Grantmaking

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Abstract

The paper develops a foundational model of the decentralized allocation of subsidies through competitive grantmaking. Casting the problem in a simple supply and demand framework, we characterize the level of applications and acceptance standard that result in equilibrium. The equilibrium success rate (grants over applications) decreases in the budget, consistent with some recent evidence, if and only if the distribution of types has decreasing hazard rate. In all stable equilibria resulting when funds are allocated across fields proportionally to applications—as well as under apportionment rules in a general class characterized in the paper—an increase in noise in the evaluation in a field perversely raises applications in that field and reduces applications in all the other fields. We characterize how the design of allocation rules can be modified to improve welfare.

Keywords: Grants, applications, grading on a curve, evaluation across fields, formula-based allocation, proportional allocation, payline, unraveling, signal noise.

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1 Introduction

To shore the losses to private businesses induced by the economic contraction generated by the Covid-19 pandemic, governments around the world have set up a number of grant-in-aid and targeted subsidy programs. Applicants interested in obtaining assistance must come forward and present evidence, which is then reviewed by program officers. More generally, governments and philanthropies commonly subsidize meritorious activities through grant programs. Applications are first solicited. Specialized review panels are then asked to evaluate and rank applications, with the aim of selecting the most worthy projects.

Over the sweep of history, artists and scientists have long relied on wealthy patrons and public support to finance their inventions and discoveries. In 1610 Galileo Galileo wrote to his former pupil Cosimo de' Medici, the Grand Duke of Tuscany, subtly asking for financial support to explore the sky with his new powerful telescope. To lure the patron, Galileo named Jupiter's moons he had just discovered the Medician stars and promised "many discoveries and such as perhaps no other prince can match." Cosimo was duly impressed and granted Galileo a full teaching buyout at the University of Pisa.[1]

A more structured process for funding talented scholars emerged in embryonic form in the first half of the nineteenth century, when science academies in France and England started offering *encouragements* and grants to support worthy projects by their members.[2] To ensure the best use of funds, learnt societies and academies began formalizing the application cycle and the review process for the selection of grant recipients. Similar selection procedures had been in place for centuries at University Colleges for assigning scholarships to promising students from families with limited means.[3]

With its roots steeped in patronage, grantmaking evolved in the modern era to become an effective method for identifying prospects worthy of funding support. As Carnegie, Rockefeller, and Russell Sage and other industrial tycoons turned philanthropists at the beginning of the twentieth century, the private foundations they endowed to "promote the wellbeing of humankind" were inundated by requests for donations. Building on their business experience, trustees of these large foundations perfected grantmaking as a systematic approach to "wholesale" giving. Modern philanthropic foundations select applications most worthy of funding with the assistance of specialized evaluation panels and delegate

[1] The quote from Galileo correspondence is reported in Westfall (1985, p. 22). For more on Galileo's patronage see also Biagioli (1990) and references therein.

[2] Thanks to the legacy of a substantial endowment of funds by Baron Montyon, around 1831–1850 the French Académie Royale des Sciences transformed research prizes for distinguished members (to recognize important discoveries made recently) into *encouragements* (to provide scientists resources for promising projects); see Crosland and Gálvez (1989). MacLeod (1971) reports parallel developments in England, such as the grant competition set up by the British Association for the Advancement of Science from 1833 and the grant scheme administered by the Royal Society with government funds from 1849.

[3] See Rushdall (1895, p. 200–204) on the examination procedures introduced to select applicants for admission to the College of Spain founded in 1367 (and still active today) at the University of Bologna from a bequest by Gil de Albornoz.

to grantees the "retail" implementation of the charitable work.[4]

As World War II drew to a close, John Maynard Keynes (1945) stewarded the creation of the Arts Council of Britain by the UK government: "A semi-independent body is provided with modest funds to stimulate, comfort and support any societies or bodies brought together on private or local initiative which are striving with serious purpose and a reasonable prospect of success to present for public enjoyment the arts of drama, music and painting."[5] Around the same time in the US, Vannevar Bush (1945), leveraging his experience as director of the war-time Office of Scientific Research and Development, forcefully argued in favor of federal support of the best curiosity-driven "basic research in the colleges, universities, and research institutes" for a wide range of sciences. The NIH in 1946 greatly expanded its extramural grants program to cover basically all areas of biomedical research, while the National Science Foundation (NSF) began operation in 1950 covering fundamental research across all scientific disciplines.

With the exponential growth in post-war government financing of science, funding organizations finessed the procedures for soliciting and evaluating grant applications. Over the last decades, the fraction of funded applications, also known as success rate, has been declining at some of the world's largest research funding organizations, prompting outcry about the escalation of resources wasted in the application process. Applicants always clamor for more funds and, especially when they do not succeed in obtaining funding or admission, complain bitterly about the cost incurred preparing the application and the low rate of success. Expert evaluators naturally favor applicants in their own fields, which they are able to evaluate, making the allocation of grants across different fields, and especially across disciplines, particularly delicate. How does grantmaking work? How well does it perform? How can we improve its design? Our foundational model of decentralized financing through grantmaking is designed to address these questions.

Grantmaking in a Single Field with Fixed Budget. To warm up, Section 2 sets the stage by analyzing the baseline specification with a single field populated by a continuum of candidates parametrized by their merit type. Submitting an application is costly, but allows the applicant to obtain a private benefit if the application is successful. The evaluator appraises the merit of each application received on the basis of a noisy signal—allowing for imperfect information is essential to justify the fact that many applicants do not succeed. Given the limited budget available for distribution in the field, grants are supplied to the applications that receive sufficiently favorable evaluation. The evaluation on the supply side, in turn, induces candidates to apply only when they perceive a chance of success sufficiently

[4] See Zunz (2012) and Leat (2016).

[5] Keynes built on his experience as chair of the war-time Council for the Encouragement of Music and the Arts. According to correspondence reported by Upchurch (2016), Keynes was partly inspired by the University Grants Committee, set up by the UK government in 1919 to finance research at British universities. The grantmaking model for supporting the arts has since been adopted by governments throughout the world, both at the local and national level. The National Endowment for the Arts was chartered by the US Congress in 1965.

high to compensate for the application cost. Because higher merit applicants receive more favorable evaluations, on the demand side candidates with a merit type above a threshold self select into applying.

By representing incentives in terms of supply and demand, even though the model does not involve prices, we can build on classic analytical tools, derive comparative statics, and explain the results in an intuitive way. For example, an anticipated increase in the budget shifts supply, as it becomes easier to obtain a grant, thus resulting in the movement along the demand to a higher level of equilibrium applications. As we show, an increase in the budget results in a reduction (increase) in the success rate—the widely reported fraction of successful applications—if the merit type distribution has decreasing (increasing) hazard rate, i.e. with a top tail thicker (thinner) than exponential. In that case, as the budget increases, the gap between the average merit of the inframarginal applicants and the merit of the marginal applicant is reduced (increased). The average winning probability must then go down toward (up away from) the marginal winning probability, which makes the marginal applicant indifferent. When the budget of the NIH was increased in 2009 as part of Obama's Stimulus Package to buffer the great recession, applications increased so much that the fraction of successful applicants ended up being reduced, in spite of the higher budget.[6] This observation is consistent with a type distribution with decreasing hazard rate and a thick top tail, which is natural for the talent of scientists and artists.[7]

Budget Apportionment Across Fields. While peer review by field experts makes the evaluation of projects relatively easy within any given field, the allocation of budget across fields is particularly thorny. Universities, and more generally knowledge organizations, face similar problems when deciding how to allocate resources and positions across departments. When raw scores are used, specialized panels in each field have an incentive to inflate scores to attract more resources to their field. To counteract the resulting grade inflation across panels, from 1988 the National Institutes of Health (NIH) started normalizing scores across evaluation panels.[8] NIH institutes introduced the payline system, whereby in each study section grants are assigned to projects that obtain percentiled scores above the so-called "payline", which is equalized across study sections. The payline system is equivalent to the proportional funding rule adopted by a number of the world's largest research grant organizations, including the Canadian Institutes of Health Research and the European Research Council (ERC).

Proportional funding allocates to each field a budget proportional to the applications received in the field relatively to the applications received in all fields.[9] Taking B to be the total budget assigned to all

[6] See Stephan (2012, p. 145).

[7] For example see Caves (2000).

[8] See Mandel's (1996, p. 182–188) historical account. For an indication of magnitude of the NIH budget, before the onset of the COVID-19 pandemic for fiscal year 2020 the U.S. Congress appropriated $41.68 billion to the NIH. The NIH allocated more than 80% of this funding to some 50,000 extramural research grants reaching more than 300,000 researchers at more than 2,500 research institutions.

[9] As explained by the European Commission (2007 p. 11) in the 2008 Work Programme for the second year of operation of the ERC, proportional allocation of budget across fields works as follows: ". . . an indicative budget will be allocated

fields $i = 1, 2, \ldots, N$, if applications received in the different fields are $a_1, a_2, \ldots, a_N$, the budget allocated to field i is then

$$B_i = \frac{a_i}{\sum_{j=1}^{N} a_j} B. \tag{PA}$$

This proportional allocation formula implies that the *success rate* in field i, defined as the fraction of successful applications in field i

$$p_i = \frac{B_i}{a_i} = \frac{B}{\sum_{j=1}^{N} a_j}, \tag{1}$$

is automatically equalized across all fields, $p_i = p$. In the context of research funding, grant applications in each field are assigned to a different panel (equivalent to an NIH study section) of evaluators with expertise in the field. Expert evaluators in each panel are then asked to select the most fund-worthy applications so as to exhaust $100 \times p$ per cent of the budget requested by the applications in the field.

The proportional rule PA by construction allocates a larger fraction of the overall budget to a field that attracts more applications. By automatically equalizing the fraction of successful projects over applications across different fields, proportional allocation appears to be fair in treating all fields in the same way. Proportional allocation also eliminates administrative discretion and political meddling in funding allocation, given that the budget allocation is determined automatically only on the basis of relative demand from applications across fields. As another important virtue, the proportional allocation scheme has the merit of flexibly responding to demand-side signals. In spite of its simplicity, we argue that proportional funding—as well as a general class of symmetric sub-proportional allocation rules we characterize—has important pitfalls when fields are heterogeneous, as they typically are.

Constant-Payline Equilibrium: Grading on a Curve. Section 3 develops the keystone of our construction, the characterization of the constant-payline equilibrium. Suppose now that grants are awarded to a constant fraction p of applicants, so that the budget of grants pa is proportional to applications. This case captures directly grading on a curve for a course that grants a given fraction of distinction grades or honors to enrolled students. Relative grading also operates under regulations such as Texas' Top 10% Plan, guaranteeing automatic admission to state-funded universities for all students who graduate in the top decile of their high school senior class.[10] The constant-payline solution can also be seen as the partial equilibrium resulting under PA for a small field. Like a small country in international trade theory takes the terms of trade as given, a small field faces a constant payline as applications increase, when this increase corresponds an infinitesimal fraction of applications relative to the applications in all fields.

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to each panel, in proportion to the budgetary demand of its assigned proposals. This indicative budget is calculated as the cumulative grant request of all proposals to the panel divided by the cumulative grant request of all proposals to the domain of the call, multiplied by the total indicative budget of the domain." The total budget allocated to the ERC for the period 2014-2020 is 13.1 billion euros.

[10]See Cullen, Long, and Reback (2013) for an empirical analysis.

The constant-payline supply curve is always downward sloping—not upward sloping like the fixed-budget supply. As applications increase the average merit types of applicants decrease, and thus a lower fraction of applicants would clear any given acceptance standard. To make sure that a constant fraction of applicants succeeds and that the payline is constant, the acceptance standard must be reduced. With both supply and demand sloping down, equilibrium multiplicity is a first critical drawback of grading on a curve. The constant-payline equilibrium is unique and stable if the constant-payline supply is always flatter than demand; we give a necessary and sufficient condition for this. Increasing hazard rate of the type distribution is sufficient for equilibrium uniqueness and stability. Multiple equilibria arise only when the type distribution has decreasing hazard rate. In that case, as applications increase the marginal type that is added to the applicants' pool gets closer to the average type of the inframarginal applicants. To keep the success rate constant at the original level, the acceptance standard must then be reduced by more than it is needed to encourage the additional demand.

Paradox of Relative Evaluation. As the evaluation signal in a field becomes noisier, applications in that field in all stable equilibria unambiguously increase—and increase more than under fixed budget. Consider the limit case in which the grantmaker can perfectly evaluate applicants merit types without noise. When information is perfect, candidates with types below the acceptance standard are sure they will not succeed and thus will want to save the application cost. However, only a fraction $p < 1$ of applicants must win, according to the constant payline. Given that only the top $100 \times p$ per cent of the applications succeed, candidates not in the top $100 \times p$ per cent of the applicants pool do not succeed and so are better off holding off their application. Iterating the logic, no candidate applies when there is no noise in the evaluation. When the evaluator signal is perfectly informative, the equilibrium always unravels: zero applications is the only outcome compatible with equilibrium. Reversing the logic leading to market breakdown in Akerlof (1970), here good types, when they are perceived as such, make competition for scarce grants tougher and thus drive out bad types. But as applications decrease the pool of grants is proportionally reduced, so that top types dig their own grave. This is the paradox of relative evaluation.

More subtly, we show that unraveling holds much more generally also if the grantmaker signal is sufficiently informative provided that the hazard rate of the type distribution is bounded, even though the hazard rate is increasing (e.g., with logistic types). When the type distribution has decreasing hazard rate (featuring a top tail thicker than exponential, as in the Pareto or Weibull distribution with shape parameter $k < 1$) there is a stable equilibrium with unraveling for *any* level of noise—and the unraveling equilibrium is *unique* when the average type is sufficiently high.

Partial Equilibrium with Responsive Payline. Section 4 proceeds to analyze the partial equilibrium in a field, once we take into account the reduction in the payline (1) as applications increase. We show that whenever the payline is reduced with an increase in applications, well beyond the PA rule, the downward adjustment in payline makes the supply curve less negatively sloped than the constant-payline supply, thus preserving uniqueness and comparative statics. The unraveling results are also preserved provided that the success rate at $a = 0$ is less than one.

Full Equilibrium Across Fields. Equipped with these building blocks, Section 5 turns to grantmaking across fields where applicants in each field are possibly characterized by different parameters: application cost and grant benefit, type and signal distributions, and noise in the evaluator signal. The equilibrium acceptance standards and applications are characterized for a general sub-proportional budget allocation rule that encompasses PA and fixed budget apportionment as special cases. The full equilibrium takes into account the supply-side interdependence through the budget allocation rule. We show that in all stable equilibria as evaluation in a field becomes more precise applications decrease in that field and increase in all the other fields. Thus, sub-proportional allocation is biased against more consensual fields.

Optimal Design of Funding Rules. The comparison of full equilibrium allocation with the optimal allocation for the grantmaker (or for a social planner maximizing the total surplus of grantmaker and candidates) is subtle. We show that in the optimal allocation applications sometimes decrease and sometimes increase in a field noise—but applications in a field always increase in the noise in other fields, contrary to what happens in the equilibrium induced by a symmetric sub-proportional allocation rule (which is not adjusted for the change in the noise parameter). With fields with symmetric parameters and starting from a symmetric allocation, we show that full equilibrium applications in any field decreases excessively in noise dispersion compared to the socially optimal allocation. Finally, we analyze how evaluator (or social) welfare can be improved by tweaking the allocation rule based on field parameters. In particular, it is optimal to increase proportionality in fields where evaluation is noisier.

Our analysis is generally relevant for a wide variety of allocation schemes that contain elements of proportionality. For examples, admission boards at universities might be tempted to equalize admission rates across different majors or degree programs. Similarly, editorial boards at academic journals exert pressure to equalize the success rate across editors who deal with different subfields. The analysis stresses the danger of giving in to the temptation to equalize success rates across heterogeneous fields.

1.1 Contribution to Literature

Economists have given short shrift to grantmaking, but there is some work on budget allocation across fields. In a pioneering application of marginal analysis, Peirce (1867) sketches the normative theory of resource allocation across research fields for a planner. As stressed at least since Arrow (1962), market forces tend to underprovide research, mostly because invention is non-rival. Governments, however, have limited information about the benefits of research in different fields. Weisbrod (1963) offers an early attempt to quantify the social benefits of medical research across diseases.[11] Weinstein and Zeckhauser (1973) link the problem of the optimal allocation of budget to fields to the decision theoretic approach underlying hypothesis testing.

At a positive level, the description of the actual process for determining NIH funding by the federal government in the early days inspires Wildavsky's (1964) formulation of the incremental nature of budget apportionment; our static model abstracts from dynamic considerations.[12] Zuckerman and Merton (1971) notice that acceptance rates at leading scholarly journals vary across academic disciplines, with higher rejection rates in social sciences and humanities compared to physical sciences; our analysis shows that the performance of allocation rules with proportional elements is particularly problematic when fields are heterogeneous.[13] Rejection rates also vary along similar lines across directorates at the National Science Foundation.[14]

Lazear (1997) outlines a lottery model of research funding (researchers can increase their chance of obtaining a grant by buying more tickets) but abstracts away from self-selection and noisy evaluation on which we focus. Scotchmer (2004, Chapter 8) formulates a simple dynamic model of demand for funding where high quality researchers self select into applying and are disciplined to deliver because they expect to be funded in the future. Building on a setting with continuous types and scale-location signal similar to ours, Leslie (2005) sketches the demand side for submissions to academic journals—in addition to a complete analysis of the demand side, we add (noisy) evaluation on the supply side and characterize the equilibrium depending on the budget allocation rule.[15] See also Stephan (2012, Chapter 6) for a broad discussion and references on science funding and Azoulay and Li (2020) for a recent overview of the fledgling empirical literature on grant funding for science.[16]

In our model the application cost, akin to what Nichols and Zeckhauser (1982) call an ordeal, induces more worthy applicants to self-select. While in their model the inconvenience cost of the ordeal differs across types, in our model the cost is the same for all applicants, but the evaluator uses an additional noisy signal about the applicant's type so that the application cost acts as an endogenous screening device. The noise in the evaluation process thus plays a key role in our model as in the

[11] In a review of the NIH, Zeckhauser (1967) also argues that disease burden should guide funding choices.

[12] See also the formalization by Davis, Dempster, and Wildavsky (1964). Savage (1999) gives a historical account of the influence process behind university earmarks in comparison to merit-based public funding of research.

[13] Zukerman and Merton (1971, page 77) write: ". . . the more humanistically oriented the journal, the higher the rate of rejecting manuscripts for publication; the more experimentally and observationally oriented, with an emphasis on rigour of observation and analysis, the lower the rate of rejection." Referee please take notice.

[14] Cole and Cole's (1981) landmark study documents differences in agreement among reviewers (as measured by inter-rater reliability) across fields at the NSF.

[15] See also Cotton (2013) and Taylor and Yildirim (2011), focusing on discrimination issues, which we skirt.

[16] Gans and Murray (2012) overviews the main funding sources available for scientists (government, private firms' internal R&D, and foundations), with a focus on comparing their different disclosure and openness requirements. Boudreau, Guinan, Lakhani, and Riedl (2016) investigate the role of the intellectual distance between evaluators' expertise and the research proposals in systematically shaping funding outcomes.

literature on statistical discrimination, pioneered by Phelps (1972) and surveyed by Moro and Fang (2001). In that strand, Cornell and Welch (1996) argue that competition for ranking in a tournament discriminates against candidates the evaluator is *less* informed about. Our model moots this channel by focusing on an evaluator who is equally informed about applicants belonging to the same field. The new effect we uncover, instead, acts across fields. Competition within a field with more noisy evaluation becomes closer to a lottery and thus encourages more applications. In turn, when the budget of grants available to a field increases in applications, candidates the evaluator is *more* informed about end up being inefficiently discriminated against—the opposite of Cornell and Welch's outcome.[17]

While our model zooms in on the noisy evaluation process of applicants, the literature on tournaments and contests—from Lazear and Rosen (1981) to O'Keeffe, Viscusi, and Zeckhauser (1984), Moldovanu and Sela (2001), Che and Gale (2003), Siegel (2009), Gross and Bergstrom (2019), and Fang, Noe, and Strack (2020)—mostly focuses on reward and elicitation of contestants' effort incentives, from which we abstract. Closer to our setting, Morgan, Sisak, and Várdy (2018) analyze the incentives of applicants to select different fields in a setting with exogenous supply. Instead, we focus on endogenously determining the supply through the budget allocation when applicants cannot pick field but can only choose whether or not to apply. Within the agency literature, Che, Dessein, and Kartik (2013), Alonso (2018), and Frankel (2020) largely focus on how to optimally constrain biased evaluators—in our model, instead, evaluators within each field are unbiased.

2 Grantmaking in a Single Field: Fixed Budget Equilibrium

Begin by considering grantmaking in a single field that features a continuum of candidates parametrized by their merit θ, corresponding to the value created for the grantmaker if the project is financed. Candidates know their merit, which follows distribution G in the population, with size normalized to one. We are completely general about G, and only assume for convenience that it admits a continuously differentiable and strictly positive density g on a connected support $\left[\underline{\theta},\overline{\theta}\right]$, possibly unbounded on either side.[18]

To be considered for a grant award, candidates must apply at cost c, the opportunity cost of the time spent preparing the application and describing the work.[19] Applicants who are awarded grants obtain a

[17]Cornell and Welch's (1996) logic can explain why the success rate is lower for clinical studies compared to basic science at the NIH, as displayed empirically by Martin, Lindquist, and Kotchen (2008). Even though the success rate is equilized across panels, clinical study applications—for which there is more noise—are at a disadvantage when competing with basic science applications in the same panel. If clinical studies were evaluated by different panels than basic science their success rate would be automatically equalized. However, according to our analysis, more applications would be submitted for clincial studies and less for basic science.

[18]If the support is unbounded below we have $G^{-1}(0)=\underline{\theta}=-\infty$ and if it is unbounded above $G^{-1}(1)=\overline{\theta}=\infty$.

[19]Application costs can well be sizeable. According to survey evidence by von Hippel and von Hippel (2015) on astronomers and social and personality psychologists who submitted applications for basic research grants to NASA, the NIH, and the NSF, principal investigators spent on average 116 hours to prepare the applications. This represent a major increase

private benefit v in terms of career advancement and kudos.[20]

In this baseline version of the model, the grantmaker allocates an exogenously given budget of B grants, expressed as a fraction of the unit-size population of candidates. The allocation of grants to applications is done by an evaluator (the review panel) who is instructed to award the B grants to the applicants that generate the most favorable noisy signal

$$x = \theta + \sigma\varepsilon \tag{2}$$

about the merit θ, where the noise ε follows distribution F with continuously differentiable and strictly positive density f and connected support $[\underline{\varepsilon}, \overline{\varepsilon}]$, possibly unbounded on either side.[21] We assume that the signal satisfies the monotone likelihood ratio (MLR) property, so that a higher signal indicates higher merit.[22] A key role in our analysis is played by the parameter $\sigma \geq 0$, which measures the dispersion and thus the accuracy of the information contained in the grantmaker signal, as defined by Lehmann (1988): the signal perfectly reveals the merit when $\sigma = 0$ and becomes completely uninformative as $\sigma \to \infty$.[23]

The timing of the baseline problem with a single field and fixed budget B is as follows:

1. Candidates observe their own type θ and decide whether to apply.

2. The evaluator awards the B grants to the applicants with most favorable signals x

Candidates and evaluator have common knowledge of the model and its parameters. Being atomistic, candidates do not take into account the negligible impact of their application.[24] Equilibria have

to the early day of science funding. For comparison, in 1921 the prominent German biochemist Otto Warburg submitted to the Notgemeinschaft der Deutschen Wissenschaft (Emergency Association of German Science, the fore runner of the Deutsche Forschungsgemeinschaft) a funding application with a single sentence: 'I require 10,000 marks'; see Koppenol, Bounds, and Dang (2011).

[20] The model can also easily accommodate the addition of an embarrassment or psychological cost d borne by the candidate when the application is turned down. The cost benefit ratio c/v, which determines demand incentives, is then replaced by $(c+d)/(v+d)$.

[21] In the special case without shifting support (when $\underline{\varepsilon} = -\infty$ and $\overline{\varepsilon} = \infty$) no signal is perfectly revealing.

[22] MLR is equivalent to logconcavity of the density $\frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right)$; see Lehmann and Romano (2005, p. 323) for a proof.

[23] Inverting the signal distribution $y = F\left(\frac{x-\theta}{\sigma}\right)$ from (2), the quantile function of the signal is $x = \theta + \sigma F^{-1}(y)$. For every percentile y, the quantile difference $\left[\theta + \sigma_2 F^{-1}(y)\right] - \left[\theta + \sigma_1 F^{-1}(y)\right]$ is decreasing in y for $\sigma_2 < \sigma_1$. Thus, an increase in σ makes the signal more dispersed and reduces Lehmann (1988) information accuracy. Equivalently, the quantile transform $\theta + \sigma_2 F^{-1}\left(F\left(\frac{x-\theta}{\sigma_1}\right)\right) = \frac{\sigma_2}{\sigma_1}x + \left(1 - \frac{\sigma_2}{\sigma_1}\right)\theta$ is increasing in θ for $\sigma_2 < \sigma_1$. As shown by Quah and Strulovici (2009), any decision maker with preferences in the general interval dominance ordered class obtains a higher expected payoff state by state when σ is reduced. This class encompass all monotone decision problems (widely used in statistics) and single-crossing preferences (common in economics) as special cases. We take signal noise as exogenous; extensions could endogenize σ based on congestion or dynamic consideration along the lines pursued by Board, Meyer-ter-Vehn, and Sadzik (2020).

[24] With a finite population of candidates, an additional application in a field would lead to a (possibly small, but non zero) impact on the success rate for that field, even holding constant the behavior of other candidates. This effect vanishes as the number of candidates increases.

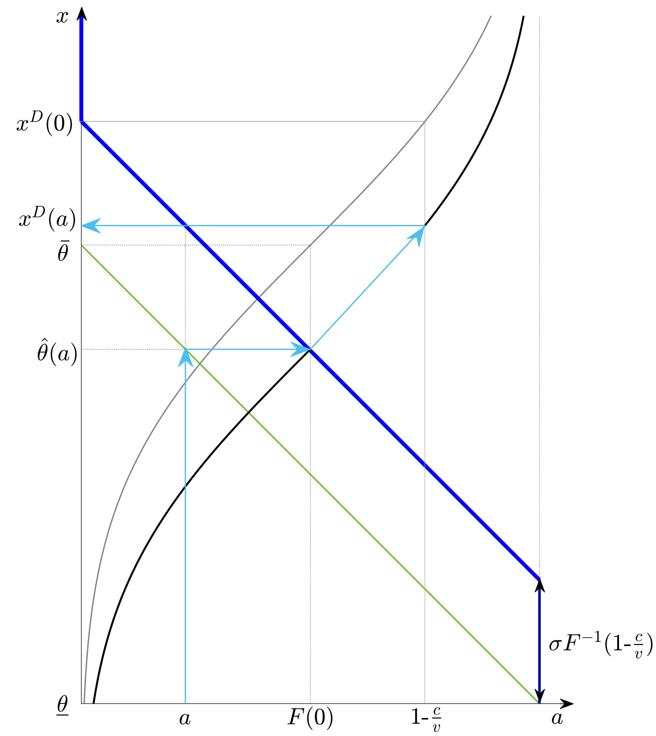

Figure 1: Application demand.

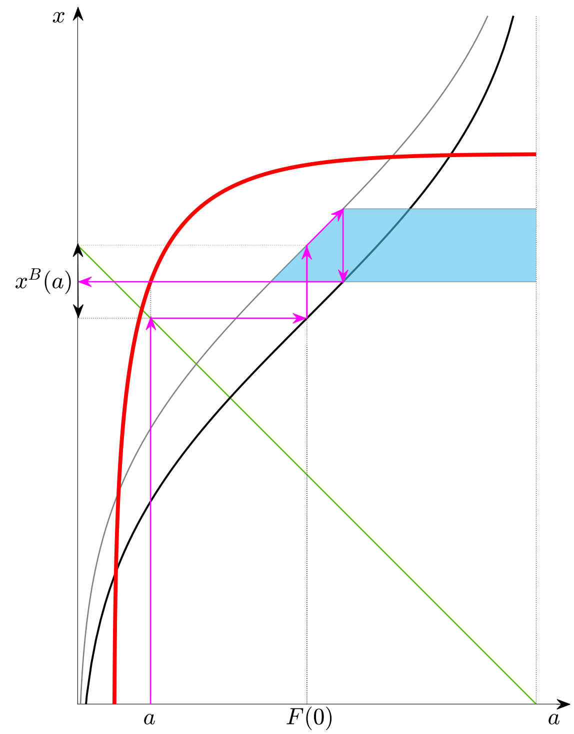

Figure 2: Fixed-budget supply.

the following monotonic structure, allowing us to solve the model through a simple representation in terms of demand and supply, even though no prices are involved. On the supply side, the evaluator awards grants to applications with $x \geq \hat{x}$, by the MLR property. On the demand side, candidates with higher merit are more likely to win, and thus apply for $\theta \geq \hat{\theta}$. The rest of this section explains this demand and supply construction and characterizes the unique fixed-budget equilibrium that results.

2.1 Application Demand: Self Selection

Expecting the evaluator to accept whenever the signal is above $\hat{x}$, candidates apply if their benefit from the grant times the expected probability of obtaining a grant outweighs the application cost

$$v\left[1-F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right] \geq c. \tag{3}$$

Given acceptance standard $\hat{x}$, candidates optimally apply if $\theta \geq \hat{x}+\sigma F^{-1}(1-c/v)$, resulting in application demand equal to

$$a^D(\hat{x}) := 1-G\left(\hat{x}+\sigma F^{-1}(1-c/v)\right).$$

Figure 1 illustrates the construction of the demand, displayed in bold dark blue. The horizontal axis reports the fraction of applicants in the population, playing the same role as quantity in classic demand theory. The marginal applicant is obtained from the counter-quantile of the type distribution $G^{-1}(1-a)$, displayed in green in this example with uniformly distributed types $G(\theta)=\theta$. The marginal applicant expects the grantmaker to observe a signal centered around its type $G^{-1}(1-a)$ and thus expects to succeed and obtain a grant with probability $1-F\left(\frac{x-G^{-1}(1-a)}{\sigma}\right)$, corresponding to the thicker black curve in the figure for an example with normal signal. Setting the success probability for the marginal applicant equal to the cost benefit ratio c/v and solving for the acceptance standard that makes the marginal applicant indifferent, we obtain the inverse demand

$$\hat{x}^{D}(a)=G^{-1}(1-a)+\sigma F^{-1}(1-c/v). \tag{4}$$

This is the acceptance standard that induces applications a. By the location structure of the signal, the inverse demand is equal to the counter-quantile function of the type distribution shifted up by $\sigma F^{-1}\left(1-\frac{c}{v}\right)$, as marked in the figure.

By the additive structure of the signal, the success probability for the highest type $\bar{\theta}=G^{-1}(1)$, $1-F\left(\frac{x-G^{-1}(1)}{\sigma}\right)$, can be found by shifting up the marginal type $G^{-1}(1-a)$ by $G^{-1}(1)-G^{-1}(1-a)$, as displayed in the gray upward sloping curve. The choke point of the demand is thus at $G^{-1}(1)+\sigma F^{-1}(1-c/v)$, when the highest type is exactly indifferent to apply. For application demand to be positive, the acceptance bar must be reduced below this level. More generally, an increase in the acceptance standard makes an award less likely and thus discourages applications:

Proposition 1 (Demand) *(a) Application demand $a^{D}(\hat{x})$ is downward sloping: if the acceptance standard is raised, less candidates apply.*

2.2 Fixed-Budget Supply: Selection through Evaluation

What is the optimal allocation of the budget B by the grantmaker on the supply side? With a applications, candidates with types above $G^{-1}(1-a)$ self select into applying. The acceptance standard $\hat{x}^{B}$ on the fixed-budget supply with a applicants is such that the average success probability of all the applications submitted is equal to the available budget of prizes

$$\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{\hat{x}^{B}-\theta}{\sigma}\right)\right]g(\theta)\,d\theta=B. \tag{5}$$

Figure 2 illustrates the construction of the fixed-budget supply (the red thick curve) for an example with uniform types $G(\theta)=\theta$ and normal signal. Given a applications (on the horizontal axis), the counter-quantile function $G^{-1}(1-a)$ (in green) gives the marginal type on the vertical axis, with success probability $1-F\left(\frac{x-G^{-1}(1-a)}{\sigma}\right)$ (in black). By MLR, the success probability decreases in the applicant's type. Thanks to the location structure of the signal, the success probability $1-F\left(\frac{x-\theta}{\sigma}\right)$ for any applicant type $\theta\in\left[G^{-1}(1-a),\bar{\theta}\right]$ (on the black segment with arrows at both ends, displayed on the vertical axis) at acceptance standard x can be obtained from the success probability of the highest type $\overline{\theta} = G^{-1}(1)$ (displayed in gray), once the acceptance standard is shifted up by $G^{-1}(1) - \theta$. According to (5), the standard $\hat{x}^B$ (on the vertical axis) such that budget B is allocated to the top a applicants is obtained by sliding the vertical segment of length $G^{-1}(1) - G^{-1}(1-a)$ (starting from the pink vertical segment with the arrow pointing up) until the (light blue) area to the right of the gray curve, weighed by the density $g(\theta)$, is equal to the budget B (ending into the pink vertical segment with the arrow pointing down).

As applications a increase, the acceptance standard $\hat{x}^B$ must be raised because otherwise the grant prizes assigned would be more than the budget available B. The increase in the standard reduces the winning probability for each applicant, thus making space for the additional applicants. Equivalently, if the acceptance standard is raised, applications must increase to exhaust the same budget.

Proposition 1 (Fixed-Budget Supply) ***(b)*** *Fixed-budget supply $\hat{x}^B(a)$ is upward sloping: the acceptance standard such that a fixed grant budget is allocated increases in the amount of applications.*

2.3 Fixed-Budget Equilibrium

An equilibrium results if one of the following three conditions hold: (i) supply and demand cross for an interior $a \in (0,1)$, (ii) supply is above demand at $a = 0$, and (iii) and demand is above supply at $a = 1$. An equilibrium is defined to be stable if any local perturbation leads back to the equilibrium under classic *tâtonnement* adjustment.[25]

Proposition 1 (Fixed-Budget Equilibrium) ***(c)*** *A fixed-budget equilibrium exists and is unique and stable.*

Continuity of demand and supply guarantees existence by Brouwer fixed-point theorem; the same argument guarantees existence for all the model specifications considered in the paper. Equilibrium applications are positive $a > 0$ whenever the budget is positive $B > 0$. Given that with fixed budget the supply is upward sloping, it must cross the downward sloping demand from below, ensuring stability and uniqueness.

2.4 Impact of Signal Dispersion

What is the impact of an increase in noise dispersion σ on demand and supply in a single field and fixed budget? With *perfect information*, $\sigma = 0$, the second term in (4) vanishes, so the inverse demand

[25] Starting from any allocation a^0, x^0 in an ε-neighborhood of an equilibrium a^E, x^E, we define the equilibrium to be stable if the *tâtonnement* supply $\hat{x}^S(a)$ and demand $a^D(\hat{x})$ adjustment process $a^{t+1} = a^D\left(\hat{x}^S(a^t)\right)$ and $x^{t+1} = \hat{x}^S\left(a^D(x^t)\right)$ leads to the equilibrium, $\lim_{t\to\infty}(a^t, x^t) \to \left(a^E, x^E\right)$. For the general case with multiple fields the conditions for stability we derive below borrow from competitive equilibrium analysis (e.g., Arrow and Hurwicz 1958).

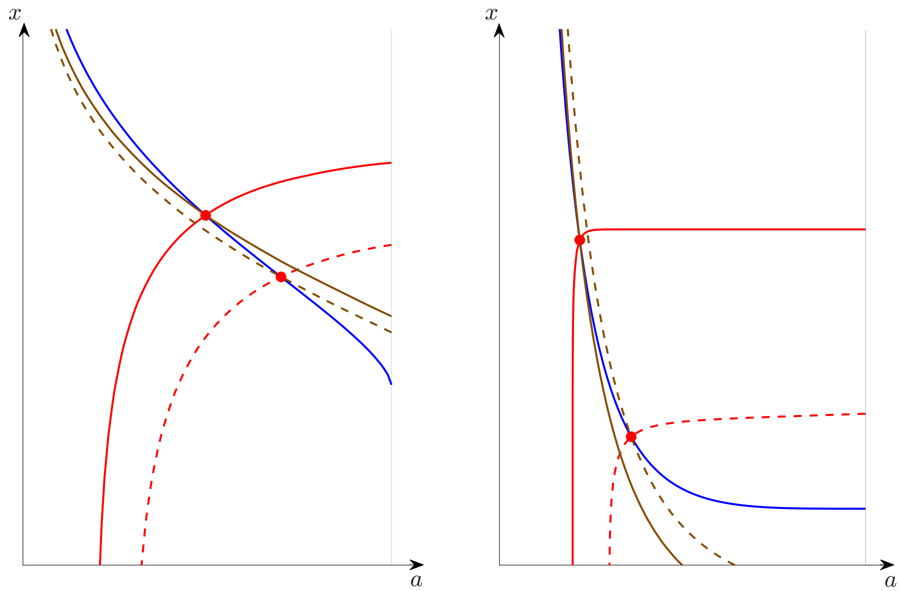

Figure 3: Comparative statics of the fixed-budget equilibrium with respect to the budget for a type distribution with increasing (decreasing) hazard rate on the left (right) panel.

is identical to the counter-quantile function of the type distribution $\hat{x}^D(a) = G(1-a)$. The supply function (5) becomes an inverted L: when $a < B$ all applicants are accepted and the standard is set at the lower bound of the support (with some budget remaining unspent), while for $a \geq B$ the acceptance standard is set at $\hat{x}^B = G^{-1}(1-B)$.[26] Thus, the equilibrium is at $a = B$ and $\hat{x}^B = \hat{\theta} = G^{-1}(1-B)$. With perfect information applicants succeed with probability one in equilibrium, so that the success rate $p = B/a$ (the fraction of successful applicants) is equal to 1.

An increase in noise dispersion shifts demand up and to the right if and only if $F^{-1}(1-c/v) < 0$, e.g., for $c/v < 1/2$ whenever the signal distribution is symmetric. An increase in noise dispersion induces an anti-clockwise rotation in the supply curve. Even though σ has an ambiguous impact on demand and supply, it always increases equilibrium applications:

Proposition 1 (Dispersion) ***(d)*** *Equilibrium applications in a single field with fixed budget always increase in signal dispersion σ.*

Thus, the equilibrium success rate monotonically decreases in signal dispersion, $\partial p/\partial\sigma < 0$. Historically, the success rate has been declining in a number of research funding organizations, reaching a level in the range $\simeq 10/15\%$ for both the ERC and the NIH.

2.5 Impact of Budget

Next, we perform comparative statics with respect to the budget B. This exercise is relevant to understand the impact of budget variation such as the drastic increase in research funding for the NIH following the American Recovery and Reinvestment Act (ARRA) of 2009, which resulted in an incremental allocation of \$8.97 billion to extramural research grants. The additional budget was allocated in two tranches.

Unanticipated Budget Increase. Part of the funds were allocated by NIH to "not ARRA solicited" applications which had been previously submitted and reviewed in recent evaluation cycles, but were marginally rejected. The budget allocated to not ARRA solicited applications corresponds to an unanticipated increase in supply. Holding fixed the amount of applications a at the pre-shock level, the model predicts that the applications funded as a result of the higher budget are of lower quality. In line with this prediction, in their empirical analysis of the impact of the not ARRA solicited allocation (corresponding to 19.3% of total ARRA budget appropriated to the NIH) Park, Lee, and Kim (2015) document that ARRA projects resulted in less high-impact articles than regular projects.

Anticipated Budget Increase. The remainder of the funding bonanza was set aside to increase the budget for "ARRA solicited" grant competitions. In this case, potential applicants were informed of the larger budget. As displayed in Figure 3, an increase in the budget shifts the supply down. This change is anticipated when applications are submitted, thus inducing a movement along the demand curve, resulting in a higher level of equilibrium applications.

Proposition 1 (Budget) ***(e)*** *An increase in the budget B shifts the fixed-budget supply $x^B(a)$ down, thus pushing up equilibrium applications.*

How does the budget impact the success rate (also known as payline at the NIH), defined as the fraction of successful applications, $p = B/a$? Differentiating with respect to B we obtain

$$\frac{dp}{dB} \gtreqless 0 \Leftrightarrow \frac{da}{dB} \lesseqgtr \frac{a}{B}.$$

Thus, the average success probability $p = B/a$ among applicants increases with the budget if and only if applications increase less than proportionally with the budget—or, equivalently, if the budget elasticity of applications is less than one. Our model has a sharp prediction for when equilibrium applications increase more than proportionally with the budget:

[26]Note that the success probabily $1 - F\left(\frac{\hat{x}^B - \theta}{\sigma}\right)$ converges to the Heaviside function as $\sigma \to 0$.

Proposition 1 (Success Rate) ***(f)*** *The equilibrium success rate increases in the budget B if and only if the type distribution has increasing hazard rate,*

$$\frac{\partial}{\partial\theta}\frac{g(\theta)}{1-G(\theta)} > 0. \tag{IHR}$$

This results relies on Proposition 2, a central result of the paper we present in the next section, which shows that the locus of points with constant success rate (displayed in ochre in Figure 3) (a) is always downward sloping, (b) shifts down when the payline increases, and (c) is flatter (or steeper) than the demand curve to which it crosses if the type distribution has increasing hazard rate as illustrated on the right-hand side panel (or decreasing hazard rate, on the right-hand side panel). Given that an increase in the budget shifts the supply down (and to the right) for given demand, at the new equilibrium the payline increases if and only if the constant success rate locus is flatter than the demand curve, i.e., if and only if the hazard rate is increasing.

This result has a simple intuition. If the type distribution has IHR (or decreasing hazard rate), it also has decreasing (or increasing) residual expectation

$$\frac{\partial E\left[\theta-\hat{\theta}|\theta\geq\hat{\theta}\right]}{\partial\hat{\theta}} < 0 \text{ (or} > 0\text{)}, \tag{DRE}$$

(or IRE); see Bagnoli and Bergstrom's (2005) Theorem 6.[27] Thus under IHR, as applications increase, the distance between the average type of the inframarginal applicants and the merit type of the marginal applicant $\hat{\theta}$ (which is reduced as a goes up) also increases. Thus, the pool of inframarginal applicants become relatively stronger than the marginal applicant. Given that along the demand curve the success probability of the marginal applicant is fixed at c/v by construction, in equilibrium the average success probability of the inframarginal applicants—the success rate p—must then increase in a. The opposite conclusion holds if the type distribution has decreasing hazard rate. In the boundary case when the type distribution is exponential (with constant hazard rate) the constant success rate locus is parallel to the demand curve, regardless of the signal distribution. Applications then increase proportionally with the budget, leaving the success rate unchanged.

Applications must increase more than proportionally with the budget for the success rate to decrease as the budget increases—and this occurs in equilibrium if and only if the type distribution has decreasing hazard rate, in which case the constant success rate locus is steeper than the demand curve. This is exactly what happened as a result of the "ARRA solicited" part of budget increase in 2009. As documented by Stephan (2012, p. 145), applications increased so much that the success rate actually decreased—this means that the constant success rate locus was steeper than the demand curve. This

[27] A logconcave density grows at a decreasing rate and declines at an increasing rate. By Prekopa's theorem, logconcavity (logconvexity) of the density $g(\theta)$ implies logconcavity (logconvexity) of the countercumulative distribtion $1-G(\theta)$, which in turn implies logconcavity (logconvexity) of the right-hand integral $H(\theta)=\int_{\theta}^{\bar{\theta}}\left[1-G\left(\tilde{\theta}\right)\right]d\tilde{\theta}$, which in turn is equivalent to the fact that the residual expectation $E\left[\theta-\hat{\theta}|\theta\geq\hat{\theta}\right]$ is decreasing (increasing).

observation indicates that the top tail of the distribution of researcher types is thicker than exponential, for which the hazard rate is constant. Note that a distribution has decreasing hazard rate whenever it is larger than the exponential distribution in van Zwet (1964) convex transform order. Thus, types with decreasing hazard rate are more right skewed and have a thicker top tail than exponential—and they can be obtained starting from an exponential by stretching the density quantile function toward the top tail through a convex transformation.[28]

3 Grading on a Curve: Constant-Payline Equilibrium

Having characterized the equilibrium allocation in a single field with constant budget, we now turn to the second building block of our analysis. What outcome results when the success rate (the fraction of successful applications) is constant, regardless of the amount of applications? According to equation (1), this feature arises when the budget of grants available to a field is proportional to applications received in the field. The analysis in this section applies to a field that is so small that an increase in applications in the field does not impact the overall success rate. This is approximately true for a small field representing a small fraction of the overall budget allocated to all fields—as in a small-country partial-equilibrium analysis in international trade theory, where the price in the international market is taken as given. The full equilibrium analysis in Section 5 with endogenous payline satisfying (1) crucially builds on the results we derive in this section.

This analysis is also directly relevant to grading on a curve, where instructors can assign a constant fraction of distinctions, top grades or honors to the class.[29] Many school practice this kind of relative grading to control grade inflation; see Johnson (2003). Relative grading can be also be induced by regulation; for example, according to Texas' "Top 10% Rule" students who graduate in the top ten percent of their high school class are guaranteed automatic admission to all state-funded universities.[30] With a constant success rate, what are the incentives to select a class or school for students who aim to graduate with honors? Do teachers or schools with more noisy grading attract more or less students? This section characterizes participation incentives under a constant payline system which awards prizes to the top $100p\%$ of applicants.

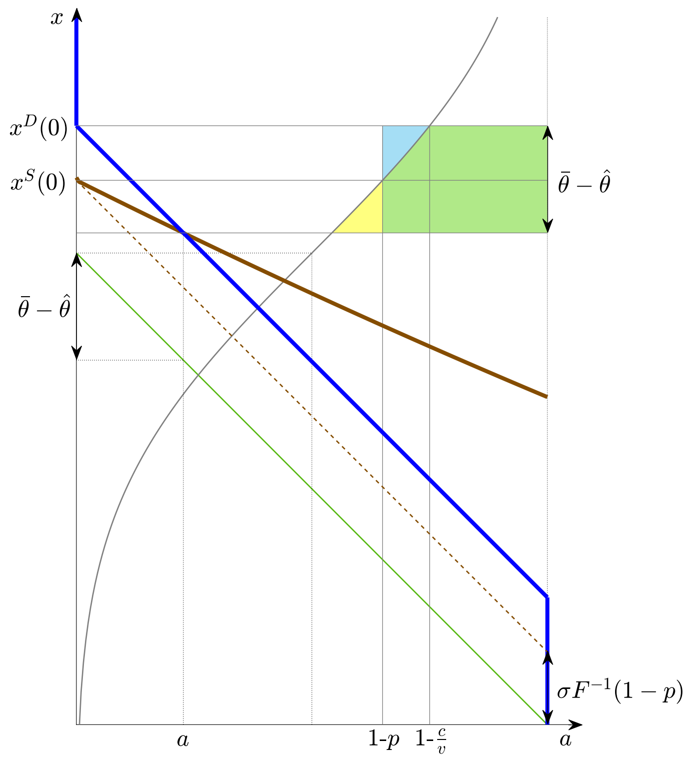

Figure 4: Constant-payline locus and constant-payline equilibrium.

3.1 Constant-Payline Locus

Consider a single field facing a constant success rate. The budget of grants (or prizes) available in the supply equation (5) is adjusted so as to keep the success rate constant at p by setting $B = pa$. The supply is then given by the constant-payline locus

$$\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{x^p-\theta}{\sigma}\right)\right]g(\theta)\,d\theta = p \tag{6}$$

Figure 4 illustrates the construction of the constant payline locus in the $(a,\hat{x})$ space. Applicants with higher types enjoy a higher success probability than weaker applicants. Given a, with proportional funding the acceptance standard $\hat{x}^p$ is set so that the average probability of winning is p across all

[28]Given two distributions G and H, van Zwet (1964) defines G to be smaller than H in the convex transform order, denoted $G \prec_c H$, whenever $H^{-1}(G(\cdot))$ is convex. As shown by van Zwet (1964), a distribution G with increasing (decreasing) hazard rate can be obtained through an increasing and concave (convex) transformation $G^{-1}(G_{\text{Exp}}(\cdot))$ of a random variable with exponential distribution. To gain intuition, visualize the random variable G^{-1} on the vertical axis as an increasing transformation of an exponential random variable G_{Exp}^{-1} on the horizontal axis through a Q–Q plot. Concavity (convexity) of $G^{-1}(G_{\text{Exp}}(\cdot))$ contracts (stretches) the top tail and makes it thinner (thicker) than the top tail of an exponential.

[29]In this case, the opportunity cost of not enrolling in another class or school than plays the role of application cost.

[30]See Cullen, Long, and Reback (2013) for an empirical analysis.

applicants, or

$$\int_{\hat{x}^p-\sigma F^{-1}(1-p)}^{\bar{\theta}} \left[\left(1-F\left(\frac{\hat{x}^p-\theta}{\sigma}\right)\right)-p\right] g(\theta)d\theta = \int_{G^{-1}(1-a)}^{\hat{x}^p-\sigma F^{-1}(1-p)} \left[p-\left(1-F\left(\frac{\hat{x}^p-\theta}{\sigma}\right)\right)\right] g(\theta)d\theta. \tag{7}$$

The argument of the integral on the left-hand side of (7) is the difference between the success probability for stronger applicants with types $\theta \in \left[\hat{x}^p - \sigma F^{-1}(1-p), \bar{\theta}\right]$ and the average success probability p, which is also the actual success probability of type $\hat{x}^p - \sigma F^{-1}(1-p)$. The proportional supply $\hat{x}^p(a)$ is such that the excess success probability (weighed by the corresponding type density) for stronger applicants on the left-hand side—the area depicted in yellow in Figure 4 for an example with uniformly distributed types, $g(\theta) = 1$—is equal to the integral of the difference between p and the acceptance probability for weaker applicants with types $\theta \in \left[G^{-1}(1-a), \hat{x}^p - \sigma F^{-1}(1-p)\right]$ on the right-hand side of (7)—the light blue area in Figure 4.

Proposition 2 (Constant-Payline Locus) *The constant-payline locus solving (6)*
***(a)** decreases in applications, $d\hat{x}^p/da \leq 0$;*
***(b)** decreases in the success rate, $d\hat{x}^p/dp \leq 0$.*
***(c)** is flatter than demand $d\hat{x}^p/da > d\hat{x}^D/da$ at a if and only if type and signal distributions satisfy* increasing mean excess success

$$\frac{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\hat{x}-\tilde{\theta}}{\sigma}\right)\left[1-G\left(\tilde{\theta}\right)\right] d\tilde{\theta}}{\int_{\theta}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\hat{x}-\tilde{\theta}}{\sigma}\right) g\left(\tilde{\theta}\right) d\tilde{\theta}} < \frac{1-G(\theta)}{g(\theta)} \tag{IMES}$$

at $\theta = G^{-1}(1-a)$. Increasing hazard rate IHR for all $\theta \geq G^{-1}(1-a)$ is sufficient for IMES.

According to part (a), the acceptance standard on the constant-payline locus $\hat{x}^S$ is a downward sloping function of applications, a. As applications increase, the average quality of applicants is reduced. To keep the success rate at the same level for a pool of applicants that is now worse, the acceptance standard must be reduced. Thus, the constant-payline locus always slopes down.

When the success rate is increased, as in part (b), the acceptance standard for any a must be reduced. Intuitively, for any given a, more applications must be accepted to exhaust the larger budget that results when the success rate is raised.

For part (c), rewrite the constant-payline locus integrating by parts the average success probability, the left-hand side of (6), as

$$1-F\left(\frac{\hat{x}-G^{-1}(1-a)}{\sigma}\right) + \frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}} \left[1-G(\theta)\right] \frac{1}{\sigma} f\left(\frac{\hat{x}-\theta}{\sigma}\right) d\theta = p, \tag{8}$$

as shown by equation (23) in the proof. The first term on the left-hand side of (8) is the same as in the demand equation (9), while on the right-hand side c/v is replaced by p—thus, without the second term, the locus would be parallel to demand, as represented by the dashed brown curve in Figure 4. The second term corresponds to the difference between the constant-payline locus in brown and this dashed brown curve. This difference increases in a under IMES (or under the stronger IHR), so that the constant-payline locus is flatter than the demand.[31]

In the boundary case where the type distribution is exponential (constant hazard rate), the constant-payline locus is parallel to the demand curve, for *any* noise distribution F and dispersion σ. The success rate is then constant along the demand curve.

3.2 Constant-Payline Equilibrium: Stability and Multiplicity

To characterize the equilibrium suppose the payline is above the cost benefit ratio, $p > c/v$; otherwise, $a = 0$ in equilibrium. The demand condition (3) requires that the acceptance probability for the marginal type $\hat{\theta}$ which generates demand $a = 1 - G(\hat{\theta})$ is exactly equal to c/v, or

$$1 - F\left(\frac{\hat{x} - G^{-1}(1-a)}{\sigma}\right) = \frac{c}{v}, \tag{9}$$

represented by the crossing of the distribution function with the vertical line at $1 - c/v$. The supply condition requires that the average success probability satisfies (6). Notice that the acceptance probability for the marginal type $\hat{\theta} = G^{-1}(1-a)$ is $1 - F\left(\frac{\hat{x}-G^{-1}(1-a)}{\sigma}\right) = c/v < p$; by the location property of the distribution, this probability can be read off the distribution function $F\left(\frac{x-\bar{\theta}}{\sigma}\right)$ in the graph by setting $x = \hat{x} + \bar{\theta} - G^{-1}(1-a)$.[32] At the other end, the acceptance probability for the top type $\theta = \bar{\theta}$ when the acceptance bar is at $\hat{x}$, $1 - F\left(\frac{\hat{x}-\bar{\theta}}{\sigma}\right)$, must necessarily be higher than p. At a constant-payline equilibrium, both the marginal demand condition (9) and the constant-payline supply condition (6) are satisfied.

Demand and constant-payline supply are both decreasing functions of the amount of applications a, raising the question of equilibrium stability and uniqueness. As illustrated by Figure 4 for a example with uniformly distributed types, when the constant-payline supply is flatter than supply, the equilibrium outcome is unique and stable. In general:

Proposition 2 (Constant-Payline Equilibria) ***(d)*** *A constant-payline equilibrium a^p, x^p is stable if and only if it satisfies IMES locally at $\theta = G^{-1}(1-a^p)$ and $x = x^p$. IHR is sufficient for IMES.*

[31] If the signal distribution is uniform $F(x) = 1/2 + x$, condition IMES is equivalent to logconcavity of the the right-hand side integral of the survival function $\int_{\theta}^{\bar{\theta}} [1 - G(\theta)]\, d\theta$, which is implied by increasing hazard rate; see Bagnoli and Bergstrom (2005).

[32] At any given acceptance bar $\hat{x}$ the upward sloping curve Figure 4 represents the distribution function $F\left(\frac{\hat{x}-\bar{\theta}}{\sigma}\right)$ corresponding to the highest type, $\theta = \bar{\theta}$. The success probability $1 - F\left(\frac{\hat{x}-\bar{\theta}}{\sigma}\right)$ is then obtained as the distance from the vertical line at $a = 1$. Thanks to the location structure of the experiment, the success probability for a candidate of type $\theta < 1$ can be read off this same curve by sliding to the right by $\bar{\theta} - \theta$, thus obtaining $1 - F\left(\frac{\hat{x}-\theta}{\sigma}\right)$.

(e) *If IMES (or a fortiori IHR) holds globally, there is a unique and stable constant-payline equilibrium.*
(f) *If IMES is violated for some θ and x, there are parameters for which there are multiple constant-payline equilibria; all equilibria at which IMES is reversed are unstable.*

According to part (d), IMES (or the stronger IHR) guarantees that demand is steeper than supply, so that their vertical distance $\hat{x}^D(a) - \hat{x}^p(a)$ decreases in a. If the equilibrium is interior, then supply crosses demand from below ensuring stability. If supply starts off for $a = 0$ above demand, we have a stable equilibrium at $a = 0$. If supply remains below demand for all $a \in [0,1]$, the equilibrium is at the corner $a = 1$, again stable.

If IMES or IHR hold globally throughout the support we also have equilibrium uniqueness, as shown in part (e). The IHR condition, satisfied by all distributions with logconcave densities (such as uniform, logistic, normal, and Weibull with shape parameter $k > 1$), is commonly assumed as a regularity condition in economics models. In our model this assumption has strong implications in terms of equilibrium stability and uniqueness. Given that IHR is unduly restrictive in the context of our application, our analysis investigates the more general case with decreasing (as well as non-monotonic) hazard rate.

Why is violation of IMES necessary for multiple equilibria? Intuitively, for multiple equilibria to result it must be that the density of types must decrease so steeply in θ that an increase in demand by low types generates such a large increase in the supply of awards that the acceptance standard (so as to keep the success rate constant at p) must be reduced by more than it is needed to encourage the additional demand. When IMES is violated, there is a range of a for which the distance between demand and supply is increasing. We can then find a combination of parameters leading to multiple equilibria, as shown in part (f). Equilibria are generically odd in number and follows an alternating stability pattern.

Multiple Equilibria Paths. To illustrate the pattern resulting when the type distribution has decreasing hazard rate, the left panel of Figure 5 displays constant payline equilibria resulting with types following the Pareto Lomax distribution $G(\theta) = 1 - (1 - \beta\theta)^{-\alpha}$ for $\alpha, \beta > 0$ and a uniform signal $F(\varepsilon) = 1/2 + \varepsilon$.[33] For $\sigma < \hat{\sigma} = E[\theta]/(p - c/v) = [\alpha\beta(p - c/v)]^{-1}$, there is unique and stable constant payline equilibrium at $a = 0$, while for $\sigma \geq \hat{\sigma}$ there are two stable equilibria at $a = 0$ and $a = 1$ and an interior unstable equilibrium at $a = [\sigma\beta(\alpha - 1)(p - \gamma)]^{-\alpha}$, with comparative statics consistent with the results presented above. The dashed black curve displays the path of the constant playline equilibrium as σ increases starting from $\hat{\sigma}$, where equilibria bifurcate, as shown by the red arrows.

[33] Given that the type distribution has decreasing hazard rate, he distance between the pseudo supply curve (displayed in dashed brown, parallel to demand as explained in the proof of Proposition) and the constant-payline supply (brown) decreases in a. Thus, there is at most one interior constant-payline equilibrium and this equilibrium is unstable.

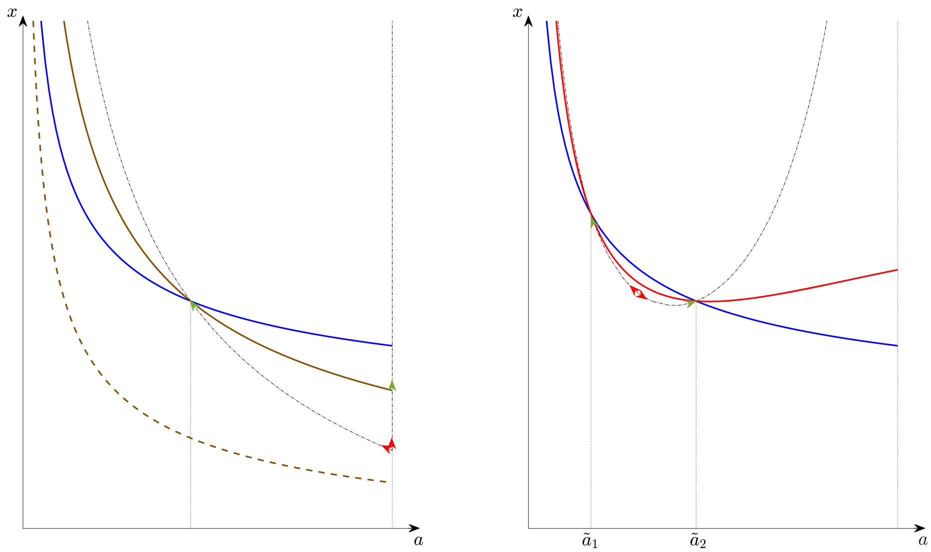

Figure 5: If the type distribution has decreasing hazard rate, supply starts off above demand so there is an unraveling equilibium for any level of signal noise. The left panel shows the path of constant-payline equilibria; the red arrows corresponds to the threshold level of signal dispersion above which there are multiple equilibria. The right panel displays the path of full equilibria in the first field competing with a second field with uniformly distributed types.

3.3 Equilibrium Unraveling: The Paradox of Relative Evaluation

We now establish that when the evaluation is based on perfect information, $\sigma = 0$, in the unique constant-payline equilibrium with $p < 1$ no candidate applies.

This striking result follows from crossing demand and constant-payling supply under perfect information, which are respectively $\hat{x}^D(a) = G^{-1}(1-a)$ and $\hat{x}^p(a) = G^{-1}(1-pa)$.[34] Given any acceptance standard x, with perfect information all applicants with $\theta \geq x$ perfectly anticipate that they will succeed and thus apply so as to obtain $v > c$. But only a fraction p of these applicants can succeed, according to the constant-payline supply equation. Thus, if $a > 0$, a fraction $(1-p)$ of applicants cannot succeed. But this means that the applicants with types below the $1-p$ quantile of the conditional type distribution among applicants, having perfect information and thus anticipating that they will not succeed, actually strictly prefer not to apply so as to save the application cost c. The constant-payline equilibrium with perfect information always unravels for $p < 1$, with $x^E = \overline{\theta}$ and zero applications are submitted $a^E = 0$.[35] This unraveling logic highlights how grading on the curve, if perfect, destroys participation incentives.

The following characterization of unraveling of the constant payline equilibrium is more subtle and plays a central role in the paper:

[34]The perfect information constant-payline supply is obtained solving $\int_{\max\langle G^{-1}(1-a),x\rangle}^{\overline{\theta}} g(\theta)\,d\theta = pa$.

[35]Or, equivalently, only the highest type $\overline{\theta}$ (measure-zero) applies and is awarded a fraction p of the grant.

Proposition 2 (Unraveling) *(g) When the noise distribution has unbounded support ($\underline{\varepsilon} = -\infty$ and $\overline{\varepsilon} = \infty$), there is a stable constant-payline equilibrium with unraveling, $a = 0$, if and only if*

$$\sigma \leq \hat{\sigma} = \frac{f\left(F^{-1}\left(1-c/v\right)\right)}{p-c/v} \Big/ \lim_{\theta \to \overline{\theta}} \frac{g(\theta)}{1-G(\theta)}. \tag{10}$$

Under this condition the constant-payline supply has a vertical intercept at $a = 0$ above the intercept of the demand curve, thus there is an equilibrium in which no candidates apply. This result has the following important implications:

- If the hazard rate of the type distribution is unbounded above $\lim_{\theta \to \overline{\theta}} g(\theta)/[1-G(\theta)] = \infty$, then the constant-payline equilibrium unravels *if and only if* the evaluator has perfect information, $\sigma = 0$.[36]

- If the type distribution has bounded hazard rate, $\lim_{\theta \to \overline{\theta}} g(\theta)/[1-G(\theta)] < \infty$, the threshold $\hat{\sigma}$ is bounded away from zero. Unraveling then results in the non-empty interval $\sigma \in [0, \hat{\sigma}]$. The thicker the tail of the type distribution, the larger $\hat{\sigma}$.

- If the hazard rate of the type distribution is not only bounded but also increasing, the unraveling equilibrium, when it results under condition 10, is *unique*.[37]

- Combining parts (g) and (c) we conclude that there is *always* a stable constant-payline equilibrium with unraveling for any σ if the type distribution has decreasing hazard rate at the top (with inequality IHR reversed in a left neighborhood of $\overline{\theta}$).

The proof in the appendix reports the general formula that also allows for noise with bounded support.[38] In the boundary case with negative exponential type distribution $g(\theta) = \alpha \exp(-\alpha\theta)$, for $\sigma < \hat{\sigma}$ unraveling $a^p = 0$ results in the unique stable equilibrium, at the boundary $\sigma = \hat{\sigma}$ there is a continuum of equilibria for *any* $a \in [0, 1]$, and for $\sigma > \hat{\sigma}$ all candidates apply $a = 1$ in the unique stable equilibrium.[39]

The mechanism that leads to unraveling in our model—with no applications being submitted in equilibrium in fields with perfect (of sufficiently precise) evaluation—is reminiscent of Akerlof's (1970) market for lemons. However, in our setting unraveling leads to breakdown of applications in fields where information is symmetric, rather than asymmetric as in Akerlof. Candidates who are able to predict how they will be evaluated prefer to hold out and save the application cost, unless they are

[36]This is the case for all distributions with bounded support, such as the uniform distribution used in the figures presented so far, as well as for the normal distribution.

[37]For example, the logistic or Gumbel extreme value distributions have increasing and bounded hazard rate.

[38]When the noise distribution is bounded, the support of the signal shifts with θ—some signals then perfectly reveal the applicant's type. Formula (26) adjusts (10) to take into account that some applicant types succeed with probability one.

[39]Equation (27) in the proof reports the expression for $\hat{\sigma}$ for an example where the noise is also exponential.

confident of being accepted. Fields with accurate evaluation are driven out by fields with noisier evaluation. Proportional allocation creates perverse incentives that allow fields where information is more asymmetric to thrive.[40]

3.4 Impact of Noise Dispersion

We are now ready for the headline result. For all parameter values, applications in all stable constant-payline equilibria increase in the amount of noise in the signal—or, equivalently, decreases in signal accuracy:

Proposition 2 (Dispersion) *In every stable constant-payline equilibrium, applications (when interior)*
(h) *(strictly) increase in noise dispersion σ and*
(i) *(strictly) increase in noise dispersion σ more than in a fixed-budget equilibrium with the same budget $B = pa$:*

$$\frac{da^p}{d\sigma} \geq \frac{da^{B=pa^p}}{d\sigma} \geq 0.$$

As shown in the proof, the sign of the impact is reversed for unstable equilibria, for which applications decrease in signal dispersion.[41] In combination with part (b), this comparative statics holds under IMES, when the constant payline supply curve is flatter than the demand. Note that an increase in signal dispersion induces contrasting effects on demand and supply: when demand increases, supply increases by less; when demand decreases, supply decreased by more. Nevertheless, the overall impact is unambiguous: constant-payline equilibrium applications necessarily increase in all stable equilibria—and they increase more than in a fixed budget equilibrium that spends the same budget.

If the evaluator signal is *completely uninformative* ($\sigma \to \infty$), the scheme becomes a lottery. Given that the signal contains no information, the evaluator selects winners randomly. If $p > c/v$ then all candidates apply, $a^p = 1$. As σ decreases, at some point some candidates at the bottom of the distribution expect that their acceptance probability is too low to justify spending the application cost. By the monotone structure of the equilibrium, only top researchers self select into applying. Within this self-selected pool, only the top p applications are successful. As σ is reduced further, better and better low-end applicants withdraw.

This drawback might explain why relative grading is much less common for elective courses where it would generate a race to the bottom—grading on a curve tends to be used for core classes for which students have no choice whether to enrol.

[40]While our model takes a static perspective, it would be interesting to analyze how grant funding dynamically impacts the career of applicants across fields. In a tractable dynamic framework, Bardhi, Guo, and Strulovici (2020) characterize environments in which costly experimentation amplifies small differences in ability.

[41]The analysis in the text naturally focuses on stable equilibria. As the definition in footnote 25 makes clear, stable equilibria are more sensible than unstable equilibria. The appendix gives a complete treatment.

4 Partial Equilibrium in a Single Field with Decreasing Payline

The analysis in Section 3 covers the case with a small field that does not affect the payline, as in a small country model in international trade. We now construct the partial equilibrium for a large field where payline is *decreasing* in *applications* in the *same* field

$$\frac{\partial p_i}{\partial a_i} \leq 0. \tag{DAS}$$

Naturally, this condition holds under the proportional allocation rule PA presented in the Introduction, as well as under more general rules analyzed in the next section. As in the previous section, this is still a partial equilibrium analysis, where we consider a field in isolation, and thus disregard the general equilibrium effects generated by the fact that the allocation in the other fields should also be adjusted to make sure the budget constraint (11) holds.

Under DAS we have:

Proposition 3 (Partial Equilibrium) ***(a)*** *If IMES (or a fortiori IHR) holds globally, there is a partial equilibrium and this equilibrium is stable.*
(b) *In any (interior) stable partial equilibrium, applications increase (strictly) in noise dispersion:* $\partial a^i / \partial \sigma_i \geq 0$.

According to parts (a) and (b), the stability and comparative statics properties of the partial equilibrium are determined by the same key conditions IMES and IHR we derived for the constant payline equilibrium in parts (e) and (h) of Proposition 2. Compared to the case with constant payline, as applications in a field increase, in a partial equilibrium the payline is now reduced according to DAS, thus shifting the supply curve up and making it less negatively sloped and thus preserving stability.[42] If the constant payline equilibrium is stable, a fortiori the partial equilibrium must also be stable. Compared to the case with constant payline, the impact of dispersion on equilibrium applications is dampened but remains positive, $da_i^p / d\sigma_i > da^i / d\sigma_i$—by the same logic driving Proposition 2.i. Once the adverse response of the payline to the increase in applications is taken into account, unraveling results a fortiori under the condition reported in Proposition 2.

5 Allocation Across Fields

We now turn to the full problem of grant allocation across fields $i = 1, ..., N$, each populated by a continuum of candidates representing the pool of potential applicants. Each field is characterized by specific

[42]Note that while the partial equilibrium supply is always downward sloping, the partial equilibrium supply can be U shaped when the payline decreases sufficiently in applications (for example, under PA when the field is sufficiently large); see the supply in red in Figure 6.

parameters, such as type distribution G_i, signal noise distribution F_i, noise dispersion σ_i, application cost c_i, and private benefit v_i from obtaining a grant.[43] As in our baseline model, candidates are atomistic and thus they do not take into account the impact of their application decision on the acceptance standard.

5.1 Formula-Based Apportionment

Given the total number of grants B available for distribution to all fields, the following budget constraint must hold across fields

$$\sum_{i=1}^{N} B_i = B, \tag{11}$$

where we can always write the budget of grants made available to field i as

$$B_i = a_i p_i \left(a_i, a_{-i}, B\right), \tag{12}$$

the product of the applications in the field and the field-specific budget per-application, $p_i\left(a_i, a_{-i}, B\right)$. We restrict attention to *sub-proportional allocation* rules for which the Jacobian matrix of the budget per-application vector $\mathbf{p}$

$$\mathbf{dp} = \left(\frac{\partial p_i}{\partial a_j}\right) \text{ is } \textit{totally negative}, \tag{SPA}$$

i.e., the determinants of any $q \times q$ square submatrix with rows $\left(i_1, ..., i_q\right)$ and columns $\left(j_1, ..., j_q\right)$ where $1 \leq i_1 \leq \cdots \leq i_q \leq N$ and $1 \leq j_1 \leq \cdots \leq j_q \leq N$ for any $q \in \{1, ..., N\}$ are nonnegative (nonpositive) if q is even (odd):

$$\operatorname{sgn}\det\begin{pmatrix} \frac{\partial p_{i_1}}{\partial a_{j_1}} & \frac{\partial p_{i_1}}{\partial a_{j_2}} & \cdots & \frac{\partial p_{i_1}}{\partial a_{j_q}} \\ \frac{\partial p_{i_2}}{\partial a_{j_1}} & \frac{\partial p_{i_2}}{\partial a_{j_2}} & \cdots & \frac{\partial p_{i_2}}{\partial a_{j_q}} \\ \vdots & & \ddots & \vdots \\ \frac{\partial p_{i_q}}{\partial a_{j_1}} & \frac{\partial p_{i_q}}{\partial a_{j_2}} & \cdots & \frac{\partial p_{i_q}}{\partial a_{j_q}} \end{pmatrix} = \operatorname{sgn}\left(-1\right)^q.$$

Note that total negativity requires that the signs alternate for *all* the minors of the matrix, non only for the principal minors (i.e., the determinants of the submatrices obtained by deleting the *same* rows and columns)—the weaker condition that defines negative semidefinite matrices.[44]

[43]The model can be easily extended to allow for fields to have different size and for the budget that each applicant can request to vary across fields, so that if fraction a_i of candidates apply in field i of size n_i the total funds requested in the field are $n_i q_i a_i$. In practice, grant calls typically set upper bounds to the size of the award applicants can ask, sometimes depending on the career stage of the applicant. The ERC sets the maximum allowed awards at the same level for all fields. Given that almost all applicants request (and successful applicants are awarded) approximately the maximum allowed, we do not model the individual choice of amount by the applicant. In the more general case in which grant applicants request awards of different size, panel i selects the projects with the highest score so as to distribute the fraction $100 \times p$ of the total funds applied for in field i.

[44]Following Karlin's (1968) traditional terminology, the sign inequality we impose is weak, allowing for equality; see also Pinkus (2010). With the alternative convention advocated by Fallat and Johnson (2011), SPA requires that the Jacobian of the payline matrix is totally nonpositive.

To illustrate consider the case with $N = 2$ fields. Condition SPA for the principal submatrix with a single row and corresponding column implies that the payline in a field must be decreasing in applications in the same field, condition DAS we considered in the partial equilibrium analysis in the previous section, or equivalently that the elasticity of the budget assigned to each field is less than one

$$\frac{\partial B_i}{\partial a_i}\frac{a_i}{B_i} \leq 1, \tag{13}$$

i.e., the budget increases less than proportionally with applications. Under the proportional allocation rule PA this condition boils down to $\sum_{j\neq i} B_i/B < 1$, which is clearly satisfied. Condition SPA for the full matrix is

$$\begin{vmatrix} \frac{\partial p_i}{\partial a_i} & \frac{\partial p_i}{\partial a_j} \\ \frac{\partial p_i}{\partial a_j} & \frac{\partial p_j}{\partial a_j} \end{vmatrix} \geq 0 \text{ for all } i, j,$$

which, combined with the budget constraint (11), equivalently requires that the sum of the elasticities of the budget assigned to each two fields is less than one

$$\frac{\partial p_i}{\partial a_i}\frac{a_i}{B_i} + \frac{\partial p_j}{\partial a_j}\frac{a_j}{B_j} \leq -1 \Leftrightarrow \frac{\partial B_i}{\partial a_i}\frac{a_i}{B_i} + \frac{\partial B_j}{\partial a_j}\frac{a_j}{B_j} \leq 1. \tag{14}$$

It is immediate to see that this condition holds with *equality* under proportional allocation PA. Condition SPA for the two submatrices with different rows and columns requires that the payline (and thus the budget allocated to a field) is *decreasing* in *applications* in the *other* field

$$\frac{\partial p_i}{\partial a_j} \leq 0 \Leftrightarrow \frac{\partial B_i}{\partial a_j}\frac{a_j}{B_i} \leq 0 \text{ for } i \neq j. \tag{DAO}$$

Condition SPA is satisfied by the general class of *quasi-proportional* budget *allocation* rules

$$p_i(a_i, a_{-i}, B) = \frac{B_i}{a_i} = \frac{a_i^{\rho_i - 1}}{\sum_{j=1}^J a_j^{\rho_j}} B \Rightarrow B_i = \frac{a_i^{\rho_i}}{\sum_{j=1}^J a_j^{\rho_j}} B \tag{QPA}$$

with *proportionality coefficients* ρ_i, encompassing a number of common rules. For $\rho_i = 1$ for all i, we obtain the PA rule used by the ERC, NIH, and Canadian research funding organizations. For $\rho_i = 0$ for all i we recover the fixed budget rule adopted by the NSF as well as by UK and Australian agencies. Under QPA the field budget elasticity with respect to field applications is $(\partial B_i/\partial a_i)(a_i/B_i) = \rho_i \left(\sum_{j\neq i} B_i\right)/B$, so that (13) and (14) hold if $\rho_i \leq 1$. The cross budget elasticity is $\left(\partial B_i/\partial a_j\right)\left(a_j/B_i\right) = -\rho_j B_j/B$, so that condition DAO is satisfied if $\rho_j \geq 0$. More generally:

Proposition 4 *Quasi-proportional allocation QPA with* $\rho_i \in [0, 1]$ *is sub-proportional SPA.*

5.2 Full Equilibrium in All Fields

Building on Section 4's analysis of the partial equilibrium in a single field for given applications in all the other fields, we now turn to the full equilibrium allocation that results once we account for the general equilibrium adjustments across fields necessary for the budget constraint (11) to hold. Extending the logic of Proposition 3, we obtain the following characterization of the full equilibrium with multiple fields for all SPA rules:

Proposition 5 (Characterization of Full Equilibrium) ***(a)*** *If the type distributions in every field satisfy IMES or IHR, the full equilibrium is unique and stable.*
(b) *In any (interior) stable full equilibria, applications in any field i* ***(i)*** *(strictly) increase in the noise dispersion in that field*

$$\frac{da_i^F}{d\sigma_i} \geq 0$$

and ***(ii)*** *(strictly) decrease in the noise dispersion in any other field*

$$\frac{da_i^F}{d\sigma_j} \leq 0.$$

The comparative statics for unstable equilibria is reversed. To illustrate the logic of the result, consider $N = 2$ fields with proportional allocation PA. Holding constant applications in field 2, by Proposition 3 stable partial equilibrium applications in field 1 increase in σ_1. In general equilibrium, by PA the increase in applications raises the field budget B_1 at the expense of the other field. As B_2 is reduced, equilibrium applications in the other fields decrease. This, in turn, increases the budget available for field 1, thus dampening the initial reduction in budget created by the increase in applications.

To illustrate the result, the right panel of Figure 5 displays the full supply (red) resulting in field 1 with Pareto Lomax distributed types that competes with field 2 where types are uniformly distributed, with uniform signals in both fields. As for the constant-payline equilibrium reported in the left panel and explained at the end of Section 3.2, there exists a critical level of the signal noise $\tilde{\sigma}_1$ (corresponding to the red arrows) below which field 1 unravels with $a_1 = 0$ in all full equilibria.[45] For $\sigma_1 > \tilde{\sigma}_1$, in addition to the unraveling equilibrium, there two interior equilibria, with applications in the stable (unstable) increasing (decreasing) in σ_1, as shown by the green arrows along the dashed black path.

5.3 Full Equilibrium Unraveling under Proportional Allocation

When allocation is proportional PA, we argue that applications necessarily unravel in all fields i with perfect evaluation $\sigma_i = 0$, provided that there is at least one field j with noisy evaluation $\sigma_j > 0$ and that the total amount of applications outstrips the budget, so that the equilibrium payline is less than one.

[45] Note that $\tilde{\sigma}_1 > \hat{\sigma}_1$ characterized in the example at the end of Section 3.2.

First, note that with perfect evaluation in field i, demand is $a_i^D(\hat{x}) = 1 - G(\hat{x})$ and supply is $\hat{x}_i^S(a_i, a_{-i}) = G^{-1}\left(1 - \frac{Ba_i}{a_i + a_{-i}}\right)$ for given $a_{-i} := \sum_{j \neq i} a_j$. The partial equilibrium for a field with $\sigma_i = 0$ given a_{-i} is $a_i(a_{-i}) = \max\langle B - a_{-i}, 0\rangle$.

Second, with perfect evaluation in *all* fields $i = 1, ..., N$, there is a large set of multiple equilibria with $p = 1$. Any $\mathbf{a}$ such that $\sum_{i=1}^N a_i = B$ where all applicants are sure to win is an equilibrium, but the winning applicants can be from any of the fields. In particular, there is a symmetric equilibrium in which $a_i = B/N$ for all i. There are also extreme equilibria in which applications in a field are zero, provided applications in the other fields is sufficiently large to scoop up all the available funds, B.

Finally, suppose that there is at least one field j with $\sigma_j > 0$ and n_i fields with perfect evaluation $\sigma_i = 0$. If the total budget $B < N - n_i$ so that the equilibrium payline $p < 1$, applications necessarily unravel in all the fields with $\sigma_i = 0$, as claimed.

Unraveling across Fields with Constant (or Decreasing) Hazard Rate. When types are exponentially distributed $G_i(\theta) = 1 - \exp(-\alpha_i\theta)$ and the signal is also exponential in every field $F_i(\varepsilon) = 1 - \exp(-\varepsilon)$ with σ_i, the full equilibrium takes a particularly simple form. Order fields $i = 1, ..., N$ by the index

$$\iota_i = \frac{1 - \alpha_i\sigma_i}{(c_i/v_i)^{\alpha_i\sigma_i} - \alpha_i\sigma_i c_i/v_i}, \tag{15}$$

from lowest to highest. The index increases in dispersion σ_i, and decreases in the expectation of the prior type distribution $1/\alpha_i$ in the field, and decreases in the cost-benefit ratio c_i/v_i. For example if $B < 1$ (so that budget of grants is lower than the size of a single field, here normalized to one), in the unique full equilibrium all grants are scooped up by a single field, the field with highest index ι_i. If all other parameters are identical, this is the field with noisier evaluation, worse average type, and lower cost-benefit ratio.

With exponential types, according to Proposition 2 the constant-payline supply is parallel to demand, so that the constant-payline equilibrium features unraveling $a_i^p = 0$ with no candidates applying if $p < 1/\iota_i$ (when supply is above demand), full coverage $a_i^p = 1$ with all candidates applying if $p > 1/\iota_i$ (supply below demand), and extreme multiplicity $a \in [0, 1]$ for $p = 1/\iota_i$ (supply coinciding with demand). When all fields have exponential types and parameters are symmetric across fields, any allocation, including the symmetric allocation $a = B/N$, is a full equilibrium.[46] In this case, even the slightest deviation from symmetric parameters results in a unique equilibrium with unraveling across all fields—where the field with highest index ι_i obtains all the grants. This exponential example confirms the general pattern according to which equilibrium applications in a field increase in dispersion relative to other fields. In addition, as the cost benefit ratio decreases or the type distribution becomes

[46]The multiplicity resulting under symmetric exponential types is similar to the multiplicity that holds under perfect information for any distribution.

less positively skewed in a field, with density more steeply decreasing, field candidates apply more aggressively.[47]

6 Optimal Funding Apportionment and Design

To illustrate how inefficient proportional allocation can be, consider two fields 1 and 2, identical other than for the fact that evaluation is perfect in field 1 but completely uninformative in field 2. The evaluator's value from awarding a grant is equal to the candidate's type θ. Suppose the total budget is equal to $B < 1$. The optimal policy for the evaluator is to allocate the entire budget to field 1 if $G^{-1}(1-B) \geq E[\theta]$, given the two fields have the same type distribution. In the unique PA equilibrium we clearly have $a_1 = 0$ and $a_2 = B$, yielding evaluator surplus of $BE[\theta]$. In this admittedly extreme scenario, proportional allocation is actually the worst possible allocation system.

More generally, we now compare the full equilibrium resulting under PA to the optimal allocation that maximizes the payoff of the evaluator

$$\sum_{i=1}^{N} V_i(a_i) := \sum_{i=1}^{N} \int_{G_i^{-1}(1-a_i)}^{\bar{\theta}_i} \theta \left[1 - F_i\left(\frac{x^D(a_i) - \theta}{\sigma_i}\right)\right] g_i(\theta)\, d\theta$$

subject to the demand system $x_i^D(a_i) = G_i^{-1}(1-a_i) + \sigma_i F_i^{-1}(1 - c_i/v_i)$ under the budget constraint (11)

$$\sum_{i=1}^{N} b_i(a_i) := \sum_{i=1}^{N} \int_{G_i^{-1}(1-a)}^{\bar{\theta}_i} \left[1 - F_i\left(\frac{x_i^D(a_i) - \theta}{\sigma}\right)\right] g_i(\theta)\, d\theta = B,$$

with Lagrangian

$$\mathscr{L}(\mathbf{a}, \lambda) = \sum_{i=1}^{N} V_i(a_i) + \lambda \left(B - \sum_{i=1}^{N} b_i(a_i)\right).$$

To facilitate the comparison we consider a small deviation from a symmetric scenario in which the equilibrium allocation is also optimal:

Proposition 6 *Suppose that (i) the designer problem is strictly concave, $\partial^2 \mathscr{L}/\partial a_i^2 < 0$, and (ii) the shadow value of the budget decreases in noise dispersion, $\partial \lambda / \partial \sigma_i < 0$. Consider a scenario with symmetric parameters in all fields and focus on the unique symmetric stable full equilibrium resulting under proportional allocation PA. As noise dispersion σ_i increases in a field, PA full equilibrium applications in that field increase more than in the designer optimal allocation:*

$$\frac{\partial a_i^{PA}}{\partial \sigma_i} > \frac{\partial a_i^{*}}{\partial \sigma_i}.$$

[47] This pathological result holds more generally for distributions with decreasing hazard rate, such as Weibull with shape parameter $k < 1$. As explained in footnote 28, these distributions are smaller than the exponential distribution in van Zwet (1964) convex transform order. More generally, when all field have the same type distributions with a top tail thicker than exponential the field, if all other parameters are the same, the field with more dispersed signal obtains all the funding.

Condition (i) guarantees that the evaluator optimal allocation is interior and characterized by the first-order condition. Concavity holds under broad conditions; for example, it holds strictly under full information and thus by continuity also when the noise level is low. Concavity also holds strictly when the type distribution is exponential and the signal distribution is also exponential, and thus by continuity it necessarily holds strictly for closeby type distributions with IHR (such as Weibull with $k > 1$) as well as decreasing hazard rate (Weibull with $k < 1$). Condition (ii) is natural, given that an increase in noise dispersion decreases information and thus the payoff of the evaluator.[48]

The argument follows from two claims. First, equilibrium applications in a field under proportional allocation PA increase in noise dispersion more than under fixed budget, corresponding to SPA with $\rho_i = 0$ for all fields—this result is essentially a full equilibrium version of Proposition 2.i. Second, equilibrium applications under fixed budget increase in noise dispersion more than in the designer optimal allocation. As noise dispersion increases in field i, the shadow value of the budget decreases in noise dispersion, which makes it optimal to increase the budget allocated to all the other fields and thus to decrease the budget allocated to field i. We conclude that under fixed budget—and a fortiori under PA—applications increase too much compared to the optimal allocation.

Given the inefficiency of the allocation resulting under PA when fields differ in the informativeness of the evaluation, it is natural to wonder how the designer can improve upon PA. For illustration, deviate from a symmetric scenario by increasing information accuracy for field i, i.e., by decreasing noise dispersion σ_i. By Proposition 6, PA induces too few applications in field i and too many applications in the other fields for the evaluator. Within the SPA class, we can implement the optimal allocation by reducing proportionality (and thus containing the reduction in applications) for the field with lower noise dispersion:

Proposition 7 *Under the same assumptions as Proposition 6, as noise dispersion σ_i decreases in a field, departing from proportional allocation PA the designer optimal allocation can be implemented by reducing proportionality ρ_i in that field.*

As illustrated by Figure 6, start from the evaluator optimal symmetric allocation coinciding with the PA equilibrium marked as A, at the crossing of demand (in blue) and supply (in red).[49] When noise dispersion is reduced to $\sigma_1' < \sigma_1$, demand and supply in field 1 (on the left panel) shift to the dashed curves to the new equilibrium marked as B, with lower applications by Proposition 5.b.i. The payline is decreased, thus shifting down the supply and increasing applications in field 2 (on the left panel) by Proposition 5.b.ii. The optimal allocation for the new parameters at C is implemented by reducing field 1 responsiveness to $\rho_1' < 1$, so as to increase (decrease) supply curves (in fuchsia) and thus applications

[48]See footnote 23.

[49]The supply here is upward sloping, as explained in footnote 42.

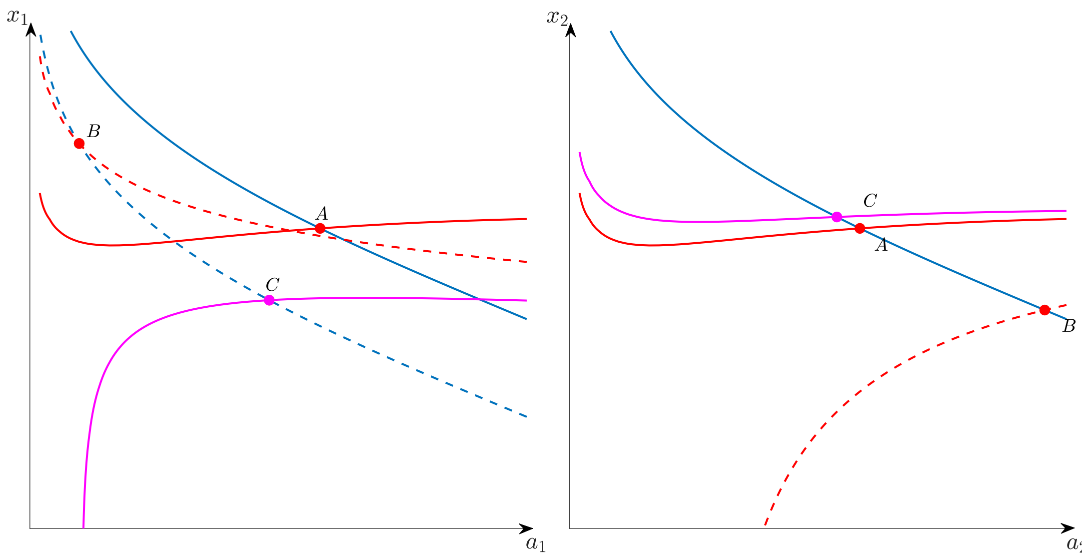

Figure 6: Optimal design of responsiveness of allocation rule.

in field 1 (2). The result follows from the fact that (stable) equilibrium applications in a field decrease in the responsiveness ρ_i in the same field and increase in the responsiveness in the other fields ρ_{-i}.

While we considered the case of a designer who chooses the allocation rule that maximizes evaluator welfare, similar results hold for a social planner who maximizes the sum of evaluator welfare and applicant surplus.

7 Conclusion

Our analysis of proportional allocation immediately applies also to large research fellowships programs, such as the EU-wide Marie Skłodowska-Curie Action (MSCA) scheme that assigns its total budget (6.16 billion euros for the period 2014-2020) in proportion to applications across all disciplines.[50] The macro evolution of funding patterns there, as well as at the ERC in its first twelve years of activity, seem to be broadly consistent with our key comparative statics. For a confirmation, it would help to have data about the agreement across reviewers (inter-rater reliability) in different fields, following on the footsteps of Cole and Cole (1981). The drawbacks our analysis highlights are particularly severe for mechanisms that equalize the success rate among very heterogeneous fields, as is the case for the ERC and MSCA, but perhaps less problematic for funders (like the NIH) that focus research in the same area (medicine, even though NIH study sections cover a wide variety of disciplines, methodologies, and topics).[51]

[50]The Canadian SSHRC Doctoral Fellowships program (covering all humanities and social sciences) also follows PA.

[51]While the great majority of NIH institutes/centers adopt the payline system and publish paylines, it is only understandable that some institutes/centers at the NIH prefer not to publish their paylines, thus retaining some flexibility when treating proposals from different panels.

The bottom-up formula-based approach to funding apportionment analyzed here can be contrasted to alternative top-down approaches, such as those prevailing at the NSF, in the UK, and Australia, where the funding allocated across programs is approved by Parliament through appropriations legislation, following a consultation process and a detailed proposal by the directors of the research funding organizations. Even at agencies that do adopt proportional allocation, success rates for different programs and across fields are regularly published and closely monitored. While differences in success rates across fields in non-proportional systems persist over time, there is an implicit pressure to reduce the budget for fields with higher success rates in favor of fields with lower success rates.

General-interest academic journals are subject to a similar pressure to allocate space to different subfields in proportion to submissions. When co-editors are given a common target acceptance rate, fields with less accurate (or consensual) evaluation will attract more submissions.[52] Similarly, university admission boards are tempted to admit students to different programs in proportion to applications—or to increase slots available in areas that attract more applications. Giving in to this temptation may spark a race to the bottom in terms of quality of admitted students.

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[52] See also Akerlof's (2020) discussion of how a bias toward hardness can arise in science. Our analysis suggests a mechanism through which hardness prevails within a discipline, even though it is detrimental in the competition across disciplines. In our model, individual disciplines tend to be dominated by harder subfields and investigations with more accurate evaluation. When elements of proportionality are present in the allocation of resources across disciplines, disciplines with more accurate evaluation are destined to obtain less resources and thus become less attractive.

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8 Appendix: Proofs

Proof of Proposition 1. (a) Differentiating the demand equation

$$D(x,a;\sigma) = 1 - F\left(\frac{x - G^{-1}(1-a)}{\sigma}\right) - \frac{c}{v} = 0 \tag{16}$$

gives

$$D_x = -\tfrac{1}{\sigma} f\left(\tfrac{x-G^{-1}(1-a)}{\sigma}\right) < 0 \quad D_a = -\tfrac{1}{\sigma} f\left(\tfrac{x-G^{-1}(1-a)}{\sigma}\right) \tfrac{1}{g\left(G^{-1}(1-a)\right)} < 0 \tag{17}$$

so that $da^D/dx = -D_x/D_a = -g\left(G^{-1}(1-a)\right) < 0$.

(b) Differentiating the supply equation

$$S(x,a;\sigma,p)=\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{x-\theta}{\sigma}\right)\right]g(\theta)\,d\theta-B=0, \tag{18}$$

we have

$$S_x=-\int_{G^{-1}(1-a)}^{\bar{\theta}}\tfrac{1}{\sigma}f\left(\tfrac{x-\theta}{\sigma}\right)g(\theta)\,d\theta<0 \quad S_a=1-F\left(\tfrac{x-G^{-1}(1-a)}{\sigma}\right)>0 \tag{19}$$

so that $d\hat{x}^S/da=-S_a/S_x>0$.

(c) Existence follows from standard arguments by applying Brouwer's fixed-point theorem, given that demand and supply are both continuous function. Uniqueness follows from $d\hat{x}^D/da<0<d\hat{x}^S/da$.

(d) Noise dispersion has ambiguous impact on demand, $da^D/d\sigma=-D_\sigma/D_a \gtreqless 0$. From

$$D_\sigma=\frac{\hat{x}-G^{-1}(1-a)}{\sigma^2}f\left(\frac{\hat{x}-G^{-1}(1-a)}{\sigma}\right), \tag{20}$$

the sign of the comparative statics depends on whether the marginal applicant is below or above the acceptance standard on the demand curve, $G^{-1}(1-a)=\hat{\theta}\lesseqgtr\hat{x}$. From (4) this holds whenever $F^{-1}(1-c/v)\gtreqless 0 \Leftrightarrow 1-c/v \gtreqless F(0)$. If the signal distribution satisfies $F(0)=1/2$, which always holds if the signal distribution is symmetric, $F(\varepsilon)=1-F(-\varepsilon)$, we have $da^D/d\sigma \gtreqless 0 \Leftrightarrow c/v \lesseqgtr 1/2$. By Cramer's rule comparative statics of the fixed budget equilibrium with respect to σ is

$$\frac{da}{d\sigma}=-\begin{vmatrix} D_x & D_\sigma \\ S_x & S_\sigma \end{vmatrix}\Bigg/\begin{vmatrix} D_x & D_a \\ S_x & S_a \end{vmatrix}.$$

The determinant at the denominator is negative by $d\hat{x}^D/da<0<d\hat{x}^S/da$. Using (17), (19), (20), and

$$S_\sigma=\int_{G^{-1}(1-a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)\frac{x-\theta}{\sigma}g(\theta)\,d\theta,$$

the determinant at the numerator $D_xS_\sigma-D_\sigma S_x$ has the same sign as

$$\int_{G^{-1}(1-a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)\frac{\theta-G^{-1}(1-a)}{\sigma}g(\theta)\,d\theta>0,$$

thus $da^B/d\sigma>0$.

(e) The claim follows from $d\hat{x}^S/dB=-S_B/S_x>0$, using (19) and $S_B=-1$.

(f) An increase in the budget shifts the (upward sloping) fixed-budget supply left and to the right, along the same (downward sloping) demand curve, thus resulting in an increase in equilibrium applications and in a reduction in the acceptance standard, as displayed in Figure 3. Given that the constant-payline locus shifts the constant-payline locus down and to the left as the payline increases, we conclude that the success rate increases if and only if the constant-payline locus is flatter than demand, which by Proposition 2.c. holds if and only if the hazard rate of the type distribution is increasing.

Proof of Proposition 2. (a) Differentiating the constant-payline locus

$$P(x,a;\sigma,p)=\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{x-\theta}{\sigma}\right)\right]g(\theta)\,d\theta}{a}-p=0, \tag{21}$$

we have

$$\begin{aligned}
&P_x=-\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)g(\theta)d\theta}{a} && P_a=\frac{1}{a}\left[1-F\left(\frac{x-G^{-1}(1-a)}{\sigma}\right)-\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{x-\theta}{\sigma}\right)\right]g(\theta)d\theta}{a}\right]\\
&P_p=-1 && P_\sigma=\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)\frac{x-\theta}{\sigma}g(\theta)d\theta}{a}.
\end{aligned} \tag{22}$$

Clearly, $P_x<0$. Next, we establish that $P_a<0$. Integrating by parts the left-hand side of the constant-payline locus (6) we have

$$\begin{aligned}
&\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right]g(\theta)\,d\theta\\
=\ &\frac{1}{a}\left[1-F\left(\frac{\hat{x}-\theta}{\sigma}\right)\right]G(\theta)\Bigg|_{G^{-1}(1-a)}^{\bar{\theta}}-\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{\hat{x}-\theta}{\sigma}\right)G(\theta)\,d\theta\\
=\ &1-F\left(\frac{\hat{x}-G^{-1}(1-a)}{\sigma}\right)+\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-G(\theta)\right]\frac{1}{\sigma}f\left(\frac{\hat{x}-\theta}{\sigma}\right)d\theta.
\end{aligned} \tag{23}$$

The first term on the left-hand side of (8) is the same as in the demand equation (9), while on the right-hand side c/v is replaced by p. Thus, the *pseudo-supply locus*—defined as function $x(s)$ that equates to p the right-hand side of (23) without the second term—is parallel to demand, as represented by the dashed brown curve in Figure 4. The second term in (23) determines the difference between the constant-payline locus (brown) and the pseudo-supply locus (dashed brown). Substituting (23) into the expression for P_a in (6) and simplifying we find

$$P_a=-\frac{1}{a}\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-G(\theta)\right]\frac{1}{\sigma}f\left(\frac{\hat{x}-\theta}{\sigma}\right)d\theta<0.$$

By implicit differentiation we conclude that the constant-payline locus slopes down,

$$\frac{\partial\hat{x}^p}{\partial a}=-\frac{P_a}{P_x}=-\frac{\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-G(\theta)\right]\frac{1}{\sigma}f\left(\frac{\hat{x}-\theta}{\sigma}\right)d\theta}{\int_{G^{-1}(1-a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{\hat{x}-\theta}{\sigma}\right)g(\theta)\,d\theta}<0. \tag{24}$$

(b) From $P_x<0$ and $P_p=-1<0$ we conclude $dx^p/dp=-P_p/P_x=1/P_x<0$.

(c) Comparing (24) and $\partial\hat{x}^D/\partial a=-1/g\left(G^{-1}(1-a)\right)$ at $a=1-G(\theta)$ we conclude that the constant budget supply is less negatively sloped than demand, $\frac{\partial\hat{x}^p}{\partial a}>\frac{\partial\hat{x}^D}{\partial a}$, whenever IMES holds or equivalently under

$$\int_{\hat{\theta}(a)}^{\bar{\theta}}\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)\left[\frac{g(\theta)}{g\left(\hat{\theta}(a)\right)}-\frac{1-G(\theta)}{1-G\left(\hat{\theta}(a)\right)}\right]d\theta>0.$$

Thus, IHR is sufficient for IMES.

(d) The result follows from part (c) and the discussion in the text.

(e) If IMES (or a fortiori IHR) holds not only locally but also globally, the constant-payline supply being always flatter than demand either (i) starts and stays always below demand (in which case the equilibrium is at $a^p = 1$) or (ii) starts below and crosses demand once from below, with a single interior crossing at $a^p > 0$, or (iii) starts above and stays above demand so that unraveling results $a^p = 0$ (under the conditions characterized by Proposition 2). In all cases the equilibrium outcome is stable and unique.

(f) The difference between the inverse demand (4) and the pseudo supply $G^{-1}(1-a)+\sigma F^{-1}(1-p)$ we constructed in the proof of Proposition 2 is equal to $\sigma\left[F^{-1}(1-c/v)-F^{-1}(1-p)\right]$. If the *mean excess success* function

$$\frac{\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-G(\theta)\right]\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)d\theta}{\int_{G^{-1}(1-a)}^{\bar{\theta}}g(\theta)\frac{1}{\sigma}f\left(\frac{x-\theta}{\sigma}\right)d\theta}$$

is non-monotonic we can then choose parameters σ, c/v, and p to obtain multiple equilibria.

(g) The result follows from comparing the vertical intercept of demand (x_0^D) with the vertical intercept of supply (x_0^p) in the limit at $a \to 0$. (I) First, if the type distribution have bounded support, $G^{-1}(1) = \bar{\theta} < \infty$, we have

$$x_0^D = \lim_{a\to 0} G^{-1}(1-a) + \sigma F^{-1}(1-c/v) = \bar{\theta} + \sigma F^{-1}(1-c/v).$$

From (23), the vertical intercept of supply solves

$$1-F\left(\frac{x_0^p-\bar{\theta}}{\sigma}\right)+\lim_{a\to 0}\frac{1}{a}\int_{G^{-1}(1-a)}^{\bar{\theta}}\left[1-G(\theta)\right]\frac{1}{\sigma}f\left(\frac{x_0^p-\theta}{\sigma}\right)d\theta = p \tag{25}$$

where the second term on the right-hand side is zero. Thus, if $\sigma > 0$ we have $x_0^p = \bar{\theta}+\sigma F^{-1}(1-p) \lesseqgtr x_0^D \Leftrightarrow p \gtreqless c/v$. If, instead, $\sigma = 0$ we have $x_0^p = x_0^D$. We conclude that when the support is bounded there is an unraveling equilibrium ($a^p = 0$) for all σ if $p \leq c/v$ but only for $\sigma = 0$ if $p < c/v$. (II) Turning to the case with type distribution with unbounded support $G^{-1}(1) = \bar{\theta} = \infty$, we now show that if the hazard rate is unbounded we obtain the same conclusion as with bounded support; when instead the hazard rate is bounded, equilibrium unraveling results for a larger set of parameters. When the support is unbounded, by l'Hôpital's rule the second term of (25) converges to

$$\lim_{\bar{\theta}\to\infty}\frac{1}{\sigma}f\left(\frac{x_0^p-\bar{\theta}}{\sigma}\right)\Big/\frac{g(\bar{\theta})}{1-G(\bar{\theta})}.$$

Computing this at $x_0^p = x_0^D = \bar{\theta}+\sigma F^{-1}(1-c/v)$ we find the boundary level $\hat{\sigma}$ given in (10). In conclusion, when $\sigma < \hat{\sigma}$ the supply starts off above demand in the right neighborhood of $a = 0$ so that there is a stable unraveling equilibrium.

Following the same steps, formula (10) can be generalized to cover noise distributions with bounded support, as discussed in footnote 38. There is a stable constant-payline equilibrium with unraveling, $a = 0$, if and only if $\sigma \leq \hat{\sigma}$ where $\hat{\sigma}$ solves

$$\lim_{\theta \to \underline{\theta}} \frac{1}{\hat{\sigma}} \left(\frac{f\left(F^{-1}(1-c/v)\right)}{\frac{g(\theta)}{1-G(\theta)}} - \frac{f(\underline{\varepsilon})}{\frac{g(\theta)}{1-G(\hat{x}^D - \sigma\underline{\varepsilon})}} \right) - \lim_{\theta \to \underline{\theta}} \int_{\theta}^{\hat{x}^D - \hat{\sigma}\underline{\varepsilon}} \frac{\partial \frac{1}{\hat{\sigma}} f\left(\frac{\hat{x}^D - \tilde{\theta}}{\hat{\sigma}}\right)}{\partial x^D} \frac{1-G(\tilde{\theta})}{g(\theta)} d\tilde{\theta} = p - c/v \tag{26}$$

where $\hat{x}^D = \theta + \sigma F^{-1}(1-c/v)$. For example, when types are exponential, $G(\theta) = 1 - \exp(-\alpha\theta)$ and the noise is also exponential $F(\varepsilon) = 1 - \exp(-\varepsilon)$ with $\underline{\theta} = \underline{\varepsilon} = 0$ and $\overline{\theta} = \overline{\varepsilon} = \infty$, applying (26) we obtain that $\hat{\sigma}$ solves

$$p = \frac{(c/v)^{\alpha\hat{\sigma}} - \alpha\hat{\sigma}c/v}{1 - \alpha\hat{\sigma}}. \tag{27}$$

At this parameter boundary, the demand is exactly identical to the constant-payline supply, so that any $a^p \in [0,1]$ constitutes a constant-payline equilibrium.

(h) Applying the implicit function theorem to the system (16) and (21) gives

$$\frac{da}{d\sigma} = -\begin{vmatrix} D_x & D_\sigma \\ P_x & P_\sigma \end{vmatrix} \Big/ \begin{vmatrix} D_x & D_a \\ P_x & P_a \end{vmatrix} \tag{28}$$

The determinant at the denominator is negative (positive) if and only if the constant payline equilibrium is stable—i.e., the supply is flatter (steeper) than the demand. From (17) and (22), the determinant at the numerator of (28) is always positive

$$\begin{aligned} J_{[a]\sigma} := \begin{vmatrix} D_x & D_\sigma \\ P_x & P_\sigma \end{vmatrix} &= -\frac{1}{a\sigma} f\left(\frac{x - G^{-1}(1-a)}{\sigma}\right) \\ &\left[\int_{G^{-1}(1-a)}^{\overline{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \frac{x-\theta}{\sigma} g(\theta)\, d\theta - \frac{x - G^{-1}(1-a)}{\sigma} \int_{G^{-1}(1-a)}^{\overline{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta)\, d\theta\right] \\ = \; & -\frac{1}{\sigma} f\left(\frac{x - G^{-1}(1-a)}{\sigma}\right) \frac{1}{a} \int_{G^{-1}(1-a)}^{\overline{\theta}} \frac{G^{-1}(1-a) - \theta}{\sigma} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) g(\theta)\, d\theta \geq 0. \end{aligned} \tag{29}$$

We conclude that $da^p/d\sigma \geq 0$ for all stable equilibria and $da^p/d\sigma \leq 0$ for all unstable equilibria, with strict inequalities for interior equilibria. Finally, $da^p/d\sigma \geq da^{B=pa}/d\sigma$ holds because the two expressions have the same numerator, while the denominator of $da^p/d\sigma$ (which remains positive when the constant payline equilibrium is stable) is equal to the denominator of $da^{B=pa}/d\sigma$ minus $p > 0$.

Proof of Proposition 4. We distinguish (i) principal from (ii) nonprincipal submatrices of the Jacobian of the payline matrix. (i) Consider first the principal submatrices constructed by eliminating from the payline matrix Jacobian the same rows $\{i_{1,\ldots,}i_q\} =: \iota_q$ and columns $\{j_1 = i_1, \ldots, j_q = i_q\}$, thus retaining rows and columns with indexes in the set $\mathcal{N} \setminus \iota_q = \{1, \ldots, N\} \setminus \{i_{1,\ldots,}i_q\}$. Computing from

QPA the own effect $\partial p_i/\partial a_i = -B[1-\rho_i(1-p_i a_i)]\, p_i/a_i$ and the cross effect $\partial p_i/\partial a_j = -B\rho_j p_j p_i$ for $j \neq i$, the determinant of these submatrices is

$$(-1)^{N-q} B^{N-q} \frac{\prod_{i\in\mathcal{N}\setminus\iota_q} a_i^{\rho_i-2}}{\left(\sum_{i\in\mathcal{N}} a_i\right)^{1+N-q}} \left[\sum_{i\in\mathcal{N}\setminus\iota_q} a_i^{\rho_i} \prod_{j\in\mathcal{N}\setminus\iota_q,\neq i} \left(1-\rho_j\right) + \prod_{i\in\mathcal{N}\setminus\iota_q} (1-\rho_i) \sum_{k\in\iota_q} a_k^{\rho_k}\right].$$

Given that the factor in brackets is always positive when $\rho_i \leq 1$ in all fields i, the sign of this minor is negative (positive) whenever $N-q$ is odd (even) if $\rho_j \in [0,1]$ for all $i,j \in \{1,\ldots N\}$.

(ii) Next, non-principal submatrices can be seen as submatrices of a matrix obtained by first eliminating a row i and a different column $j \neq i$. We distinguish two types of non-principal submatrices. (a) First, consider non-principal submatrices constructed by eliminating, in addition to row i and column j, the same set of rows and columns. The determinant of a submatrices of $\mathbf{dp}[\mathcal{N}\setminus i, \mathcal{N}\setminus j]$, to which we subtracted rows $\{i_{1,\ldots,}i_q\} \not\supseteq i,j$ rows and the same columns $\{j_1 = i_1,\ldots,j_q = i_q\}$ resulting in rows $\mathcal{N}\setminus i\setminus\iota_q$ and columns $\mathcal{N}\setminus j\setminus\iota_q$, is

$$(-1)^{N-q-1}\rho_j \prod_{k\in\mathcal{N}\setminus\iota_q,\neq i,j} \frac{1-\rho_k}{a_k} \frac{\prod_{i\in\mathcal{N}\setminus\iota_q} a_i^{\rho_i-1}}{\left(\sum_{i=1}^N a_i^{\rho_i}\right)^{N-q}} B^{N-q-1},$$

which is negative (or positive) whenever $N-q-1$ is odd (or even) if $\rho_j \in [0,1]$ for all $i,j \in \{1,\ldots N\}$. (b) Second, consider non-principal submatrices constructed by eliminating at least two different rows and columns, say rows $i = i'$ and $i = i''$ and columns $j = j$ and $j = j''$. All these submatrices have determinant equal to zero—by Laplace expansion these determinants can be calculated as the weighted sum of determinants of 2×2 submatrices taking the form

$$\frac{\partial p_k}{\partial a_l}\frac{\partial p_m}{\partial a_n} - \frac{\partial p_k}{\partial a_n}\frac{\partial p_m}{\partial a_l} = 0 \text{ with } k \neq l \neq m \neq n,$$

which are zero because $\partial p_i/\partial a_j = -B\rho_j p_j p_i$ for $i \neq j$.

Proof of Proposition 3. (a) Denoting the determinant of the demand and constant-payline supply system by

$$J^{ip} := \begin{vmatrix} \frac{\partial D_i}{\partial x_i} & \frac{\partial D_i}{\partial a_i} \\ \frac{\partial P_i}{\partial x_i} & \frac{\partial P_i}{\partial a_i} \end{vmatrix},$$

the determinant of the system with partial equilibrium supply, once we take into account the dependence of the payline on applications, becomes

$$J^i := \begin{vmatrix} \frac{\partial D_i}{\partial x_i} & \frac{\partial D_i}{\partial a_i} \\ \frac{\partial P_i}{\partial x_i} & \frac{\partial P_i}{\partial a_i} - \frac{\partial p_i}{\partial a_i} \end{vmatrix} = J^{ip} - \frac{\partial D_i}{\partial x_i}\frac{\partial p_i}{\partial a_i} < J^{ip}, \tag{30}$$

where the inequality follows from $\partial D_i/\partial x_i < 0$ and $\partial p_i/\partial a_i < 0$ by assumption (DAS). By IMES we have $J^{ip} < 0$, so that $J^i < 0$. Thus, the partial equilibrium is unique and stable.

(b) The result follows by implicit differentiation and part (a) given that $J_{[a_i]\sigma_i}$ is the same as under constant payline, $da^i/d\sigma_i = -J_{[a_i]\sigma_i}/J^i > 0$.

Proof of Proposition 5. (a) If $B = 0$ there is unique equilibrium at the corner $p = a_i = 0$ for all i. For $B > 0$, there is an equilibrium with $a_i > 0$ for some i. Condition IMES guarantees that any given p determines a unique vector of field-level application rates $a_1, a_2, ..., a_N$; given that the right hand side of (1) is decreasing in p, the overall equilibrium is unique. Turning to stability, recall that the full equilibrium solves the system of $2N$ demand and supply equations obtained by replacing the budget (1) into the supply equations. To compute the determinant of the Jacobian of this system

$$J^{1,\dots,N} := \begin{vmatrix} \frac{\partial D_1}{\partial x_1} & \frac{\partial D_1}{\partial a_1} & & 0 & 0 & & 0 & 0 \\ \frac{\partial S_1}{\partial x_1} & \frac{\partial P_1}{\partial a_1} - \frac{\partial p_1}{\partial a_1} & & 0 & -\frac{\partial p_1}{\partial a_i} & & 0 & -\frac{\partial p_1}{\partial a_N} \\ & & \ddots & & & & & \\ 0 & 0 & & \frac{\partial D_i}{\partial x_i} & \frac{\partial D_i}{\partial a_i} & & 0 & 0 \\ 0 & -\frac{\partial p_i}{\partial a_1} & & \frac{\partial P_i}{\partial x_i} & \frac{\partial P_i}{\partial a_i} - \frac{\partial p_i}{\partial a_i} & & 0 & -\frac{\partial p_i}{\partial a_N} \\ & & & & & \ddots & & \\ 0 & 0 & & 0 & 0 & & \frac{\partial D_N}{\partial x_N} & \frac{\partial D_N}{\partial a_N} \\ 0 & -\frac{\partial p_N}{\partial a_1} & & 0 & -\frac{\partial p_N}{\partial a_i} & & \frac{\partial P_N}{\partial x_N} & \frac{\partial P_N}{\partial a_N} - \frac{\partial p_N}{\partial a_N} \end{vmatrix} \tag{31}$$

we introduce some notation. Given a set $\mathscr{S}$ of integers define by $I_q^{\mathscr{S}}$ the set of all elements $(i_1, ..., i_q)$ in the power set of $\mathscr{S}$ with q elements such that $i_1 \leq \cdots \leq i_q$, with each $i_j \in \mathscr{S}$. For any positive integer N, denote the set of integers up to N as $\mathscr{N} := \{1, ..., N\}$. In particular, the set $I_q^{\mathscr{N}} := \{(i_1, ..., i_q) : 1 \leq i_1 \leq \cdots \leq i_q \leq N\}$ in $\mathbb{N}_+^q$ is the set of strictly increasing sequences of q integers in $\{1, ..., n\}$, as in Pinkus (2010, p. 1). Denoting a generic element as $\iota_q = (i_1, ..., i_q) \in I_q^N$, we then have

$$J^{1,\dots,N} = \prod_{i \in N} \underbrace{J^{ip}}_{-} + \sum_{q=0}^{N-1} (-1)^{N-q} \sum_{\iota_q \in I_q^N} \underbrace{\det \mathbf{dp}_{\mathscr{N} \setminus \iota_q}}_{\text{sign}(-1)^{N-q}} \prod_{j \in \mathscr{N} \setminus \iota_q} \underbrace{\frac{\partial D_j}{\partial x_j}}_{-} \prod_{i \in \iota_q} \underbrace{J^{ip}}_{-}, \tag{32}$$

where the signs indicated below the terms follow from IMES, SPA, and (17). Note that if the partial equilibrium with fixed payline in each field i is stable, $J^{ip} < 0$ (as guaranteed by IMES) for all $i = 1, \dots, N$, this determinant $J^{1,\dots,N}$ has a negative sign when the number N of markets is odd and a positive sign for N even. Thus, the full equilibrium is stable.

Turn to the comparative statics of the full equilibrium. Given that $J^{1,\dots,N}$ and $J^{1,\dots,N \setminus i}$ have opposite sign, we obtain that for any locally stable selection of the partial equilibrium, full equilibrium demand in any field i increases in the dispersion of the evaluation in that field

$$\frac{da_i^F}{d\sigma_i} = -\frac{\overbrace{J_{[a_i]\sigma_i}}^{+} J^{1,\dots,N \setminus i}}{J^{1,\dots,N}} > 0, \tag{33}$$

where $J_{[a_i]\sigma_i} > 0$ follows from Proposition 2.

(b) Next, we compute by Cramer's rule the cross impact of the dispersion in field j on applications in field i. The determinant of the Jacobian obtained by replacing the $2i$-th column of (31) with the column vector

$$\left(\frac{\partial D_i}{\partial \sigma_j} = 0 \quad \frac{\partial P_i}{\partial \sigma_j} = 0 \quad \cdots \quad \frac{\partial D_j}{\partial \sigma_j} \quad \frac{\partial P_j}{\partial \sigma_j} \quad \cdots \quad \frac{\partial D_N}{\partial \sigma_j} = 0 \quad \frac{\partial P_N}{\partial \sigma_j} = 0 \right)^T$$

is

$$J^{1,\dots,N}_{[a_i]\sigma_j} = -\underbrace{J_{[a_j]\sigma_j}}_{+}\underbrace{\frac{\partial D_i}{\partial x_i}}_{-}\left(\sum_{q=0}^{N-2}(-1)^{N-q-1}\sum_{\iota_q \in I_q^{\mathcal{N}\setminus\{i,j\}}}\underbrace{\det \mathbf{dp}_{\mathcal{N}\setminus\iota_q\setminus j,\mathcal{N}\setminus\iota_q\setminus i}}_{\text{sign}(-1)^{N-q-1}}\prod_{h\in\mathcal{N}\setminus\iota_q\setminus\{i.j\}}\underbrace{\frac{\partial D_h}{\partial x_h}}_{-}\prod_{k\in\iota_q}\underbrace{J^{kp}}_{-}\right),$$

which is positive (negative) if N is even (odd), where the signs indicated below the terms follow from (29), (17), SPA, and IMES. By

$$\frac{da_i^F}{d\sigma_j} = -\frac{J^{1,\dots,N}_{[a_i]\sigma_j}}{J^{1,\dots,N}} < 0,$$

we conclude that for any stable equilibrium, for which $J^{1,\dots,N}$ is negative (positive) if N is even (odd), full equilibrium demand in any field i decreases in the dispersion of the evaluation in any other field j. The comparative statics for unstable equilibria is reversed.

Proof of Proposition 6. The result follows from the following two claims:

Claim 1 *Starting from the unique symmetric stable full equilibrium resulting under proportional allocation PA ($\rho_i = 1$ in all fields) with symmetric parameters in all fields, as noise dispersion σ_i increases in a field, full equilibrium applications in that field increase more under equal fixed budget ($\rho_i = 0$ in all fields).*

Proof of Claim 1. Using (33), we now establish the following comparison between $\partial a_i/\partial\sigma_i$ under PA for $\rho_i = 1$ and for $\rho_i = 0$ and

$$\frac{\partial a_i^{\rho=1}}{\partial \sigma_i} = -\frac{J_{[a_i]\sigma_i}J^{1,\dots,N\setminus i}_{\rho=1}}{J^{1,\dots,N}_{\rho=1}} > -\frac{J_{[a_i]\sigma_i}J^{1,\dots,N\setminus i}_{\rho=0}}{J^{1,\dots,N}_{\rho=0}} = \frac{\partial a_i^{\rho=0}}{\partial \sigma_i}. \tag{34}$$

Given that under PA with $\rho_i = 1$ we have the payline $p_i = p$ is across fields and has a common derivative $\partial p_i/\partial a_j = \partial p/\partial a$ (so that the Jacobian of the payline is a constant matrix), the determinant (32) of the system of $2N$ demand and supply equations becomes

$$J^{1,\dots,N} = \prod_{i=1}^{N} J_{ip} - \frac{\partial p}{\partial a}\sum_{i=1}^{N}\frac{\partial D_i}{\partial x_i}\prod_{j\neq i} J_{jp},$$

which in a setting with symmetric fields boils down to

$$J_{\rho=1}^{1,\ldots,N} = J_p^N - \frac{\partial p}{\partial a} N \frac{\partial D}{\partial x} J_p^{N-1} = J_p^N + \frac{B}{Na^2}\frac{\partial D}{\partial x} J_p^{N-1}, \tag{35}$$

where $\partial p/\partial a = -B/(N^2a^2)$ in a symmetric equilibrium. With fixed budget, $\rho_i = 0$, the Jacobian of the payline is a diagonal matrix with diagonal elements equal $-B/(Na_i^2)$ and the resulting determinant (32) can be computed to be equal to

$$J_{\rho=0}^{1,\ldots,N} = \sum_{k=0}^{N} \binom{N}{k} \left(\frac{B}{Na^2}\frac{\partial D}{\partial x}\right)^k J_p^{N-k} = \left(J_p + \frac{B}{Na^2}\frac{\partial D}{\partial x}\right)^N. \tag{36}$$

Combining (35) and (36), claim (34) is equivalent to

$$-\frac{1}{J_p}\frac{J_p + \frac{B}{(N-1)a^2}\frac{\partial D}{\partial x}}{J_p + \frac{B}{Na^2}\frac{\partial D}{\partial x}J_p} > -\frac{\left(J_p + \frac{B}{(N-1)a^2}\frac{\partial D}{\partial x}\right)^{N-1}}{\left(J_p + \frac{B}{Na^2}\frac{\partial D}{\partial x}\right)^N},$$

and, after rearranging, to

$$\left(-\frac{B}{Na^2}\frac{\partial D}{\partial x}\right)^{N-1} + (-1)^N \sum_{k=1}^{N-2}\left(\frac{B}{a^2}\frac{\partial D}{\partial x}\right)^k \left[\binom{N-2}{k}\frac{1}{(N-1)^k} - \binom{N-1}{k}\frac{1}{N^k}\right] J_p^{N-1-k} > 0,$$

which always holds.

Claim 2 *Starting from symmetric parameters in all fields, fixed-budget equilibrium applications in field i increase in noise dispersion σ_i more than in the designer optimal allocation.*

Proof of Claim 2. The Lagrangian is strictly quasi-concave if and only if the sign of the determinant of its Hessian is

$$(-1)^r \begin{vmatrix} \frac{\partial^2 \mathscr{L}}{\partial a_1^2} & 0 & 0 & 0 & 0 & -\frac{\partial b_1}{\partial a_1} \\ 0 & \ddots & 0 & 0 & 0 & \vdots \\ 0 & 0 & \frac{\partial^2 \mathscr{L}}{\partial a_i^2} & 0 & 0 & -\frac{\partial b_i}{\partial a_i} \\ 0 & 0 & 0 & \ddots & 0 & \vdots \\ 0 & 0 & 0 & 0 & \frac{\partial^2 \mathscr{L}}{\partial a_r^2} & -\frac{\partial b_r}{\partial a_r} \\ -\frac{\partial b_1}{\partial a_1} & \ldots & -\frac{\partial b_i}{\partial a_i} & \ldots & -\frac{\partial b_r}{\partial a_r} & 0 \end{vmatrix} = (-1)^r \sum_{i=1}^{r}\left(-\frac{\partial b_i}{\partial a_i}\right)^2 \prod_{j\neq i}\frac{\partial^2 \mathscr{L}}{\partial a_j^2} > 0$$

for all $r = 2, \ldots, N$; see MasColell, Whinston, and Green's (1995) Theorems M.C.4 and M.D.3.i. Implicit differentiation of the system of first-order conditions

$$\frac{\partial \mathscr{L}}{\partial a_i} = \frac{\partial V_i}{\partial a_i} - \lambda \frac{\partial b_i}{\partial a_i}$$

and Cramer's rule give

$$\frac{\partial\lambda^*}{\partial\sigma_i}=\frac{\prod_{j\neq i}\frac{\partial^2\mathscr{L}}{\partial a_j^2}}{\sum_{k=1}^{N}\left(-\frac{\partial b_k}{\partial a_k}\right)^2\prod_{j\neq k}\frac{\partial^2\mathscr{L}}{\partial a_j^2}}\left[\frac{\partial^2\mathscr{L}}{\partial a_i^2}\left(-\frac{\partial b_i}{\partial\sigma_i}\right)-\frac{\partial^2\mathscr{L}}{\partial\sigma_i\partial a_i}\left(-\frac{\partial b_i}{\partial a_i}\right)\right], \tag{37}$$

where the first factor is always positive, so that $\partial\lambda^*/\partial\sigma_i<0$ whenever the term in brackets is negative. Similarly, we have

$$\frac{da_i^*}{d\sigma_i}=-\frac{\left(-\frac{\partial b_i}{\partial\sigma_i}\right)\left(-\frac{\partial b_i}{\partial a_i}\right)\prod_{j\neq i}\frac{\partial^2\mathscr{L}}{\partial a_j^2}+\left[\sum_{k\neq i}\left(-\frac{\partial b_k}{\partial a_k}\right)^2\prod_{j\neq k,i}\frac{\partial^2\mathscr{L}}{\partial a_j^2}\right]\frac{\partial^2\mathscr{L}}{\partial\sigma_i\partial a_i}}{\sum_{k=1}^{N}\left(-\frac{\partial b_k}{\partial a_k}\right)^2\prod_{j\neq k}\frac{\partial^2\mathscr{L}}{\partial a_j^2}}, \tag{38}$$

where by $\partial^2\mathscr{L}/\partial a_j^2<0$ the denominator is negative for N even and positive for N odd, while at the numerator the first term is positive for N even and negative for N odd because

$$\begin{aligned}\frac{\partial b_i}{\partial\sigma_i}&=-\int_{G_i^{-1}(1-a_i)}^{\bar{\theta}_i}\frac{1}{\sigma_i}f_i\left(\frac{x_i^D(a_i)-\theta}{\sigma_i}\right)\frac{\theta-G_i^{-1}(1-a_i)}{\sigma_i}g_i(\theta)\,d\theta<0,\\ \frac{\partial b_i}{\partial a_i}&=\frac{c_i}{v_i}+\int_{G^{-1}(1-a_i)}^{\bar{\theta}_i}\frac{1}{\sigma_i}f_i\left(\frac{x_i^D(a_i)-\theta}{\sigma_i}\right)\frac{g_i(\theta)}{g\left(G^{-1}(1-a_i)\right)}d\theta>0,\end{aligned}$$

the bracket in the second term is also positive for N even and negative for N odd, and the sign of the last factor $\partial^2\mathscr{L}/\partial\sigma_i\partial a_i$ is in general ambiguous. In an interior symmetric equilibrium, (38) boils down to

$$\left.\frac{da_i^*}{d\sigma_i}\right|_{a_i=a_i^*=a^*}=-\frac{1}{N}\left(\frac{\partial b_i}{\partial\sigma_i}\Big/\frac{\partial b_i}{\partial a_i}\right)-\frac{N-1}{N}\left(\frac{\partial^2\mathscr{L}}{\partial\sigma_i\partial a_i}\Big/\frac{\partial^2\mathscr{L}}{\partial a_i^2}\right). \tag{39}$$

Finally, we compare (39) to the right-hand side of (34). Given that the fixed budget equilibrium solves a system of N equations (field by field) of the form $B/N-b_i(a)=0$, we have

$$\left.\frac{\partial a_i^{\rho=0}}{\partial\sigma_i}\right|_{a_i=a_i^{\rho=0}}=-\frac{\partial b_i}{\partial\sigma_i}\Big/\frac{\partial b_i}{\partial a_i} \tag{40}$$

With symmetric parameters in all fields, the unique stable fixed budget (full) equilibrium (Proposition 1.c) is clearly symmetric and thus coincides with the designer optimal solution, $a_i^{\rho=0}=a_i^*=a^*$ in all fields. From (39) and (40) we have

$$\left.\frac{\partial a_i^{\rho=0}}{\partial\sigma_i}\right|_{a_i^{\rho=0}=a^*}>\left.\frac{da_i^*}{d\sigma_i}\right|_{a_i^*=a^*}\Leftrightarrow\frac{\partial b_i}{\partial\sigma_i}\Big/\frac{\partial b_i}{\partial a_i}<\frac{\partial^2\mathscr{L}}{\partial\sigma_i\partial a_i}\Big/\frac{\partial^2\mathscr{L}}{\partial a_i^2},$$

which holds whenever $\partial\lambda^*/\partial\sigma_i<0$.

Proof of Proposition 7. We now show that $\partial a_i/\partial\rho_i < 0 < \partial a_i/\partial\rho_j$ at any stable full equilibrium with SPA. Under SPA with $\rho_i \in [0,1]$ and $\rho_j = 1$ for $j \neq i$, we have $\partial p_i/\partial a_j = -Ba_i^{\rho_i-1}/\left(a_i^{\rho_i} + \sum_{j\neq i} a_j\right)^2 < 0$, $\partial p_i/\partial\rho_i = -Ba_i^{\rho_i-1}(\sum_{j\neq i} a_j)\ln a_i/\left(a_i^{\rho_i} + \sum_{j\neq i} a_j\right)^2 < 0$, $\partial p_j/\partial\rho_i = -Ba_i^{\rho_i}\ln a_i/\left(a_i^{\rho_i} + \sum_{j\neq i} a_j\right)^2 > 0$, and $\partial p_j/\partial a_j = -B/\left(a_i^{\rho_i} + \sum_{j\neq i} a_j\right)^2 = \partial p_j/\partial a_k < 0$ for $j,k \neq i$. Implicit differentiation of the equilibrium system gives

$$J_{a_i\rho_i} = \frac{\partial p_i}{\partial\rho_i}\frac{\partial D_i}{\partial x_i}\prod_{j\neq i} J_{jp} + \frac{\partial D_i}{\partial x_i}\sum_{k\neq i}\frac{\partial D_k}{\partial x_k}\left(\frac{\partial p_i}{\partial a_k}\frac{\partial p_k}{\partial\rho_i} - \frac{\partial p_i}{\partial\rho_i}\frac{\partial p_k}{\partial a_k}\right)\prod_{j\neq i,k} J_{jp}$$

which is negative for N even and positive for N odd and thus has an opposite sign to $J^{1,\ldots,N}$ (also using $J_{jp} < 0$ from stability). We conclude that equilibrium applications decrease in the responsiveness of the payline (for any N),

$$\frac{\partial a_i}{\partial\rho_i} = \frac{J_{a_i\rho_i}}{J^{1,\ldots,N}} < 0. \tag{41}$$

Proceeding similarly, we have

$$\frac{\partial a_j}{\partial\rho_i} = -\frac{\frac{\partial D_j}{\partial x_j}\prod_{k\neq i,j} J_{kp}\left[\frac{\partial D_i}{\partial x_i}\left(\frac{\partial p_i}{\partial a_i}\frac{\partial p_k}{\partial\rho_i} - \frac{\partial p_k}{\partial a_i}\frac{\partial p_i}{\partial\rho_i}\right) - \frac{\partial p_k}{\partial\rho_i}J_{ip}\right]}{J^{1,\ldots,N}} > 0 \tag{42}$$

Thus, by reducing ρ_i applications in field i are increased and in the other fields j reduced, thus allowing the designer to implement the optimal allocation. The result follows combining (41) and (42) with the discussion in the text.