Measuring Tax Multipliers. The Narrative Method in Fiscal VARs.

By Carlo Favero and Francesco Giavazzi *

This paper argues in favour of empirical models built by including in fiscal VAR models structural shocks identified via the narrative method. We first show that "narrative" shocks are orthogonal to the relevant information set a fiscal VAR. We then derive impulse responses to these shocks. The use of narrative shocks does not require the inversion of the moving-average representation of a VAR for the identification of the relevant shocks. Therefore, within this framework, fiscal multipliers can be identified and estimated even when, in the presence of "fiscal foresight", the MA representation of the VARs is not invertible.
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This paper argues in favour of estimating fiscal multipliers by including in a VAR shocks to taxes and government revenues identified via the narrative methods (Christina Romer and David Romer 2010, from here onwards R&R). If shocks identified via the narrative method are orthogonal to the relevant information set in the VAR (a testable proposition), then fiscal multipliers are naturally computed via the impulse responses to such shocks generated by the VAR. An essential advantage of the procedure we propose is that it does not require the inversion of the moving-average representation of a VAR. This means that fiscal multipliers can be validly identified and estimated even when the MA representation of the VAR is not invertible — which is the case when agents receive information on the tax changes they will face in the future, *i.e.* in the presence of fiscal foresight.

Using this approach we solve an apparent puzzle in the measurement of tax multipliers. In the empirical literature tax multipliers estimated analyzing the effects of shocks identified within a VAR are surprisingly different from multipliers computed using shocks identified via the narrative method. R&R, using U.S. data and studying the post World War II period, find a tax multiplier significantly

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greater than one: a tax increase equivalent to 1 per cent of GDP reduces output over the next three years by nearly 3 per cent. Instead, authors who analyze tax shocks identified within a VAR (Olivier Blanchard and Roberto Perotti, 2002 and Perotti, 2008, from here onwards B&P) typically find a multiplier of about one. We show that this difference is not explained by a difference in the shocks (VAR *vs.* narrative) but by the different models used to estimate their effects on macro variables. If the effects of shocks identified by the narrative method are analyzed in the context of a multivariate VAR (rather than using a limited information, single-equation approach), then the multiplier is not different from that obtained in the traditional fiscal VAR approach.

The paper starts replicating the apparently contradictory results delivered by the two main approaches to the estimation of tax multipliers. R&R, as we said, identify tax shocks from the "narrative" of Presidential speeches and Congressional records: this analysis allows them to separate legislated changes in those they consider endogenous (induced by short-run counter-cyclical concerns, or adopted as a response to changes in government spending) and those they judge exogenous (associated with a political shift, or adopted in response to the state of government debt, or in the attempt to raise long-run economic growth). Analyzing the post World War II period they find, as we mentioned, a multiplier significantly greater than one. The fiscal VAR approach identifies tax shocks either (as in B&P) exploiting the fact that it typically takes longer than one quarter for discretionary fiscal policy to respond to news in macroeconomic variables and using institutional information on the elasticities of tax revenues and government spending to macro variables, or imposing restrictions on the sign of impulse responses (Andrew Mountford and Harald Uhlig 2002), or relying on a Choleski ordering (Antonio Fatàs and Ilian Mihov 2001). These different identification schemes deliver similar tax multipliers — typically close to one.

There are also important differences in the structural stability of these estimates in the post World War II period. The B&P results for the sample 1960 to 2001 average very different multipliers before and after 1980. In the first part of the sample tax cuts have a positive and significant effect on output, with a multiplier only slightly smaller compared with R&R (around 2.6 at a three year horizon). After 1980 the multiplier turns negative and significant. On the contrary, the tax multiplier estimated by R&R is stable over the two sub-samples.

This contrasting evidence is summarized in Figure 1, where we reproduce the effect on output of an exogenous shift in U.S. Federal tax liabilities equivalent to 1 per cent of U.S. GDP computed using the two approaches: the R&R method and the B&P identification scheme applied to a closed-economy fiscal VAR which includes government expenditure, government receipts, output growth, inflation, and the average interest cost of the public debt.

Could these differences be due to the fact that structural VAR's fail to identify truly exogenous shifts in taxes? Figure 2 shows that indeed the exogenous tax shocks identified by the two alternative methods are quite different. Their correlation over the entire sample is 0.22 and the two identification strategies lead to a substantial disagreement as to when the largest shifts in tax policy occurred. "Shocks" measure exogenous shifts in government revenues, but there is no reason why such measure should be unique. Different identification approaches could produce different time series of tax shocks, each exogenous and thus each legitimate. In other words, alternative instruments, both valid, although different, could deliver the same estimate of the tax multiplier.

The point that alternative ways of identifying tax shocks could be the reason why estimated multipliers differ, is reminiscent of the debate on the identification of monetary policy shocks. Glenn Rudebusch (1998) criticizes the VAR-based analysis of the effects of monetary policy shocks observing that shocks identified from structural VAR's — typically from a regression of the Federal funds rate on an assortment of macro variables, and therefore via a recursive identification scheme between macroeconomic variables and monetary policy — bear little correlation with shocks to the Federal funds rate derived from forward-looking financial markets (the Fed funds future). He thus concludes that monetary policy shocks identified from a VAR make no sense. Christopher Sims (1998) replies observing that in a multivariate framework measures of the same variables that bear little correlation with one another can produce identical transmission mechanisms. He suggests, as an example, the measurement of the effects of supply shocks in a simple demand-supply model. Two variables can shift the supply function, weather and insect density. Consider two alternative instrumenting strategies: each excludes one instrument. As both supply shifters are valid instruments, the two models will produce valid and equivalent estimates of the structural parameters, despite the fact that they use different instruments and thus identify different shocks to the supply function.

This observation suggests that in order to compare tax multipliers obtained from shocks estimated in a VAR with those obtained using "narrative" shocks, the obvious thing to do consists in including narrative shocks in a VAR model and then compare the two impulse responses. This paper shows why this can be done and argues that it is a better way to estimate tax multipliers. In the case of monetary policy shocks Fabio Bagliano and Carlo Favero (1999), using the same approach, have shown that VAR and non-VAR shocks deliver the same description of the monetary transmission mechanism.

The paper is organized as follows. We start by showing that the R&R narrative shocks are valid shocks from the point of view of a fiscal VAR, *i.e.* they are orthogonal to the information set used to construct the conditional distribution of the variables included in a fiscal VAR (output, tax revenues, government spending, inflation and the cost of debt service). In the light of this evidence we estimate the tax multiplier treating the shocks identified via the narrative method as structural shocks in the VAR. We argue that this is the natural way to estimate the tax multiplier because one of the variables included in a fiscal VAR is government revenue, and what the narrative method does is precisely to identify exogenous shocks to government revenue. Using this approach we obtain estimates of the tax multiplier that are very similar to those obtained identifying tax shocks within a VAR and we thus conclude that the apparent puzzle in the measurement of tax multipliers depends on the limited information approach used by R&R. We then consider the possibility that R&R shocks are measured with error due to the presence of implementation lags in fiscal policy. This allows us to show another advantage of using narrative shocks in a fiscal VAR: using the subdivision (proposed by Karel Mertens and Morten Ravn 2010) of the R&R shocks into anticipated and unanticipated we compute fiscal multipliers associated to unanticipated tax shocks and to anticipated tax shocks, that is shifts in taxes announced at time t with some implementation lag. Before closing, we address the issue of the validity of the VAR specification for the empirical analysis of the effects of fiscal policy.

I. VAR-based and Narrative Measures of the tax multiplier

This section proposes a way to estimate tax multipliers based on combining the fiscal VAR approach with the narrative identification of shocks to government revenue. We start by describing how Figures 1 and 2 were constructed.

A. The VAR Approach

We first consider the structural VAR estimated in the B&P approach. Tax multipliers are obtained from a vector autoregression of the form [1]:

$$\begin{aligned} \mathbf{Z}_t &= \mathbf{C}_1 \mathbf{Z}_{t-1} + \mathbf{u}_t \\ \mathbf{Z}_t^{'} &= \left[\begin{array}{ccccc} i_t & y_t & \Delta p_t & t_t & g_t \end{array} \right] \end{aligned} \tag{1}$$

where i_t is the average nominal cost of the public debt, y_t is level of real GDP, Δp_t is inflation, t_t and g_t are, respectively, (the logs of) government revenues and government expenditures net of interest. We regard this as a natural choice for a minimal set of variables to be included in the analysis of the effects of fiscal policy in that it allows to fully recover the debt-deficit dynamics. Being able to track the debt dynamics when estimating fiscal multipliers is of crucial importance to avoid analyses of "unsustainable fiscal policies" and to make sure that the question "What is the fiscal multiplier" is not asked along a path for the debt dynamics that is at odds with the beliefs of government bond-holders (see Eric Leeper 2010)[2].

[1] For simplicity we consider a first order VAR. VARs of any order can be re-parametrized as a first order VAR, using a stacked representaton.

[2] Our choice of variables is very close to that of Perotti (2008) but it is wider than that of the first Blanchard and Perotti (2002) paper in which the analysis of fiscal policy is conducted within a three-

Structural VAR's identify fiscal shocks imposing restrictions that allow to recover uniquely the structural shocks of interest from the reduced form residuals, $\mathbf{u}_t$. The innovations in the reduced form equations for taxes and government spending, u_t^g and u_t^t, contain three terms: *(i)* the response of taxes and government spending to fluctuations in macroeconomic variables, such as output and inflation, that is implied by the presence of automatic stabilizers; *(ii)* the discretionary response of fiscal policy to news in macro variables, and *(iii)* truly exogenous shifts in taxes and spending, the shocks we wish to identify. B&P exploit the fact that it typically takes longer than a quarter for discretionary fiscal policy to respond to news in macroeconomic variables: at quarterly frequency the contemporaneous discretionary response of fiscal policy to macroeconomic data can thus be assumed to be zero. To identify the component of u_t^g and u_t^t which corresponds to automatic stabilizers they use institutional information on the elasticities of tax revenues and government spending to macroeconomic variables. They thus identify the structural shocks to g and t by imposing on the matrices $\mathbf{A}$ and $\mathbf{B}$ that determine the mapping from the VAR innovations $\mathbf{u}$ to the structural shocks $\mathbf{e}$ ($\mathbf{A}\mathbf{u}_t = \mathbf{B}\mathbf{e}_t$) the following restrictions

$$\begin{bmatrix} 1 & 0 & a_{gy} & a_{g\Delta p} & a_{gi} \\ 0 & 1 & a_{ty} & a_{t\Delta p} & a_{ti} \\ a_{31} & a_{32} & 1 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & 1 & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & 1 \end{bmatrix} \begin{bmatrix} u_t^g \\ u_t^t \\ u_t^y \\ u_t^{\Delta p_t} \\ u_t^i \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 \\ 0 & 0 & 0 & b_{44} & 0 \\ 0 & 0 & 0 & 0 & b_{55} \end{bmatrix} \begin{bmatrix} e_t^g \\ e_t^t \\ e_t^1 \\ e_t^2 \\ e_t^3 \end{bmatrix}$$

where e_t^i $(i = 1, 2, 3)$ are non-fiscal shocks and have no structural interpretation. Since a_{gy}, $a_{g\Delta p}$, a_{gi}, a_{ty}, $a_{t\Delta p}$ and a_{ti} are identified using external information [3], there are only 15 parameters to be estimated. As there are also 15 different elements in the variance-covariance matrix of the 5-equation VAR innovations, the model is just identified. The e_t^i $(i = 1, 2, 3)$ are derived by imposing a recursive scheme on the bottom three rows of $\mathbf{A}$ and $\mathbf{B}$, but the identification of the two fiscal shocks—the only ones that we shall use to compute impulse responses—is independent of this assumption. Finally, the identification assumption imposes

variable VAR for government expenditure, government receipts and GDP. All the results we report in this section based on the B&P approach are robust when a simpler three-variable specification is adopted instead of our five-variables VAR. Results from a three-variable VAR are available upon request. . The Data Appendix describes how our data-set was constructed.

[3] The elasticities of taxes and government spending with respect to output, inflation and interest rates used in the identification have been updated in Perotti (2008) and are

Elasticities of government revenues and expenditures						
	a_{gy}	$a_{g\Delta p}$	a_{gi}	a_{ty}	$a_{t\Delta p}$	a_{ti}
Entire sample	0	-0.5	0	1.85	1.25	0
1960:1-1979:4	0	-0.5	0	1.75	1.09	0
1980:1-2006:2	0	-0.5	0	1.97	1.40	0

$b_{12} = 0$.[4]

Figure 1 reports, under the label *B&P impulse responses* the responses of the level of output to a one-period shock in e_t^t of the size of 1 per cent of GDP. In Figure 2 we report under the label *Blanchard-Perotti VAR shocks* the time-series of e_t^t.

B. *The Narrative Approach*

R&R construct a time-series of shocks to government revenues using an approach that does not require the estimation of a model. They consult the narrative record, such as Presidential speeches and Congressional reports, to identify the size, timing, and principal motivation for all major postwar tax policy actions. They then classify legislated tax changes into *endogenous* (those induced by short-run countercyclical concerns and those taken in response to a change in government spending) and *exogenous* (those that are responses to the state of government debt or to concerns about long-run economic growth). Having constructed a time series of exogenous shifts in taxes, e_{t-i}^{RR} — where each e_{t-i}^{RR} measures the impact of a tax change at the time it was implemented $(t-i)$ on tax liabilities at time t — R&R measure their effect on output estimating, using quarterly data and ordinary least squares, a single equation of the form

$$\Delta y_t = a + \sum_{j=0}^{M} b_j e_{t-j}^{RR} + v_t \tag{2}$$

where Δy_t is real GDP growth. Exogenous tax shocks are measured as a percentage of GDP. So the response of the level of output at time $t+i$ to a one-period shock of the size of 1 per cent of GDP is measured by the sum of the b_i coefficients. This is what we report in Figure 1 under the label *R&R.* As in R&R we have chosen $M = 12$. Figure 2 reports the time series of e_{t-i}^{RR}. Note that the correlation between e_t^t and e_t^{RR} is not very high (0.22), although it is statistically different from zero ($t - stat$, 3.22). Moreover the evidence from important (in quantitative terms) episodes is mixed in the sense that we have both matches and mis-matches.

C. *Understanding the difference*

To understand the difference between the narrative and the VAR approaches we need a common "encompassing" framework. We construct it starting from

[4] B&P provide robustness checks for this assumption by setting $b_{21} = 0$ and estimating b_{12}. We have also experimented with this alternative assumption. In practice, as the top left corner of the $\mathbf{B}$ matrix is not statistically different from a diagonal matrix, the assumption $b_{12} = 0$ is irrelevant to determine the shape of impulse response functions.

the structural representation of the VAR in (1)

$$\mathbf{A}\mathbf{Z}_t = \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{B}\mathbf{e}_t. \tag{3}$$

The MA representation of (3) is

$$\mathbf{Z}_t = \mathbf{\Gamma}(L)\mathbf{e}_t \tag{4}$$

where $\mathbf{\Gamma}(L) \equiv \frac{\mathbf{A}^{-1}\mathbf{B}}{\mathbf{I}-\mathbf{A}^{-1}\mathbf{C}L}$. The MA representation is not directly estimated in the VAR, but it can be derived by inversion, after having estimated (3) .

We re-write (4) as follows

$$\begin{aligned} \mathbf{Z}_t &= \sum_{j=0}^{M} \mathbf{\Gamma}_0^j \mathbf{\Gamma}_1 \mathbf{e}_{t-i} + \mathbf{\Gamma}_1^{M+1} \mathbf{Z}_{t-(M+1)} \\ \mathbf{\Gamma}_0 &\equiv \mathbf{A}^{-1}\mathbf{B},\ \mathbf{\Gamma}_1 \equiv \mathbf{A}^{-1}\mathbf{C}. \end{aligned}$$

and extract from the above system the equation for output growth

$$\begin{aligned} \Delta y_t &= \sum_{j=0}^{M} \gamma_j^{y,t} e_{t-j}^t + \sum_{j=0}^{M} \gamma_j^{y,g} e_{t-j}^g + \sum_{j=0}^{M} \gamma_j^{y,y} e_{t-j}^y \\ &\quad + \sum_{j=0}^{M} \gamma_j^{y,\Delta p} e_{t-j}^{\Delta p} + \sum_{j=0}^{M} \gamma_j^{y,i} e_{t-j}^i + \mathbf{\Gamma}_1^{M+1} \mathbf{Z}_{t-(M+1)} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \gamma_j^{y,x} &= \mathbf{s}^x \mathbf{\Gamma}_0 \mathbf{\Gamma}_1^i \mathbf{s}^{t\prime} \qquad x = t, g, y, \Delta p, i \\ \mathbf{s}^g &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix},\ \mathbf{s}^t = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \\ \mathbf{s}^y &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{s}^{\Delta p} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \\ \mathbf{s}^i &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

To compare (5) with (2), the equation estimated by R&R, we need to spell out the relation between the structural tax shocks identified from the VAR and those constructed using the narrative method. Assume

$$
\begin{aligned}
e_t^t &= e_t^{RR} + \varepsilon_t \\
\varepsilon_t &\sim i.i.d.\left(0, \sigma_\varepsilon^2\right)
\end{aligned}
\tag{6}
$$

i.e. assume that the difference between the shocks identified from the VAR and those identified via the narrative method is some error ε_t. This assumption has a number of testable implications, in particular e_t^{RR} should be orthogonal to all the lags of all the variables included in the VAR. We shall test this in the next sub-section of the paper.

Substituting (6) into (5) we obtain a specification for Δy_t that encompasses the two alternative models

$$
\begin{aligned}
\Delta y_t = & \sum_{j=0}^{M} \gamma_j^{y,t} e_{t-j}^{RR} \\
& + \sum_{j=0}^{M} \gamma_j^{y,t} \varepsilon_{t-i} \\
& + \sum_{j=0}^{M} \gamma_j^{y,g} e_{t-j}^{g} + \sum_{j=0}^{M} \gamma_j^{y,y} e_{t-j}^{y} + \sum_{j=0}^{M} \gamma_j^{y,\Delta p} e_{t-j}^{\Delta p} + \sum_{j=0}^{M} \gamma_j^{y,i} e_{t-j}^{i} \\
& + \mathbf{\Gamma}_1^{M+1} \mathbf{Z}_{t-(M+1)}
\end{aligned}
\tag{7}
$$

(7) makes clear that the limited information approach adopted by R&R — in which the tax multiplier is estimated from a specification including only the first term in (7) — has the advantage of allowing a direct identification of the tax shocks, but at the cost of omitting several sources of information included in the system approach adopted in a VAR. The question is how large are the costs and benefits of the two strategies. We provide an assessment starting by asking under what conditions the two approaches will deliver the same estimate of the tax multiplier.

D. Evaluating the difference

Comparing (7) with (2) reveals that there are three conditions that need to be satisfied for the two approaches to deliver the same estimates of the impulse response of output to a tax shock e_t^{RR}

1) the tax shocks e_{t-j}^{RR} must be orthogonal to the *noise term* ε_{t-j} introduced by the VAR

2) the tax shocks e_{t-j}^{RR} must be orthogonal to all other shocks in the VAR that

might influence output growth: e_{t-j}^{g}, e_{t-j}^{y}, $e_{t-j}^{\Delta p}$, e_{t-j}^{i}

3) the tax shocks e_{t-j}^{RR} must be orthogonal to $\mathbf{Z}_{t-(M+1)}$.

Given the specification of the VAR, the first condition can be tested by assessing the orthogonality of the R&R shocks to the information set that is used in the VAR to measure innovations and therefore shocks.

Orthogonality of e_{t-j}^{RR} to e_{t-j}^{g}, e_{t-j}^{y}, $e_{t-j}^{\Delta p}$ and e_{t-j}^{i} is the identifying assumption in R&R: from an analysis of the extensive discussion in the narrative record of why each e_{t-j}^{RR} action was taken, R&R conclude that *"most actions had a single predominant motivation, and that some of those motivations are unrelated to other factors likely to have important effects on output growth (and to any other tax responses policymakers may have been making to those factors at around the same time)"*.

The third condition, however is unlikely to be satisfied, for the following reason. R&R classify as exogenous those legislated tax changes that are "*responses to the state of government debt or to concerns about long-run economic growth*". Since the variables that in a fiscal VAR are normally included in **Z** fully determine the dynamics of debt, the orthogonality of e_{t-j}^{RR} to $\mathbf{Z}_{t-(M+1)}$ does not seem to be satisfied by the R&R identification strategy because it is unlikely that the orthogonality condition would apply to distant lags of **Z**. To see this, consider the government intertemporal budget constraint

$$d_t = \frac{1+i_t}{(1+x_t)} d_{t-1} + \frac{\exp(g_t) - \exp(t_t)}{\exp(y_t)} \tag{8}$$

where $x_t \equiv \Delta p_t + \Delta y_t + \pi_t \Delta y_t$. From (8) it is immediately obvious that the path of the debt-GDP ratio is fully determined at any point in time by the dynamics of the variables normally included in the vector **Z**. Therefore, the orthogonality of the "tax shocks" e_{t-j}^{RR} to $\mathbf{Z}_{t-(M+1)}$ is violated if government receipts and expenditures respond to the level of the debt-GDP ratio. This consideration remains valid when (8) is linearized and the debt feedback in the fiscal reaction function is captured by the distributed lags of macroeconomic variables in the VAR. [5]

[5] Debt and the intertemporal government budget constraint are typically omitted from empirical investigations of the effects of fiscal shocks—not only by R&R, but essentially by the entire empirical literature. This omission is inconsistent with the empirical evidence in Henning Bohn (1998, 2008). If fiscal variables respond to the level of the debt, the analysis of the impact of fiscal shocks should be conducted by explicitly recognizing a role for debt and the stock-flow identity linking debt and deficits, since the response of the economy to fiscal shocks will depend on the dynamic impact on the debt of such shocks. One justification for omitting debt is that the effects of this variable are captured by all other variables included (linearly) in a fiscal VAR. The debt dynamics equation, however, is non-linear. Whether or not including debt directly in the VAR makes a difference thus depends on how good an approximation the linear version of (8) is. Hess Chung and Eric Leeper (2007) have analyzed an empirical model that explicitly considers the government intertemporal budget constraint via cross-equation restrictions derived from a log-linearized version of (8).

The empirical relevance of this point is illustrated by the results reported in Table 1a and 1b where we investigate the significance of the variables included in $\mathbf{Z}_{t-M+1}$ for explaining e_{t-j}^{RR} ($j = 1...M$ where, consistently with the R&R specification, we set $M = 12$).

[**Insert Table 1 here**]

The evidence in Table 1 shows that while for high values of j there is no correlation between the variables in $\mathbf{Z}_{t-M+1}$ and e_{t-j}^{RR}, the correlation has a clear pattern negatively related to j and becomes sizeable (and statistically significant) for j between zero and three (*i.e.* for distant lags of $\mathbf{Z}$). Two observations are in order here:

- First, this evidence does not reject the hypothesis of orthogonality between the narrative shocks and short lags of the variables included in $\mathbf{Z}$. As a consequence, the e_t^{RR} can be validly treated as structural shocks in a VAR for $\mathbf{Z}_t$, where the lag length is four to six quarters, as normally assumed in fiscal VARs estimated on quarterly data. The point is visually illustrated in Figure 3 where we report the R&R shocks, e_t^{RR}, along with the residuals of the regression of e_t^{RR} on the first four lags of the variables included in $\mathbf{Z}$. The two series are nearly perfectly correlated with the only difference being some noise that it is added by the regression in all periods when e_t^{RR} takes a value of zero (absence of shocks identified by the narrative method). This statistical evidence speaks clearly in favour of treating the R&R shocks as structural shocks in a fiscal VAR.

- Second — and most relevant to the R&R approach to measuring fiscal multipliers — a truncated MA representation delivers biased estimates of the effect of tax shocks from the two-year horizon onward. The source of the bias is the correlation between the distant lags of government receipts and of GDP and the narrative shocks. In terms of equation (7) the bias in the R&R approach arises from the truncation of the MA representation, that becomes relevant when one considers the relation between the narrative shocks and the long run dynamics of the variables included in the VAR. In particular, the most significant correlation occurs between the narrative shocks and the distant lags of output and taxation.

II. Using Narrative Shocks in a Fiscal VAR

The "encompassing" framework discussed in the previous section suggests the following empirical specification for estimating tax multipliers

$$
\begin{aligned}
(9) \qquad \mathbf{Z}_t &= \sum_{i=1}^{k} \mathbf{C}_i \mathbf{Z}_{t-i} + \boldsymbol{\delta} e_t^{RR} + \boldsymbol{\gamma} \left(d_{t-1} - d^* \right) + \mathbf{u}_t \\
d_t &= \frac{1 + i_t}{\left(1 + \Delta p_t\right)\left(1 + \Delta y_t\right)} d_{t-1} + \frac{\exp\left(g_t\right) - \exp\left(t_t\right)}{\exp\left(y_t\right)} \\
\mathbf{Z}_t' &= \left[\begin{array}{ccccc} i_t & y_t & \Delta p_t & t_t & g_t \end{array} \right]
\end{aligned}
$$

where $\mathbf{Z}_t$ includes the five variables present in a fiscal VAR. We explicitly introduce debt in the VAR. As already stated in the introduction, we believe that it is important to conduct empirical investigations on the size of the fiscal multiplier within a model like (9) . This allows to track the dynamics of the debt-GDP ratio in response to fiscal shocks, thus checking that fiscal multipliers are not computed along "unsustainable debt paths". However, since empirically the estimation of (9) on U.S. data never delivers "unsustainable debt paths" [6], to allow a direct comparison with traditional fiscal VARs, we shall report two sets of results: those obtained by imposing $\boldsymbol{\gamma} = \mathbf{0}$ and those obtained by relaxing this assumption.

The main advantage of (9) is that this specification allows us to address all the points discussed in the previous section. We reinterpret the narrative shocks e_t^{RR} as observed structural shocks to one of the variables included in the fiscal VAR, namely t_t. The validity of the assumption $e_t^t = e_t^{RR} + \varepsilon_t$ can be directly checked by assessing the orthogonality of e_t^{RR} to the information set used in the VAR. If the hypothesis $e_t^{RR} \perp \mathbf{Z}_{t-i}$ is not rejected, then the e_t^{RR} can be considered as observable structural shocks to t_t.

For $\boldsymbol{\gamma} = \mathbf{0}$, impulse responses to e_t^{RR} are obtained in a full-information framework whose underlying MA representation is infinite, rather than truncated as in R&R. For $\boldsymbol{\gamma} \neq \mathbf{0}$ we extend the fiscal VAR framework explicitly allowing for a response of all variables in the VAR to the distance of the debt-to-GDP ratio from a target debt level d^*. Such a debt-feedback mirrors that estimated in Bohn (1998); following Bohn we take 0.35, as the target value for d^* (as shown in Figure A1, this is also the average debt level in our sample.) As we introduce the debt level into the VAR, we need to make it endogenous, otherwise impulse response functions would be computed assuming a constant debt ratio, thus ruling out the reason why debt is included in the first place — namely to allow macro variables to respond to the effect of the fiscal shock on the level of the debt. The way to make the debt ratio endogenous is to append to the model the equation that describes how it evolves over time as a function of the path of all other variables, *i.e.* the government's intertemporal budget constraint (IGBC). [7]

[6] We have checked this by tracking the debt dynamics in response to fiscal shocks. We do not report these results but we make them available upon request.

[7] Note that the budget constraint is an identity: it does not add new parameters to be estimated, nor

The introduction of the IGBC makes (9) non-linear: constructing an MA representation of $\mathbf{Z}_t$ is thus no longer possible. However, impulse responses can still be computed going through the following steps

1) generate a baseline simulation for all variables by solving (9) dynamically forward (this requires setting to zero all shocks for a number of periods equal to the horizon up to which impulse responses are needed),
2) generate an alternative simulation for all variables by setting to one–just for the first period of the simulation–the structural shock of interest, and then solve dynamically forward the model up to the same horizon used in the baseline simulation,
3) compute impulse responses to the structural shocks as the difference between the simulated values in the two steps above. (Note that these steps, if applied to a standard VAR, would produce standard impulse responses. In our case they produce impulse responses that allow for both the feedback from d_{t-i} to $\mathbf{Z}_t$ and for the endogeneity of d_t modelled via (8),
4) compute confidence intervals via bootstrap methods.[8]

III. Estimating the tax-multiplier

If the assumption $e_t^{RR} \perp \mathbf{Z}_{t-i}$ is satisfied, then estimating the tax-multiplier by including e_t^{RR} in the VAR — as done in (9) — is very natural. First, the VAR contains an equation describing the endogenous evolution of tax changes, the equation for t_t. Moreover, including e_t^{RR} directly in the VAR no longer requires the identification of exogenous tax shocks from the VAR residuals, since these shocks are identified using information outside the VAR. Figure 3 clearly shows that constructing tax shocks from the residuals of a regression of e_t^{RR} on $\mathbf{Z}_{t-i}$ would not do justice to the careful and thorough narrative identification of the tax shocks performed by R&R: such a regression would only contaminate the tax shocks with spurious noise for all data points where the narrative method identifies no shock.

R&R analyze the robustness of the results obtained from the estimation of (2) computing impulse responses to shocks to exogenous tax changes identified within a two-variable VAR with log-output and the exogenous tax changes. Shocks to exogenous tax changes are identified as the residuals from a regression of e_t^{RR} on twelve lags of each of the two variables included in the VAR. We report in Figure 4 e_t^{RR} and the residuals from the R&R bivariate VAR. Figure 4 resembles

new shocks to be identified.

[8] Bootstrapping requires saving the residuals from the estimated VAR and then iterating the following steps: a) re-sample from the saved residuals and generate a set of observation for $\mathbf{Z}_t$ and d_t, b) estimate the VAR and identify strucutral shocks, c) compute impulse responses going thorough the steps described in the text, d) go back to step 1. By going thorugh 1,000 iterations we produce bootstrapped distributions for impulse responses and compute confidence intervals.

Figure 3 very closely. This fact has two important consequences. First, the shocks generated by the bi-variate VAR are a noisy measure of the tax shocks. Second, the impulse responses generated from the VAR are virtually equivalent to those obtained from the truncated MA representation (given that e_t^{RR} and the residuals of the regression of this variable on its own lags and the lags of log output are virtually identical). This suggests that the robustness analysis conducted within the two-variables VAR model is not a useful benchmark against which to assess the original empirical evidence.[9] A more useful benchmark is obtained by introducing — as discussed above in (9) — shocks identified outside the VAR into the relevant VAR.

The first important evidence that emerges from the estimation of (9) is that the coefficient on e_t^{RR} is not statistically different from zero only in the equation for t_t, thus confirming that e_t^{RR} are valid shocks to taxes..

The tax multipliers obtained from the estimation of (9) are illustrated in Figures 5 and 6. Figure 5 compares the effect on output of an e_t^{RR} shock equivalent to one per cent of U.S. GDP estimated using, alternatively, (2), model (9) without the IGBC ($\boldsymbol{\gamma}_i = 0$), and model (9) with the IGBC. Estimating the effect of R&R tax shocks using a VAR rather than a single equation framework delivers a response of output that is much smaller than that reported by R&R and very similar to that delivered by a traditional fiscal VAR and reported in Figure 1. The impact of a tax shock on output growth estimated within a VAR never exceeds one per cent. The VAR also highlights the instability of the tax multiplier before and following 1980: the multiplier in the first sub-sample is larger and significantly different from the estimated value in the second sub-sample, where it is not significantly different from zero.

The results in Figure 5 show that the differences between the impulse responses obtained from the estimation of a single equation and those obtained within a system framework only appear after a few quarters, and not on impact. This is consistent with the evidence reported in Table 1.

A. Debt and the non-linear debt dynamics

The model augmented with debt and the non-linear debt dynamics equation produces results which are very similar to those obtained by including the R&R shocks in a traditional fiscal VAR. Figure 5 has shown that when the R&R measure of tax shocks is analyzed within a multiple equation model, rather than in a single equation framework, the estimated multipliers are much smaller. While simultaneity is important, we find no major empirical difference between a non-linear model with an explicit debt dynamics equation and a linearized model where the effect of debt is captured by its components. However, despite the

[9] This approach is also used in Romer and Romer (2009) to assess the response of expenditure to tax shocks. The importance of using a VAR to capture simultaneity is clearly illustrated in that paper where tax shocks are used as the dependent variable in four-variables VAR.

fact that non-linearities are not significant over our sample of U.S. data, the non-linear specification might become of crucial importance for analyzing cases in which the debt-GDP ratio is very persistent and fiscal shocks are large. U.S. data are drawn from a sustainable fiscal regime: within this regime it is likely that the feedback between fiscal variables and the (linearized) debt dynamics is captured in a linear VAR specification that includes all the variables that enter in the debt-deficit relationship. Nevertheless, having the possibility of checking that fiscal multipliers are computed along a sustainable path is an important step, mostly overlooked in the empirical analysis of fiscal multipliers.

B. *Augmenting the R&R regression with further lags of* Z_t

Figure 6 completes our evidence by reporting the results obtained when re-running the R&R regression augmented with $\mathbf{Z}_{t-(M+1)}$

$$\Delta y_t = a + \sum_{i=0}^{M} b_i e_{t-i}^{RR} + \mathbf{C}_i^{M+1} \mathbf{Z}_{t-(M+1)} + e_t \tag{10}$$

This is a robustness check R&R do not perform, since the one they report only uses information dated up to time M. Figure 6 reports the effect of tax shocks as originally computed by R&R, along with those based on the augmented regression (10) over the full sample 1950:1-2007:1. The Figure shows that, consistently with the results reported in Table 1, the truncation affects the size of the multiplier after the eight quarter. From lag eigth onward, the multiplier estimated using the augmented equation is much closer to the one delivered by the inclusion of the R&R shocks in a fiscal VAR.[10] Interestingly, the R^2 increases from 0.09 in the original R&R specification to 0.17 in the augmented specification.

IV. Observations

The empirical analysis conducted in the previous section by including narrative shocks in a fiscal VAR leads naturally to two obervations, the first on the relation between our proposed approach and "fiscal foresight", the second on the validity of the VAR specification.

A. *VAR's, the Narrative Identification and Fiscal Foresight*

Using the narrative record to identify tax shocks does not require the inversion of the moving average representation of a VAR. In this section we illustrate that coupling a VAR specification with the identification of structural fiscal shocks

[10] The small remaining difference between the impulse responses can be rationalized by the additive noise that drives a wedge between the VAR shocks and the R&R shocks.

independent from the VAR is particularly advantageous in the presence of fiscal foresight.

Structural VAR shocks and narrative shocks are different instruments for the true underlying unobservable tax shocks. The main difference between the two instruments arises from the fact that the narrative shocks are derived independently from any statistical model. Instead the VAR-based evidence is obviously model dependent and its validity relies on the assumption that the agents' and the econometrician's information sets are aligned. Leeper et al. (2008) point out that fiscal foresight could cause a misalignment of the two information sets, thus making it impossible to extract meaningful shocks to taxes from statistical innovation in the VAR.

Fiscal foresight happens when agents, at some point in time, receive signals on the taxes they will face in the future. This is very likely given legislative and implementation lags in tax policy. To understand the implication of fiscal foresight consider, as an example, the simplest RBC model, adapted from Leeper et al. (2008). The model is log-linearized, with log preferences, inelastic labour supply and complete depreciation of capital. A proportional tax is levied against income and used for lump-sum transfers on a period by period basis. There is no government spending. The economy is subject to two shocks: an exogenous technological shock e_t^A and a tax shock, $e_{t,t+p}^{\tau}$. The tax shock features an implementation lag of p periods, *i.e.* news about changes in taxes arrive p periods before they are implemented.

The equilibrium conditions of the model are

$$\begin{aligned}
\frac{1}{C_t} &= \alpha\beta E_t \left(1-\tau_{t+1}\right) \frac{1}{C_{t+1}} \frac{Y_{t+1}}{K_t} \\
C_t + K_t &= Y_t = A_t K_{t-1}^{\alpha} \\
\tau_t &= \tau \exp\left(e_{t-p,t}^{\tau}\right) \\
A_t &= \exp\left(e_t^A\right)
\end{aligned}$$

Log-linearizing — so that $\theta E_t k_{t+1} - (1+\alpha\theta) k_t + \alpha k_{t-1} = \rho E_t \tau_{t+1} - e_t^A$ where $\theta = \alpha\beta(1-\tau)$ and $\rho = \tau(1-\theta)/(1-\tau)$ — these reduce to a bivariate model for capital and technology

$$\begin{aligned}
k_t &= \alpha k_{t-1} + e_t^A - \rho \sum_{i=0}^{\infty} \theta^i E_t e_{t+1+i-p,t+1+i}^{\tau} \\
a_t &= e_t^A
\end{aligned}$$

Consider estimating a bivariate VAR in a_t, k_t and retrieving the two shocks from the VAR innovations. As the equilibrium looks different for different degrees of fiscal foresight, the outcome of this procedure would clearly be affected by it.

- in the case of no fiscal foresight the $(p = 0)$ the equilibrium is

$$\begin{aligned} k_t &= \alpha k_{t-1} + e_t^A \\ a_t &= e_t^A \end{aligned}$$

  and a VAR in a_t, k_t would feature stochastic singularity, as only one shock will drive the two variables;

- in the case of one-period fiscal foresight, $p = 1$, the equilibrium is

$$\begin{aligned} a_t &= e_t^A \\ k_t &= \alpha k_{t-1} + e_t^A - \ \rho e_{t,t+1}^{\tau} \end{aligned}$$

  and a Choleski identification for the innovations in the VAR in a_t, k_t would allow to correctly identify the structural shocks of interest;

- in the case of two-periods fiscal foresight, $p = 2$, the equilibrium is

$$\begin{aligned} a_t &= e_t^A \\ k_t &= \alpha k_{t-1} + e_t^A - \ \rho \left(e_{t-1,t+1}^{\tau} + \theta e_{t,t+2}^{\tau} \right) \end{aligned}$$

  and it would not be possible to identify the structural shocks of interest from the VAR innovations. In fact, for any $p \geqq 2$ we have non-invertibility of the moving average component of the time series of k_t (see Lars Peter Hansen and Thomas Sargent, 1991, Marco Lippi and Lucrezia Reichlin, 1994).

Note that in the presence of fiscal foresight, the VAR identification is hopeless, while the narrative approach is still able to identify tax shocks, as e_t^t is constructed independently from the VAR and the estimation of a VAR augmented directly with the relevant combination of tax shocks is clearly feasible. Moreover, the narrative approach naturally delivers a classification of tax shocks into anticipated and unanticipated, where the relevant information set to identify anticipations is clearly larger than that normally considered in a fiscal VAR. To show how narrative shocks can be included in a fiscal VAR to deal separately with the effects of unanticipated and anticipated fiscal policy we take from Mertens and Ravn (2010) [11] the classification of the R&R shocks into anticipated and unanticipated. Mertens and Ravn consider the following decomposition of the R&R shocks

$$e_t^{RR} = \tau_t^u + \tau_{t,0}^a$$

[11] We are grateful to the two authors for having provided us with their dataset.

where τ_t^u are the unanticipated tax shocks occurring at time t while $\tau_{t,0}^a$ are tax shocks that are implemented at time t , having been legislated and therefore announced at a date earlier than t. The notation is different from that of our simple illustrative model to reflect the fact that the implementation lag is not fixed. In the data the difference between announcement and implementation dates features a twin peaked distribution with the peaks occurring one at 0-30 days and the other beyond 151 days; the median implementation lag is six quarters. Mertens and Ravn (2010) define a tax change as anticipated if the implementation lags exceeds 90 days (1-quarter). To address the anticipation effect of tax shocks a series of new variables is constructed, $\tau_{t,i}^a$, that measures the sum of all anticipated tax changes known at date t to be implemented at date $t+i$.

The taxonomy of the R&R shocks introduced by Mertens and Ravn makes clear that the tax multipliers derived by interpreting all e_t^{RR} shocks as unanticipated would be difficult to interpret in the presence of implementation lags. However, the idea of using the narrative shocks identified independently from a VAR within the VAR is still applicable. In fact, the output effects of anticipated and unanticipated U.S. tax policy shocks can be derived by estimating the following system that includes in an appropriate way all different tax shocks [12]

$$
\begin{aligned}
(11) \qquad t_t &= \sum_{i=1}^{k} \mathbf{C}_{1i}\mathbf{Z}_{2t-i} + \sum_{i=1}^{k} \mathbf{c}_{2i}\mathbf{t}_{t-i} + \delta_{11}\tau_t^u + \delta_{12}\tau_{t,0}^a + u_{1t} \\
\mathbf{Z}_{2t} &= \sum_{i=1}^{k} \mathbf{C}_{2i}\mathbf{Z}_{2t-i} + \boldsymbol{\delta}_{21}\tau_t^u + \boldsymbol{\delta}_{22}\tau_{t,0}^a + \sum_{i=1}^{6} \mathbf{G}_{2i}\tau_{t,i}^a + \mathbf{u}_{2t} \\
\mathbf{Z}_{2t}^{'} &= \left[\begin{array}{cccc} i_t & y_t & \Delta p_t & g_t \end{array} \right]
\end{aligned}
$$

Note that in (11) tax shocks implemented at time t enter the equation for (log) government revenue t_t with a different coefficient according to their status of unanticipated or anticipated. Tax shocks announced at time t to be implemented in the future do not enter this equation. However, tax shocks announced at time t to be implemented with all implementation lags up to six quarters are allowed to affect all other variables included in a fiscal VAR.

The specification in (11) generates different impulse responses for anticipated and unanticipated tax shocks. We report in Figure 7 two tax multipliers. The first is associated with an unanticipated tax shocks at time t, while the second describes the effects on output of a tax shock announced at time t with an implementation lag of six periods. Note that the output effect of the unanticipated tax shock is very similar to that of the R&R shocks with a long-run multiplier of about one. Interestingly, as in Mertens and Ravn, the announcement of a

[12] In the light of the results of our previous section we only consider the specification without debt-feedback.

positive tax shock has a *positive* impact on output before the implementation, that becomes negative only after the implementation date.[13]

B. Is a Fiscal VAR a valid specification?

The objective of this paper was to show that tax multipliers can be estimated using a VAR even if fiscal shocks are not identified within the VAR, but from narrative methods. Some recent papers have questioned the optimality of a VAR as empirical approximations. Perotti (2010) shows that a VARMA is a better specification than a VAR if the discretionary component of taxation is allowed to have different effects on output than the automatic response of tax revenues to macro-economic variables. Mertens and Ravn (2010) also argue in favour of a VARMA specification on the ground of consistency with a fiscal DSGE model. Charour et al (2010) take a fiscal DSGE as the Data Generating Process (DGP) and show that the impulse responses generated by the R&R approach and our suggested approach are virtually identical. The authors show that, under the null hypothesis of the validity of the model adopted, both the VAR and the truncated MA representation are approximation to the true DGP and deliver similar fiscal multipliers.

Our paper has nothing to say on the optimality of a VAR as an empirical specification, as we do not interpret the VAR necessarily as the reduced form of a DSGE model. In fact, while VAR models can be intepreted as approximated solution of DSGE models (as in Ryan Chahrour, Stephanie Schmitt-Grohé and Martín Uribe 2010), it is perfectly possible that a VAR is a valid empirical representation for data that are not generated by a DSGE. In the previous sections we have shown that the truncated MA representation adopted by R&R to measure fiscal multipliers might introduce a bias if the narrative shocks are correlated with distant lags of macroeconomic and fiscal variables. (as we already mentioned, such correlation is effectively induced by the identification choice adopted by R&R). Charour et al (2010) are very clear in stating that their investigation cannot be informative on the difference between impulse responses generated by different identification strategies: in fact the exogenous shocks identified by R&R do not exist in the structural DSGE model adopted. A DSGE-based intepretation of our results is a useful exercise but cannot be informative on situations in which the DGP is different from the DSGE model adopted, and identification is the main source of discrepancy between the alternative results .

V. Conclusions

This paper proposes an empirical strategy for the estimation of tax multpliers which combines the estimation of a VAR model with non-VAR based measures

[13]Blanchard (1981) finds a similar theoretical result analyzing anticipated and unanticipated fiscal shocks in a model with sticky prices and perfect foresight.

of shocks to government revenues. First, we find that shocks identified via the narrative approach are orthogonal to the relevant information set in traditional fiscal VAR's: therefore they are valid structural shocks in such VAR's. We thus estimate the multiplier associated with tax shocks identified via the narrative method, by including them directly in a fiscal VAR. We find a multiplier of about one. The advantage of using the narrative record to identify tax shocks is that it does not require the inversion of the moving average representation of a VAR for their identification. Exploiting this property of narrative shocks — and the classification, proposed by Mertens and Ravn (2010), of R&R shocks into anticipated and unanticipated — we have estimated the multipliers associated with unanticipated and anticipated tax shocks: shocks announced at time t with an implementation lag of six periods. We find that the muliplier associated with unanticipated tax shocks (and estimated within a a fiscal VAR) is very similar to that of the R&R shocks. Anticipated tax shocks, instead, have opposite effects in the pre-implementation and the post-implementation periods.

We have also estimated the multiplier keeping track of the effect of tax shocks on the level of the debt-GDP ratio. We have done this allowing for the non-linearity which arises from the government budget constraint. We find no significant empirical difference between a non-linear model with an explicit debt dynamics equation, and a linearized model where the effect of debt is captured by its components. Despite the fact that non-linearities do not appear to be important over our sample of U.S. data, the non-linear specification might become of crucial importance for analyzing cases in which the debt-to-GDP ratio is very persistent and large fiscal shocks happen.

The methodology we have developed to analyze the impact of tax shocks by keeping track of the non-linear government budget constraint, could be used in other settings. For instance, the discussions on the importance of including capital as a slow-moving variable to capture the relation between productivity shocks and hours worked (see e.g. Lawrence Christiano, Martin Eichenbaum and Robert Vigfussonet 2005 and Varadarajan Chari, Patrick Kehoe and Ellen McGrattan 2005) could benefit from an estimation technique that tracks the dynamics of the capital stock generated by the relevant shocks. The same applies to open economy models that study, for instance, the effects of a productivity shock on the current account and that typically omit a feedback from the stock of external debt to macroeconomic variables.

This approach could also be extended to the analysis of the effects of tax shocks on debt sustainability, an issue which cannot be addressed in the context of a VAR that fails to keep track of the debt dynamics.

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VII. Data Appendix

y_t is (the log of) real GDP per capita, Δp_t is the log difference of the GDP deflator. Data for the stock of U.S. public debt and for population are from the FRED database (available on the Federal Reserve of St.Louis website,also downloaded on December 7th 2006). Our measure for g_t is (the log of) real per capita primary government expenditure: nominal expenditure is obtained subtracting from total Federal Government Current Expenditure (line 39, NIPA Table 3.2) net interest payments at annual rates (obtained as the difference between line 28 and line 13 on the same table). Real per capita expenditure is then obtained by dividing the nominal variable by population times the GDP chain deflator. Our measure for t_t is (the log of) real per capita government receipts at annual rates (the nominal variable is reported on line 36 of the same NIPA Table).

The R&R tax shocks start in 1947, while our data only start in 1950:1 because data for total governemnt spending are available on a consistent basis only from 1950:1. We thus exclude the exogenous shocks that occurred between January 1947 and December 1949.

Our approach requires that the debt-dynamics equation in (9) tracks the path of d_t accurately: we thus need to define the variables in this equation with some care. The source for the different components of the budget deficit and for all macroeconomic variables are the NIPA accounts (available on the Bureau of Economic Analysis website, downloaded on December 7th 2006). The average cost servicing the debt, i_t, is obtained by dividing net interest payments by the federal government debt held by the public (FYGFDPUN in the Fred database) at time $t-1$. The federal government debt held by the public is smaller than the gross federal debt, which is the broadest definition of the U.S. public debt. However, not all gross debt represents past borrowing in the credit markets since a portion of the gross federal debt is held by trust funds–primarily the Social Security Trust Fund, but also other funds: the Trust Fund for Unemployment Insurance, the Highway Trust Fund, the pension fund of federal employees, etc.. The assets held by these funds consist of non-marketable debt.[14] We thus exclude it from our definition of federal public debt. We are unable to build the debt series back to 1947:1, the start of the Romer and Romer sample, because, as mentioned above, data for total governemnt spending, needed to buld the debt series, are available on a consistent basis only from 1950:1

Figure A-1 reports, starting in 1970:1 (the first quarter for which the debt data are available in FRED), this measure of the debt held by the public as a fraction of GDP (this is the dotted line). We have checked the accuracy of the debt dynamics equation in (9) simulating it forward from 1970:1 (this is the continuous line in Figure A-1). The simulated series is virtually super-imposed to the actual one:

[14] Cashell (2006) notes that "this debt exists only as a book-keeping entry, and does not reflect past borrowing in credit markets."

the small differences are due to approximation errors in computing inflation and growth rates as logarithmic differences, and to the fact that the simulated series are obtained by using seasonally adjusted measures of expenditures and revenues. Based on this evidence we have used the debt dynamics equation to extend d_t back to 1950:1.

Table 1.a: Testing orthogonality of tax shocks $e^{RR}_{t:t+5}$ to $\mathbf{Z}_{t-1:t-4}$.

	e^{RR}_{t}	e^{RR}_{t+1}	e^{RR}_{t+2}	e^{RR}_{t+3}	e^{RR}_{t+4}	e^{RR}_{t+5}
i_{t-1}	0.036 0.46	−0.032 −0.4	0.067 0.83	−0.04 −0.49	−0.01 −0.12	0.042 0.51
i_{t-2}	−0.081 −0.68	0.061 0.5	−0.118 −0.97	0.018 0.15	0.014 0.11	−0.091 −0.72
i_{t-3}	0.061 0.51	−0.073 −0.61	0.023 0.19	0.059 0.47	−0.081 −0.64	−0.144 −1.13
i_{t-4}	−0.019 −0.23	0.045 0.54	0.03 0.36	−0.037 −0.44	0.078 0.92	0.198 2.31
y_{t-1}	0.008 0.41	−0.041 −1.98	−0.022 −1.07	−0.007 −0.36	−0.001 −0.04	−0.001 −0.07
y_{t-2}	−0.048 −1.61	0.016 0.54	0.011 0.35	0.02 0.68	−0.001 −0.05	0.018 0.6
y_{t-3}	0.013 0.42	0.025 0.8	0.026 0.86	−0.007 −0.22	0.038 1.25	−0.012 −0.4
y_{t-4}	0.029 1.43	0.002 0.12	−0.009 −0.43	0.001 0.05	−0.029 −1.47	0.003 0.13
Δp_{t-1}	0.090 1.75	−0.049 −0.94	−0.001 −0.01	0.032 0.65	−0.108 −2.21	0.017 0.36
Δp_{t-2}	−0.121 −2.22	0.04 0.74	0.043 0.8	−0.112 −2.09	0.055 1.08	0.048 0.95
Δp_{t-3}	0.002 0.04	0.077 1.38	−0.098 −1.76	0.034 0.65	0.095 1.81	−0.031 −0.6
Δp_{t-4}	0.019 0.36	−0.09 −1.72	0.043 0.87	0.05 1	−0.027 −0.55	−0.014 −0.28
t_{t-1}	−0.002 −0.3	0.016 2.42	0.005 0.71	−0.005 −0.74	0.008 1.25	−0.004 −0.6
t_{t-2}	−0.014 2.01	−0.006 −0.69	−0.008 −1.02	0.012 1.42	−0.01 −1.25	−0.002 −0.3
t_{t-3}	0.017 −0.42	−0.015 −1.79	0.01 1.17	−0.008 −0.94	−0.007 −0.85	0.006 0.8
t_{t-4}	−0.014 −2.04	0.002 0.27	−0.012 −1.76	−0.005 −0.83	0.004 0.55	−0.006 −0.93
g_{t-1}	0.003 0.55	0.003 0.48	0 0.03	0.005 0.94	−0.004 −0.71	−0.001 −0.1
g_{t-2}	0.001 0.12	−0.001 −0.19	0.006 0.94	−0.007 −1.04	0.001 0.18	0.003 0.41
g_{t-3}	−0.002 −0.23	0.006 0.81	−0.007 −1.05	0.001 0.13	0.001 0.1	−0.003 −0.43
g_{t-4}	−0.001 −0.27	−0.006 −1.25	0.001 0.16	0.001 0.25	0.001 0.22	0 −0.01
R^2	0.068	0.071	0.069	0.069	0.071	0.077
F	0.76 (0.75)	0.80 (0.71)	0.77 (0.74)	0.77 (0.74)	0.80 (0.71)	0.86 (0.62)

Table 1.b: Testing orthogonality of tax shocks $e^{RR}_{t+6:t+12}$ to $\mathbf{Z}_{t-1:t-4}$

	e^{RR}_{t+6}	e^{RR}_{t+7}	e^{RR}_{t+8}	e^{RR}_{t+9}	e^{RR}_{t+10}	e^{RR}_{t+11}	e^{RR}_{t+12}
i_{t-1}	-0.048 -0.59	-0.23 -2.79	-0.033 -0.4	0.033 0.41	-0.017 -0.21	-0.296 -3.69	-0.016 -0.2
i_{t-2}	-0.151 -1.19	0.201 1.57	0.02 0.15	-0.079 -0.62	-0.242 -1.93	0.331 2.66	0.086 0.67
i_{t-3}	0.186 1.46	0.047 0.37	-0.073 -0.57	-0.217 -1.73	0.35 2.8	0.043 0.35	-0.136 -1.07
i_{t-4}	0.017 0.2	-0.016 -0.18	0.093 1.09	0.272 3.24	-0.082 -0.99	-0.069 -0.83	0.078 0.92
y_{t-1}	0.011 0.58	-0.004 -0.2	-0.019 -0.96	-0.018 -0.92	0.028 1.45	0.001 0.05	-0.037 -1.86
y_{t-2}	-0.013 -0.44	0 0.01	0.012 0.4	0.052 1.82	-0.025 -0.87	-0.024 -0.84	0.083 2.85
y_{t-3}	-0.006 -0.19	0.006 0.21	0.063 2.11	-0.032 -1.11	-0.034 -1.16	0.07 2.42	-0.064 -2.14
y_{t-4}	0.015 0.75	0.005 0.26	-0.048 -2.44	0.003 0.17	0.037 1.91	-0.043 2.42	0.022 1.14
Δp_{t-1}	0.041 0.88	-0.025 -0.54	-0.06 -1.29	0.01 0.22	0.006 0.12	0.03 0.66	-0.002 -0.03
Δp_{t-2}	-0.036 -0.72	-0.019 -0.37	0.044 0.89	0.005 0.11	0.026 0.53	0.015 0.32	0.005 0.1
Δp_{t-3}	-0.028 -0.56	0.05 1	0.028 0.55	0.011 0.22	-0.002 -0.04	-0.002 -0.04	-0.065 -1.28
Δp_{t-4}	0.055 1.16	0.023 0.48	-0.003 -0.06	-0.001 -0.03	-0.023 -0.5	-0.034 -0.72	0.045 0.95
t_{t-1}	-0.005 -0.77	0.003 0.51	0.003 0.53	-0.004 -0.67	-0.009 -1.42	-0.003 -0.51	0.001 0.18
t_{t-2}	0.007 0.91	0.002 0.21	-0.009 -1.16	-0.004 -0.51	0.003 0.44	0.005 0.64	-0.02 -2.43
t_{t-3}	0.002 0.28	-0.009 -1.17	-0.006 -0.79	0.003 0.41	0.011 1.36	-0.021 -2.75	0.022 2.66
t_{t-4}	-0.011 -1.68	-0.002 -0.3	0.005 0.78	0 -0.01	-0.011 -1.81	0.016 2.54	-0.008 -1.25
g_{t-1}	0.001 0.16	-0.002 -0.37	-0.002 -0.46	-0.002 -0.44	0.003 0.53	-0.002 -0.42	-0.002 -0.42
g_{t-2}	-0.002 -0.26	-0.003 -0.5	0 0	0.003 0.45	-0.005 -0.76	0 -0.04	0 0.07
g_{t-3}	-0.003 -0.43	0.003 0.46	0.002 0.29	-0.004 -0.67	0.001 0.1	0.002 0.33	-0.002 -0.38
g_{t-4}	0.003 0.66	0.002 0.34	0.001 0.17	0.003 0.67	0.002 0.39	0 0.09	0.004 0.84
R^2	0.088	0.084	0.080	0.12	0.13	0.15	0.11
F	1.00 (0.45)	0.95 (0.51)	0.90 (0.58)	1.36 (0.14)	1.56 (0.06)	1.84 (0.02)	1.25 (0.21)

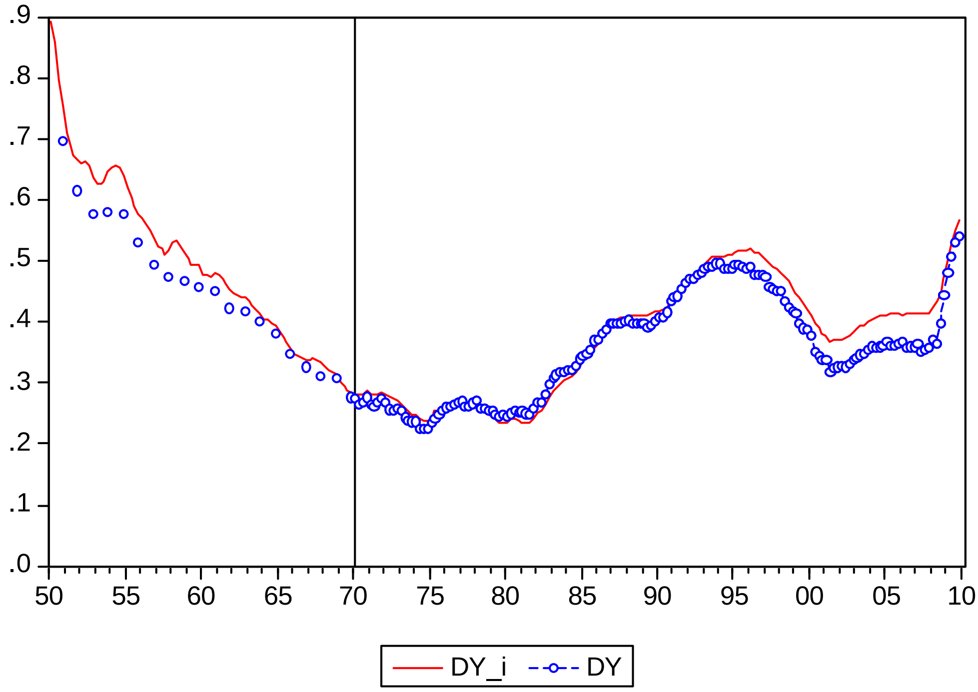

Figure A1: Actual (DY) and simulated (DY_I) (dynamically backward and forward starting in 1970:1) debt-GDP ratio. Actual data are observed at quarterly frequency from 1970 onwards and at annual frequency from 1970 backward. The simulated data are constructed using the government intertemporal budget constraint (2) with observed data and initial conditions given by the debt-to-GDP ratio in 1970:1.

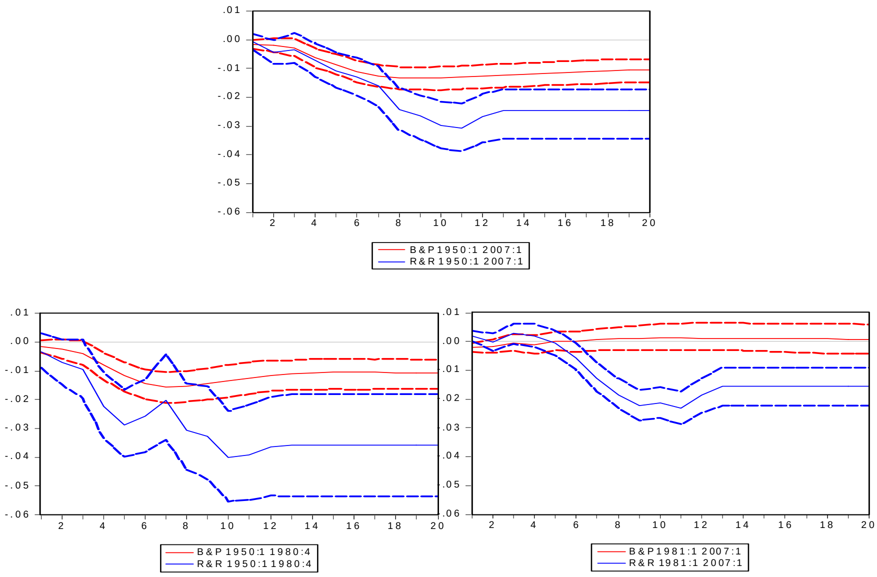

Figure 1: Different estimates of structural tax shocks in the narrative and the SVAR approaches

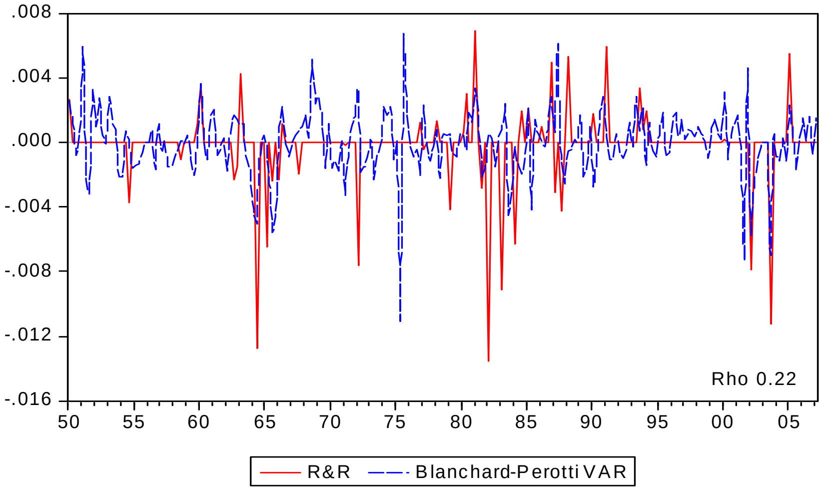

Figure 2: Different estimates of structural tax shocks in the narrative and the VAR approaches

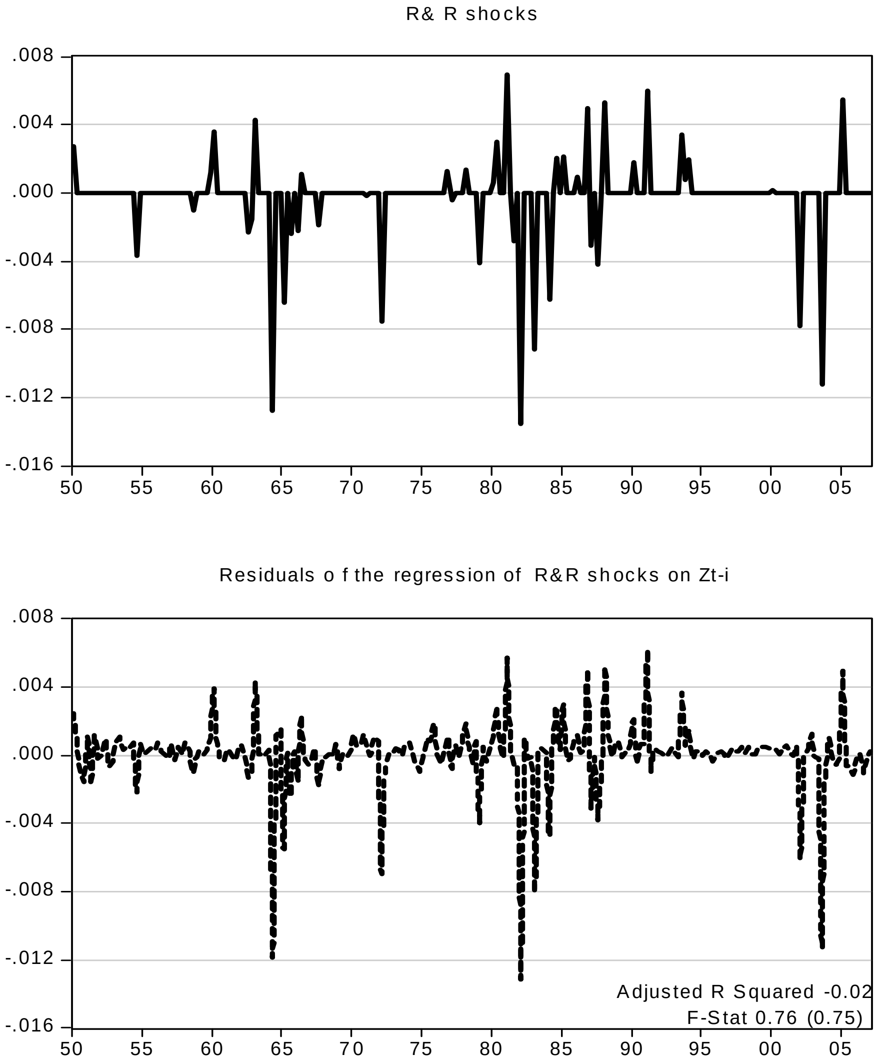

Figure 3: R&R shocks and Residuals from the regression of R&R shocks on Z_{t-i}.

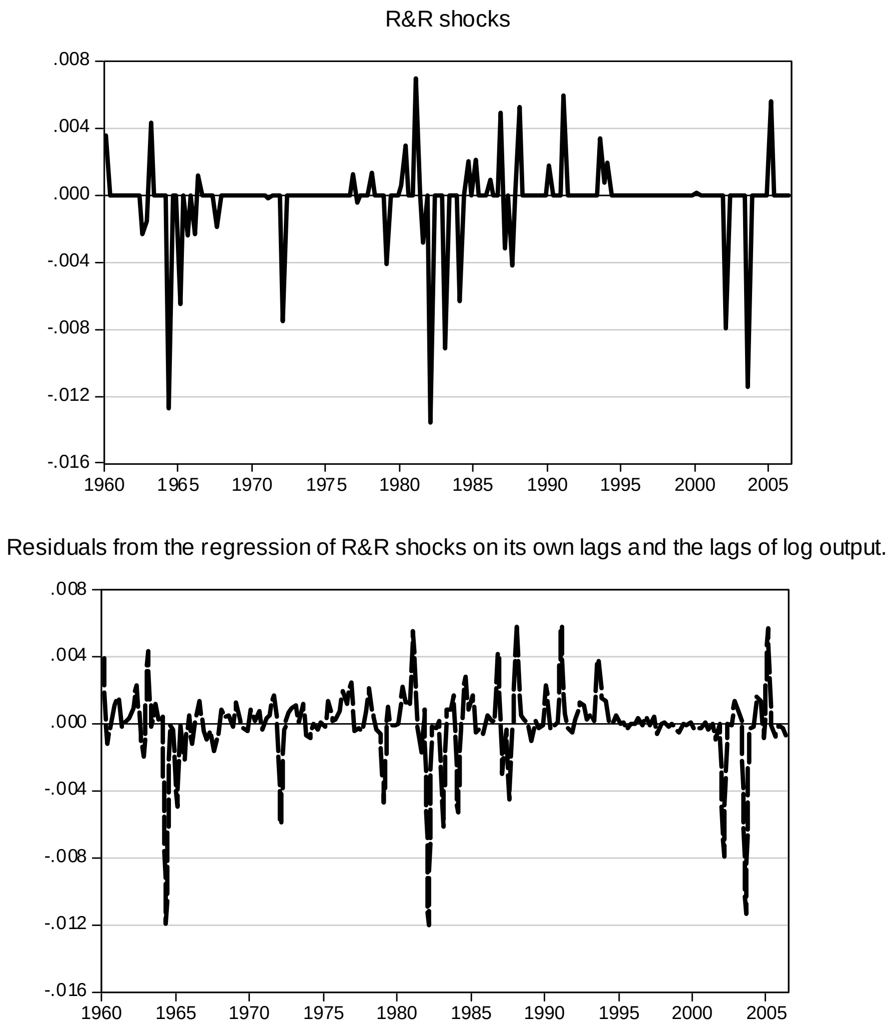

Figure 4. Assessing the R&R robustness check

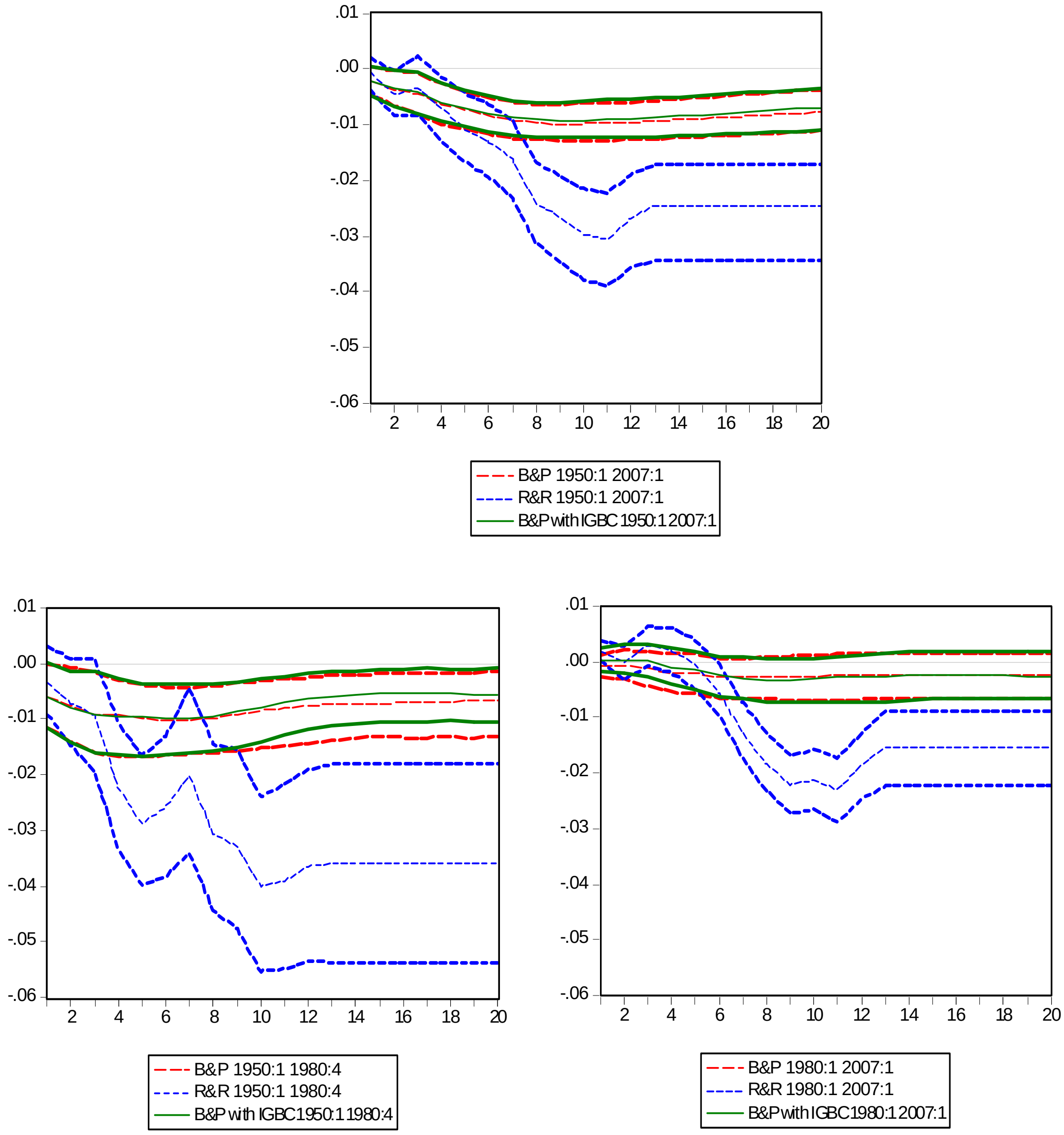

Figure 5: Estimated Impact of an Exogenous Tax Increase of 1% of GDP on GDP with the R&R framework and with the Fiscal VAR framework, with and without the IGBC

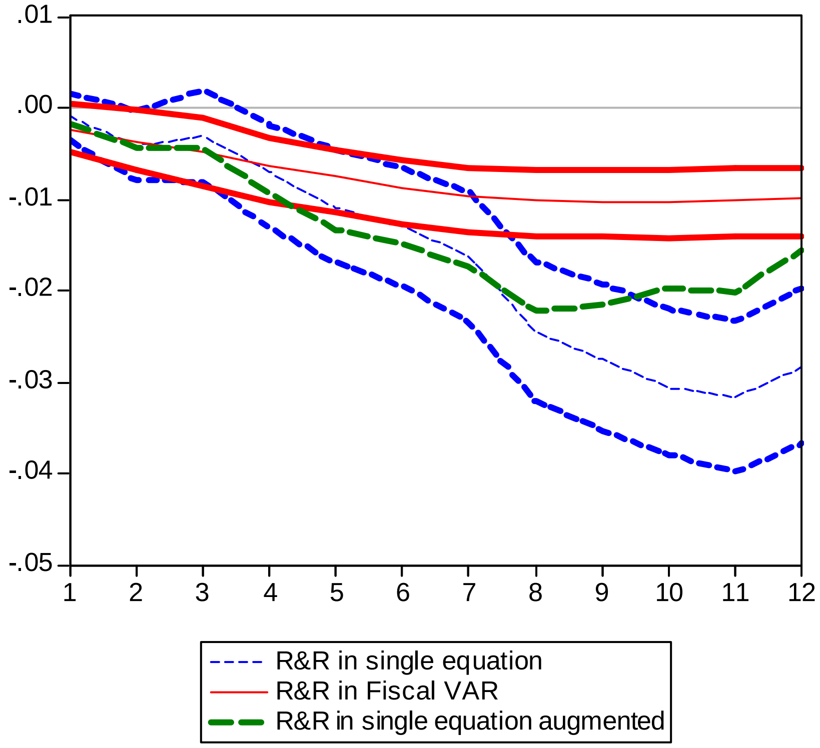

Figure 6: Different Shocks, Same Models, Same Impulse Responses

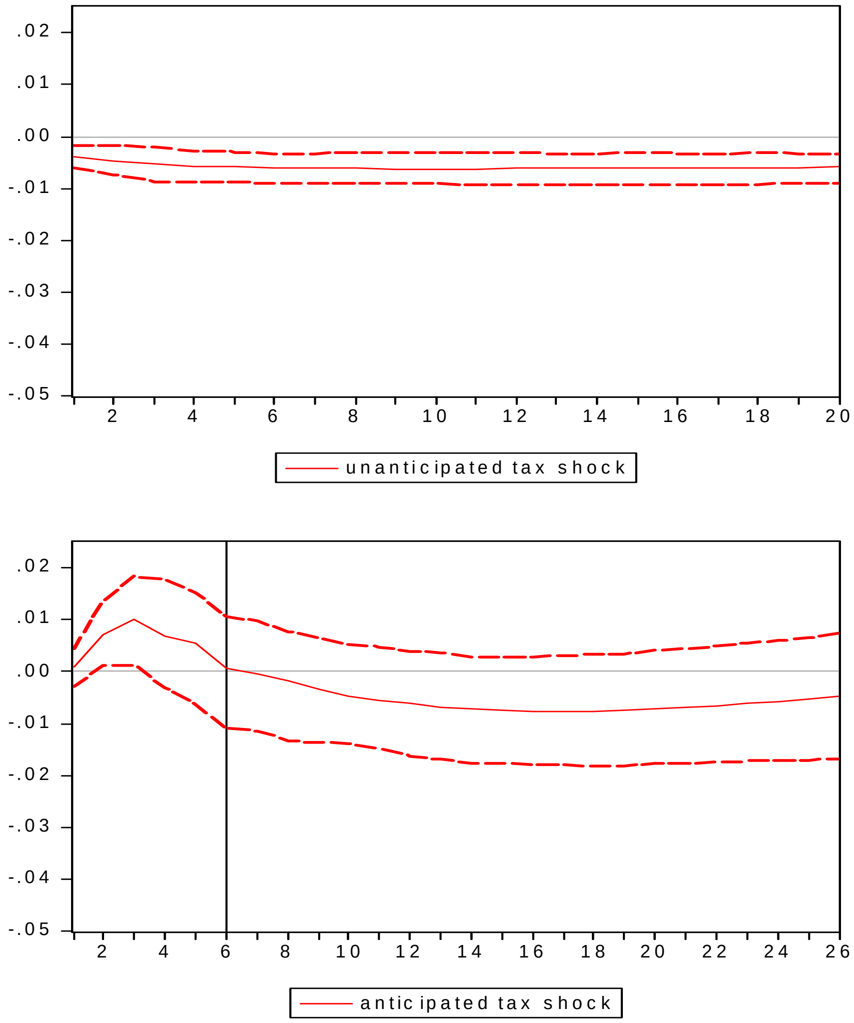

Figure 7: The output effects of unanticipated and anticipated (announced at time t for time t+6) positive tax shocks