Eric Neis, 2005, Corporate yield spreads: Default risk or liquidity? New evidence from the credit-default swap market, *Journal of Finance* 60, 2213–2253.

Maksimovic, Vojislav, and Gordon Phillips, 2001, The market for corporate assets: Who engages in mergers and asset sales and are there efficiency gains?, *Journal of Finance* 56, 2019–2065.

Maksimovic, Vojislav, and Gordon Phillips, 2002, Do conglomerate firms allocate resources inefficiently across industries? Theory and evidence, *Journal of Finance* 57, 721–767.

Mills, Lillian F., and Kaye J. Newberry, 2005, Firms' off-balance sheet and hybrid debt financing: Evidence from their book-tax reporting differences, *Journal of Accounting Research* 43, 251–282.

Nicodano, Giovanna, and Elisa Luciano, 2009, Leverage and value creation in holding-subsidiary structures, Collegio Carlo Alberto Working Paper No. 95.

Parrino, Robert, Allen Poteshman, and Michael Weisbach, 2005, Measuring investment distortions when risk-averse managers decide whether to undertake risky projects, *Financial Management* 34, 21–60.

Proschan, Frank, 1965, Peakedness of distributions of convex combinations, *Annals of Mathematical Statistics* 36, 1703–1706.

Rossi, Stefano, and Paolo F. Volpin, 2004, Cross-country determinants of mergers and acquisitions, *Journal of Financial Economics* 74, 277–304.

Rothschild, Michael, and Joseph Stiglitz, 1970, Increasing risk: I. A definition, *Journal of Economic Theory* 2, 225–243.

Sahlman, William, 1990, The structure and governance of venture-capital organizations, *Journal of Financial Economics* 27, 473–521.

Sarig, Oded, 1985, On mergers, divestments, and options: A note, *Journal of Financial and Quantitative Analysis* 20, 385–389.

Schoar, Antoinette, 2002, Effects of corporate diversification on productivity, *Journal of Finance* 57, 2379–2403.

Scott, James H. Jr., 1977, On the theory of conglomerate mergers, *Journal of Finance* 32, 1235–1250.

Shaffer, Sherrill, 1994, Pooling intensifies joint failure risk, *Research in Financial Services* 6, 249–280.

Smith, Clifford W. Jr., and Jerold B. Warner, 1979, Bankruptcy, secured debt, and optimal capital structure: comment, *Journal of Finance* 34, 247–251.

Subramanian, Krishnamurthy, Frederick Tung, and Xue Wang, 2009, Law and project finance, Working Paper, Emory University, Goizueta Business School.

Townsend, Robert M., 1979, Optimal contracts and competitive markets with costly state verification, *Journal of Economic Theory* 21, 265–293.

Villalonga, Belén, 2004, Diversification discount or premium? New evidence from the business information Tracking Series, *Journal of Finance* 59, 479–506.

Warner, Jerold B., 1977, Bankruptcy costs: Some evidence, *Journal of Finance* 32, 337–347.

Weiss, Lawrence A., 1990, Bankruptcy resolution: Direct costs and violation of priority of claims, *Journal of Financial Economics* 27, 285–314.

Winton, Andrew, 1999, Don't put all your eggs in one basket? Diversification and specialization in lending, Working Paper, University of Minnesota.

Supplementary Appendix: Debt, Equity, and Taxes

In this supplementary appendix we extend the model to allow the firm to use equity, as well as debt, to finance part of the initial investment. As in the standard tradeoff theory of capital structure, equity payments are subject to corporate taxation, whereas debt payments are tax deductible and are therefore exempt from taxes. Our framework is isomorphic to other frictional costs linked to equity financing, such as higher underwriting fees, negative signaling costs, or agency costs of excess equity.

Model Extension. Financing for each corporation can be obtained in competitive credit and equity markets. As in the basic model, the availability of a competitive credit market is equivalent to assuming that each corporation makes a take-it-or-leave-it offer to a single creditor. Corporation j, consisting of n_j projects, promises to repay $n_j r'_j$ at $t = 2$ in exchange of $n_j D_j$ at $t = 1$. Thus, the promised per-project repayment r'_j now depends on the part of the initial investment outlay of each project that is financed through debt, $D_j \leq 1$.

A competitive equity market is equivalent to assuming that each corporation makes a take-it-or-leave-it offer to a single outside equity investor. We denote the fraction of the equity sold by corporation j as α_j, and the equity value of the corporation, if it consists of n_j projects, as $n_j E_j$. For all the projects to be financed, the sum of debt and equity financing per-project must cover the initial investment outlay of each project, $D_j + \alpha_j E_j = 1$. We also assume that, while debt payments are tax deductible and therefore exempt from taxes, equity payments are subject to a corporate tax of τ, which captures the tax disadvantage (or other net costs) of equity relative to debt.[25]

[25] Leland (2007) makes the more realistic assumption that only interest expenses are tax deductible. This, however, creates an endogeneity problem. When interest only is deductible, the fraction of debt service attributed to interest payments depends on the value of the debt, which in turn depends on the fraction of debt service attributed to interest payments. Instead of relying in numerical techniques to find debt values and optimal leverage, we follow Kale, Noe, and Ramirez (1991) and assume that both interest and principle are tax deductible. We also assume away personal taxes.

Financing Conditions. For the case of separate financing, we now need to distinguish two cases, because there are situations in which it is possible to obtain a rate r_i' that avoids bankruptcy altogether, $r_i' \leq r_L$, by selling a fraction α of the corporation. If this rate exists, it should satisfy

$$\alpha(1-\tau)\left[p\left(r_H - r_i'\right) + (1-p)\left(r_L - r_i'\right)\right] = \alpha E_i \quad \text{and} \quad r_i' = D_i.$$

Since there is no bankruptcy, the net interest rate is zero and the principal is equal to the debt value. Substituting into the total financing condition, $D_i + \alpha E_i = 1$, this rate can be obtained if and only if

$$r_i'(\alpha) := \frac{1-\alpha(1-\tau)\left[pr_H + (1-p)r_L\right]}{1-\alpha(1-\tau)} \leq r_L. \tag{27}$$

If the firm uses no equity ($\alpha = 0$), then $r_i' = 1$ and the condition is never satisfied ($r_L < 1$), as in the baseline debt-only case. But, as more equity is offered, the debt repayment is lower ($r_i'(\alpha)$ is decreasing) and, if taxes are low, the condition can be satisfied. Equity, however, is costly because of taxes. It is optimal for the firm to sell the lowest equity stake α_i' satisfying condition (27), $r_i'(\alpha_i') = r_L$. Still, if taxes are high enough, it is not possible to obtain this rate, not even by selling all the equity.

Following the same procedure, a rate such that $r_i'' < r_H$ can be obtained if and only if

$$r_i''(\alpha) := \frac{1-\alpha(1-\tau)pr_H - (1-p)\gamma r_L}{\left[1-\alpha(1-\tau)\right]p} \leq r_H, \tag{28}$$

which generalizes condition (1) of the baseline setup to $\alpha > 0$, as $r_i''(0) = r_i^* \leq r_H$. This condition is satisfied precisely as long as condition (1) is satisfied, independently of the level of equity sold. Given that the firm prefers to sell the lowest possible fraction of equity, no equity at all is sold in the optimum, $\alpha_i'' = 0$. In this case, equity does not help in reducing the probability of bankruptcy.

The following proposition characterizes which of these two rates is optimally chosen when they are both available.

Proposition 12 (Equity and taxes: separate financing) *Suppose that both rates r_i' and r_i'' are available. There exists $\overline{\tau}_i$ such that the optimal rate and fraction of equity sold are, respectively, r_L and $\alpha_i' > 0$ if $\tau \leq \overline{\tau}_i$, and r_i^* and $\alpha = 0$ if $\tau > \overline{\tau}_i$.*

If taxes are sufficiently high, the projects are financed at the same rate as in the baseline case without equity. Moreover, it is then optimal to finance the projects entirely with debt. When taxes are lower, however, it becomes optimal to finance the projects at a rate that avoids bankruptcy altogether ($r_i'(\alpha_i') = r_L$) by selling a positive amount of equity, $\alpha_i' > 0$.

For the case of joint financing, there are three potential rates. The first rate, which avoids bankruptcy altogether, $r_m' \leq r_L$, is the same as (and can be obtained under the same circumstances as) the rate resulting with separate financing, $r_m' = r_i'$. Indeed, if bankruptcy can be avoided, then the corporate structure does not matter.

Second, a rate that avoids bankruptcy if one realized return is high and the other is low can be obtained if and only if

$$r_m''(\alpha) := \frac{1 - \alpha(1-\tau)\left[p^2 r_H + 2p(1-p)\frac{r_H + r_L}{2}\right] - (1-p)^2 \gamma r_L}{\left[1 - \alpha(1-\tau)\right]\left[1 - (1-p)^2\right]} \leq \frac{r_H + r_L}{2}, \tag{29}$$

which is again a generalization for $\alpha \geq 0$ of condition (2) of the baseline case, $r_m''(0) = r_m^* \leq (r_H + r_L)/2$. Again, it is optimal for the firm to choose the minimum amount of equity that satisfies condition (29). If condition (2) is satisfied, the firm does not need to sell any equity at all, $\alpha_m'' = 0$. If condition (2) is not satisfied, this rate can still be obtained, however, by selling some equity.

Third, a rate that avoids bankruptcy only if both realized returns are high can be obtained as long as

$$r_m'''(\alpha) := \frac{1 - (1-p)^2 \gamma r_L - 2p(1-p)\gamma \frac{r_H + r_L}{2} - \alpha(1-\tau)p^2 r_H}{\left[1 - \alpha(1-\tau)\right] p^2} \leq r_H, \tag{30}$$

which again, generalizes the condition of the baseline case for $\alpha \geq 0$, i.e. $r_m'''(0) = r_m^{**} \leq r_H$. As in the highest rate for separate financing, this condition is satisfied precisely as long as condition (3) is satisfied, independently of the equity sold. Given that the firm prefers to sell the lowest possible fraction of equity, the resulting level is $\alpha_m''' = 0$.

Proposition 13 (Equity and taxes: joint financing) *Suppose more than one rate (r_m', r_m'', r_m''') is available. There exist $\overline{\tau}_m^a$, $\overline{\tau}_m^b$, and $\overline{\tau}_m^c$ such that:*

(i) If condition (2) is satisfied, the optimal rate and fraction of equity sold are, respectively, r_L *and* $\alpha'_m > 0$ *if* $\tau \leq \overline{\tau}^a_m$*, and* r^*_m *and* $\alpha = 0$ *if* $\tau > \overline{\tau}^a_m$.

(ii) If condition (2) is not satisfied, the optimal rate and fraction of equity sold are, respectively, r_L *and* $\alpha'_m > 0$ *if* $\tau \leq \overline{\tau}^b_m$, $\frac{r_H+r_L}{2}$ *and* $\alpha''_m > 0$ *if* $\overline{\tau}^b_m < \tau \leq \overline{\tau}^c_m$*, and* r^{**}_m *and* $\alpha = 0$ *if* $\tau > \overline{\tau}^c_m$.

Good and Bad Conglomeration. The profitability of joint financing depends on the cases identified in Proposition 13. In case (i), joint financing is always profitable at least weakly. This case is equivalent to the case of good conglomeration in the baseline model. The condition is exactly the same as the condition enabling the firm to obtain r^*_m in Section 3. In case (ii), conglomeration is bad in the baseline model. And, if taxes are sufficiently high, conglomeration is still bad here. If taxes are lower, however, financing with equity allows the firm to finance the projects with rates that avoid bankruptcy in the case with intermediate returns and even with rates that avoid bankruptcy altogether.

Proposition 14 (Equity and taxes: joint v. separate financing) *When both separate and joint financing are feasible:*

(i) If condition (2) is satisfied, both financing regimes are equally profitable if $\tau \leq \overline{\tau}^a_m$*, whereas joint financing dominates if* $\tau > \overline{\tau}^a_m$.

(ii) If condition (2) is not satisfied, both financing regimes are equally profitable if $\tau \leq \overline{\tau}^b_m$*, joint financing dominates if* $\overline{\tau}^b_m < \tau \leq \overline{\tau}_i$*, and separate financing dominates if* $\tau > \overline{\tau}_i$.

In sum, if taxes or other equity costs are sufficiently high, only debt is used and the same situation analyzed in the baseline model arises. That is, joint financing is profitable in case (i) and separate financing is profitable in case (ii). The condition setting apart joint and separate financing is exactly the same as in the baseline model without equity. If taxes are intermediate, joint financing can be profitable in cases in which it is not profitable in the baseline model with only debt (case ii). This is because, by financing jointly and using equity, it becomes possible to obtain a rate that avoids intermediate bankruptcy or bankruptcy altogether. Finally, if taxes are sufficiently low, joint financing is inconsequential because bankruptcy can be avoided altogether with joint as well as with separate financing.

The exclusive use of debt in separate finance is consistent with the many empirical studies that find that a disproportionate proportion of funding in project finance is in the form of debt. Kleimeier and Megginson (2000), for example, find that projects funded with project finance loans have an average loan-to-project value ratio of 67%. Esty (2003) shows that the average (respectively median) project company has a book value debt-to-total capitalization ratio of 70% (respectively 70%) compared to 33.1% (respectively 30.5%) for similar-sized firms. Our result is also consistent with the almost exclusive use of debt financing in securitization structures, where little if any external equity is issued.

Proofs for Supplementary Appendix. *Proof of Proposition 12:* We proceed by computing the payoff obtained when using each of the two rates and then we compare the payoffs. If the firm uses $r_i'(\alpha)$ (specified in (27)), the firm obtains, substituting into $(1-\alpha)E_i$,

$$\frac{(1-\alpha)(1-\tau)}{\tau+(1-\alpha)(1-\tau)}\left[pr_H+(1-p)r_L-1\right]. \tag{31}$$

This payoff is decreasing in α, as the firm obtains a fraction of the net present value that corresponds to the (after-tax) equity holding; the remaining part is retained by the government through taxes. Therefore the firm should use the smallest level of equity possible. But, as explained in the text, the firm should use a positive level of equity to satisfy condition (27). Optimally, we have

$$\alpha_i' := \frac{(1-r_L)}{(1-\tau)\left[pr_H+(1-p)r_L-1+(1-r_L)\right]}.$$

Provided that $\alpha_i' \leq 1$ ($r_i'(\alpha)$ can be obtained), the firm obtains, substituting into (31),

$$\left[pr_H+(1-p)r_L-1\right]-\tau p\left(r_H-r_L\right). \tag{32}$$

As argued in the text, if the firm uses $r_i''(\alpha)$ (specified in (28)), the optimal amount of equity is

$\alpha_i'' = 0$. The borrower then obtains

$$(1-\tau)\left[pr_H + (1-p)\gamma r_L - 1\right]. \tag{33}$$

Comparing the payoffs in each case, (32) and (33), it is optimal for the firm to choose the first over the second rate if and only

$$\tau < \overline{\tau}_i := 1 - \frac{(1-r_L)}{(1-\gamma)\,(1-p)r_L + (1-r_L)},$$

that is if bankruptcy costs $(1-\gamma)$ are high enough and/or taxes are small. Q.E.D.

Proof of Proposition 13: Following the same procedure as in the proof of Proposition 12, we first compute the per-project payoff of the firm when using each of the three types of rate and then we compare these payoffs. If the projects are financed at a rate that avoids bankruptcy altogether, $r_m'(\alpha)$, which is equal to $r_i'(\alpha)$ (as specified in (27)), the firm obtains the same payoff as in the case of separate financing, equal to (32). As in the case of separate financing, the firm needs to use a positive level of equity to obtain this rate, and therefore uses the minimum amount $\alpha_m' > 0$ such that $r_m'(\alpha_m') = r_L$.

If projects are financed at a rate $r_m''(\alpha)$ (as specified in (29)), the firm obtains

$$(1-\tau)\left[p^2 r_H + 2p(1-p)\left(\frac{r_H + r_L}{2}\right) + (1-p)^2\gamma r_L - 1\right] \tag{34}$$

if condition (2) is satisfied and

$$(1-\tau)\left(p^2 r_H - p^2\frac{r_H + r_L}{2}\right) + \left[1-(1-p)^2\right]\frac{r_H + r_L}{2} + (1-p)^2\gamma r_L - 1 \tag{35}$$

if condition (2) is not satisfied. If condition (2) is satisfied, the firm does not need to use any equity to obtain r_m'', and therefore $\alpha_m'' = 0$ and $r_m''(0) = r_m^*$. If condition (2) is not satisfied, the firm needs to use a positive level of equity to obtain r_m'', and therefore uses the minimum amount $\alpha_m'' > 0$ such that $r_m''(\alpha_m'') = \frac{r_H + r_L}{2}$.

Finally, if projects are financed at a rate $r'''_m(\alpha)$ (specified in (30)), the firm obtains

$$(1-\tau)\left[p^2 r_H + 2p(1-p)\gamma\frac{r_H + r_L}{2} + (1-p)^2\gamma r_L - 1\right], \tag{36}$$

and no equity is used, $\alpha'''_m = 0$, as it does not help to reduce the probability of bankruptcy.

We now compare the payoffs in each case. Suppose first that condition (2) is satisfied (part (i) in the statement of the proposition). Then, the payoff when using r''_m is given by (34). It can be easily checked that this is always greater than the payoff that can be obtained when using r'''_m, (36). Comparing the payoffs when using r'_m with those of using r''_m, r'_m is optimal if and only if

$$\tau < \overline{\tau}^a_m := 1 - \frac{(1-r_L)}{(1-r_L) + (1-p)^2(1-\gamma)r_L}.$$

Suppose second that the condition (2) is not satisfied (part (ii) in the statement of the proposition). Then, the payoff when using r''_m is given by (35). In this case, r'_m is preferred to r''_m as long as

$$\tau < \overline{\tau}^b_m := 1 - \frac{\left[1-(1-p)^2\right]\frac{r_H+r_L}{2} + (1-p)^2\gamma r_L - r_L}{(1-\frac{p}{2})p\left[r_H - r_L\right]},$$

r''_m is preferred to r'''_m as long as

$$\tau < \overline{\tau}^c_m := 1 - \frac{1 - \left[1-(1-p)^2\right]\frac{r_H+r_L}{2} - (1-p)^2\gamma r_L}{1 - p^2\frac{r_H+r_L}{2} - 2p(1-p)\gamma\frac{r_H+r_L}{2} - (1-p)^2\gamma r_L},$$

and r'_m is preferred to r'''_m as long as

$$\tau < \overline{\tau}^d_m := 1 - \frac{(1-r_L)}{(1-\gamma)\left[2p(1-p)\frac{r_H+r_L}{2} + (1-p)^2 r_L\right] + (1-r_L)}.$$

It can be easily shown that the order of these cutoffs is given by $\overline{\tau}^b_m < \overline{\tau}^d_m < \overline{\tau}^c_m$. Therefore, we have the optimal choices claimed in the text. Q.E.D.

Proof of Proposition 14: In this proof, we need to compare the payoffs of joint and separate financing. Suppose first that condition (2) is satisfied (part (i) in the statement of the proposition). If r'_m is used for joint financing ($\tau < \overline{\tau}^a_m$), then the payoff with joint and separate financing are the same. If r''_m is used ($\tau > \overline{\tau}^a_m$), then the payoff with joint financing is larger than the payoff of separate financing.

Suppose now that condition (2) is not satisfied (part (ii) in the statement of the proposition). Comparing the cutoffs for joint and separate financing, it is easy to show that $\overline{\tau}_m^b < \overline{\tau}_i < \overline{\tau}_m^c$. Then, we can compare the payoffs under joint and separate financing. First, when r_m' is optimal with joint financing ($\tau < \overline{\tau}_m^b$), the payoffs under joint and separate financing are the same. When r_m'' is optimal with joint financing ($\overline{\tau}_m^b < \tau < \overline{\tau}_m^c$), it is straightforward to check that the payoffs are higher under joint financing if r_i' is obtained with separate financing, i.e. $\tau < \overline{\tau}_i$, but are lower if r_i'' is obtained with separate financing, i.e. $\tau > \overline{\tau}_i$. Finally, when r_m''' is optimal with joint financing ($\tau > \overline{\tau}_m^c$), joint financing yields a lower payoff than separate financing (under separate financing r_i'' would be optimal because $\overline{\tau}_i < \overline{\tau}_m^c$). As in the baseline model, the bankruptcy costs are higher under joint financing because of the risk-contamination effect. Q.E.D.