New Keynesian Models, Durable Goods, and Collateral Constraints

Tommaso Monacelli *
Università Bocconi, IGIER and CEPR

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Abstract

Econometric evidence suggests that, in response to monetary policy shocks, durable and non-durable spending co-move positively, and durable spending exhibits a much larger sensitivity to the shocks. A standard two-sector New Keynesian model with perfect financial markets is at odds with these facts. The introduction of a borrowing constraint, where durables play the role of collateral assets, helps in reconciling the model with the empirical evidence.

Keywords: durable goods, sticky prices, collateral constraint.

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*Correspondence to: IGIER Bocconi, Via Salasco 5, 20136, Milan, Italy. Email: tommaso.monacelli@unibocconi.it., URL: http://www.igier.uni-bocconi.it/monacelli. I thank the editor and an anonymous referee for their extremely useful comments. I also would like to thank Tim Fuerst, Zvi Hercowitz, Alessandro Notarpietro, and Daniele Terlizzese for valuable insights. All errors are my own responsibility only. An Appendix containing supplementary material is available via Science Direct.
1 Introduction

New Keynesian (NK henceforth) models of the last generation, featuring imperfect competition and price stickiness as central building blocks, have recently become a workhorse reference for the analysis of business cycles and monetary policy.\footnote{To name a few, Goodfriend and King (1997), Rotemberg and Woodford (1997), Clarida et al. (1999), Woodford (2003).} Surprisingly, most of these models have largely ignored the role played by durable goods.

In the data, the evolution of durable spending in response to monetary shocks is characterized by two main features. First, durable spending co-moves positively with non-durable spending. Second, the sensitivity of durable spending to monetary shocks is significantly larger than the one of non-durable spending.

A baseline two-sector NK model with perfect financial markets is generally at odds with those facts: if price stickiness is asymmetric in the two sectors, whenever consumption contracts in one sector it tends to expand in the other. The intuition for this theoretical anomaly lies in a distinctive feature of durable goods under perfect financial markets: namely, that their shadow value (which corresponds to the discounted stream of marginal utilities of the durables) is almost constant.\footnote{See also Barsky et al. (2007).} This is due to the stock-flow ratio of durables being particularly high, so that a unit of durables does not add much to overall utility at the margin. As a result, durable consumption is very sensitive to variations in the user cost of durables. Hence if durable prices are flexible (sticky) and non-durable prices sticky (flexible), a monetary contraction lowers (increases) the relative price of durables, and almost invariably the user cost, leading consumption to rise in the flexible-price sector and to fall in the sticky-price one. In a nutshell, a positive correlation between the user cost and the relative price of durables is at the heart of the co-movement problem.

This paper shows that the presence of credit market frictions can reconcile an otherwise standard NK model with the empirical evidence of monetary policy shocks on durable and...
non-durable spending. In our economy, agents with heterogenous discount factors trade nominal private debt in equilibrium, with the borrowers being subject to a collateral constraint. As a result, the latter do not act as full consumption smoothers, but exhibit preferences tilted towards current consumption. Importantly, their borrowing limit is endogenously tied to the (expected future) value of the stock of durables. This feature has a twofold implication: first, the shadow value of durables is now linked to the shadow value of borrowing (a marginal unit of durables provides an additional service: it allows to expand borrowing); second, the ability of borrowing depends also on the evolution of the asset price, i.e., the relative price of durables.

To understand the implications of a borrowing constraint for the transmission mechanism, consider a monetary policy contraction, and let (for the sake of exposition) durable prices be more flexible than non-durable prices, so that the relative price of durables falls in response to the shock. By increasing the shadow value of borrowing, an interest rate hike alters the dynamics under perfect financial markets in two main respects: first, it breaks the quasi-constancy of the shadow value of durables. This happens because the latter is now a function not only of the (current and future) marginal utility of durables (which is roughly constant, as under perfect financial markets), but also of the shadow value of borrowing, which varies in response to shocks. Second, a policy rate hike increases the user cost of durables, producing a substitution towards non-durable consumption. In fact, the user cost and the relative price of durables are negatively correlated under credit market imperfections, in stark contrast with the case of perfect financial markets. The latter effect also helps in reconciling the model with the evidence that durable consumption is a more sensitive component of spending to monetary policy shocks.

Asset price movements reinforce the collateral-constraint channel described above. When a monetary policy contraction lowers the relative price of durables, it also lowers the collateral value of the durable stock, thereby affecting the borrowing capability also on the extensive margin. The latter effect is at work, for instance, when durable goods
prices are assumed to be relatively more flexible than non-durable prices.

The role of durable goods in New Keynesian models has only recently received some attention. Erceg and Levin (2006) study optimal monetary policy in a sticky price model with durable and non-durable goods, but without a borrowing constraint. In a similar environment, Barsky et al. (2007) analyze the transmission of monetary shocks and argue that it is largely affected by the assumption on the degree of stickiness of durable goods prices. Our analysis is related to their work, in that it shows that the critical role played by the stickiness (or lack thereof) of durable goods prices can be de-emphasized by the introduction of credit market imperfections. Campbell and Hercowitz (2005) study the role of collateralized debt in a business cycle model, but their analysis is confined to a one-sector, real business cycle model.3

2 Monetary Shocks and Durable Spending: the Evidence

In this section we document two stylized features that characterize the dynamic evolution of durable and non-durable spending in response to (identified) monetary policy shocks. First, durable spending co-moves positively with non-durable spending in response to those shocks. Second, the sensitivity of durable spending to policy shocks is significantly larger than the one of non-durable spending. This evidence complements the one in Erceg and Levin (2006) and Barsky et al. (2007) by documenting also the behavior of household debt.

To assess the impact of monetary policy shocks we estimate a quarterly VAR model for the U.S. economy specified as follows:

3Most recently a paper by Carlstrom and Fuerst (2006) addressing the co-movement problem, although written independently, has been brought to my attention. That paper differs from mine in that it emphasizes the possibility that sticky wages, coupled with adjustment costs in the durable sector, may be a candidate explanation for the co-movement problem.
\[ Y_t = \sum_{j=1}^{L} A_j Y_{t-j} + B E_t \]  

(1)

where \( E_t \) is a vector of contemporaneous disturbances. The vector \( Y_t \) comprises six variables: (i) real GDP, (ii) real durable consumption, (iii) real non-durable consumption and services, (iv) the GDP deflator, (v) total real household debt (the sum of mortgage and consumer credit debt), and (vi) the federal funds rate. Except for the funds rate, all variables are in logs and have been deflated by the GDP deflator.\(^4\) The VAR system features a constant, a time trend, and four lags, and is estimated over the sample 1952:1-2007:2.

To identify a monetary policy shock, we resort to a standard recursive identification scheme (Christiano et al. 1999). Figure 1 displays estimated responses of real GDP, real non-durable spending, real durable spending, and total private debt to a one-standard-deviation innovation in the federal funds rate. Dashed lines represent two-standard error bands. Hence we see that both components of spending and GDP react negatively to the policy tightening. The smooth and persistent response of these variables is in line with a recent widespread empirical evidence (Rotemberg and Woodford 1997, Christiano et al. 1999). Household debt is also observed to fall very gradually in response to the shock. Importantly, the fall in durable spending peaks earlier than the one of non-durables, and is almost three times larger at the peak. These results are robust to the specification of alternative orderings, less or additional lags, and to the introduction of alternative variables.

3 The Model

The economy is composed of a continuum of households in the interval \((0, 1)\). There are two types of households, named borrowers and savers, of measure \( \omega \) and \( 1 - \omega \) respectively. Each individual household’s time endowment is normalized to 1. There are also

\(^4\) The source of data are FRB Flow of Funds and FRED.
two sectors (producing durable and non-durable goods respectively), each populated by a large number of monopolistic competitive firms. The two types of households have heterogeneous preferences, with the borrower being more impatient than the saver. All households derive utility from consumption of a non-durable final good and from services of a durable final good. Debt accumulation reflects intertemporal trading between the borrowers and the savers. The borrowers are subject to a collateral constraint, with the borrowing limit tied to the value of the expected future value of the stock of durables. Noticeably, in our context, the borrowers differ from so-called "rule-of-thumb" (or Keynesian) consumers (see, e.g., Galí et al. 2007). While the latter are, by assumption, myopic agents who consume only out of their current income, the former are rational forward-looking agents whose ability to smooth consumption intertemporally is limited by the constraint on borrowing (Chah et al. 1995).

3.1 Final Good Producers

In each sector \( j = c, d \) a perfectly competitive final good producer purchases \( Y_{j,t}(i) \) units of intermediate good \( i \). Each producer in sector \( j \) operates the production function:

\[
Y_{j,t} = \left( \int_0^1 Y_{j,t}(i) \left( \frac{\varepsilon_{j-1}}{\varepsilon_j} \right) \right)^{\frac{\varepsilon_j}{\varepsilon_j-1}} \varepsilon_j > 1, \ j = c, d
\]  

(2)

where \( Y_{j,t}(i) \) is the quantity demanded of the intermediate good \( i \) by final good producer \( j \), and \( \varepsilon_j \) is the elasticity of substitution between differentiated varieties in sector \( j \). Notice, in particular, that in the durable good sector \( Y_{d,t}(i) \) refers to expenditure in the new durable intermediate good \( i \) (rather than services). Maximization of profits yields demand functions for the typical intermediate good \( i \) in sector \( j \):

\[
Y_{j,t}(i) = \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t} \quad j = c, d
\]  

(3)

---

for all \(i\). In particular, \(P_{j,t} \equiv \left( \int_0^1 P_{j,t}(i)^{1-\epsilon_j} \, di \right)^{\frac{1}{1-\epsilon_j}}\) is the price index consistent with the final good producer in sector \(j\) earning zero profits.\(^6\)

### 3.2 Borrowers

A typical borrower consumes an index of consumption services of durable and non-durable final goods, defined as:

\[
X_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} (C_t)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_t)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}
\]  
(4)

where \(C_t\) denotes consumption of the final non-durable good, \(D_t\) denotes services from the stock of the final durable good at the end of period \(t\), \(\alpha > 0\) is the share of durable goods in the composite consumption index, and \(\eta \geq 0\) is the elasticity of substitution between services of non-durable and durable goods.\(^7\)

The borrower maximizes the following utility program:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \right\}
\]  
(5)

subject to the sequence of budget constraints (in nominal terms):

\[
P_{c,t} C_t + P_{d,t} (D_t - (1 - \delta) D_{t-1}) + R_{t-1} B_{t-1} = B_t + W_t N_t + T_t
\]  
(6)

where \(E_t \{ \}\) denotes (conditional) expectations at any given period \(t\), \(B_t\) is end-of-period \(t\) nominal one-period debt, \(R_{t-1}\) is the nominal lending rate on loan contracts stipulated at time \(t - 1\), \(W_t\) is the nominal wage, \(N_t\) is total labor supply, and \(T_t\) are lump-sum government transfers/taxes. Labor is assumed to be perfectly mobile across sectors, implying that the nominal wage rate is common across sectors.

In real terms (units of non-durable consumption), (6) reads:

\[^6\text{Hence the problem of the final good producer } j \text{ is: } \max P_{j,t} Y_{j,t} - \int_0^1 P_{j,t}(i) Y_{j,t}(i) \, di \text{ subject to (2).}\]

\[^7\text{Throughout the paper we will refer to } D_t \text{ as durable services (i.e., the end-of-period stock of durables) and define durable investment as the flow variable } D_t - (1 - \delta) D_{t-1}.\]
\[ C_t + q_t(D_t - (1 - \delta)D_{t-1}) + R_{t-1} \frac{b_{t-1}}{\pi_{c,t}} = b_t + \frac{W_t}{P_{c,t}} N_t + \frac{T_t}{P_{c,t}} \]  

where \( q_t \equiv P_{d,t}/P_{c,t} \) is the relative price of the durable good, \( \pi_{c,t} \equiv P_{c,t}/P_{c,t-1} \) is non-durable good (gross) inflation, and \( b_t \equiv B_t/P_{c,t} \) is real debt (in units of non-durables).

Below we will specialize the form of the utility function as follows:

\[ U(X_t, N_t) = \log(X_t) - \frac{\nu N_t^{1+\varphi}}{1 + \varphi} \]

where \( \varphi \) is the inverse elasticity of labor supply, and where \( \nu \) is a parameter that indexes the preference for hours worked of each agent.

Private borrowing is subject to an endogenous limit. At any time \( t \), the amount that the borrower agrees to repay in the following period, \( R_t B_t \), is tied to the expected future value of the durable stock (after depreciation):

\[ R_t B_t \leq (1 - \chi)(1 - \delta)E_t \{ D_t P_{d,t+1} \} \]

where \( \chi \) is the fraction of the durable good value that cannot be used as a collateral. Notice that expected movements in the relative price of durables affect the ability of borrowing directly. This channel will be important in evaluating the transmission of monetary policy shocks in the model. The form of constraint (9) can be rationalized in terms of limited enforcement. Although debt repudiation is in principle feasible for the borrower, this option would entail losing the entire current value of the asset. Hence the provision of collateral acts against that temptation.

We assume that, in a neighborhood of the deterministic steady state, equation (9) is always satisfied with equality. Hence we can rewrite the collateral constraint in real
terms (i.e., in units of non-durable consumption) as follows:

\[ b_t = (1 - \chi)(1 - \delta)E_t \left\{ \frac{D_t q_{t+1}}{R_t / \pi_{c,t+1}} \right\} \]  \hspace{1cm} (10)

Given initial values \{b_{-1}, D_{-1}\}, the borrower chooses \{N_t, b_t, D_t, C_t\} to maximize (5) subject to (7) and (10). By defining \lambda_t and \lambda_t \psi_t as the multipliers on constraints (7) and (10) respectively, and \psi_{c,t} as the marginal utility of variable \( t = C, N, D \), efficiency conditions for the above program read:

\[ \frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}} \] \hspace{1cm} (11)

\[ U_{c,t} = \lambda_t \] \hspace{1cm} (12)

\[ q_t U_{c,t} = U_{d,t} + \beta(1 - \delta)E_t \left\{ U_{c,t+1}q_{t+1} \right\} + (1 - \chi)(1 - \delta)U_{c,t}q_t \psi_t E_t \left\{ \pi_{d,t+1} \right\} \] \hspace{1cm} (13)

\[ R_t \psi_t = 1 - \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{R_t}{\pi_{c,t+1}} \right\} \] \hspace{1cm} (14)

Equation (11) is a standard condition linking the real wage (in units of non-durables) to the borrower’s marginal rate of substitution between consumption and leisure. In (12), the borrower’s marginal utility of consumption is equated to the shadow value of relaxing the flow budget constraint (7). Equation (13) requires the borrower to equate the marginal utility of non-durable consumption to the shadow value of durable services. The latter depends on three components: (i) the direct utility gain of an additional unit of durable; equality, at least locally, variations in its tightness will still be measurable in terms of its corresponding shadow value.
(ii) the expected utility stemming from the possibility of expanding future consumption by means of the realized resale value of the durable purchased in the previous period; (iii) the marginal utility of relaxing the collateral constraint, which is proportional to \( \psi \) (recall that the impatient agent can purchase new debt only by acquiring durables). Notice that, in the case in which the borrowing constraint is not binding (\( \psi = 0 \) for all \( t \)), the shadow value of a unit of durable good comprises only the terms (i) and (ii).

Equation (14) is a modified version of a typical Euler equation. Indeed it reduces to a standard Euler condition in the case of \( \psi = 0 \) for all \( t \). Consider, for the sake of argument, \( \psi \) rising from zero to a positive value (for any given \( R_t \)). This implies, from (14), that \( U_{c,t} > \beta E_t \{ U_{c,t+1} R_t / \pi_{c,t+1} \} \). In other words, the marginal utility of current consumption exceeds the marginal gain of shifting one unit of consumption intertemporally. The higher \( \psi \), the higher the net marginal benefit of acquiring today a unit of the durable asset which in turn allows, by relaxing the collateral constraint at the margin, to purchase additional current consumption. Hence a rise in \( \psi \) signals a tightening of the collateral constraint.

**User Cost** An alternative interpretation of condition (13) is that it requires to equate the marginal rate of substitution between durable and non-durable consumption, \( U_{d,t}/U_{c,t} \), to the *user cost* of durables \( (Z_t) \), which in this case reads:

\[
Z_t \equiv q_t \left[ 1 - (1 - \chi) (1 - \delta) \psi_t E_t \{ \pi_{d,t+1} \} \right] - \beta (1 - \delta) E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} q_{t+1} \right\}
\]  

(15)

Under perfect financial markets (\( \psi = 0 \) for all \( t \)), movements in the user cost are typically dominated by (current and expected) variations in the asset price \( q_t \) (Erceg and Levin 2006). A typical feature of the model with a borrowing constraint is instead that movements in the user cost are also affected by the shadow value \( \psi \).

A de-linking between the user cost \( Z_t \) and the asset price \( q_t \) will be a defining feature of the dynamics under credit market imperfections (see more below). To understand this point, consider a log-linear approximation of (13) and (14) around the deterministic
steady state. Using the symbol "\( \hat{\text{^\text{\textcircled{}}}} \)" to denote percent deviations from corresponding steady-state values, we can write the following expression for the user cost of durables:

\[
\hat{Z}_t = \Phi^{-1}(1 - \delta) \left[ \Gamma \hat{q}_t - \beta E_t \{ \hat{q}_{t+1} \} + \gamma \hat{R}_{r,t} + (\gamma - \beta) \left( \chi \hat{\psi}_t - \hat{\zeta}_t \right) \right]
\]

where

\[
\Gamma \equiv \left[ \frac{1 - (1 - \chi)(1 - \delta)(\gamma - \beta)}{(1 - \delta)} \right]
\]

\[
\Phi \equiv 1 - (1 - \delta) [\beta + (1 - \chi)(\gamma - \beta)]
\]

\[\hat{R}_{r,t} \equiv \hat{R}_t - E_t \{ \hat{\pi}_{c,t+1} \} \]

is the (ex-ante) real interest rate in units of non-durables, and

\[\hat{\zeta}_t \equiv E_t \{(1 - \chi)\hat{\pi}_{d,t+1} - \hat{\pi}_{c,t+1}\} \]

is a composite term in sectoral inflation.

Notice that in the case \( \gamma = \beta \), i.e., when heterogeneity in patience rates vanishes (and the collateral constraint becomes locally non-binding, see more below), the expression for the user cost simplifies to:

\[
\hat{Z}_t \equiv [1 - \beta(1 - \delta)]^{-1} \left[ \hat{q}_t + \beta(1 - \delta) \left( \hat{R}_{r,t} - E_t \{ \hat{q}_{t+1} \} \right) \right]
\]

Under perfect financial markets, movements in \( \hat{Z}_t \) depend positively on the current relative price of durables, but negatively on the expected future price of durables. Intuitively, current demand for durables rises when the expected future price rises, due to the expected asset appreciation. This feature vanishes for \( \delta \to 1 \), i.e., when durability disappears. Also, the user cost depends inversely on the real interest rate, for the latter reflects the opportunity cost of investing in the durable good. Finally, depreciation rises the user cost, because it physically erodes the investment in the durable good.

In the presence of a collateral constraint (equation (16)), the expression for the user cost is affected by an additional element, namely the multiplier \( \psi_t \). As hinted above, a rise
in \( \psi_t \) signals that the collateral constraint is tighter, for the higher would be the marginal value for the borrower of tilting the consumption plan towards current consumption. By inspecting equation (16) it is clear that a rise in \( \hat{\psi}_t \) induces (\textit{ceteris paribus}) a rise in \( \hat{Z}_t \) if

\[
\Phi^{-1}(1 - \delta)(\gamma - \beta)\chi > 0
\]

which in turn requires \( \Phi > 0 \).

Notice that condition (18) is always satisfied in the limit case of \( \delta = 0 \). In that case, in fact, we have

\[
\Phi_{\delta=0} \equiv (1 - \gamma) + \chi(\gamma - \beta) > 0
\]

In the more general case of \( \delta \) small but positive, the same condition is more easily satisfied: (i) the lower the depreciation rate \( \delta \) (higher durability); (ii) the higher the inverse LTV ratio \( \chi \) (therefore the lower the ability to translate the value of the collateral into new debt); (iii) the higher the saver’s patience rate \( \gamma \) (for any given borrower’s patience rate \( \beta \)), and therefore the stronger the heterogeneity in patience rates. For instance, condition (18) will be always satisfied under our baseline parameterization (see below).

### 3.3 Savers

The economy is composed of a second category of consumers, labeled savers. We assume that the typical saver is the owner of the monopolistic firms in each sector. He/she maximizes the utility program \( E_0 \left\{ \sum_{t=0}^{\infty} \gamma^t U(X_t, N_t) \right\} \). The key feature that distinguishes the saver’s behavior is the discount factor. We assume that the saver is \textit{more patient} than the borrower, implying \( \gamma > \beta \). Since the saver’s optimal program is standard we refrain from presenting it here and refer the interested reader to the supplementary material in the Appendix.
3.4 Production and Pricing of Intermediate Goods

A typical intermediate good firm $i$ in sector $j$ hires labor (supplied by the borrowers) to operate a linear production function:

$$Y_{j,t}(i) = N_{j,t}(i)$$  \hspace{1cm} (19)

where $N_{j,t}(i)$ is total demand for labor by firm $i$ in sector $j$, and where, for simplicity, labor productivity is assumed to be constant and normalized to 1 in both sectors. Each firm $i$ has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost proportional to output, and equal to $(\vartheta_j/2) (P_{j,t}(i)/P_{j,t-1}(i) - 1)^2 Y_{j,t}$, where the parameter $\vartheta_j$ measures the degree of sectoral nominal price rigidity. In the particular case of $\vartheta_j = 0$, prices are flexible.

The problem of each monopolistic firm in sector $j$ is similar to the one presented, e.g., in Ireland (2003) for a one-sector economy. Up to a first order approximation, that problem leads to a forward-looking ("New Keynesian") Phillips curve in each sector.\footnote{See also Galí and Gertler (1999).}

Here we simply recall that in the particular case of flexible prices (in both sectors), the real marginal cost must be constant and equal to the inverse steady-state markup $(\varepsilon_j - 1)/\varepsilon_j$. In this case, the pricing condition reads:

$$- \frac{U_{n,t}}{U_{c,t}} q_t^{-1} = \frac{\varepsilon_d - 1}{\varepsilon_d} \hspace{1cm} \text{if } j = d$$  \hspace{1cm} (21)

$$- \frac{U_{n,t}}{U_{c,t}} = \frac{\varepsilon_c - 1}{\varepsilon_c} \hspace{1cm} \text{if } j = c$$  \hspace{1cm} (20)

Notice that, in the durable sector, the real marginal cost is directly affected by movements in the relative price.
3.5 Monetary Policy

We assume that monetary policy is conducted by means of a simple Taylor-type rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{\bar{\pi}_t}{\bar{\pi}}\right)^{\phi_{\pi}} \varepsilon_t \quad \phi_{\pi} > 1$$

(22)

where $R$ is the steady-state gross nominal interest rate, $\bar{\pi}_t \equiv \pi_{c,t}^{1-\omega} \pi_{d,t}^\omega$ is a composite inflation index, and $\varepsilon_t$ is a policy shock which is assumed to evolve according to

$$\exp (\varepsilon_t) = \exp(\varepsilon_{t-1})^\rho u_t$$

with $u_t \sim iid$ and $0 < \rho < 1$.

3.6 Market Clearing

Equilibrium in the goods market of sector $j = c, d$ requires that the production of the final good be allocated to total households’ expenditure and to resource costs originating from the adjustment of prices:

$$Y_{c,t} = \omega c_t + (1 - \omega) \tilde{C}_t + \frac{\partial_c}{2} (\pi_{c,t} - 1)^2 Y_{c,t}$$

(23)

$$Y_{d,t} = \omega (D_t - (1 - \delta)D_{t-1}) + (1 - \omega) \left(\tilde{D}_t - (1 - \delta)\tilde{D}_{t-1}\right) + \frac{\partial_d}{2} (\pi_{d,t} - 1)^2 Y_{d,t}$$

(24)

where $Y_{j,t} \equiv \int_0^1 Y_{j,t}(i) \, di = \int_0^1 N_{j,t}(i) \, di = N_{j,t}$ for $j = c, d$.

Equilibrium in the debt and labor market requires respectively

$$\omega B_t + (1 - \omega) \tilde{B}_t = 0$$

(25)

$$\sum_j N_{j,t} = \omega N_t + (1 - \omega) \tilde{N}_t$$

(26)
Finally, we abstract from redistribution via fiscal policy. Hence we set the transfer structure as:

\[ T_t = \tilde{T}_t = 0 \]

\section*{4 Deterministic Steady State}

In the deterministic steady state we assume that inflation is zero in both sectors. Hence \( R \) corresponds to the real (gross) rate of interest, and is pinned down by the savers’ discount rate \( R = \gamma^{-1} \) via their (standard) consumption Euler condition. Due to the assumed heterogeneity in discount rates (i.e., \( \beta < \gamma \)), the shadow value of borrowing is always positive. In other words, the borrower will always choose to hold a positive amount of debt. To show that, we evaluate (14) in the steady state (under the assumption of zero inflation) and obtain:

\[ \psi = (\gamma - \beta) > 0 \]  

Notice that, to insure a well-defined steady state, both heterogeneity in patience rates \textit{and} a borrowing limit are required. In fact, if discount rates were equal, the steady-state level of debt would be indeterminate (Becker, 1980, Becker and Foias, 1987). In this case, in fact, it would hold \( \beta / \gamma = \beta R = 1 \), and the economy would display a well-known problem of dependence of the steady state on the initial conditions.\(^{12}\) With different discount rates, and yet still perfect financial markets, the consumption path of the borrower would be tilted downward, and the ratio of consumption to income would asymptotically shrink to zero.\(^{13}\) Hence a binding collateral constraint allows a constant consumption path to be

\(^{12}\)In other words, under \( \beta = \gamma \), the economy would constantly replicate the initial (arbitrary) distribution of wealth forever. This problem is analogous to the typical one that attains to small open economies with incomplete markets.

\(^{13}\)In this case the assumption \( \beta < \gamma \) is equivalent to \( \beta R < 1 \). In the absence of exogenous growth, this implies that the (gross) growth rate of consumption (\( \beta R \)) is below the (gross) growth rate of income (which is 1). Hence, the ratio of consumption to output must shrink over time.
compatible with heterogeneity in discount rates. One can show that, under the assumption \( \beta < \gamma \), the steady-state level of debt \( b \) is stable, i.e., the economy converges to a unique positive finite value \( \bar{b} \) starting from any initial value different from \( \bar{b} \) (see Appendix).

By evaluating (13) in the steady state, and combining with (27), we obtain the borrower’s relative consumption of durables:

\[
\frac{D}{C} = \frac{\alpha}{(1 - \alpha)} \left\{ q \left[ 1 - (1 - \delta) (\beta + (1 - \chi)(\gamma - \beta)) \right] \right\}^{-\eta}
\] (28)

If \( \delta \to 1 \) (no durability), and/or \( \gamma = \beta \) (which implies \( \psi = 0 \) and therefore a non-binding collateral constraint), the durable/non-durable margin depends only on the relative price \( q \). Notice that a rise in the down-payment parameter \( \chi \) induces a fall in the relative demand for durables. Intuitively, if the ability of transforming the collateral into new debt is diminished, this makes durables less attractive.

5 Calibration and Solution Method

The steady-state real rate of interest is pinned down by the saver’s discount factor \( \gamma \). We choose an annual real rate of return of 4 percent. This implies \((1/\gamma)^4 = 1.04\), and in turn \( \gamma = 0.99 \). As in Krusell and Smith (1998), we set the borrower’s discount factor \( \beta = 0.98 \). We choose an annual depreciation rate for durable goods of 4 percent, hence \( \delta = 0.04/4 \). We set \( \chi = 0.25 \), corresponding to a loan-to-value ratio of 70 percent. The share of durable consumption in the aggregate spending index, defined by \( \alpha \), is set in such a way that the steady-state share of durable spending in total private spending, is 0.2 (see NIPA Tables). The elasticity of substitution between varieties \( \varepsilon_j \) is set equal to 6 in both sectors, implying a steady-state mark-up of 20\%. The elasticity of substitution between non-durable and durable services is set \( \eta = 1 \), implying a Cobb-Douglas consumption index \( X_t \).\(^{14}\) The inverse elasticity of labor supply \( \varphi \) is set equal to 1.

\(^{14}\)Ogaki and Reinhart (1998) estimate values for \( \eta \) slightly above unity. Qualitatively, however, our results will not hinge on the assumed value for the elasticity of substitution \( \eta \).
We set the degree of nominal rigidity in non-durable prices $\vartheta_c$ to generate a frequency of price adjustment of about four quarters. To pin down this value we proceed as follows. Let $\theta$ be the probability of not resetting prices in the standard Calvo-Yun model.\textsuperscript{15} We parameterize $1/(1 - \theta) = 4$, which implies $\theta = 0.75$, and therefore an average frequency of price adjustment of one year. Log-linearization of the optimal pricing condition in each sector yields a slope of the Phillips curve equal to $(\varepsilon_j - 1)/\vartheta_j$, whereas the slope of the Phillips curve in the Calvo-Yun model reads $(1 - \theta)(1 - \gamma \theta)/\theta$. After setting the elasticity $\varepsilon_j$, the resulting stickiness parameter satisfies $\vartheta_j = \theta(\varepsilon_j - 1)/(1 - \theta)(1 - \gamma \theta)$. A price rigidity of four quarters is a standard calibration in the recent literature. Bils and Klenow (2004) document that prices of durable goods are generally more flexible than those of non-durable goods. Nakamura and Steinsson (2008), however, do not report any systematic evidence of larger flexibility of durable prices, and estimate an average frequency of adjustment of about four quarters regardless of durability. In our simulations, then, we experiment with alternative values for the degree of stickiness in durables, ranging from full flexibility ($\vartheta_d = 0$) to sizeable stickiness ($\vartheta_d = \vartheta_c$). As for the monetary policy rule, we set $\phi_\pi = 1.5$, which is a standard value in the literature on Taylor rules, and the shock persistence parameter $\rho = 0.5$.

When we analyze the economy with heterogeneous agents we normalize the steady-state level of hours worked such that each individual agent chooses to work $1/3$ of her time endowment. This does not entail that "effective" hours worked are the same for the two agents, for the preference parameter $\nu$ will endogenously differ across agents to ensure that both categories choose to work exactly that share of their time endowment.

Our solution method consists in taking a log-linear approximation of the equilibrium conditions in the neighborhood of the deterministic steady state, in which condition (27) holds, and therefore equation (10) is satisfied with equality at least locally. This local approximation method is accurate to the extent that we limit the exogenous process \{$\varepsilon_t$\}.

\textsuperscript{15}See Woodford (2003).
to be bounded in the neighborhood of the steady state, an assumption which appears reasonable at least in the case of monetary policy shocks.

6 Co-movement Problem under Perfect Financial Markets

We start by studying the benchmark case of a standard NK model with perfect financial markets and simply augmented by the presence of a durable goods sector. This version of the model is obtained by evaluating the system of first order conditions (11)-(14) in the particular case of $\psi_t = 0$. A twofold anomaly emerges. First, when durable prices are flexible, the response of durable spending to a policy shock is countercyclical (and co-moves negatively with non-durable spending). Second, when durable prices are assumed to be sticky, durable consumption correctly contracts in response to a policy tightening, but still exhibits a wrong co-movement with consumption in the non-durable sector. Both results are at odds with the empirical evidence reported in the early part of the paper.

Let $V_t \equiv U_{c,t} q_t$ denote the shadow value of one unit of durables. To understand the effect of the policy shock, it is important to recall that a key property of durability is that, under perfect financial markets, $V_t$ is almost a constant. In fact, after iterating (13) forward, we can write:

$$V_t \equiv U_{c,t} q_t = E_t \left\{ \sum_{j=0}^{\infty} [\beta (1 - \delta)]^j U_{d,t+j} \right\} \simeq \text{const.} \quad (29)$$

Hence the right hand side of (29) depends only on (current and expected future values of) the marginal utility of durables $U_{d,t}$. Given that, for sufficiently small $\delta$, the stock-flow ratio is high for durables, $U_{d,t}$ is a very smooth process. This entails that for $V_t$ to be constant, any variation in the relative price of durables must be matched by a variation in the marginal utility of non-durable consumption, $U_{c,t}$, of the opposite sign, and therefore by a variation in non-durable consumption of the same sign. In the particular case in
which prices are equally flexible in both sectors, but also in the case in which they are equally sticky, non-durable consumption is (almost) invariant to monetary shocks. We will see later that the introduction of a borrowing constraint will alter the quasi-constancy of the shadow value $V_t$.

**Sticky Non-Durable Prices** Figure 2 displays the effect on selected variables of a 25 basis points innovation in the policy rule (22) in the model with perfect financial markets. Three limit cases are described: (i) sticky non-durable prices (and flexible durable prices), (ii) sticky durable prices (and flexible durables); and (iii) prices equally sticky in both sectors. In all cases price stickiness is the equivalent of four quarters.

When non-durable goods prices are sticky, the relative price $q_t$ falls substantially in response to the shock. This is caused by the price of durable goods falling relatively more than the price of non-durables. Notice the one-to-one co-movement between the relative price of durables $q_t$ and non-durable consumption, consistent with equation (29). Why, then, do consumption and production (employment) both rise in the durable sector? It is useful to rewrite the condition driving the consumption-leisure margin (for a generic agent) as follows:

$$\frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}} = \frac{W_t}{P_{d,t}} q_t$$

(30)

Price flexibility in the durable sector implies that the marginal cost is constant in that sector, i.e., $-U_{n,t}/U_{c,t}q_t = \text{const}$. Hence, given that the denominator $U_{c,t}q_t$ is (quasi) constant as a result of (29), both the product wage $W_t/P_{d,t}$ in the durable sector and $U_{n,t}$ must be constant. In turn, this implies that total employment must be constant in equilibrium, $N_t \approx \bar{N}$. Yet if employment falls in the non-durable sector as a result of the monetary tightening, it must necessarily rise in the durable sector (as we observe in figure 2) to keep total employment unchanged. Hence output and expenditure both contract in the non-durable sector, whereas they simultaneously expand in the durable sector.
Sticky Durable Prices  When durable prices are sticky (and yet non-durable prices are flexible), the co-movement problem arises again. Notice that the relative price of durables now rises, thereby dictating a rise also in non-durable consumption (and employment). The reason for why $q_t$ rises is just symmetric to the previous case: now, prices fall relatively more in the non-durable sector. At the same time, flexibility in the non-durable sector implies a constant real marginal cost: $-U_{n,t}/U_{c,t} = \text{const}$. Using condition (30) we can write:

$$-U_{n,t} = \overline{U}_c q \frac{W_t}{P_{d,t}} \quad (31)$$

where an upper bar indicates that $\overline{U}_c q$ is a constant (again consistent with equation (29)). Sticky durable prices imply that the product wage must fall in that sector (to accompany a fall in the sectoral real marginal cost). From (31), this implies that $U_{n,t}$ must rise, and therefore total employment must fall. But if consumption and employment in the non-durable sector both rise, then necessarily employment and expenditure must fall in the durable sector.

In the case of sticky durable prices, the model exhibits a further anomaly: the nominal interest rate falls (see also the case of equal stickiness below). As noted in Barski et al. (2007) this is due to the real return in units of durables being quasi-constant (once again a consequence of the quasi-constancy of the shadow value of durables), so that the nominal rate tracks expected inflation in durables almost one to one.

Equal Stickiness  Finally, figure 2 shows that the case of equal price stickiness in both sectors generates a completely flat response of the relative price of durables. As a result, also non-durable consumption is basically constant. This (almost) perfect correlation between non-durable consumption and the relative price follows once again from the property of constant shadow value of durables, as from equation (29). At the same time, however, we observe that durable consumption still falls. In fact, despite a constant
relative price of durables, the user cost rises, due to the rise in the real interest rate (there is no distinction in this case between durable vs. non-durable-based real interest rate). In other words, even in the absence of relative price movements, the model is still not capable of generating a clear positive co-movement between consumption in the two sectors.

7 The Role of Credit Market Imperfections

This section argues that the introduction of credit market imperfections can help in reconciling an otherwise standard NK model with the empirical evidence on the sectoral transmission of monetary shocks. In addition to the standard effect traditionally related to price stickiness, the presence of a collateral constraint on borrowing produces a tightening of the constraint, i.e., a rise in $\psi_t$. We will label this collateral-constraint effect. The rise in the shadow value $\psi_t$ has a twofold implication. First, it breaks the quasi-constancy of the shadow value of durables, which is a key ingredient of the co-movement problem under perfect financial markets; second, it increases the user cost of durables, producing a substitution towards non-durables.

To understand the first point, recall that the shadow value of durables must satisfy (from (13)):

$$V_t = \frac{U_{d,t} + \beta(1 - \delta)E_t\{V_{t+1}\}}{K_t}$$

(32)

where

$$K_t = [1 - (1 - \chi)(1 - \delta)\psi_tE_t\{\pi_{d,t+1}\}]$$

is a composite term that depends (negatively) on the multiplier $\psi_t$ and on the expected rate of inflation in durables. Notice that, under perfect financial markets ($\psi_t = 0$ for all $t$), the composite term $K_t$ is equal to 1 for all $t$.

Log-linearizing (32) around the deterministic steady-state, and iterating forward one
obtains:

\[
\hat{V}_t = \sum_{j=0}^{\infty} \left( \frac{\beta}{1-\gamma} \right)^j E_t \{ \Theta \hat{U}_{d,t+j} - \hat{K}_{t+j} \}
\]  

(33)

where \( \Theta \equiv \Phi / \Gamma (1 - \delta) \). Notice that (33) reduces to (29) in the case \( \beta = \gamma \). Under a collateral constraint, the shadow value of durables depends not only on the marginal utility of durables (as under perfect financial markets), but also on the current and expected future values of the shadow value of borrowing (via the term \( K_t \)), which does fluctuate in response to shocks.

The additional implication stemming from the presence of a collateral constraint is that tighter credit conditions generate (via a rise in \( \psi_t \)) a rise in the user cost of durables (see our previous analysis). This induces a substitution from durables to non-durable consumption, i.e., from the flexible towards the sticky sector. Importantly, this effect generates a de-linking between the user cost and the relative price of durables (see equation 15 above). A tight co-movement between the user cost and the relative price of durables was instead a defining feature of the baseline model with perfect financial markets.

It is important to emphasize, though, that the collateral-constraint channel described above is at work for any assumed relative degree of sectoral price stickiness: in other words, this effect is independent of \( q_t \). Hence movements in the relative price of durables can strengthen the collateral-constraint effect by altering the value of the collateral asset. In turn, the implied variation in the demand for durables will feedback onto the behavior of relative prices, all in a self-reinforcing fashion.

Figure 3 depicts impulse responses to a monetary policy tightening (25 basis point innovation) in the model with a binding collateral constraint. Solid and dashed lines denote the typical borrower’s and saver’s variables respectively (with the behavior of the saver being illustrative also of the dynamics under perfect financial markets). In this experiment we assume that non-durable prices feature a standard four-quarter stickiness, whereas durable prices are more flexible (two-quarter stickiness), so that the relative
price of durables tends to move in the right direction (i.e., fall). We further assume that
the elasticity of substitution $\eta$ equals 1 and that the share of borrowers is equal to $1/2$.

Consider now the effect of introducing a collateral constraint. The monetary policy
tightening induces a rise in the marginal value of borrowing $\psi_t$. Notice that while the
shadow value of durables ($V_t$) is almost constant under perfect financial markets it rises
sharply in the case in which the collateral constraint is binding (as a result of the rise in
$\psi_t$). As in the case of perfect financial markets, the result of the policy shock is a fall in
the relative price of durables. However, the dynamics of the user cost and of the relative
price of durables are now de-linked: the relative price of durables falls whereas the user
cost rises in response to the shock, in sharp contrast with the baseline economy under
perfect financial markets.

With a collateral constraint the fall in the price $q_t$ has an additional effect: the one of
reducing directly the collateral value, further contributing to a tightening of the borrowing
conditions. As a result, real debt falls, the demand for durables drops on impact and then
starts to gradually revert back towards the steady state as the user cost gradually falls over
time. In addition, the observed rise in the user cost produces a substitution effect from
durables to non-durables. Hence the peak impact on durable consumption is larger than
the one on non-durable consumption. Notice also that the fall in real debt in response
to a policy tightening is qualitatively in line with our empirical evidence discussed in the
early part of the paper.

Simultaneously, the borrower reduces also the demand of non-durable goods, and to
a larger extent relative to the saver. This is the result of two effects. First, prices are
sticky in that sector, so the real interest rate on non-durables rises. Second, and most
importantly, the reduced ability of borrowing (due to tighter borrowing conditions as
well as to the fall in the relative price of durables) affects negatively also the borrower’s
demand for non-durables. In this vein, the presence of an endogenous collateral constraint
generates a complementarity between durable and non-durable demand.
As it is clear from figure 3 the borrower’s durable consumption response contrasts sharply to the one of the saver’s. Hence it is important to understand whether the co-movement problem disappears also for aggregate consumption. Aggregation requires an understanding of the savers’ consumption responses to the policy shock (see figure 3 again). Recall that the savers are standard permanent-income agents. Two competing effects drive their demand. For one, a positive income shock, which is the counterpart of the negative income shock for the borrowers. This effect leads the savers to increase both categories of consumption. However, the rise in the real interest rate makes them substitute consumption intertemporally, so that, on balance, savers’ non-durable consumption is observed to fall initially. At the same time, since the relative price of durables falls, the savers increase their demand for durables. For these agents, in fact, the relevant user cost is the one prevailing in the absence of any collateral constraint, and therefore it depends heavily on the behavior of the relative price.

Figure 4 illustrates how aggregate consumption responds to the policy shock under alternative values of the frequency of adjustment in durable prices (stickiness in non-durables is kept constant at four quarters). A positive co-movement generally arises under the condition that durable prices display a minimal degree of stickiness. The required degree of stickiness in durables, however, is well below the standard value of four quarters estimated, for instance, in Nakamura and Steinsson (2008). In the baseline case of prices equally sticky for four quarters, the model generates a strong positive co-movement. The latter result contrasts sharply with the one obtained in the baseline NK model with perfect credit markets. Notice that a negative co-movement arises also between non-durable consumption and durable investment (expenditure). The latter falls sharply (in accordance with our introductory empirical evidence) but its fall is short-lived. This is the result of the dynamic of durable services lacking persistence (i.e., being not hump-shaped), thereby durable services fall but then starts almost immediately to revert back to the steady state (recall that the household’s preferences require to smooth the
response of $D_t$ and not the one of investment). It would be straightforward, however, to amend the model to allow for a more persistent dynamic in durable investment.\footnote{There are at least three features that might help in this direction: first, some form of segmentation in the labor market, thereby sectoral labor is either not perfectly mobile across sectors (as in Erceg and Levin 2006) or not perfectly substitutable in the production function; second, adjustment costs in (the rate of change of) durable investment; third, habit persistence in durable consumption.}

The same figure displays the behavior of the nominal interest rate. As in the case of frictionless credit markets the degree of stickiness in durable prices is relevant in determining the sign of the response: if the stickiness of durable prices is sufficiently high, and as high as the one of non-durables, the interest rate slightly falls. It is interesting, however, that there exist values of durable stickiness for which the model delivers a positive co-movement between durable and non-durable consumption and simultaneously the nominal interest rate is observed to rise, as conventional wisdom would predict.

\section{8 Conclusions}

This paper has shown that credit market imperfections on the household’s side can be relevant in accounting for the transmission of monetary policy shocks on durable and non-durable spending. The key idea is that, by affecting credit conditions for constrained agents, monetary policy can have an impact on the intertemporal relative price of durables (the user cost), and therefore on the sectoral allocation of demand.

Our conclusions bear implications on two further grounds. First, Barsky et al. (2007) have recently highlighted that the presence of durable goods, despite their smaller relative share in total spending, can substantially alter the transmission of monetary shocks within a standard NK sticky-price model. In particular, if durable prices are flexible, their model exhibits monetary neutrality, while if durable prices are sticky, the model behaves as a standard sticky-price model even if non-durable prices are flexible. Our paper shows that the assumption on the degree of stickiness of durables becomes significantly less crucial once a collateral constraint on borrowing is introduced in the model.
Second, a recent research program has tried to assess the empirical validity of dynamic stochastic general equilibrium (DSGE) models via structural estimation methods (Smets and Wouters 2003, Christiano et al. 2005). In that research program, credit markets are usually assumed to be frictionless. An extension of estimated DSGE models to include a role for collateral constraints (both on the household’s and the firm’s side) seems of paramount importance in order to improve the ability of such models to provide an adequate representation of the data.
References


Figure 1. Estimated Impulse Responses to a Monetary Policy Tightening (sample 1954:1 2007:2, dashed lines are two standard error bands, see text for VAR model specification).
Figure 2. Co-movement Problem under Free Borrowing: Impulse Responses to a Monetary Policy Tightening.
Figure 3. Impulse Responses to a Monetary Policy Tightening: Model with Collateral Constraint.
Figure 4. Impulse Responses to a Monetary Policy Tightening: Effect of Varying the Degree of Stickiness in Durable Prices (model with collateral constraint).