Demographic Trends, the Dividend/Price Ratio and the Predictability of Long-Run Stock Market Returns[*]

Carlo A. Favero
Bocconi University, IGIER & CEPR

Arie E. Gozluklu
Bocconi University

Andrea Tamoni
Bocconi University

This Version: December, 2009

Abstract

This paper documents the existence of a slowly evolving trend in the dividend/price determined by a demographic variable, MY: the ratio of middle-aged to young population. Deviations of the dividend/price from this slowly evolving long-run component explain transitory but persistent long-horizon fluctuations in aggregate stock market returns and excess returns. The significance of MY in the explanation of low-frequency fluctuations in the dividend-price ratio is a prediction of an overlapping generation model in which the demographic structure mimics the pattern of live births in the US, that have featured alternating twenty-year periods of boom and busts. Our evidence explains the mixed evidence on the ability of the price-dividend ratio as predictor of stock market returns and it also provides a model based interpretation of statistical corrections for breaks in the mean of the dividend-price recently proposed in the literature.

KEYWORDS: dynamic dividend growth model, long run returns predictability, demographics.

J.E.L. CLASSIFICATION NUMBERS: G14, G19, C10, C11, C22,C53.

[*]Favero acknowledges financial support from Bocconi University and from MIUR. We thank Sami Alpanda, Philippe Andrade, Henrik Bessembinder (the editor), Francesco Billari, Michael Halling, Joachim Inkmann, Ralph Koijen, Massimiliano Marcellino, Nicola Pavoni, Sasson Bar-Yosef, Gianluca Violante, two anonymous referees and seminar participants at the conference "Financial and Real Activity", Paris, the EFA 2009 meetiings in Bergen, ICEEE 2009 in Ancona, Collegio Carlo Alberto, Tilburg, LBS, University of York, for helpful comments. Any errors or omissions are the responsibility of the authors. We are grateful to Andrew Mason and Guillaume Vandenbroucke for having provided us with some of the data used in this paper.

1 Introduction

The empirical relevance of the dividend-price ratio for predicting long-run stock market returns is one of the most debated issues in financial econometrics. In fact, this variable regularly plays an important role in the recent empirical literature that has replaced the long tradition of the efficient market hypothesis (Fama, 1970) with a view of predictability of returns (see, for example, Cochrane, 2007). However, there is an ongoing debate on the robustness of the predictability evidence and its potential use from a portfolio allocation perspective (Boudoukh et al., 2008; Goyal&Welch, 2008).

Most of the available evidence on predictability can be framed within the dynamic dividend growth model proposed by Campbell and Shiller (1988). This model uses a loglinear approximation to the definition of returns on the stock market. Under the assumption of stationarity of the log of price-dividend ratio and of the validity of a standard transversality condition, $(p-d)_t$ is expressed as a linear function of the future discounted dividend growth, Δd_{t+j} and of future returns, h^s_{t+j} :

$$(p-d)_t = \overline{(p-d)} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[(\Delta d_{t+j} - \bar{d}) - (h^s_{t+j} - \bar{h})] \quad (1)$$

where $\overline{(p-d)}$, the mean of the price-dividend ratio, $\bar{d}$, the mean of dividend growth rate, $\bar{h}$, the mean of log return and ρ are constants.

The dynamic dividend growth is based on the assumption of stationarity of the (log) dividend-price ratio. Consistently with such an assumption, under the maintained hypothesis that stock market returns, and dividend-growth are covariance-stationary, Eq. (1) says that the log of the price-dividend ratio is stationary (the log of price and the log of dividend are cointegrated with a (-1,1) cointegrating vector), and that deviations of (log) prices from the common trend in (log) dividends summarize expectations of either stock market returns, or dividend growth or some combination of the two.

The empirical investigation of the dynamic dividend growth model has established a few empirical results :

(i) $(p-d)_t$ is a very persistent time-series and forecasts stock market returns and excess returns over horizons of many years (Fama and French (1988), Campbell and Shiller (1988), Cochrane (2001, Ch. 20), and Cochrane(2007)).

(ii) $(p-d)_t$ does not have important long-horizon forecasting power for future discounted dividend-growth (Campbell (1991), Campbell, Lo and McKinlay(1997) and Cochrane(2001)).

(iii) the very high persistence of $(p-d)_t$ has led some researchers to question the evidence of its forecasting power for returns, especially at short-horizon. Careful statistical analysis that takes full account of the persistence in $(p-d)_t$ provides little evidence in favour of predictability of stock-market returns and excess returns based on the log-dividend-price ratio (Nelson and Kim, 1993; Stambaugh, 1999; Ang and Bekaert, 2007; Valkanov, 2003; Goyal and Welch, 2003 and Goyal and Welch 2008). Structural breaks hava also been found in the relation between $(p-d)_t$ and future returns (Neely and Weller(2000) and Paye and Timmermann(2006), Rapach and Wohar(2006)).

(iv) More recently, Lettau and Ludvigson (2001, 2005) have found that dividend growth and stock returns are predictable by long-run equilibriium relationships derived from a linearized version of the consumer's intertemporal budget constraint. The excess consumption with respect to its long run equilibrium value is defined by the authors alternatively as a linear combination of labour income and financial wealth, *cay*, or as a linear combination of aggregate dividend payments on human and non-human wealth, *cdy*. *cay* and *cdy* are much less persistent time-series than $(p-d)_t$, they are predictors of dividend-growth and, when included in a predictive regression relating stock market returns to $(p-d)_t$, they swamp the significance of this variable. Lettau and Ludvigson(2005) interpret this evidence in the light of the presence of a common component in dividend growth and stock market returns. Such a component cancels out from (1), *cay* and *cdy* are instead able to capture it, as the linearized intertemporal consumer budget constraint delivers a relationship between excess consumption and expected dividend growth or future stock market returns that is independent from their difference.

A recent strand of the empirical literature has related the contradictory evidence on the dynamic dividend growth model to the potential weakness of its fundamental hypothesis that log dividend-price ratio is a stationary process (Lettau&Van Nieuwerburgh, 2008, LVN henceforth). LVN use a century of US data to show evidence on the breaks in the constant mean $\overline{(p-d)}$. We report the time series of a century of US data on $(p-d)_t$ analyzed by LVN in Figure 1. As a matter of fact, the evidence from univariate test for non-stationarity and bivariate cointegration tests does not lead to the rejection of the null of the presence of a unit-root in $(p-d)_t$ [1].

As shown in Figure 1, LVN identify two statistically significant breaks in the mean and date them 1954 and 1991. They then provide evidence that deviations of $(p-d)_t$ from its time-varying mean have a much stronger forecasting power for stock market returns than deviations of $(p-d)_t$ from a constant mean[2]. This evidence for time-variation in the mean of the dividend-price ration has been also confirmed by Johannes et al.(2008), who estimate the process for log dividend price ratio within a particle filtering framework.

[1] The Dickey-Fuller test for the null of non-stationarity delivers an observed statistics of -2.34 when computed over the full sample 1911-2008 and a value of -1.72 when computed over the sample 1955-2008. This evidence is confirmed by the implementation of the Johansen(1991) test on a bivariate VAR for p_t and d_t, that does npot lead to the rejection of the null hypothesis of at most zero cointegrating vectors over the full-sample and the poet-war subsample.

[2] These results are confirmed by the search for possible structural breaks in the cointegrating relationship based on the application of the recursive test based on the non zero-eigenvalues suggested in Hansen and Johansen (1999). The eigenvalue shows a remarkable amount of variability over the examination period with indication of three break points around 1950, 1980, 2000.

The evidence for a slowly evolving mean in $(p-d)_t$ has been so far reported as a pure statistical fact. LVN give some hints on possible causes for the breaks arising from economic fundamentals due to technology innovations, changes in expected return, etc. but do not explore further the possible effects of fundamentals. The idea of correcting $(p-d)_t$ to reduce its persistency has been also pursued by an alternative strand of research that relates the apparent non-stationary of this variable to a shift in corporate payout policies. Boudoukh et al. (2007) provide a new measure of the cash flow based net payout yield (dividends plus repurchases minus issuances are used instead of dividends to construct the relevant ratio) that is much more quickly mean reverting than the dividend-price ratio[3].

The aim of our paper is investigate the possibility that the slowly evolving mean in the log price-dividend is related to demographic trends. We first illustrate how the theoretical model by Geanakoplos, Magill and Quinzii (2004, henceforth, GMQ) predicts that a specific demographic variable, MY, the ratio of middle-aged to young population, explains fluctuations in the dividend yield.

GMQ consider an overlapping generation model in which the demographic structure mimics the pattern of live births in the US, that have featured alternating twenty-year periods of boom and busts. They conjecture that the life-cycle portfolio behaviour (Bakshi&Chen, 1994), which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off from their investment once they are retired, plays important role in determining equilibrium asset prices. Consumption smoothing by the agents given the assumed demographic structure requires that when the MY ratio is small (large), there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets and therefore the dividend-price ratio should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. We take the GMQ model to the data via the conjecture that fluctuations in MY could capture a slowly evolving mean in $(p-d)_t$ within the dynamic dividend growth model. Demographic trends should capture the slowly evolving mean in $(p-d)_t$ and then, deviations of $(p-d)_t$ from $\overline{(p-d)_t}$ could be used as a potential predict for long-term stock market returns and dividend growth. Our empirical strategy has the potential for identifying separately the importance of demographic variables for high-frequency and low-frequency fluctuations in asset prices. Investigations conducted in the literature on the interaction between asset prices and demographic variables have traditionally concentrated either on high-frequency or low frequency fluctuations but have never considered an empirical framework based on the dynamic dividend growth model, capable of accommodating both of them, with a

[3] The procedure is not uncontroversial, in fact Lettau et al. (2006) argue these shifts are unlikely to explain the full decrease in this financial ratio: other financial valuation ratios such as earning-price ratios witness similar declines.

different role (see Poterba(2001), Goyal(2004), Erb et al., (1996); DellaVigna&Pollet(2006), Ang&Maddaloni(2005)) .

We use first long-run predictive regressions and cointegration analysis to assess the statistical significance of MY in a dynamic dividend growth model. The robustness of our results is evaluated by comparing the predictive power of the dividend-price ratio corrected for demographics with that of the dividend-price ratio, the dividend-price ratio corrected for breaks in mean(LVN) and the cash flow based net payout yield (Boudoukh et al.(2007). The role of MY is then further investigated against different alternative specifications, in particular those based on *cay* and *cdy*. Finally, the availability of very long-run projections for MY is exploited to derive predictions of long-run equity returns up to 2050.

2 Demography and the Dividend/Price Ratio: The GMQ Model

GMQ analyze an overlapping generation model in which the demographic structure mimics the pattern of live births in the US. Live births in the US have featured alternating twenty-year periods of boom and busts. They consider an OLG exchange economy with a single good (income) and three periods; young, middle-aged, retired. Each agent (except retirees) has an endowment, labor income, w= (w^y,w^m,0) and there are two types of financial instruments, riskless bond and risky equity, which allows agents to redistribute income over time. In the simplest version of the model, dividends and wages are deterministic, hence bond and equities are perfect substitutes. GMQ assume that in *odd (even)* periods a large (small) cohort N(n) enters the economy, therefore in every odd (even) period there will be $\{N, n, N\}(\{n, N, n\})$ cohorts living. They conjecture that the life-cycle portfolio behaviour (Bakshi&Chen, 1994) which suggests that agents should borrow when young, invest for retirement when middle-aged, and live off from their investment once they are retired, plays important role in determining equilibrium asset prices.

Let $q_o(q_e)$ be the bond price and $\{c_y^o,c_m^o,c_r^o\}(\{c_y^e,c_m^e,c_r^e\})$ the consumption stream in the odd (even) period. The agent born in odd period then faces the following budget constraint

$$c_y^o + q_o c_m^o + q_o q_e c_r^o = w^y + q_o w^m \tag{2}$$

and in even period

$$c_y^e + q_e c_m^e + q_o q_e c_r^e = w^y + q_e w^m \tag{3}$$

Moreover, in equilibrium the following resource constraint must be satisfied

$$Nc_y^o + nc_m^o + Nc_r^o = Nw^y + nw^m + D \quad (4)$$
$$nc_y^e + Nc_m^e + nc_r^e = nw^y + Nw^m + D \quad (5)$$

where D is the aggregate dividend for the investment in financial markets. If q_o were equal to q_e, the agents would choose to smooth their consumption, i.e. $c_y^i = c_m^i = c_r^i$ for $i = o, e$, but then for values of wages and aggregate dividend calibrated from US data the equilibrium condition above would be violated leading to excess demand either for consumption or saving. To illustrate this point we refer to the calibration provided by GMQ; take $N = 79, n = 69$ as the size (in millions) of Baby Boom (1945-64) and Baby Bust (1965-84) generations (thus, we obtain in even period a high MY ratio of $MY = \frac{N}{n} = 1.15$, and in odd period $MY = \frac{n}{N} = 0.87$ (See Figure 3a)). and $w^y = 2, w^m = 3$ to match the ratio (middle to young cohort) of the average annual real income in US. We can calculate the total wage in even and odd periods using $Nw^y + nw^m$ for odd periods and $nw^y + Nw^m$ for even periods, and then given the average ratio (0.19) of dividend to wages we compute the aggregate dividends. Assuming an annual discount factor of 0.97, which translates to a discount of 0.5 in the model of 20-year periods, if $q_o = q_e = 0.5$ were to hold and agents smooth their consumption, from the budget constraint (eq. 6-7) we obtain $c_y^i = c_m^i = c_r^i = \bar{c} = 2$, but then the resource constraint (eq. 8-9) above would have been violated. For instance, an agent from Baby Bust generation would enter in an even period in the model, i.e. (n, N, n) and high MY ratio, and faces the following aggregate resource constraint: $n(c_y^e - w^y) + N(c_m^e - w^m) + nc_r^e - D = 69 \times (2-2) + 79(2-3) + 69 \times 2 - 70 = -11$, where $D = 0.19(\frac{375+365}{2}) = 70$. This leads to excess saving in the economy. For equilibrium conditions to hold, the model implies that asset prices should increase and hence discourage saving in the economy (the experience we observed during 90's in US). When the MY ratio is small (large), i.e. an odd (even) period, there will be excess demand for consumption (saving) by a large cohort of retirees (middle-aged) and for the market to clear, equilibrium prices of financial assets should adjust, i.e. decrease (increase), so that saving (consumption) is encouraged for the middle-aged. Thus, letting q_t^b be the price of the bond at time t, in a stationary equilibrium, the following holds

$$q_t^b = q_o \text{ when period odd}$$
$$q_t^b = q_e \text{ when period even}$$

together with the condition $q_o < q_e$. Moreover the model predicts a positive correlation between MY and market prices, consequently a negative correlation with the dividend yield.

So, since the bond prices alternate between q_o and q_e, then the price of equity must also alternate between q_t^e and q_t^e as follows

$$\begin{aligned} q_o^{eq} &= Dq_o + Dq_o q_e + Dq_o q_e q_o + ... \\ q_e^{eq} &= Dq_e + Dq_e q_o + Dq_e q_o q_e + ... \end{aligned}$$

which implies

$$\begin{aligned} DP_o &= \frac{D}{q_o^{eq}} = \frac{1 - q_o q_e}{q_o q_e + q_o} \\ DP_e &= \frac{D}{q_e^{eq}} = \frac{1 - q_o q_e}{q_o q_e + q_e} \end{aligned}$$

where DP_o (DP_e) is the dividend price ratio implied by low (high) MY in the model for the odd (even) periods.

3 Testing the GMQ Model

GMQ model provides a foundation for a long-run relationship between the price-dividend ratio and demography. GMQ define the empirical counterpart of the MY ratio as the proportion of the number of agents aged 40-49 to the number of agents aged 20-29, which serves as a sufficient statistic for the whole population pyramid. We report the MY ratio in Figure 2. Interestingly, this variable shows an highly persistent dynamics and a twin peaked behavior, with peaks and throughs around 1950, 1980, 2000, around the break points in $(p-d)_t$.

To combine the GMQ model with the dynamic dividend growth we consider the derivation of LVN, who allow for a time varying mean in the linearization and consider MY as the potential determinant of this slowly evolving process.

$$\begin{aligned} (p-d)_t &= \overline{(p-d)_t} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[(\Delta d_{t+j} - \bar{d}) - (h_{t+j}^s - \bar{h})] \qquad (6) \\ \overline{(p-d)_t} &= \beta_0 + \beta_1 MY_t + u_t \end{aligned}$$

Inserting GMQ in the dynamic dividend growth model leads to the prediction that the (log) dividend price adjusted for demographics should be significant in long-horizon forecasting regression for the real stock market returns, the real dividend growth, and their difference. MY should also be significant in explaining the persistence of the dividend-price, and the varaible predicted to be stationary in this extended model is not the dividend-price but a combination between price, dividends and MY. We investigate the hypothetical cointegrating relation between dividend, prices and MY, by running the

Johansen(1988) procedure on a cointegrating system based on the vector of variables $\mathbf{y}_t' = \begin{bmatrix} d_t & p_t & MY_t \end{bmatrix}$.

3.1 Long-Horizon Forecasting regressions

We report in Table 1 the evidence from long-horizon forecasting regression. To make our evidence directly comparable with that reported in LL(2005) we consider predictive regressions for horizons ranging from one to six years. A full description of all data used in our empirical analysis is provided in the Data Appendix.

Table 1.1-1.3 reports the evidence for forecasting returns, dividend growth, returns adjusted for dividend growth, based on the following three models:

$$\begin{aligned}
\sum_{j=1}^{k}(h_{t+j}^{s}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
\sum_{j=1}^{k}(\Delta d_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
\sum_{j=1}^{k}(h_{t+j}^{s} - \Delta d_{t+j}) &= \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j} \\
k &= 1, ..., 6
\end{aligned}$$

We report heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994). Alternatively, we also conduct a (wild)bootstrap exercise (Davidson& Flachaire, 2008) to compute p-values. To take care of the potential effect on statistical inference in finite sample of the use of overlapping data we also report the rescaled t-statistic recommended by Valkanov (2003) for the hypothesis that the regression coefficient on the dividend/price adjusted for the effect of demographics is zero. We report test of predictability at each horizon but we also compute joint tests across horizons based on SUR estimation and report in the last row the relevant χ^2 statistics with associated p-values.

The evidence can be summarized as follows:

i) MY is always significant alongwith p_t and $d_t$in all the forecasting regressions for real stock market returns (Panel A). The adjusted R^2 of the predictive regression increases with the horizon from 0.09 at the 1-year horizon to 0.54 at the 6-year horizon. Consistently with the prediction of the GMQ model the effect of MY is negative on the slowly evolving mean of the dividend-price positive for expected returns at all horizon.

ii) MY is never significant in all the forecasting regressions for real dividend growth (Panel B). The adjusted R^2 of the predictive regression declines with the horizon from

0.15 at the 1-year horizon to 0.06 at the 6-year horizon.

iii) MY is always significant alongwith p_t and $d_t$in all the forecasting regressions for real stock market returns adjusted for real dividend growth (Panel C). The adjusted R^2 of the predictive regression increases with the horizon from 0.26 at the 1-year horizon to 0.67 at the 6-year horizon. The evidence of the strongest predictability of $\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j})$ is fully consistent with the dynamic dividend growth model. Such evidence, paired with that on the different forecastability of the two components of stock market returns adjusted for dividend growth, rules out the dominance of a common stochastic component for the determination the dynamics of dividend growth and stock market returns.

iv) MY dominates alternative approaches proposed in the literature to capture an evolving mean in the dividend-price ratio. In the last rows of each panel of the Table we report the results of augmenting the long-run forecasting regressions based on the GMQ model with alternative filtered dividend price series. In particular we consider , $(d-p)_t^{LvN}$, the (log) dividend price corrected for breaks in LVN and $(d-p)_t^{CFN}$, the cash flow based net payout yield (dividends plus repurchases minus issuances) proposed by Boudoukh et al.(2007).

Overall the long-run forecasting regressions lend strong support to the inclusion of MY in the traditional dynamic dividend growth model. The dividend-price corrected for a slowly long-run mean, determined by MY , predicts long-run stock market returns and long-run stock market returns adjusted for dividend growth, but it does not predict long-run dividend growth. The R^2 associated to the relevant predictive regressions increases with the horizon. This evidence of a positive relation between predictability and the forecasting horizon is interesting, in that both the Dynamic Dividend Growth model and the GMQ model establish a predictive relation for long-run returns. In fact, the most natural horizon for the GMQ model is one generation, i.e. about twenty years. Of course, it is difficult to establish some evidence via predictive regressions for twenty years returns, as we have available only one century of data. To give to the reader a visual impression on the relationship between real stock market returns and MY at a frequency as close as possible to that implied by the relevant models, we report in Figure 3 MY and 20-year real stock market returns. We find the graphical evidence interesting and fully consistent with the statistical evidence from the long-run regressions at higher frequencies.

3.2 Cointegration

The evidence of forecasting power of a linear combination of dividend, prices and MY for forecasting long-run returns and long-run returns adjusted for dividend growth, provides indirect evidence of stationarity of such a combination. The validity of this hypothesis can be further investigated by running the Johansen(1988, 1991) procedure on a cointegrating system based on the vector of variables $\mathbf{y}_t' = \begin{bmatrix} d_t & p_t & MY_t \end{bmatrix}$. We then test for cointegration within a three-variate VAR in which we allow for the presence of a deterministic trend (GMQ explicitly state that they "assume that the model has been detrended so that the systematic sources of growth of dividends and wages arising from population growth, capital accumulation and technical progress are factored out." (GMQ, p.6)) in the level of variables that cancels out in the long-run equilibrium relationship.[4]

We report in Table 1.1-1.2 the evidence for the full-sample and for the sub-sample 1955-2008. The results lead to the rejection of the null of at most zero cointegrating vectors, while the null of at most one cointegration vector cannot be rejected. The evidence in favour of one cointegrating vector in which all variables are always significant confirms that the high persistence of the dividend/price is matched by the high persistence of MY.Using augmented Dickey-Fuller test, the null of a unit root in MY cannot be rejected. The coefficients determining the adjustment in presence of disequilibrium in the Vector Error Correction model confirm the evidence from the forecasting regressions reported in the previous section: stock market returns adjusts in presence of disequilibrium. The significance of MY increases in the second sub-sample, where LVN found the two breaks in $(p-d)_t$.[5]

Figures 4.1-4.2 provides a graphical assessment of the capability of MY_t of capturing the slowly evolving mean of $(d-p)_t$. Figure 4.1 reports the residuals from our cointegrating vector, alongwith $(d-p)_t$, and the deviations of $(d-p)_t$ from $\overline{(d-p)}_t^{LVN}$, the shifting mean identified by LVN. Figure 4.2 reports residuals from our cointegrating vector with the cycle of $(d-p)_t$, obtained by applying an Hodrick-Prescott filter to the original series. The graphical evidence illustrates how the cointegration based correction matches the break-based correction in LVN (2008) and the cycle obtained by applying the HP filter. It is important to note that while the cointegration based analysis can be promptly used for forecasting, the same does not apply to both the HP filter and the correction for breaks.

Overall we take the evidence of long-run forecasting regressions and cointegration analysis as consistently supportive of the GMQ model. Two more remarks are in order before we move forward.

First, in the GMQ model bond and stock are perfect substitutes, therefore the evalu-

[4] See Appendix B, for the etails of the specification of our statistical model In a previous version of this paper we allows for a presence of a technology driven trend, proxied by Total Factor Productivity, in the long-run equilibium relationship. We have decidd to excude TFP from the cointegrating relationship on the basis of two arguments i) the presence of a technology driven trend in the dividend pice ratio is very hard to justify theoretically ii) the TFP trend does not attract any significance when included in the long-run forecasting regressions discussed in the previous section. We are grateful to an anonymos referee for attracting our attention on this point

[5] We have also investigated the stability of the cointegrating relationship by using the recursively calculated eigenvalues and the tests for constancy of the parameters in the cointegrating space proposed by Nyblom (1989), Hansen&Johansen, (1999) and Warne et al., (2003). The results, available upon request, show no evidence of instability.

ation of the performance of MY_t and TFP_t in forecasting yields to maturity of long-term bonds seems a natural extension of our empirical investigation. In fact, the debate on the so-called FED model (Lander et al., 1997) of the stock market, based on a long-run relation between the price-earning ratio and the long-term bond yield, brings some interesting evidence on this issue. The FED model is based on the equalization, up to a constant, between long-run stock and bond market returns. This feature is shared by the GMQ framework, and it requires a constant relation between the risk premium on long-term bonds and the risk premium on stocks. It has been shown that, although the FED model performs well in period where the stock and bond market risk premia are strongly correlated, some measure of the fluctuations in their relative premium is necessary to model periods in which volatilities in the two markets have been different (see, for example, Asness (2003)). As a consequence, to put MY_t at work to explain the bond yields, some modelling of the relative bond/stock risk premia is also in order. We consider this as an interesting extension, beyond the scope of this paper, that is on our agenda for future work.

Second, although MY_t is the GMQ model consistent measure of demographics, there are a number of different potential measures for demographic trends. We have therefore conducted robustness analysis of our cointegration results to the introduction of different measures of demographic structure of the population and productivity trends. The results, discussed in Appendix B, are supportive of our preferred specification.

4 MY, CAY and CDY

In the light of the evidence reported in the previous section it is interesting to reconsider point iv) in the introduction and evaluate the significance of the introduction of MY_t in the dynamic dividend growth model against *cay* and *cdy*. As stated in the introduction, Lettau and Ludvigson (2001, 2005) have found that dividend growth and stock returns are predictable by long-run equilibrium relationships derived from a linearized version of the consumer's intertemporal budget constraint. The excess consumption with respect to its long run equilibrium value is defined by LL alternatively as a linear combination of labour income and financial wealth, *cay*, or as a linear combination of aggregate dividend payments on human and non-human wealth, *cdy*. *cay* and *cdy* are much less persistent time-series than $(p-d)_t$, they are predictors of dividend-growth and, when included in a predictive regression relating stock market returns to $(p-d)_t$, they swamp the significance of this variable.

Evaluating the effect of the inclusion of *cay* and *cdy* in the long-run forecasting regressions that also include MY_t is important for a number of reasons. First, it is a parsimonious way of evaluating the model with MY_t against all financial ratios traditionally adopted to predict returns. In fact, Lettau and Ludvigson(2001,2005) show the superior performance in predicting long-run returns of cay and cdy with respect to all the traditionally adopted financial ratios, such as the detrended short term interest rate (Campbell, 1991; Hodrick, 1992), the log dividend earnings ratio and the log price earning ratio (Lamont, 1998), the spread of long term bond yield (10Y) over 3M treasury bill, and the spread between the BAA and the AAA corporate bond rates. Second, it would allow further investigation on the presence of a common component in dividend and stock market returns suggested by LL(2005) but not consistent with our findings in Table 1.3, that witness the significance of MY for predicting long-run returns and long-run returns adjusted for dividend growth. Third, it could shed further light on the relative importance of cay and cdy and MY_t for predicting returns and dividend growth in the dynamic dividend growth model. Note that a joint significance of cay or cdy and MY_t in long-run forecasting regressions for real stock market returns is fully consistent with the GMQ model if the significance of MY_t is interpreted in the light of its role as predictor for $\overline{(p-d)_t}$ while cay or cdy are taken as predictors of $\sum_{j=1}^{\infty} \rho^{j-1} E_t(\Delta d_{t+j} - \bar{d})$.

We report the relevant evidence in Tables 3.1-3.3. cay and cdy are estimated by LL as cointegrating residuals for the systems (c_t, a_t, y_t) and (c_t, d_t, y_t), where c_t is log consumption, y_t is log labor income, a_t is log asset wealth (net worth), d_t is log stock market dividends. We have taken the cointegrating relationship directly from LL(2005): $cay_t = c_t - 0.33a_t - 0.57y_t$, $cdy_t = c_t - 0.13d_t - 0.68y_t$. The evidence clearly indicates that the significance of $(p-d)_t$ corrected for MY_t in the long-horizon regressions is not reduced by the augmentation of the model with cay and cdy. These two variables, and in particular cdy, have strong predictive power for dividend growth. Therefore, the evidence that the best predictive model for long-horizon stock returns is the one combining dividend/price with the demographic variable and cdy is indeed fully consistent with an interpretation based on the Dynamic Dividend Growth model where MY explains the slowly evolving component of the mean of the dividend/price and cdy acts as a predictor of dividend-growth. Such an intepretation is supported by the long-horizon regressions for stock returns adjusted for dividend growth, in which both MY and cdy enter with highly significant coefficients of the opposite sign, positive for MY and negative for cdy.

4.1 Out-of-Sample Evidence

In this section we follow Goyal and Welch (2008), and analyze the performance of cdy_t and MY adjusted dividend-price ratio from the perspective of a real-time investor. We therefore consider out-of-sample evidence, for the 1-year, 2-year, and 3-year horizons, and we compare the performance of the bivariate model based on the combination of the two predictors with that of the two univariate models based on each predictor and the univariate models based on $(d-p)_t$ and $\overline{(d-p)}_t^{LVN}$.

We run rolling forecasting regressions for the one, three and five years ahead horizon by using as an initialization sample 1955-1981. The forecasting period begins in 1982 includes the anomalous period of late 90's where the sharp increase in stock market index weakens the forecasting power of financial ratios. In particular, we consider both the two univariate models and the bivariate encompassing model and compare the forecasting performance with historical mean benchmark. In the first two columns of Table 4 we report the adjusted $\bar{R}^2$ and the t-statistics using the full sample 1955-2008. Then we also report mean absolute error (MAE) and root mean square error (RMSE) calculated based on the residuals in the forecasting period, namely 1982-2008. The first column of out-of-sample panel report the out-of-sample R^2 statistics (Campbell&Thomson, 2008) which is computed as

$$R_{OS}^2 = 1 - \frac{\sum_{t=t_0}^{T}(r_t - \hat{r}_t)^2}{\sum_{t=t_0}^{T}(r_t - \bar{r}_t)^2}$$

where $\hat{r}_t$ is the forecast at $t-1$ and $\bar{r}_t$ is the historical average estimated until $t-1$. In our exercise, $t_0 = 1982$ and $T = 2008$. If R_{OS}^2 is positive, it means that the predictive regression has lower mean square error than the prevailing historical mean. In the last column, we report the Diebold-Mariano (DM) t-test for checking equal-forecast accuracy from two nested models for forecasting h-step ahead excess returns.

$$DM = \sqrt{\frac{(T+1-2*h+h*(h-1))}{T}} * \left[\frac{\bar{d}}{\widehat{se}(\bar{d})}\right]$$

where we define e_{1t}^2 as the squared forecasting error of prevailing mean, and e_{2t}^2 as the squared forecasting error of the predictive variables, $d_t = e_{1t}^2 - e_{2t}^2$, i.e. the difference between the two forecast errors, $\bar{d} = \frac{1}{T}\sum_{t=t_0}^{T} d_t$ and $\widehat{se}(\bar{d}) = \frac{1}{T}\sum_{\tau=-(h-1)}^{h-1}\sum_{t=|\tau|+1}^{T}(d_t - \bar{d}) * (d_{t-|\tau|} - \bar{d})$. A positive DM t-test statistics indicates that the predictive regression model performs better than the historical mean.

Insert here Table5

We report in Figure 5 the cumulative squared prediction errors of historical mean minus the cumulative squared prediction error of our best forecasting model

Insert here Figure 5

We use all the available data from 1909 until 1954 for initial estimation and then we recursively calculate the cumulative squared prediction errors until the sample end, namely 2008.

Overall, the results reported in Table 5 and Figure 5 confirm the evidence from the forecasting regressions, with a clear indication that the model combining cdy_t and MY adjusted dividend-price ratio dominates all alternative specifications, both within-sample and out-of-sample.

5 Long-Run Equity Premium Projections

An interesting feature of MY_t is that long-run forecast for this variable are readily available. In fact, the Bureau of Census(BoC) provide on its website projections up to 2050 for MY_t. In this section we combine long-run horizon regression with the cointegrating system estimated in section 2 to construct a model that can be simulated to generate long-run equity premium projections.

We concentrate on 5-year excess returns and estimate the following model:

$$\begin{aligned}
\sum_{j=1}^{5}(h^s_{t+j} - r_{f,t+H}) &= c_1 + c_2\,(p_t - c_3 d_t - c_4 MY_t) + u_{1t} \qquad (7) \\
\begin{bmatrix} \Delta p_{t+1} \\ \Delta d_{t+1} \end{bmatrix} &= \begin{bmatrix} c_5 \\ c_{10} \end{bmatrix} + \begin{bmatrix} c_6 \\ c_{11} \end{bmatrix} \begin{bmatrix} 1 & -c_3 & -c_4 \end{bmatrix} \begin{bmatrix} p_t \\ d_t \\ MY_t \end{bmatrix} + \\
& \begin{bmatrix} c_7 & c_8 & c_9 \\ c_{12} & c_{13} & c_{14} \end{bmatrix} \begin{bmatrix} \Delta p_t \\ \Delta d_t \\ \Delta MY_t \end{bmatrix} + \begin{bmatrix} u_{2t} \\ u_{2t} \end{bmatrix},
\end{aligned}$$

(7) Combines a long-run forecasting regression for five year excess returns, defined as the difference between returns on the S&P500 and the risk-free rate, with the equations for Δp_{t+1}, Δd_{t+1} in the cointegrated VAR estimated in Section 2. Equity premium projections are obtained by forward simulation of the first equation. This requires projections for the three right-hand side variable. We obtain them directly from the BoC for MY_t and by forward simulation of the CVAR estimated in Section 2 for p_t and d_t.Three comments are in order on the specification of (7). First, omitting an equation for MY_t from the model used for projections requires (strong) exogeneity of this variable: we believe in the validity of such an assumption. Second, we impose cross-equation restrictions in order to have the same estimates of the coefficients determining the long-run equilibrium of the system in the equation for excess-retunrs and in the equation for 1-year returns and dividend growth. Third, we did not report the results based on the inclusion of cdy in our forecasting model. In fact, the long-horizon forecast for this variable do rapidly converge to its historical mean to leave the variability of projections of the risk-premium to be dominated by projections for MY_t. Moreover, as pointed out by Goyal and Welch(2008), this

variable might suffer from look-ahead bias, as the cointegrating coefficients are computed using full-sample estimates. Therefore, we kept it in the previous section as look-ahead bias would make it a tougher competitor to our preferred model, but we do not include it in our forecasting exercise.

The estimates of the parameters and the adjusted R^2 for each equation are reported in Table 6.

Insert here Table 6

The estimates are fully consistent with those reported in the previous sections[6] and do require any further comments. Figure 6 illustrates the results from the projection of the model.

Insert here Figure 6

Over the sample up to 2008 we report (pseudo) out-sample 5-year annualized equity premium forecasts and its realizations. The model does consistently very well with only two exceptions: the 1929 crisis and the boom market at the end of the millennium. We then conduct the out-of-sample exercise by estimating the model with data up to 2008, and then by solving it forward stochastically to obtain out-of-sample forecasts until 2050. Our simulation predicts a rapid stock market recovery for the next two years followed by a fluctuations of the risk premium around an mean of 5.02 per cent, just below the historical average. The width of the 95 per cent confidence intervals points to the existence of a sizeable amount of uncertainty around point estimates. Interestingly, the model does not foresee a dramatic market meltdown, a "doomsday" scenario, due to a collective exit from the stock market by retired the baby boomers. The GMQ model relies on the cyclicality of young and middle aged cohorts, and the projections for MY up to 2050 do not imply any meltdown scenario.

6 Conclusions

This paper has documented the existence of a slowly evolving trend in the mean dividend/price determined by a demographic variable, MY , the ratio of middle-age to young population. We have shown that MY captures well a slowly evolving component in the mean dividend/price ratio and it is strongly significant in long-horizon regressions for real stock market returns.

A model including MY overperforms all alternative models for forecasting returns.

[6] All the evidence reported for long-run forecasting regressions based on real equity returns, the dependent variable consitent with the DDG model, is robust when excess returns are used as a dependent variable instead of real returns.

The best forecasting model for real stock market returns found in our work is the one combining MY with cdy, a variable constructed by LL(2005) to capture excess consumption with respect to its long run equilibrium value. We take this evidence as strongly supportive of the Dynamic Dividend Growth model with an evolving mean, determined by MY. In fact, the model predicts that long-horizon returns should depend on the deviations of the dividend-price ratio from its mean and by long-run dividend growth. We show that MY models the mean of the dividend-price ratio while cdy is a predictor of long-horizon dividend-growth (see LL(2005)). Therefore, our results confirm those in LL(2005), that an important component of time-varying expected returns is not captured by fluctuations of the dividend-price ratio around a constant mean, but we also find that the variability of time-varying expected returns not explained in long-horizon regressions variable falls dramatically when the mean is allowed to change as a function of MY.

The empirical results we have reported should be of special relevance to the strategic asset allocation literature, in which the log dividend-price ratio is often used in VAR models as a stationary variable capturing time-variation in the investment opportunity set, and as an input into the optimal asset allocation decision of a long-horizon investor. Allowing for the presence of MY in the VAR models used to estimate the time profile of returns and their volatility might cast new light on the hot debate on the safety of stock market investment for the long-run (see Campbell and Viceira (2002), Pastor and Stambaugh(2009)). This topic is on our agenda for future work.

Finally, by exploiting the exogeneity and the predictability of MY, we have also provided projections for equity risk premia up to 2050. Our simulations point to an average equity risk premium of about five per cent for the period 2010-2050.

References

[1] Abel, Andrew B., 2003, The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security, Econometrica, 71, 2, 551-578.

[2] Abel, Andrew B., 2001, Will Bequests Attenuate the Predicted Meltdown in Stock Prices When Baby Boomers Retire?, Review of Economics and Statistics, 83, 4, 589-595.

[3] Ang, Andrew and Geert Bekaert, 2007, Stock Return Predictability: Is It There? The Review of Financial Studies, 20, 651–707.

[4] Ang, Andrew and Angela Maddaloni, 2005, Do Demographic Changes Affect Risk Premiums? Evidence from International Data, Journal of Business, 78, 341-380.

[5] Asness, Clifford, 2003, Fight the Fed Model: the Relationship between Future Returns and Stock and Bond Market Yields, Journal of Portfolio Management, Fall 2003, 30(1), pp. 11-24.

[6] Bakshi, Gurdip S., and Zhiwu Chen, 1994, Baby Boom, Population Aging, and Capital Markets, Journal of Business, 67, 2, 165-202.

[7] Beaudry, Paul and Franck Portier, 2006, Stock Prices, News and Economic Fluctuations, American Economic Review, 96, 1293-1307.

[8] Bloom, David E., David Canning, and Jaypee Sevilla, 2003, The Demographic Dividend. A new Perspective on the Economic Consequences of Population Change, Rand Corporation, Santa Monica.

[9] Boudoukh, Jacob, Richardson, Matthew and Robert F. Whitelaw, 2008, The Myth of Long-Horizon Predictability, The Review of Financial Studies, 21, 4, 1577-1605.

[10] Boudoukh, J., Michaely, R., Richardson, M. and M. Roberts, 2007, On the Importance of Measuring Payout Yield: Implications for Empirical Asset Pricing, Journal of Finance, forthcoming.

[11] Brooks, Robin J.,2002, Asset-Market Effects of the Baby Boom and Social-Security Reform, American Economic Review, 92, 2, 402-406.

[12] Brooks,Robin J., 2000, What Will Happen to Financial Markets When The Baby Boomers Retire?, Computing in Economics and Finance, 92, Society for Computational Economics.

[13] Campbell, J. Y., 1991. A variance decomposition for stock returns. Economic Journal 101, 157–179.

[14] Campbell, J. Y., Lo, A. W., MacKinlay, C., 1997. The Econometrics of Financial Markets. Princeton University Press, Princeton, NJ.

[15] Campbell, John Y., and Samuel B. Thomson, 2008, Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?, The Review of Financial Studies, 21, 1509-1531.

[16] Campbell, John. Y., 2001, A Comment on James M. Poterba's "Demographic Structure and Asset Returns", The Review of Economics and Statistics, 83, 4, 585-588.

[17] Campbell, John Y., and Robert Shiller, 1988, Stock Prices,Earnings, and Expected Dividends, Journal of Finance, 43, 661-676.

[18] Cochrane, J. H., 2001. Asset Pricing. Princeton University Press, Princeton, NJ.

[19] Campbell, John Y., and Luis M. Viceira, 2002, Strategic Asset Allocation: Portfolio Choice for Long-Term Investors (Oxford University Press).

[20] Cochrane, John H., 2007, The Dog that Did Not Bark: A Defense of Return Predictability, Review of Financial Studies, 20, 5.

[21] Dalla Vigna S. and J.Pollet, 2007, Demographics and Industry Returns, American Economic Review, Vol 97, pp. 1167-1702.

[22] Davidson, Russell and Emmanuel Flachaire, 2008, The Wild bootstrap, Tamed at Last, Journal of Econometrics,146, 1, 162-169.

[23] Erb, Claude B., Campbell R. Harvey, and Tadas E. Viskanta, 1996, Demographics and International Investment, Financial Analysts Journal (July/August), 14-28.

[24] Fama, Eugene and Kenneth R. French, 1988, Dividend Yields and Expected Stock Returns, Journal of Financial Economics, 22, 3-26.

[25] Geanakoplos, John, Magill, Michael and Martine Quinzii, 2004, Demography and the Long Run Behavior of the Stock Market, Brookings Papers on Economic Activities, 1: 241-325.

[26] Goyal, Amit, and Ivo Welch, 2008, A Comprehensive Look at the Empirical Performance of Equity Premium Prediction. The Review of Financial Studies, 21-4, 1455-1508.

[27] Goyal, A., Welch, I., May 2003. Predicting the equity premium with dividend ratios. Management Science 49 (5), 639–654.

[28] Goyal, Amit, 2004, Demographics, Stock Market Flows, and Stock Returns, Journal of Financial and Quantitative Analysis, 39, 1, 115-142.

[29] Hansen, Henrik and Soren Johansen, 1999, Some Tests for Parameter Constancy in cointegrated VAR-models, The Econometrics Journal, 2, 306-333.

[30] Hodrick, Robert, and Edward C. Prescott, 1997, Postwar U.S. Business Cycles: An Empirical Investigation, Journal of Money, Credit, and Banking, 1-16.

[31] Hodrick, Robert, 1992, Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement, Review of Financial Studies, 5, 357–386.

[32] Jaimovich, Nir and Henry E. Siu, 2008, The Young, the Old, and the Restless: Demographics and Business Cycle Volatility, NBER working paper, 14063.

[33] Johannes, Michael, Korteweg, Arthur and Nicholas Polson, 2008, Sequential Learning, Predictive Regressions, and Optimal Portfolio Returns, working paper.

[34] Johansen, Soren, 1988, Statistical analysis of cointegrating vectors, Journal of Economic Dynamics and Control,12, 231–254.

[35] Johansen, Soren, 1991, Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, Econometrica, 56, 1551–1580.

[36] Juselius, Katarina, 2006, The Cointegrated VAR Model: Methodology and Applications, Oxford University Press.

[37] Lander, Joel, Athanasios Orphanides, and Martha Douvogiannis, 1997, Earnings Forecasts and the Predictability of Stock Returns: Evidence from trading the S&P, Journal of Portfolio Management, 23(4), 24-35.

[38] Lettau, Martin, and Sydney Ludvigson, 2005, Expected Returns and Expected Dividend Growth, Journal of Financial Economics, 76, 583-626.

[39] Lettau, Martin, and Sydney Ludvigson, 2001, Consumption, Aggregate Wealth and Expected Stock Returns, Journal of Finance, 56, 3, 815-849.

[40] Lettau, Martin, and Stijn Van Nieuwerburgh, 2008, Reconciling the Return Predictability Evidence, Review of Financial Studies, 21, 4, 1607-1652.

[41] Lewellen, Jonathan, 2004, Predicting Returns with Financial Ratios. Journal of Financial Economics 74:209–35.

[42] Macunovich, Diane J., 2002, Birth Quake, Chicago University Press.

[43] Mason, Andrew, and Ronald Lee, 2005, Reform and Support Systems for the Elderly in Developing Countries: Capturing the Second Demographic Dividend, *Genus*, 2, 11-35.

[44] Nelson, C. C., Kim, M. J., 1993. Predictable stock returns: The role of small sample bias. Journal of Finance 43, 641–661.

[45] Neely C.J., Weller P.A.(2000). Predictability in international asset returns: a reexamination. Journal of Financial and Quantitative Analysis, 35, 601-620

[46] Newey, Whitney K. and Kenneth D. West, 1987, A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, Econometrica, 55, 3, 703-08.

[47] Newey, Whitney K. and Kenneth D. West, 1994, Automatic Lag Selection in Covariance Matrix Estimation, Review of Economic Studies, 61, 631-653.

[48] Nyblom, Jukka, 1989, Testing for the Constancy of Parameters over Time. Journal of the American Statistical Association 84, 223-230.

[49] Paye, B.S., Timmermann A.(2006), Instability of return prediction models. Journal oF Empirical Finance, 13, 274-315

[50] Pastor, Lubos and Robert F. Stambaugh, 2009, Are Stocks Really Less Volatile in the Long Run?,Working Paper 14757, http://www.nber.org/papers/w14757

[51] Poterba, James M., 2001, Demographic Structure and Asset Returns, The Review of Economics and Statistics, 83, 4, 565-584.

[52] Rapach D.E., Wohar M.E., 2006, In-sample vs. out-of-sample tests of stock return predictability in the context of data-mining, Journal of Empirical Finance, 13, 231-247

[53] Shiller, Robert J., 2005, Irrational Exuberance, second edition, Princeton University Press.

[54] Stambaugh, R. F., 1999. Predictive regressions. Journal of Financial Economics 54, 375–421.

[55] Valkanov, R., 2003. Long-horizon regressions: Theoretical results and applications. Journal of Financial Economics 68, 201–232. 33

[56] Yoo, Peter S., 1997, Population Growth and Asset Prices, Federal Reserve Bank of St. Louis working paper no.1997-016A.

Table 1.1: Long-horizon regressions (1910 -2008)

j-period regressions for real stock returns

$$\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 (t-stat): 15.63 (0.000) — horizon k in years

	1	2	3	4	5	6
β_1 (t-stat)	−0.27 (−4.000)	−0.28 (−6.287)	−0.23 (−6.563)	−0.21 (−9.077)	−0.19 (−9.352)	−0.16 (−7.824)
β_2 (t-stat)	0.30 (3.679)	0.32 (5.748)	0.26 (5.801)	0.24 (7.974)	0.21 (8.291)	0.17 (7.065)
β_3 (t-stat)	0.44 (4.065)	0.45 (5.875)	0.37 (6.509)	0.34 (7.627)	0.29 (7.865)	0.26 (7.710)
$\beta_1 = -\beta_2$ (t-stat / w.b.p-values)	−0.19 (−3.645 / 0.002)	−0.20 (−5.439 / 0.000)	−0.16 (−6.016 / 0.000)	−0.16 (−7.652 / 0.000)	−0.14 (−8.584 / 0.002)	−0.13 (−7.740 / 0.008)
$t/\sqrt{T} - test$	{0.30**}	{0.49***}	{0.59***}	{0.75***}	{0.89***}	{0.95***}
$\beta_3 \mid (\beta_1 = -\beta_2)$ (t-stat)	0.42 (3.447)	0.44 (4.636)	0.38 (5.059)	0.36 (6.006)	0.33 (6.578)	0.29 (6.915)
$adjR^2$	0.09	0.25	0.33	0.44	0.52	0.54
$adjR^2(\beta_1 = -\beta_2)$	0.07	0.20	0.27	0.37	0.46	0.50
F-statistic	4.10	12.18	16.74	26.62	36.74	39.48

Testing MY against alternative models:

$$\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
$\overline{(d-p)}_t^{LVN}$	β_3 (t-stat)	0.44 (4.149)	0.45 (6.138)	0.39 (7.242)	0.36 (9.009)	0.32 (10.383)	0.28 (10.449)
	β_4 (t-stat)	0.07 (0.665)	0.09 (1.128)	0.05 (0.665)	0.03 (0.626)	0.01 (0.140)	−0.03 (−0.636)
$\overline{(d-p)}_t^{BMRW}$	β_3 (t-stat)	0.44 (2.850)	0.46 (4.849)	0.39 (6.229)	0.35 (8.561)	0.31 (10.286)	0.27 (10.925)
	β_4 (t-stat)	0.51 (3.602)	0.37 (2.945)	0.21 (2.443)	0.09 (1.496)	0.02 (0.407)	−0.03 (−1.103)

t-tests are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1}MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson& Flachaire, 2008) to compute p-values and report for β_1 $t/\sqrt{T} - test$ suggested in Valkanov (2003) in curly brackets. Significance at the 10%, 5%, and 1% level of the $t\ /\sqrt{T}$ test using Valkanov's (2003) critical values is indicated by *, **, and ***, respectively. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

Table 1.2: Long-horizon regressions (1910-2008)

j-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

χ^2 (t-stat): 2.66 (0.015) — horizon k in years

	1	2	3	4	5	6
β_1	0.19	0.10	0.05	0.03	0.02	0.01
(t-stat)	(2.097)	(1.827)	(1.400)	(1.222)	(1.006)	(0.803)
β_2	−0.23	−0.13	−0.07	−0.04	−0.03	−0.02
(t-stat)	(−2.196)	(−1.996)	(−1.585)	(−1.441)	(−1.308)	(−1.153)
β_3	−0.15	−0.03	0.04	0.07	0.08	0.08
(t-stat)	(−1.250)	(−0.371)	(0.780)	(1.863)	(2.428)	(2.755)
$\beta_1 = -\beta_2$	0.10	0.05	0.02	0.01	−0.00	−0.00
(t-stat / w.b.p-values)	(1.580 / 0.240)	(1.118 / 0.433)	(0.638 / 0.353)	(0.298 / 0.171)	(−0.018 / 0.096)	(−0.234 / 0.106)
$\beta_3 \mid (\beta_1 = -\beta_2)$	−0.12	−0.03	0.03	0.06	0.06	0.06
(t-stat)	(−1.118)	(−0.366)	(0.714)	(1.700)	(2.067)	(2.134)
$adjR^2$	0.15	0.09	0.05	0.04	0.05	0.06
$adjR^2(\beta_1 = -\beta_2)$	0.06	0.03	0.02	0.02	0.03	0.03
F-statistic	6.87	4.05	2.73	2.52	2.66	3.04

Testing MY against alternative models:

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
$\overline{(d-p)}_t^{LVN}$	β_3	−0.09	−0.00	0.05	0.07	0.08	0.09
	(t-stat)	(−0.877)	(−0.033)	(0.993)	(1.886)	(2.511)	(2.886)
	β_4	−0.24	−0.14	−0.08	−0.05	−0.02	0.00
	(t-stat)	(−2.412)	(−2.094)	(−1.520)	(−0.942)	(−0.321)	(0.024)
$\overline{(d-p)}_t^{BMRR}$	β_3	−0.18	−0.05	−0.00	0.03	0.04	0.06
	(t-stat)	(−1.304)	(−0.697)	(−0.001)	(0.772)	(1.752)	(2.509)
	β_4	0.09	0.14	0.16	0.16	0.15	0.08
	(t-stat)	(0.879)	(1.981)	(2.122)	(2.206)	(2.977)	(2.304)

t-tests are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1}MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson& Flachaire, 2008) to compute p-values. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

Table 1.3: Long-horizon regressions (1910-2008)

j-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \varepsilon_{t,t+j}$$

	horizon k in years					
χ^2 (t-stat) 27.94 (0.000)	1	2	3	4	5	6
β_1	−0.46	−0.39	−0.28	−0.24	−0.20	−0.17
(t-stat)	(−6.159)	(−8.285)	(−9.393)	(−10.344)	(−14.944)	(−12.523)
β_2	0.53	0.45	0.33	0.28	0.24	0.20
(t-stat)	(6.127)	(8.234)	(9.157)	(10.022)	(13.150)	(10.940)
β_3	0.59	0.47	0.33	0.27	0.22	0.18
(t-stat)	(4.631)	(5.313)	(5.297)	(5.697)	(6.202)	(6.361)
$\beta_1 = -\beta_2$	−0.28	−0.24	−0.18	−0.16	−0.14	−0.12
(t-stat / w.b.p-values)	(−4.961/0.001)	−6.603 / 0.000)	(−7.345 / 0.000)	(−8.516 / 0.000)	(−13.217 / 0.000)	(−12.976 / 0.002)
$t/\sqrt{T}-test$	{0.47***}	{0.67***}	{0.77***}	{0.91***}	{1.04***}	{1.12***}
$\beta_3 \mid (\beta_1 = -\beta_2)$	0.55	0.47	0.35	0.31	0.27	0.23
(t-stat)	(3.610)	(4.206)	(4.204)	(4.646)	(5.160)	(5.406)
$adjR^2$	0.24	0.45	0.51	0.60	0.66	0.67
$adjR^2(\beta_1 = -\beta_2)$	0.16	0.29	0.35	0.43	0.50	0.53
F-statistic	11.49	27.74	35.07	50.25	65.08	67.56

Testing MY against alternative models:

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

$z_t =$		1	2	3	4	5	6
$\overline{(d-p)}_t^{LVN}$	β_3	0.53	0.45	0.34	0.29	0.24	0.19
	(t-stat)	(4.111)	(5.041)	(5.453)	(6.172)	(7.314)	(8.584)
	β_4	0.31	0.23	0.13	0.08	0.02	−0.03
	(t-stat)	(2.618)	(2.739)	(1.915)	(1.512)	(0.513)	(−0.719)
$\overline{(d-p)}_t^{BMRR}$	β_3	0.62	0.51	0.39	0.32	0.27	0.21
	(t-stat)	(4.100)	(6.105)	(6.490)	(7.313)	(8.837)	(9.256)
	β_4	0.42	0.22	0.05	−0.07	−0.13	−0.11
	(t-stat)	(2.294)	(1.881)	(0.704)	(−1.734)	(−4.349)	(−4.035)

t-tests are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994). For the univariate model with the restriction $(\beta_1 = -\beta_2)$, $\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1(p_t - d_t + \frac{\beta_3}{\beta_1} MY_t) + \varepsilon_{t,t+j}$, we also conduct a (wild) bootstrap exercise (Davidson& Flachaire, 2008) to compute p-values and report for β_1 $t\ /\sqrt{T} - test$ suggested in Valkanov (2003) in curly brackets. Significance at the 10%, 5%, and 1% level of the $t\ /\sqrt{T}$ test using Valkanov's (2003) critical values is indicated by *, **, and ***, respectively. The χ^2 statistics with associated p-values is for the joint tests of significance across all different horizon within a SUR estimation framework.

Table 2.1: Estimates from a cointegrated VAR (1911-2008)

Cointegrating vector	p_t	d_t	MY_t	C
$\boldsymbol{\beta}$	-1.00	1.21	1.107	2.16
(s.e.)		(0.035)	(0.25)	
Error Correction Model	Δp_t	Δd_t	ΔMY_t	
$\boldsymbol{\alpha}$	0.29	-0.12	0.007	
(s.e.)	(0.096)	(0.046)	(0.007)	
Adj. R^2	0.126	0.43	0.63	
Cointegration Test	Trace	p-value	Max eigen	p-value
Hypothesized No of CE(s)				
None	29.68	0.05	22.86	0.028
At Most 1	6.82	0.59	6.75	0.51

Table 2.2: Estimates from a cointegrated VAR (1955-2008)

Cointegrating vector	p_t	d_t	MY_t	C
$\boldsymbol{\beta}$	-1.00	1.248	1.14	2.07
(s.e.)		(0.035)	(0.14)	
Error Correction Model	Δp_t	Δd_t	ΔMY_t	
$\boldsymbol{\alpha}$	0.63	-0.02	0.03	
(s.e.)	(0.16)	(0.035)	(0.015)	
Adj. R^2	0.22	0.40	0.75	
Cointegration Test	Trace	p-value.	Max eigen	p-value
Hypothesized No of CE(s)				
None	28.71	0.06	19.48	0.08
At Most 1	9.22	0.34	8.81	0.30

The reported p-values for the relevant null to test for cointegration are McKinnon-Haugh-Michelis(1999) p-values. The lag length in the VAR model is chosen according to optimal information criteria, i.e. sequential LR test, Akaike (AIC), Schwarz (SIC), Hannan-Quinn (HQ) information criterion.

Table 3.1: Long-horizon regressions with CAY and CDY (1948-2008)

k-period regressions for real stock returns

$$\sum_{j=1}^{k}(h^s_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

		horizon k in years					
		1	2	3	4	5	6
β_1 $(t-stat)$	cay_t	-0.41 (-4.050)	-0.29 (-4.694)	-0.21 (-5.570)	-0.20 (-7.222)	-0.19 (-8.257)	-0.15 (-8.790)
	cdy_t	-0.51 (-6.487)	-0.40 (-6.442)	-0.29 (-6.840)	-0.24 (-8.592)	-0.21 (-11.549)	0.19 (-12.403)
β_2 $(t-stat)$	cay_t	0.50 (3.994)	0.36 (4.492)	0.25 (5.230)	0.24 (6.373)	0.22 (7.067)	0.17 (7.384)
	cdy_t	0.63 (6.210)	0.48 (6.134)	0.34 (6.287)	0.28 (7.143)	0.25 (9.064)	0.22 (9.832)
β_3 $(t-stat)$	cay_t	0.66 (4.194)	0.48 (5.077)	0.36 (6.327)	0.32 (8.823)	0.29 (10.967)	0.24 (13.000)
	cdy_t	0.781 (5.467)	0.57 (5.774)	0.42 (6.146)	0.35 (7.756)	0.31 (9.394)	0.26 (10.067)
β_4 $(t-stat)$	cay_t	2.06 (1.336)	2.41 (3.329)	1.94 (3.111)	0.87 (1.192)	0.71 (1.471)	1.15 (3.739)
	cdy_t	-0.51 (-0.718)	0.25 (0.490)	0.27 (1.102)	0.04 (0.225)	0.06 (0.498)	0.44 (3.246)
$adjR^2$	cay_t	0.35	0.61	0.70	0.75	0.82	0.87
	cdy_t	0.34	0.56	0.65	0.74	0.81	0.86

t-tests are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994).

Table 3.2: Long-horizon regressions with CAY and CDY (1948-2008)

k-period regressions for real dividend-growth

$$\sum_{j=1}^{k}(\Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_{t+j} + \beta_4 z_t + \varepsilon_{t,t+j}$$

	horizon k in years						
		1	2	3	4	5	6
β_1 $(t-stat)$	cay_t	0.10 (1.649)	0.06 (1.481)	0.03 (0.977)	0.01 (0.451)	0.01 (0.268)	0.01 (0.251)
	cdy_t	−0.03 (−0.528)	−0.04 (−0.981)	−0.03 (−1.392)	−0.02 (−1.353)	−0.01 (−1.353)	−0.01 (−0.608)
β_2 $(t-stat)$	cay_t	−0.14 (−1.733)	−0.09 (−1.574)	−0.04 (−1.109)	−0.02 (−0.626)	−0.01 (−0.466)	−0.01 (−0.463)
	cdy_t	0.01 (0.233)	0.03 (0.698)	0.03 (1.071)	0.02 (0.940)	0.01 (0.586)	0.01 (0.284)
β_3 $(t-stat)$	cay_t	−0.07 (−0.838)	−0.02 (−0.363)	0.02 (0.542)	0.04 (1.286)	0.05 (1.806)	0.05 (2.406)
	cdy_t	0.02 (0.188)	0.04 (0.669)	0.05 (1.479)	0.05 (2.033)	0.04 (2.238)	0.04 (2.416)
β_4 $(t-stat)$	cay_t	2.92 (2.503)	2.49 (3.176)	1.62 (4.377)	0.99 (3.359)	0.61 (2.084)	0.39 (1.244)
	cdy_t	1.16 (1.561)	1.00 (2.115)	0.69 (4.500)	0.59 (4.853)	0.56 (4.272)	0.52 (3.670)
$adj R^2$	cay_t	0.33	0.42	0.40	0.28	0.20	0.19
	cdy_t	0.20	0.27	0.30	0.31	0.33	0.35

t-tests are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994).

Table 3.3: Long-horizon regressions with CAY and CDY (1948-2008)

k-period regressions for real stock returns adjusted for dividend growth

$$\sum_{j=1}^{k}(h^s_{t+j} - \Delta d_{t+j}) = \beta_0 + \beta_1 p_t + \beta_2 d_t + \beta_3 MY_t + \beta_4 z_t + \varepsilon_{t,t+j}$$

	horizon k in years						
		1	2	3	4	5	6
β_1 $(t\text{−}stat)$	cay_t	−0.51 (−4.796)	−0.36 (−5.417)	−0.24 (−7.799)	−0.22 (−9.769)	−0.19 (−9.506)	−0.15 (−10.203)
	cdy_t	−0.49 (−6.547)	−0.36 (−8.500)	−0.25 (−9.022)	−0.21 (−9.930)	−0.20 (−13.824)	−0.18 (−15.141)
β_2 $(t\text{−}stat)$	cay_t	0.65 (4.713)	0.45 (5.209)	0.29 (7.635)	0.26 (9.596)	0.23 (8.871)	0.18 (9.070)
	cdy_t	0.62 (6.220)	0.45 (7.653)	0.31 (7.887)	0.26 (8.109)	0.24 (10.452)	0.21 (11.669)
β_3 $(t\text{−}stat)$	cay_t	0.72 (4.487)	0.50 (5.255)	0.34 (6.670)	0.29 (9.758)	0.24 (10.237)	0.19 (10.029)
	cdy_t	0.77 (6.020)	0.53 (6.801)	0.37 (6.419)	0.31 (7.152)	0.27 (8.580)	0.21 (8.883)
β_4 $(t\text{−}stat)$	cay_t	−0.86 (−0.508)	−0.08 (−0.079)	0.329 (0.501)	−0.12 (−0.158)	0.10 (0.169)	0.757 (2.278)
	cdy_t	−1.67 (−2.428)	−0.74 (−1.295)	−0.422 (−2.141)	−0.55 (−3.636)	−0.50 (3.818)	−0.08 −0.722)
$adjR^2$	cay_t	0.28	0.50	0.63	0.72	0.79	0.87
	cdy_t	0.32	0.52	0.64	0.74	0.81	0.85

t-tests are based on heteroskedastic and autocorrelated consistent (HAC) covariance matrix estimators using Bartlett kernel weights as described in Newey & West (1987) where the bandwidth has been selected following the procedure described in Newey & West (1994).

Table 4: Predictive Performance

	In-Sample				Out-of-Sample			
Panel A (k=1)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	3.03	1.64	12.92	16.17	−11.22	14.54	18.60	−17.43
dp_t^{LVN}	6.36	2.64	11.93	16.20	−5.25	13.58	18.09	−8.09
cdy_t	−1.47	0.48	12.03	14.00	−16.11	13.24	15.12	−19.97
dp_t^{DT}	19.48	4.37	10.08	15.32	11.20	10.91	16.27	4.68
$dp_t^{DT} + cdy_t$	38.64	$5.97/-0.32$	8.71	10.97	28.61	9.83	11.86	14.36
Hist.Mean	–	–	12.92	16.70	–	13.40	17.63	–
Panel B (k= 2)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	5.46	1.70	15.72	20.71	−55.77	24.21	30.94	−4.01
dp_t^{LVN}	15.74	2.45	14.20	19.91	−11.72	19.99	26.20	−3.30
cdy_t	2.92	1.13	16.00	21.93	−55.19	20.04	27.45	−1.33
dp_t^{DT}	48.32	7.88	12.20	17.52	35.52	14.39	19.90	3.11
$dp_t^{DT} + cdy_t$	62.63	6.32/1.40	10.58	14.17	40.85	13.16	16.94	6.08
Hist. Mean	–	–	16.19	21.91	–	18.65	24.79	–
Panel C (k= 3)	R^2	t-stat	MAE	RMSE	R^2_{OS}	MAE	RMSE	DM
dp_t	6.34	1.99	18.26	24.87	−88.56	33.98	43.59	1.32
dp_t^{LVN}	10.76	1.70	17.91	24.81	−25.89	28.40	35.61	−4.41
cdy_t	5.91	1.41	19.45	26.94	−41.43	26.64	33.70	−1.97
dp_t^{DT}	49.73	6.70	13.22	19.00	43.95	18.15	23.32	2.90
$dp_t^{DT} + cdy_t$	64.89	7.24/2.99	13.67	16.99	48.54	17.06	20.33	2.50
Hist. Mean	–	–	19.38	26.65	–	25.26	31.74	–

This table presents statistics on k-year ahead forecast errors (in-sample and out-of-sample) for stock returns. The sample starts in 1955 and we construct first forecast in 1982. All numbers are in percent. RMSE is the root mean square error, MAE is the mean absolute error. DM is the Diebold and Mariano (1995) t-statistic for difference in MSE of the unconditional forecast and the conditional forecast. The out-of-sample R^2_{OS} compares the forecast error of the historical mean with the forecast from predictive regressions.

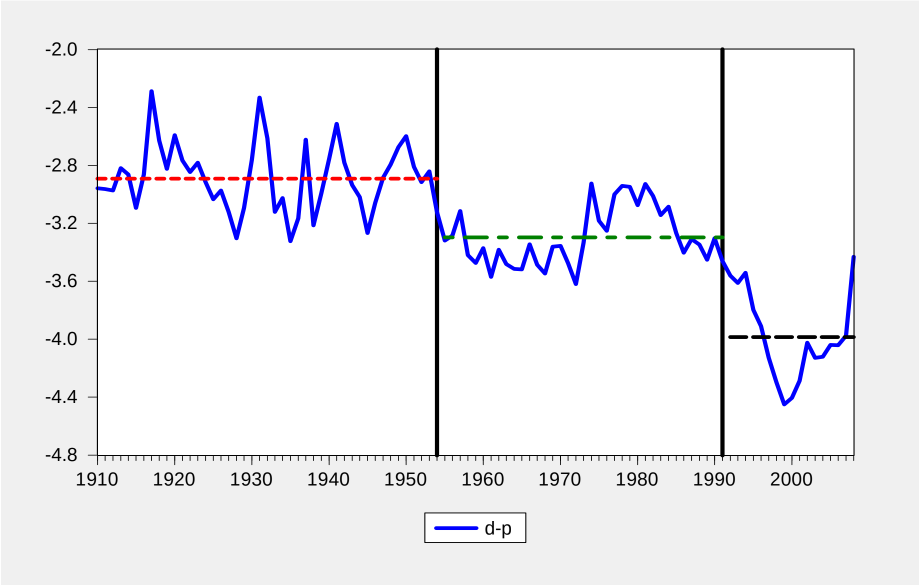

Figure 1. The time series of log dividend price ratio ($d_t - p_t$). Annual data from 1909 to 2008.

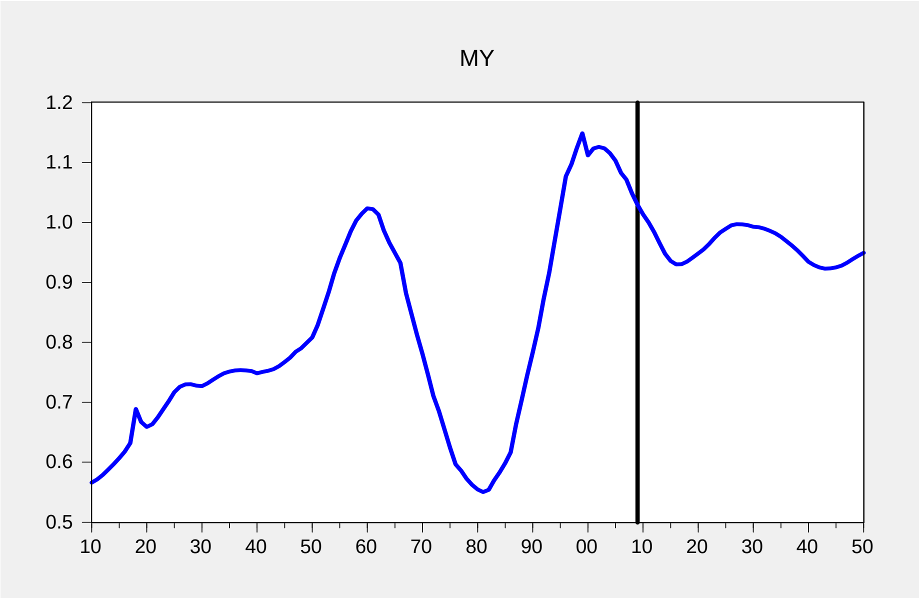

Figure 2. The time series of Middle-Young (MY) ratio.

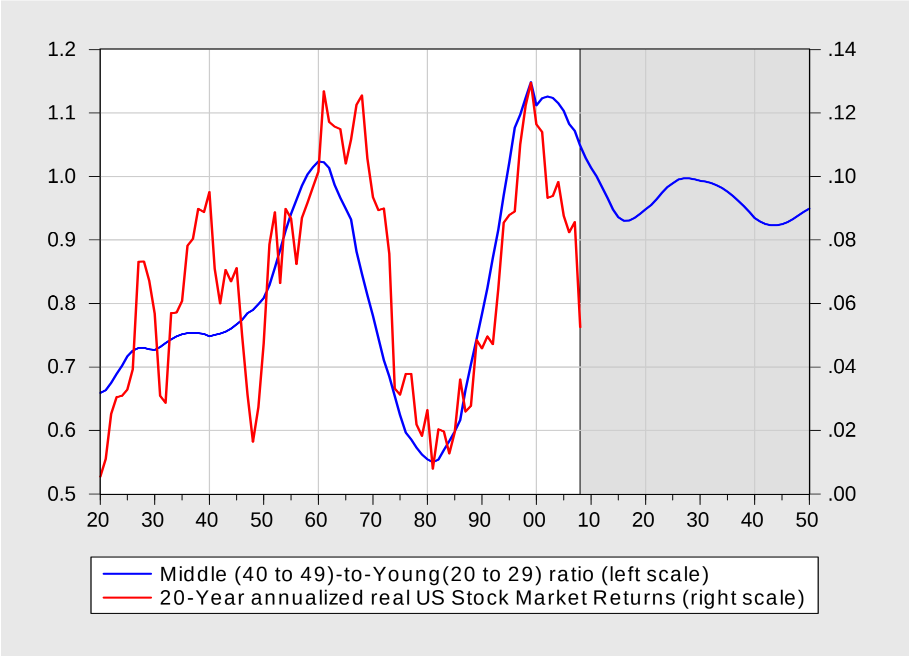

Figure 3: MY and 20-year annualized real US stock market returns

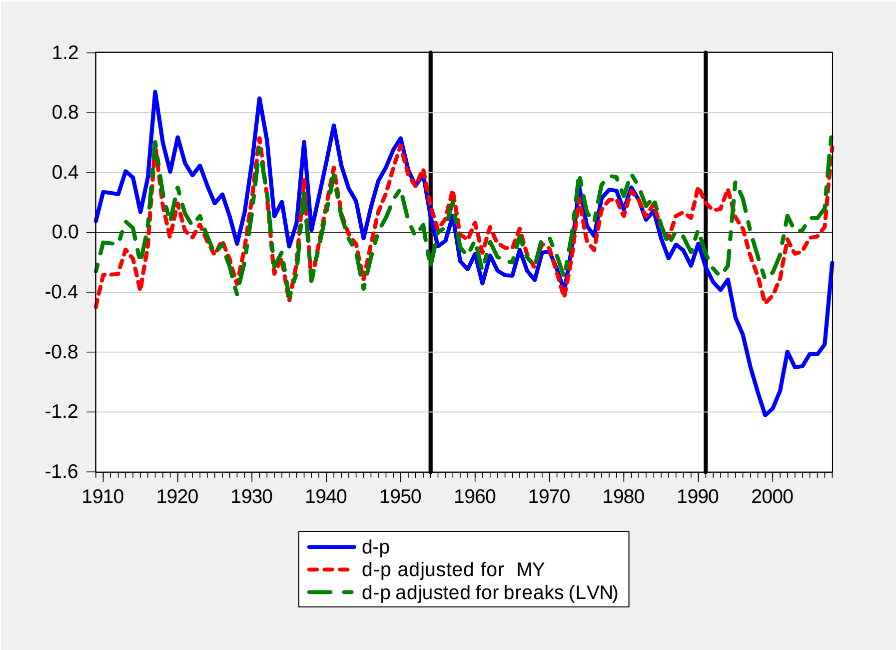

Figure 4.1: (d-p), (d-p) adjusted for breaks (LVN) and fluctuatins of (d-p) around a time-varying mean determined by MY

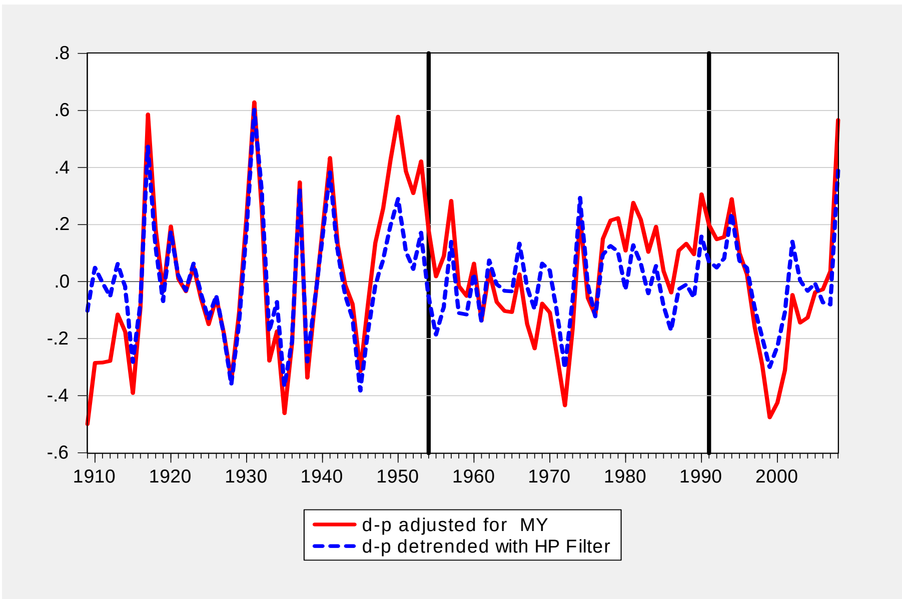

Figure 4.2: Two alternative (and consitent) measure of the cycle in (d-p)

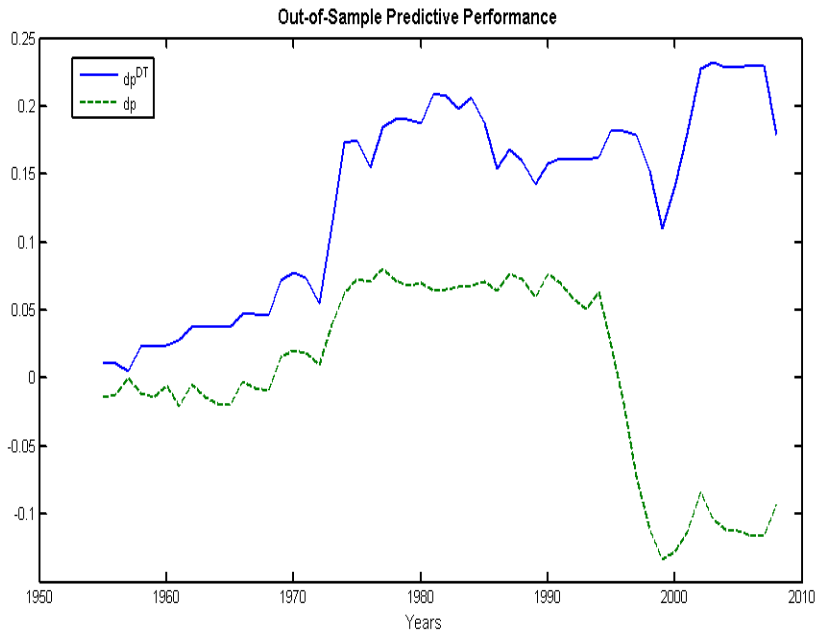

Figure 5: differences of cumulative RMSE of forecasts based on the historical prevaling mean and RMSE of forecasting models based on $(d-p)_t$ and on $(d-p)_t$ corrected for MY.

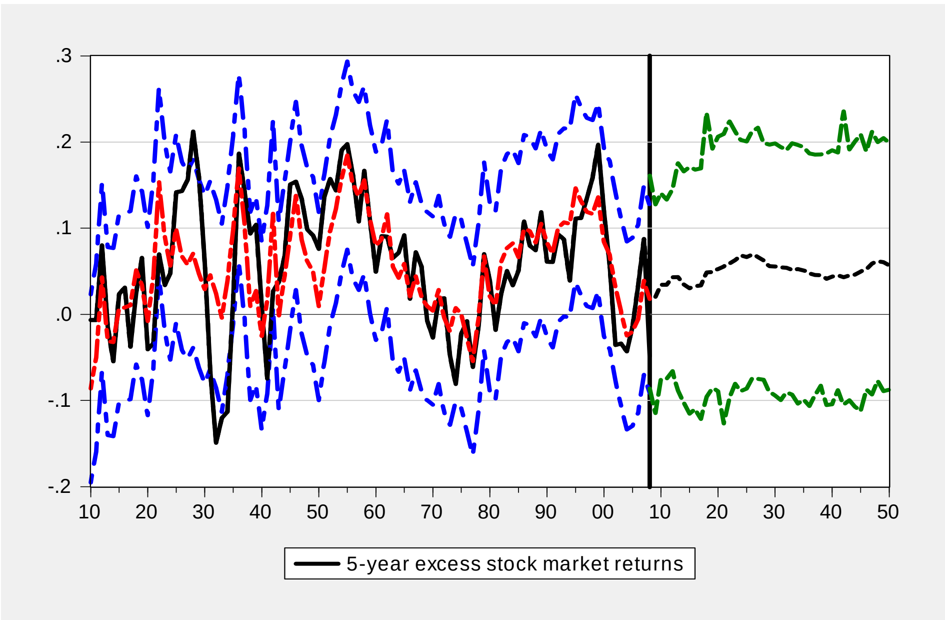

Figure 6: within sample and out-of-sample projections for 5-year stock market excess returns.

APPENDIX A: The Statistical Model for Cointegration Analysis

We consider the following statistical model:

$$\mathbf{y}_t = \sum_{i=1}^{n} \mathbf{A}_i \mathbf{y}_{t-i} + \mathbf{u}_t \tag{8}$$

$$\mathbf{y}_t \text{ is an } mx1 \text{ vector of variables} \tag{9}$$

This model can be re-written as follows

$$\begin{aligned} \Delta \mathbf{y}_t &= \mathbf{\Pi}_1 \Delta \mathbf{y}_{t-1} + \mathbf{\Pi}_2 \Delta \mathbf{y}_{t-2} + ...\mathbf{\Pi}_{n-1} \Delta \mathbf{y}_{t-n+1} + \mathbf{\Pi} \mathbf{y}_{t-1} + \mathbf{u}_t \\ &= \sum_{i=1}^{n-1} \mathbf{\Pi}_i \Delta \mathbf{y}_{t-i} + \mathbf{\Pi} \mathbf{y}_{t-1} + \mathbf{u}_t, \end{aligned} \tag{10}$$

where:

$$\begin{aligned} \mathbf{\Pi}_i &= -\left(I - \sum_{j=1}^{i} \mathbf{A}_j \right), \\ \mathbf{\Pi} &= -\left(I - \sum_{i=1}^{n} \mathbf{A}_i \right). \end{aligned}$$

Clearly the long-run properties of the system are described by the properties of the matrix Π. There are three cases of interest:

1. rank $(\Pi) = 0$. The system is non-stationary, with no cointegration between the variables considered. This is the only case in which non-stationarity is correctly removed simply by taking the first differences of the variables;

2. rank $(\Pi) = m$, full. The system is stationary;

3. rank $(\Pi) = k < m$. The system is non-stationary but there are k cointegrating relationships among the considered variables. In this case $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$, where α is an $(m \times k)$ matrix of weights and $\boldsymbol{\beta}$ is an $(k \times m)$ matrix of parameters determining the cointegrating relationships.

Therefore, the rank of Π is crucial in determining the number of cointegrating vectors. The Johansen procedure is based on the fact that the rank of a matrix equals the number of its characteristic roots that differ from zero. The Johansen test for cointegration is based on the estimates of the two characteristic roots of $\mathbf{\Pi}$ matrix. Having obtained estimates for the parameters in the $\mathbf{\Pi}$ matrix, we associate with them estimates for the m characteristic roots and we order them as follows $\lambda_1 > \lambda_2 > ...\lambda_m$. If the variables are not cointegrated, then the rank of Π is zero and all the characteristic roots equal zero. In this case each of the expression $\ln(1 - \lambda_i)$ equals zero, too. If, instead, the rank of Π

is one, and $0 < \lambda_1 < 1$, then $\ln(1-\lambda_1)$ is negative and $\ln(1-\lambda_2) = \ln(1-\lambda_3) = ... = \ln(1-\lambda_m) = 0$. The Johansen test for cointegration in our bivariate VAR is based on the two following statistics that Johansen derives based on the number of characteristic roots that are different from zero:

$$\begin{aligned} \lambda_{\text{trace}}(k) &= -T \sum_{i=k+1}^{m} \ln\left(1-\widehat{\lambda}_i\right), \\ \lambda_{\max}(k, k+1) &= -T \ln\left(1-\widehat{\lambda}_{k+1}\right), \end{aligned}$$

where T is the number of observations used to estimate the VAR. The first statistic tests the null of at most k cointegrating vectors against a generic alternative. The test should be run in sequence starting from the null of at most zero cointegrating vectors up to the case of at most m cointegrating vectors. The second statistic tests the null of at most k cointegrating vectors against the alternative of at most $k+1$ cointegrating vectors. Both statistics are small under the null hypothesis. Critical values are tabulated by Johansen (1991) and they depend on the number of non-stationary components under the null and on the specification of the deterministic component of the VAR.

APPENDIX B: Robustness analysis for the cointegrating evidence

To assess the robustness of our cointegrating relationship in identifying the low frequency relation between stock market and demographics, we evaluate the effect of augmenting our baseline relation with an alternative demographic factor. Research in demography has recently concentrated on the economic impact of the *demographic dividend* (Bloom et al., 2003; Mason&Lee, 2005). The demographic dividend depends on a peculiar period in the demographic transition phase of modern population in which the lack of synchronicity between the decline in fertility and the decline in mortality typical of advanced economies has an impact on the age structure of population. In particular a high support ratio is generated, i.e. a high ratio between the share of the population in working age and the share of population economically dependent. Empirical evidence has shown that the explicit consideration of the fluctuations in the support ratio delivers significant results in explaining economic performance (see Bloom et al., 2003). The concept of *Support Ratio (SR)* has been precisely defined by Mason and Lee (2005) as the ratio between the number of effective number of producers, L_t, over the effective number of consumers, N_t (Mason&Lee, 2005). In practice we adopt the following empirical proxy:

$$SR = a2064/(a019 + a65ov)$$

where $a2064$: Share of population between age 20-64, $a019$: Share of population between age 0-19, $a65ov$: Share of population age 65+[7].

[7] We have checked robustness of our results by shifting the upper limit of the producers to the age of

SR did not attract a significant coefficient when we augmented our cointegrating specification with this variable.

75. This is consistent with the evidence on the cross-sectional age-wealth profile from Survey of Consumer Finances, provided in Table 1 of Poterba(2001), which shows that the population share between 64-74 still holds considerable amount of common stocks. Results are available upon request.

APPENDIX C: In Appendix we give an exact description of all time-series used in our empirical investigation.

Stock Market Prices: S&P 500 index yearly prices from 1909 to 2008 are from Robert Shiller's website, we take end-of-period observations.

Stock Market Returns: For S&P 500 index, to construct the continuously compounded return r_t, we take the ex-dividend price P_t add dividend D_t[8] over P_{t-1} and take the natural logarithm of the ratio.

Risk-free Rate: We download secondary market 3-Month Treasury Bill rate from St.Louis (FRED) from 1934-2008. The risk-free rate for the period 1920 to 1933 is from New York City from NBER's Macrohistory data base. Since there was no risk-free short-term debt prior to the 1920's, we estimate it following Goyal&Welch (2007). We obtain commercial paper rates for New York City from NBER's Macrohistory data base. These are available for the period 1871 to 1970. We estimate a regression for the period 1920 to 1971, which yielded

$$T - billRate = -0.004 + 0.886 \times \ CommercialPaperRate.$$

Therefore, we instrument the risk-free rate for the period 1909 to 1919 with the predicted regression equation.

Hence we build our dependent variable which is the equity premium $(r_{m.t} - r_{f,t})$, i.e., the rate of return on the stock market minus the prevailing short-term interest rate in the year $t-1$ to t.

Second, we construct the independent variables commonly used in the long horizon stock market prediction literature; namely

Log Dividend-Price Ratio $(d - p_t)$: is the difference between the log of dividends and the log of prices. Data refer to the S&P 500 index, and they are taken from Robert Shiller's website.

Log Dividend-Earnings (payout) ratio: Both annual dividend and earning series are taken from Robert Shiller's website. The variable is constructed by taking the natural logarithm of D_t over E_t (Lamont,1998).

Log Earnings Price ratio: Both annual price and earning series are taken from Robert Shiller's website. The variable is constructed by taking the natural logarithm of E_t over P_t (Lamont,1998).

RREL: This variable, the stochastically detrended riskless rate, is constructed using monthly 3-Month Treasury Bill yield data from NBER Macrohistory Data Base (from 1920 to 1933) and 3-Month Treasury Bill: Secondary Market Rate from FRED St. Loius (1934-2008); i.e. we define RREL for month t, $RREL_t$ is r_t minus the average of r_t from

[8] In Robert Shiller's database, Prices are beginning of period, i.e. January prices, whereas dividends are distributed at the end of the period. In the last section, we simulated our models with december prices.

months $t - 12$ to $t - 1$. Yearly $RREL_t$ is the last observation at the end of the year (Campbell,1991; Hodrick,1992). The data is available from 1921-2008.

TERM: is the difference between the long-term government bond yield (10year) from Robert Shiller's Website and 3-Month T-Bill yield from NBER Macrohistory Data Base (from 1920 to 1933) and 3-Month Treasury Bill: Secondary Market Rate from FRED St. Loius (1934-2008) and available from 1920 to 2008.

DEFAULT: is the difference between the BAA and the AAA corporate bond rates. Both series are collected from St.Louis (FRED) and available from 1919 to 2008.

Consumption, wealth, income ratio (cay): taken from Lettau and Ludvigson (2001). Data are available from Sydney Ludvigson's website at annual frequency from 1948 to 2001. Lettau-Ludvigson estimates are described in equation (4) in their paper, where two lags are used in annual estimation (k = 2). This variable is called $cayp(post)$ by Goyal&Welch (2008), as they illustrate the possibility of look-ahead bias, we also consider their variable $caya(ante)$ that eliminates the bias, but report the results using $cayp$, to heve dirct comparability with LL.

Consumption, dividend, income ratio (cdy): taken from in LL (2005). Data are available from Sydney Ludvigson's website at annual frequency from 1948 to 2001. Lettau-Ludvigson estimates are described in equation (4) in their paper, where two lags are used in annual estimation (k = 2).

Demographic Variables

The U.S annual population estimates series are collected from U.S Census Bureau and the sample covers estimates from 1900-2050.

DATA SOURCES

Robert Shiller's Website: http://www.econ.yale.edu/~shiller/

NBER Macrohistory Data Base:

http://www.nber.org/databases/macrohistory/contents/chapter13.html.

Martin Lettau's Website: http://faculty.haas.berkeley.edu/lettau/

WRDS: http://wrds.wharton.upenn.edu/

US Census Bureau: http://www.census.gov/popest/archives/

Andrew Mason's Website: http://www2.hawaii.edu/~amason/

Bureau of Labor Statistics Webpage: http://www.bls.gov/data/

FRED: http://research.stlouisfed.org/fred2/

Michael R. Roberts' Website: http://finance.wharton.upenn.edu/~mrrobert/