Optimal Interest Rate Rules, Asset Prices, and Credit Frictions *

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Abstract

We study optimal Taylor-type interest rate rules in an economy with credit market imperfections. Our analysis builds on the agency cost framework of Carlstrom and Fuerst (1997), which we extend in two directions. First, we embed monopolistic competition and sticky prices. Second, we modify the stochastic structure of the model in order to generate a countercyclical premium on external finance. This is achieved by linking the mean distribution of investment opportunities faced by entrepreneurs to aggregate total factor productivity. We model monetary policy in terms of simple welfare-maximizing interest rate rules. We find that monetary policy should respond to increases in asset prices by lowering interest rates. However, when monetary policy responds strongly to inflation, the marginal welfare gain of responding to asset prices vanishes. Within the class of linear interest rate rules that we analyze, a strong anti-inflationary stance always attains the highest level of welfare.

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1 Introduction

In this paper we study optimal Taylor-type interest rate rules in an economy with nominal rigidities and credit market imperfections. Our interest is twofold. First, we aim at driving the attention of the recent literature on a typology of market distortions whose role has been largely neglected in the normative analysis of monetary policy. This is surprising, considering the increasing emphasis - starting with Bernanke and Gertler (1989) - placed on financial factors in the study of business cycles. Second, we aim at assessing - from a welfare-based perspective - the role that asset prices and/or other financial indicators should play in the optimal setting of monetary policy rules.

We model credit frictions following the agency cost framework of Carlstrom and Fuerst (1997) (CF henceforth). Financial imperfections originate from a problem of asymmetric information between an entrepreneur (the borrower) and a financial intermediary (the lender). The entrepreneur engages in a risky investment activity and seeks resources in excess of internal funds. Since the outcome of the investment activity is private information to the entrepreneur, the latter has an incentive to misreport the same outcome. This moral hazard problem induces the lender to monitor only the defaulting entrepreneurs, and transfer this implicit cost onto the average cost of credit. The appeal of this framework is that agency costs are endogenous over the business cycle and default emerges as an equilibrium phenomenon.\(^1\)

We depart from CF in two main respects. First, we introduce monopolistic competition and sticky prices, a necessary step in order to study monetary policy. Second, and in order for the model to better comply with the empirical evidence on the cyclical behavior of the cost of credit, we modify the stochastic structure of the model to generate a countercyclical behavior of the finance premium, i.e., the cost of external finance in excess of the safe rate of return. This is accomplished by assuming that the mean distribution of investment outcomes across entrepreneurs is linked to the state of aggregate productivity in the economy. Importantly, this variation allows us to overcome an often-cited limitation of the CF framework, which generates a procyclical cost of external finance, a feature usually

\(^1\)In this respect, the agency cost framework differs from the credit cycle model of Kiyotaki and Moore (1997), in which the collateral constraint always binds but default never occurs.
considered at odds with the empirical evidence.\textsuperscript{2} As a result, in our model, credit frictions generate both an effect of persistence (i.e., hump-shaped dynamics of investment and output, as in CF) and magnification in response to shocks. To clarify, while persistence is obtained relative to a nested model with frictionless credit markets, amplification is obtained relative to a model with agency costs but with time-invariant mean distribution of investment outcomes for entrepreneurs.

The issue of whether monetary policy should respond to asset prices has been recently the object of an intense debate, within both policy and academic circles.\textsuperscript{3} However, the theoretical literature linking asset prices, monetary policy and financial frictions in business cycle models has been scant. Bernanke and Gertler (2001) and Gilchrist and Leahy (2002) argue that there is negligible stabilization gain from including asset prices as independent arguments in monetary policy rules. However, Cecchetti et al. (2002) argue that this gain is likely to depend on the underlying source of shocks.\textsuperscript{4}

The common shortcoming of this literature is that it completely abstracts from strict welfare considerations. The metric adopted for the evaluation of the relative performance of policy rules is typically an output-inflation volatility frontier. This makes it hard to correctly rank alternative specifications for monetary policy, and to safely draw any conclusion about the desirability for monetary policy to react to asset price movements. It is this consideration that essentially motivates the present paper.

Our baseline economy will feature three types of distortions. Monopolistic competition in goods markets, sticky prices (both common to the recent stream of New Keynesian business cycle models) and endogenous agency costs. Agency costs, per se, have a twofold effect. In the long run, they produce an inefficiently low level of capital and investment, and hence output, since the economy suffers a deadweight loss associated to the monitoring activity of the lender. In the short-run, the presence of an endogenous premium on external finance distorts the dynamic allocation of capital and investment. In practice, agency costs act as an implicit - and time-varying - tax on investment.

\textsuperscript{2}See for instance Levin et al. (2004).
\textsuperscript{3}See Gilchrist and Leahy (2002) for a survey.
\textsuperscript{4}Iacoviello (2005) analyzes monetary policy in a model with credit cycles a la Kiyotaki and Moore (1997) and housing, and concludes that reacting to asset prices does not improve macroeconomic stability.
In this context, asset price movements are genuinely a symptom of financial distortions, for the relative price of capital goods is determined by the interaction of demand and supply in a lending market characterized by moral hazard. Hence reacting to asset prices may in principle bear a public finance motivation for monetary policy. The cyclicality of asset prices being strictly related to the presence of financial imperfections contrasts with a large business cycle literature in which asset prices merely reflect the shadow price of investment in the presence of physical adjustment costs on capital.

From a methodological viewpoint, our approach, as in Kollmann (2003a, 2003b) and Schmitt-Grohé and Uribe (2006), and unlike much of the so-called New Keynesian literature, allows to study policy rules in a dynamic economy that evolves around a steady state which remains inefficient. Importantly, in our context, the steady state will be distorted also by the presence of monitoring costs in credit markets. As emphasized by Kim et al. (2003) and Schmitt-Grohé and Uribe (2006), this strategy requires that an accurate evaluation of welfare be based on a higher order approximation of all the conditions that characterize the competitive equilibrium of the economy.

Our main finding is that monetary policy should react to increases in the asset price by lowering the nominal interest rate. The intuition lies in the fact that, as hinted above, the asset price (or relative price of capital goods) is akin to a tax which distorts the dynamic evolution of investment. In response to positive productivity shocks (which are the chief source of variability in our model) such tax evolves procyclically, thereby restraining investment.

However, responding (negatively) to the asset price in the interest rate rule improves welfare only for values of the inflation feedback coefficient which are low and in the range of those typically adopted in the specification of Taylor-type monetary policy rules (i.e., roughly between 1 and 2). In general, the marginal welfare gain of responding to the asset price flattens out when monetary policy responds more aggressively to inflation. In fact, a strong response to inflation always attains the highest level of welfare. On the other hand, though, the behavior of asset prices is only barely affected by the degree to which monetary policy stabilizes inflation. Thus, under a policy rule inducing strict stability of inflation, credit frictions generated by agency costs remain operative. This is
suggesstive of a trade-off faced by the monetary authority, which confronts two distortions (price stickiness and agency costs) with only one instrument, as well as of the fact that, in this context, the marginal gain of dampening the distortion induced by price stickiness quantitatively outweighs the benefit of neutralizing the distortion induced by agency costs and asset price fluctuations.

The remainder of the paper is as follows. Section 2 presents the model. Section 3 describes the calibration strategy. Section 4 discusses the implications of financial frictions for equilibrium dynamics. Section 5 analyzes the welfare effect of alternative interest rate rules. Section 6 concludes.

2 A Sticky-Price Economy with Agency Costs

The economy is populated by two types of firms, final goods and intermediate goods producers, and by two types of consumers, named households and entrepreneurs. We describe their respective problems below.

2.1 Final Good Producers

The aggregate final good is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by $i \in (0, 1)$, via the production function:

$$Y_t = \left( \int_{0}^{1} Y_t(i) \frac{di}{i} \right)^{\frac{\varepsilon}{1-\varepsilon}} \quad \varepsilon > 1 \quad (1)$$

Maximization of profits yields typical demand functions:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \quad (2)$$

for all $i$, where $P_t \equiv \left( \int_{0}^{1} P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.
2.2 Production of Intermediate Goods

There is a continuum of firms producing differentiated intermediate goods. The shares of these firms are owned by the households. Each firm $i \in [0, 1]$ assembles labor and capital (supplied by both households and entrepreneurs) to operate a constant-return-to-scale production function for the variety $i$ of the intermediate good:

$$Y_t(i) = A_t F(N_t(i), N^e_t(i), K_t(i))$$

where $A_t$ is a total factor productivity shifter common to all firms, $N_t(i)$ and $N^e_t(i)$ are firm $i$’s total demand for households’ and entrepreneurs’ labor respectively, and $K_t(i)$ is firm $i$’s total demand for capital (which includes capital owned by both types of agents).

We defer to a later section the analysis of the price setting problem of these firms.

2.3 Households

There is continuum of households with measure $1 - \eta$. Households derive income from renting labor and capital to intermediate goods firms, and from receiving profits of the same monopolistic firms. They use their income to purchase final consumption goods and capital goods from the entrepreneurs. Capital goods are originally final consumption goods transformed via the entrepreneurial activity to be described later.

The representative household’s sequence of budget constraints reads (in nominal terms):

$$P_tC^h_t + Q_t [K^h_{t+1} - (1 - \delta)K^h_t] + D_{t+1} \leq (1 + R_t)D_t + W^h_t N^h_t + Z_t K^h_t + \Gamma_t + T_t$$

where $K^h_t$ is capital held by the households, $Q_t$ is the price of capital goods, $Z_t$ is the nominal rental rate of capital, $\Gamma_t$ is profits earned on the shares of the monopolistic competitive firms, $W^h_t$ is the nominal wage, $N^h_t$ is labor hours, $R_t$ is the nominal interest rate on one-period risk-free bonds $D_t$, and $T_t$ are net lump-sum transfers.\(^6\)

\(^5\)An alternative ownership structure could be explored, in which the entrepreneurs directly own the shares of the intermediate goods firms that employ capital in production. In this case monopolistic profits would be part of capital income, as in Cook (2002).

\(^6\)Notice that we are implicitly assuming that each household can have access also to a full set of
Let \( z_t \equiv \frac{\tilde{z}_t}{P_t} \) be the real rental rate of capital and \( q_t \equiv \frac{\tilde{q}_t}{P_t} \) be the price of capital goods in units of consumption goods. Households maximize the utility program:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t^h, N_t^h) \right\}
\]

subject to (4). Efficiency conditions require:

\[
U_{c,t}^h = \beta(1 + R_t)E_t \left\{ U_{c,t+1}^h \frac{P_t}{P_{t+1}} \right\} 
\]

(5)

\[
U_{c,t}^h \frac{W_t^h}{P_t} = -U_{n,t}^h 
\]

(6)

\[
U_{c,t}^h = \beta E_t \left\{ U_{c,t+1}^h \left( \frac{q_{t+1}(1-\delta) + z_{t+1}}{q_t} \right) \right\} 
\]

(7)

where \( U_{c,t}^h \) is the marginal utility of households’ consumption and \( U_{n,t}^h \) is the marginal disutility of households’ labor.

Equations (5) and (6) are standard conditions on bonds investment and labor supply. Equation (7) is an intertemporal investment demand condition, with \( \frac{q_{t+1}(1-\delta) + z_{t+1}}{q_t} \) being the real rate of return on capital owned by the households. Notice that fluctuations in \( q_t \) are similar to a time-varying tax on investment. Equivalently, one can interpret this model as one with endogenous adjustment costs on capital (see CF).

### 2.4 Entrepreneurs

The second set of consumers in the economy (of measure \( \eta \)), are the *entrepreneurs*. Their activity consists in purchasing \( I_t \) units of the final consumption good and transform them into \( \omega_I I_t \) units of *capital* goods via an instantaneous risky technology. The random variable \( \omega_I \) has positive support with cumulative distribution \( \Phi(x, A_t) \equiv \text{prob}(\omega_I \leq x) \), with mean \( \omega_{m,t} \), variance \( \sigma_{\omega_I}^2 \), and density function \( \varphi(\omega_t, A_t) \). Notice that the average productivity of state contingent securities. This implies that each household will face the same intertemporal budget constraint, and therefore the same level of expenditure. However, the specification of those assets would be redundant.
each entrepreneur is *time-varying*. We assume that each entrepreneur is *on average* more productive when total factor productivity $A_t$ increases. In particular we specify:

$$\omega_{m,t} = V(A_t)$$

where the function $V(\cdot)$ is increasing and convex. Hence, in our context, aggregate productivity shocks will have the additional effect of shifting the distribution of the risky invest outcome $\omega_t$. As will appear more clear below, this feature will be key in driving the cyclical properties of the cost of external finance.

At the beginning of time $t$ an entrepreneur with nominal net worth $NW_t$ seeks for a loan (in excess of internal funds) $P_t I_t - NW_t$. The nominal lending rate on the loan is $R^L_t$. Default occurs when the return from the investment activity falls short of the amount that needs to be repaid. Hence the *default space* is implicitly defined as that range for $\omega_t$ such that:

$$\omega_t < \overline{\omega}_t \equiv \frac{(1 + R^L_t)(P_t I_t - NW_t)}{Q_t I_t}$$

where $\overline{\omega}_t$ is the cut-off value (determined endogenously in general equilibrium) for the idiosyncratic productivity shock. When default occurs, the entrepreneur cannot repay the amount $(1 + R^L_t)(P_t I_t - NW_t)$, and can only repay what is available, namely $\omega_t I_t Q_t$. To avoid misreporting by the defaulting entrepreneur, the lender verifies this, but needs to expend a nominal amount $\mu I_t Q_t$ for monitoring, with $\mu \in [0, 1]$.

At this stage, we can compute the average *nominal income of the entrepreneur* at each point in time:

$$\int_{\overline{\omega}_t}^{\infty} [\omega_t Q_t I_t - (1 + R^L_t)(P_t I_t - NW_t)] \ d\Phi(\omega_t, A_t)$$

Using (9) we obtain:

$$Q_t I_t \left\{ \int_{\overline{\omega}_t}^{\infty} (\omega_t - \overline{\omega}_t) \ d\Phi(\omega_t, A_t) \right\}$$

$$= Q_t I_t \left\{ \int_{\overline{\omega}_t}^{\infty} \omega_t d\Phi(\omega_t, A_t) - \overline{\omega}_t(1 - \Phi(\overline{\omega}_t, A_t)) \right\}$$

$$= Q_t I_t f(\overline{\omega}_t, A_t)$$
where \( f(\bar{\omega}_t, A_t) \) can be interpreted as the fraction of the expected investment outcome that accrues to the entrepreneur.

Furthermore, the average nominal income of the lender (net of monitoring costs) can be computed as:

\[
Q_t I_t \left\{ \int_0^{\bar{\omega}_t} \omega_t \ d\Phi(\omega_t, A_t) - \mu \Phi(\bar{\omega}_t, A_t) \right\} + (1 - \Phi(\bar{\omega}_t, A_t)) (1 + R_t^L)(P_t I_t - NW_t)
\]

\[
= Q_t I_t \left\{ \int_0^{\bar{\omega}_t} \omega_t \ d\Phi(\omega_t, A_t) - \mu \Phi(\bar{\omega}_t, A_t) + \bar{\omega}_t(1 - \Phi(\bar{\omega}_t, A_t)) \right\}
\]

\[
\equiv Q_t I_t g(\bar{\omega}_t, A_t) \tag{11}
\]

where \( g(\bar{\omega}_t, A_t) \) is the fraction of the expected investment outcome that accrues to the lender.

Notice that, according to the definitions above, we have

\[
f(\bar{\omega}_t, A_t) + g(\bar{\omega}_t, A_t) = 1 - \mu \Phi(\bar{\omega}_t, A_t) \tag{12}
\]

Hence a fraction of the investment income is lost in the monitoring activity. This fraction depends on the parameter \( \mu \) and on the frequency of defaulting entrepreneurs \( \Phi(\bar{\omega}_t, A_t) \).

### 2.5 Financial Contract

The financial contract assumes the form of an optimal risky debt contract à la Townsend (1979) and Gale and Hellwig (1985). The contract specifies a pair \((I_t, \bar{\omega}_t)\) that maximizes entrepreneur’s expected nominal income

\[
\max Q_t I_t f(\bar{\omega}_t, A_t) \tag{13}
\]

subject to the lender’s participation constraint:

\[
Q_t I_t \ g(\bar{\omega}_t, A_t) \geq (P_t I_t - NW_t) \tag{14}
\]

Efficiency requires that (14) holds with equality. Due to the lenders being perfectly competitive, \( P_t \) and \( Q_t \) are taken as given in the maximization problem. It is easy to show
that the solution to the problem above implies the following two first order conditions (expressed in real terms, i.e., units of final consumption goods):

\[ q_t f(\omega_t, A_t) = \frac{f'(\omega_t, A_t)}{f'(\omega_t, A_t)} \left[ q_t g(\omega_t, A_t) - 1 \right] \]  

\[ I_t = \frac{nw_t}{1 - q_t g(\omega_t, A_t)} \]  

where \( nw_t \equiv \frac{NW_t}{P_t} \). Notice that, for given \( q_t \), (15) pins down the threshold value \( \omega_t \) uniquely. Given \( \omega_t \), (16) is then a linear relation between investment and net worth, where the factor of proportionality \( \frac{1}{1-q_t g(\omega_t, A_t)} \) is independent of any idiosyncratic element. Hence (16) allows immediate aggregation.\(^7\)

Equation (16) is a crucial condition in the model. It relates the supply of capital to the relative price of capital goods \( q_t \) and to the (aggregate) financial conditions proxied by the level of net worth \( nw_t \). In the space \((q_t, I_t)\) capital supply is positively sloped, and movements in net worth shift the investment supply schedule, in turn affecting the relative price of capital goods. This is the feature that genuinely signals the presence of monitoring imperfections in the lender-borrower relationship. Hence, as already hinted above, the relative price of capital is endogenous for reasons directly related to credit market imperfections, rather than to the eventual presence of adjustment costs on capital.

By combining (16) with (9) we can derive an expression for the real gross lending rate \((1 + \frac{R^L_t}{P_t})\) as a function of the cutoff \( \omega_t \). Since the loan contract is intra-period, the lending rate measures also the cost of credit in excess of the safe rate of return (in turn equal to 1), or \textit{premium} on external finance:

\[ \psi_t \equiv \frac{(1 + R^L_t)}{P_t} - 1 = \frac{\omega_t}{g(\omega_t, A_t)} - 1 \]  

Notice that if the distribution of investment outcomes \( \Phi(\cdot) \) has a constant mean (e.g., \( \omega_{m,t} = 1 \), as usually assumed in the agency cost framework of CF), it is easy to show that the finance premium \( \psi_t \) is a monotonic increasing function only of the threshold value

\(^7\)In turn, this is a consequence of both the capital production technology and the monitoring technology being constant return to scale.
However, in the case in which the mean $\omega_{m,t}$ varies with aggregate productivity, the lender’s income share $g(\bar{\omega}_t, A_t)$ also depends on aggregate productivity. The behavior of the income share $g(\cdot)$ relative to the threshold value $\bar{\omega}_t$ is critical in driving the cyclical properties of the finance premium.

### 2.6 Entrepreneurs in General Equilibrium

Entrepreneurs inelastically supply one unit of labor (hence their total labor supply is $\eta$) and are risk-neutral. They maximize the following utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} (\beta \gamma)^t C_t^e \right\}, \quad 0 < \gamma < 1$$

(18)

Notice that the entrepreneurs discount utility at a higher rate, $\beta \gamma$, than the households (i.e., they are more impatient). This assumption insures that the entrepreneurs never hold enough wealth to overcome the financing constraints. Entrepreneurs earn income also from renting their capital stock $K_t^e$ to intermediate goods firms. Hence their nominal net worth at the beginning of time $t$ can be written:

$$NW_t = W_t^e + [Z_t + Q_t(1 - \delta)] K_t^e$$

Notice that risk-neutrality induces entrepreneurs to invest all their net worth in the investment technology.

The problem of a non-defaulting entrepreneur is to maximize (18) subject to the sequence of budget constraints:

$$Q_t K_{t+1}^e + P_t C_t^e \leq Q_t I_t f(\bar{\omega}_t, A_t)$$

(19)

The left-hand side of (19) denotes uses of funds (expenditure in new capital goods and consumption goods), whereas the right-hand side is nominal expected income, as from (10).

This intertemporal problem must satisfy the Euler equation:
\[ 1 = \beta \gamma E_t \left\{ \left( \frac{z_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right) \frac{q_{t+1} f(A_{t+1})}{1 - q_{t+1} g(A_{t+1})} \right\} \]  

(20)

The left hand side of (20) is the marginal utility of entrepreneurs’ consumption. The right hand side is the expected discounted rate of return for an entrepreneur who is not bankrupt in period $t$. The latter term has two components. The first is the safe rate of return on capital (i.e., the one gained by the households). The second component is the return on internal funds, which can be shown to strictly exceed unity, i.e., \[ \frac{q_{t+1} f(A_{t+1})}{1 - q_{t+1} g(A_{t+1})} > 1 \] for all $t$ (see CF).

### 2.7 Pricing of Intermediate Goods

Each intermediate good firm $i$ has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to \[ \frac{\vartheta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2, \] where $\pi$ is the steady state inflation rate and where the parameter $\vartheta$ measures the degree of nominal price rigidity. The higher $\vartheta$ the more sluggish is the adjustment of nominal prices. In the particular case of $\vartheta = 0$, prices are flexible.

Each monopolistic firm chooses the sequence \{ $K_t(i)$, $N_t^h(i)$, $P_t(i)$ \}, taking nominal wage rates $W_t^h$ and $W_t^e$ as given, in order to maximize expected discounted nominal profits:

\[ E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ P_t(i) Y_t(i) - (W_t^h N_t^h(i) + W_t^e \eta + Z_t K_t(i)) - \frac{\vartheta}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 P_t \right] \right\} \]  

subject to the constraint $A_t F_t(\bullet) \leq Y_t(i)$ and to (2), where $\Lambda_{0,t}$ is the household’s stochastic discount factor.

Let’s denote by \{ $mc_t$ \} the sequence of Lagrange multipliers on the above demand constraint, and by $\tilde{p}_t \equiv \frac{P_t(i)}{P_t}$ the relative price of variety $i$. The first order conditions of

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8Recall that, in our framework, $\pi > 0$. An alternative formulation may feature adjustment costs penalizing the deviation of the rate of change of prices from the past inflation rate $\pi_{t-1}$. However, Schmitt-Grohé and Uribe (2004b) show that the latter formulation (similar to the one employed in Christiano et al. (2005)) biases the optimal policy towards generating an inflation volatility significantly different from zero.
the above problem read:

\[
\frac{W_t^h}{P_t(i)} = mc_t A_t F_{n,t}^h
\]  
(22)

\[
\frac{W_t^e}{P_t(i)} = mc_t A_t F_{n,t}^e
\]  
(23)

\[
\frac{Z_t}{P_t(i)} = mc_t A_t F_{k,t}
\]  
(24)

\[
\Lambda_{0,t} Y_t \left\{ (1 - \varepsilon) \tilde{P}_t^{\varepsilon} + \varepsilon \frac{W_t}{A_t P_t} \tilde{P}_t^{\varepsilon} \right\} = \Lambda_{0,t} P_t \vartheta \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{1}{P_{t-1}(i)}
\]  
(25)

where \( F_{n,t}^h \) is the marginal product of households’ labor, \( F_{n,t}^e \) is the marginal product of entrepreneurs’ labor, and \( \pi_t \equiv \frac{P_t}{P_{t-1}} \) is the gross aggregate inflation rate. Notice that all firms employ an identical capital/labor ratio in equilibrium. The Lagrange multiplier \( mc_t \) plays the role of the real marginal cost of production. In a symmetric equilibrium it must hold that \( \vartheta = 1 \). This allows to rewrite (25) in the following form:

\[
U_{c,t}^h (\pi_t - \pi) \pi_t = \beta E_t \left\{ U_{c,t+1}^h (\pi_{t+1} - \pi) \pi_{t+1} \right\} + U_{c,t}^h A_t F_t(\bullet) \frac{\varepsilon}{\vartheta} \left( mc_t - \frac{\varepsilon - 1}{\varepsilon} \right)
\]  
(26)

The above equation has the form of a (non-linear) forward-looking New-Keynesian Phillips curve, in which deviations of the real marginal cost from its desired steady state value are the driving force of inflation.\(^9\)

### 2.7.1 Market Clearing

Equilibrium in the final good market requires that the production of the final good be allocated to total private consumption by households and entrepreneurs, investment, public spending, and to resource costs that originate from the adjustment of prices:

\[^{9}\text{Galí and Gertler (1999), Woodford (2003).}\]
\[ Y_t = (1 - \eta)C_t^h + \eta (C_t^e + I_t) + G_t + \frac{\eta}{2} (\pi_t - \pi)^2 \]  

(27)

In the above equation, \( G_t \) is government consumption of the final good which evolves exogenously and is assumed to be financed by means of lump sum taxes.

Using (12), total capital accumulation can be written

\[ K_{t+1} = (1 - \delta)K_t + \eta I_t [1 - \mu \Phi (\tilde{\omega}_t, \tilde{A}_t)] \]  

(28)

where \( K_t \equiv (1 - \eta)K_t^h + \eta K_t^e \). Notice that the presence of monitoring costs endogenously distorts aggregate capital accumulation.

Finally, equilibrium in the labor market requires

\[ \int_0^1 N_t(i)di \equiv N_t = (1 - \eta)N_t^h \]  

(29)

\[ N_t^e = \eta \]  

(30)

In the Appendix we describe the complete form of the recursive competitive equilibrium.

### 2.8 Monetary Policy

We assume that monetary policy is conducted by means of an interest rate reaction function, constrained to be linear in the logs of the relevant arguments:

\[
\ln \left( \frac{1 + R_t}{1 + \hat{R}} \right) = (1 - \phi_r) \left( \phi_\pi \ln \left( \frac{\pi_t}{\pi} \right) + \phi_q \ln \left( q_t \right) + \phi_y \ln \left( \frac{Y_t}{Y} \right) \right) + \phi_r \ln \left( \frac{1 + R_{t-1}}{1 + \hat{R}} \right)
\]

(31)

with \( \{\phi_\pi\} \in (1, \infty], \{\phi_q\} \in (-\infty, \infty], \{\phi_y\} \in (0, \infty] \) and \( \{\phi_r\} \in [0, 1] \), and where \( \pi, Y, R \) correspond to deterministic steady state values. A few observations on the specification of (31) are in order. First, as in Schmitt-Grohè and Uribe (2003), we model monetary
policy in terms of an implementable rule, whereby the central bank sets the short-run nominal interest rate in response to observable variables only. This assumption explains why - as an argument of the rule - we are including output in deviations from steady state rather than a measure of the output gap. The latter specification would in fact require, in accordance with some practice in the literature, a computation of either the efficient or the natural level of output (i.e., the level of output under flexible prices and/or frictionless credit markets). In both cases, such computation would presume that the central bank be able to observe the stochastic joint distribution of the shocks. Second, this general specification allows for a reaction of the monetary policy instrument to deviations of the relative price of capital goods $q_t$ from 1. Incidentally, the latter corresponds also to the hypothetical efficient value of $q_t$, i.e., its value in the absence of credit frictions (regardless of whether nominal rigidities are built into the model). Hence, in (31), monetary policy is also implicitly reacting to an asset price gap. This choice addresses the issue raised by Gilchrist and Saito (2006) according to which the implications of responding to asset prices may vary depending on whether the monetary authority is reacting to an asset price level as opposed to an asset price gap. In our case, responding to an asset price gap does not require any knowledge of the behavior of the efficient/natural level of the asset price. Any movement in $q_t$, in fact, can be interpreted as deviations from its efficient level.

Our approach consists in finding the policy specification $\{\phi_n, \phi_y, \phi_q, \phi_r\}$ that maximizes private agents’ welfare (see more below). While we will allow $\phi_n$ to vary only on a range of values exceeding 1 (to insure equilibrium determinacy), $\phi_q$ will potentially span both negative and positive values. We will return on this point below.

3 Calibration and Solution Strategy

The time unit is a quarter. We employ a period utility function for households $U(C_h^t, N_h^t) = \log(C_h^t) + \nu \log(1 - N_h^t)$, with parameter $\nu$ chosen in such a way to generate a steady-state level of households’ employment $N_h^0 = 0.3$. We assume that the annualized steady-state inflation rate is 4%, close to the historical average for the U.S.. We set the discount factor $\beta = 0.99$, so that the annual real interest rate is equal to 4%. We assume
that the production function for intermediate goods has the Cobb-Douglas form \( F(\cdot) = K_t^n (N_t)^{1-\alpha-\chi} \), with \( \alpha = 0.3 \) and \( \chi \) close to zero (see CF). The quarterly aggregate capital depreciation rate \( \delta \) is 0.025, the elasticity of substitution between varieties is 8. In line with the evidence reported in CF, we set \( \mu \) equal to 0.25. Notice that, in the steady state, the presence of non-zero monitoring costs \( (\mu > 0) \) implies that relative price of capital exceeds one \( (q > 1) \).

We calibrate the steady state to imply an annual (average) external finance premium \( \psi = 1.02 \) (two hundred basis points), and to generate an average bankruptcy rate of three percent \( (\Phi(\overline{\omega}, 1) = 0.03) \). We assume that \( \omega_t \) has a uniform distribution, with positive support and mean \( \omega_{m,t} = V(A_t) = A_t^{1+\sigma_{\omega}} \), with \( \sigma_{\omega} = 2 \).

Total factor productivity is assumed to evolve as:

\[
A_t = A_{t-1}^\alpha \exp(\varepsilon_t^a)
\]

where the steady-state value \( A \) is normalized to unity (which in turn implies \( \omega_m = 1 \)) and \( \varepsilon_t^a \) is an iid shock with standard deviation \( \sigma_a \). In line with the real business cycle literature (King and Rebelo, 1999), we set \( \rho_a = 0.95 \) and \( \sigma_a = 0.0056 \). Log-government consumption is assumed to evolve according to the following process:

\[
\ln \left( \frac{G_t}{G} \right) = \rho_g \ln \left( \frac{G_{t-1}}{G} \right) + \varepsilon_t^g
\]

where \( G \) is the steady-state share of government consumption (set in such a way that \( G = 0.25 \)) and \( \varepsilon_t^g \) is an iid shock with standard deviation \( \sigma_g \). We follow the empirical evidence for the U.S. in Perotti (2004) and set \( \sigma_g = 0.008 \) and \( \rho_g = 0.9 \).

In order to parameterize the degree of price stickiness \( \vartheta \), we observe that, by log-linearizing equation (26) we can obtain an elasticity of inflation to real marginal cost (normalized by the steady-state level of output)\(^{10}\) that takes the form \( \frac{\varepsilon-1}{\vartheta} \). This allows a direct comparison with empirical studies on the New Keynesian Phillips curve such as Galí and Gertler (1999) and Sbordone (2002) using a Calvo-Yun approach. In those

\(^{10}\)To produce a slope coefficient directly comparable to the empirical literature on the New Keynesian Phillips curve this elasticity needs to be normalized by the level of output when the price adjustment cost factor is not explicitly proportional to output, as assumed here.
studies, the slope coefficient of the log-linear Phillips curve can be expressed as \( \frac{(1-\hat{\vartheta})(1-\beta \hat{\vartheta})}{\hat{\vartheta}} \), where \( \hat{\vartheta} \) is the probability of not resetting the price in any given period in the Calvo-Yun model. For any given values of \( \varepsilon \), which entails a choice on the steady-state level of the markup, we can thus build a mapping between the frequency of price adjustment in the Calvo-Yun model \( \frac{1}{1-\hat{\vartheta}} \) and the degree of price stickiness \( \vartheta \) in the Rotemberg setup. The recent New Keynesian literature has usually considered a frequency of price adjustment of four quarters as realistic. Recently, Bils and Klenow (2004) have argued that the observed frequency of price adjustment in the U.S. is higher, and in the order of two quarters. As a benchmark, we parameterize \( \frac{1}{1-\hat{\vartheta}} = 4 \), which implies \( \hat{\vartheta} = 0.75 \). Given \( \varepsilon = 8 \), the resulting stickiness parameter satisfies \( \vartheta = \frac{Y \hat{\vartheta} (\varepsilon-1)}{(1-\hat{\vartheta})(1-\beta \hat{\vartheta})} \approx 20 \), where \( Y \) is steady-state output.

We solve the model by computing a second order approximation of the policy functions around the non-stochastic steady state (with positive average inflation, market power and monitoring costs). In the Appendix, we provide more details on the solution strategy.

4 Equilibrium Dynamics

In this section we study the dynamic behavior of the model. We first illustrate the role that financial frictions play in shaping the equilibrium response to shocks in contrast to a model in which the same frictions are absent. Next, we compare the standard model with agency costs (in practice, the CF model augmented with sticky prices) with our model in which the distribution of investment opportunities is assumed to be endogenous to aggregate productivity.

4.1 The Role of Financial Frictions

Figure 1 depicts impulse responses of selected variables to a positive total factor productivity shock in two cases: (i) a baseline sticky price model with capital and frictionless credit markets; (ii) the agency cost model of CF (i.e., with constant mean distribution of investment outcomes) augmented with sticky prices. The latter model nests the former for values of parameter \( \mu \) close to zero. All numbers are in percent deviations from steady-state values. This illustrative exercise is conducted under the temporary assumption that
monetary policy is specified via a simplified Taylor rule \( \ln \left( \frac{1+R_t}{1+\pi} \right) = 1.5 \ln \left( \frac{\pi_t}{\pi} \right) \).

The initial effect of the rise in productivity is to shift investment demand outward, and therefore to produce, for any given level of net worth, a rise in the price of capital. The critical element to notice is that in the agency cost model the rise in investment is paralleled by a slow response in net worth. In fact, in the short-run, net worth is mostly composed of entrepreneurial capital, and its dynamic is sluggish. The result is an initial rise in borrowing needs (i.e., \( I_t \) rises relatively more than \( nw_t \)) and a consequent rise in the marginal cost of investment, an effect that tends to dampen the impact of the shock. In turn, this generates a rise in the default threshold \( \zeta_t \). Since in the baseline agency-cost model the finance premium \( \psi_t \) is a strictly increasing function of \( \zeta_t \), the result is a rise in the external finance premium. However, over time, net worth accumulates, and its response shifts the investment supply schedule outward and to such an extent that the asset price starts to revert downward. It is this delayed response of net worth that induces a subsequent (supply-driven) boost to investment, thereby generating the observed hump-shaped dynamics in both investment and output. Noticeably, this feature of endogenous serial correlation is absent in a standard sticky-price model (with frictionless credit markets).

Interestingly, the reactivity of net worth is inversely proportional to the degree of financial frictions summarized by \( \mu \). Hence here, as in the baseline CF framework, financial frictions are synonymous with persistence. Even more subtly, and as emphasized in Carlstrom and Fuerst (1998), magnification and persistence seem to be related by a trade-off.

4.2 Distribution of Investment Outcomes and Countercyclical Finance Premium

An often-cited limitation of the CF framework is that it generates a procyclical behavior of the external finance premium, which in the model is directly proportional to the behavior of the asset price. Levin et al. (2004) report micro-based evidence showing that, during the boom of the late 1990s in the U.S., credit spreads on corporate debt were historically low. Gomes et al. (2003) show that, in the data, alternative measures of the premium on
external funds display a negative correlation with total factor productivity.\footnote{This evidence is robust to alternative measures of the finance premium: (i) spread between Baa and Aaa corporate bonds; (ii) spread between prime bank loan rate and three-month commercial paper; (iii) spread between more and less financially constrained firms (as in Lamont et al. (2001)); (iv) default likelihood indicators, as in Vassalou and Xing (2002).}

Figure 2 compares responses to a rise in total factor productivity in two versions of the agency cost model: (i) with constant mean distribution of investment outcomes: $\omega_{m,t} = \omega_m = 1$ for all $t$ (solid line); (ii) with endogenous mean distribution of investment outcomes: $\omega_{m,t} = V(A_t)$ (dashed line).

The rise in aggregate productivity once again produces an outward shift of investment demand, and therefore a rise in investment and in the price of capital. The crucial point is that in the model with endogenous $\omega_{m,t}$ a rise in aggregate productivity makes also each entrepreneur on average more productive. This effect is akin to an instantaneous shift of the investment supply schedule. Hence a rise in aggregate productivity produces now a simultaneous shift in both the investment demand and in the investment supply schedules. In contrast, in the baseline model with constant $\omega_m$ (as in CF), the shift in investment supply is delayed due to the slow accumulation of net worth.

Notice that, in the model with endogenous $\omega_{m,t}$, the price of capital is still procyclical in response to productivity shocks, a feature that appears empirically realistic. This happens because the contemporaneous shift in the investment supply schedule is initially not sufficient to produce an equilibrium fall in the price of capital. Hence, on impact, we still observe a positive excess demand of investment. However, because of the instantaneous supply effect, the price of capital rises less than in the baseline model with constant $\omega_{m,t}$, and even starts to fall below baseline after two quarters. At the same time, the share of income going to the lender $g(\pi_t, A_t)$ (which depends positively on aggregate productivity) rises proportionally more than the threshold value $\pi_t$, thereby producing a fall in the external finance premium (see equation (17)).

The fall in the finance premium and the more muted rise in the price of capital both induce investment and output to rise more on impact in the model with endogenous $\omega_{m,t}$ (relative to the model with constant $\omega_{m,t}$), generating a magnification effect. At the same time, though, the sluggish response of net worth still produces an effect of persistence.
in investment and output (although diminished relative to the case with constant \( \omega_{m,t} \)). Hence, our modified model is able to simultaneously generate both an effect of persistence and an effect of magnification. To clarify, the effect of persistence is to be understood relative to a baseline sticky-price model with frictionless credit markets, whereas the effect of magnification is to be understood relative to a model with credit frictions but with constant \( \omega_{m,t} \).

5 Welfare Evaluation

The critical feature of our analysis consists in the assessment of alternative interest rate rules based on social welfare evaluation. Some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first-order approximation methods to compare the relative welfare associated to each monetary policy arrangement. In fact, in an economy like ours, in which distortions exert an effect both in the short-run and in the steady state, stochastic volatility affects both first and second moments of those variables that are critical for welfare. Since in a first-order approximation of the model’s solution the expected value of a variable coincides with its non-stochastic steady state, the effect of volatility on the mean value of any endogenous variable is by construction neglected. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions.\(^{12}\)

This last observation also suggests that our welfare metric needs to be correctly chosen. In particular, one needs to focus on the conditional expected discounted utility of the representative agent. This is necessary to account for the transitional effects from the deterministic steady state (which defines the initial condition of the model) to the different stochastic steady states implied by each alternative policy rule, and conditional on the same distribution of the shocks.

Finally, it is important to recall that our framework features heterogeneity of consumers. However, entrepreneurs are risk-neutral agents. This implies that their mean

\(^{12}\)See Kim and Kim (2003) for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies. See Kim et al. (2003) and Schmitt-Grohé and Uribe (2004a) for a more general discussion.
level of consumption is unaffected by the underlying sources of stochastic volatility. Hence, alternative interest rate rules not only will imply the same (deterministic) steady-state level of all variables, but they will also imply the same stochastic mean consumption for entrepreneurs. This implies that a correct measure of conditional welfare needs simply to be amended by adding the mean level of entrepreneurial consumption in the deterministic steady state. However, since this will be invariant to the alternative specifications of the monetary policy rule (31), we will measure conditional welfare simply by including households’ utility:

\[ W_{0,t} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t^h, N_t^h) \right\} \]  
(32)

or in recursive form:

\[ W_{0,t} = U(C_t^h, N_t^h) + \beta E_t \{ W_{0,t+1} \} \]  
(33)

We refer to the Appendix for more details on the correct computation of the conditional expected value \( W_{0,t} \).

5.1 Comparing Simple Rules

We begin by comparing the welfare performance of alternative (ad hoc) specifications of the monetary policy rule. The rules are the following: (i) Strict Inflation Stabilization (IT); (ii) Taylor rule with inflation only (TRI); (iii) Standard Taylor rule (TR, including output) (iv) TRI with negative response to the asset price; (v) TRI with positive response to the asset price; (vi) TR with negative response to the asset price. Table 1 reports the specific parameter configuration that characterizes each rule\(^{13}\).

We compare the rules both in terms of conditional welfare \( W_{0,t} \) and in terms of a compensating measure given by the fraction \( \Omega \) of households’ consumption that would be needed to equate conditional welfare \( W_{0,t} \) under a generic interest rate policy to the level of welfare \( W_{0,t}^* \) implied by the optimal rule. Hence \( \Omega \) should satisfy the following equation:

\(^{13}\)We have also experimented with rules featuring a reaction to lagged as opposed to current inflation. Results were qualitatively unaltered.
\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1 + \Omega)C_t^p, N_t) \right\} = \mathcal{W}_{0,t}
\]

Under our specification of utility one can solve for \(\Omega\) and obtain:

\[
\Omega = \exp \left\{ (\mathcal{W}_{0,t}^* - \mathcal{W}_{0,t}) (1 - \beta) \right\} - 1
\]

(34)

The results are displayed in Table 2. All rules are evaluated with and without interest rate smoothing (\(\phi_r = 0\) and \(\phi_r = 0.9\) respectively). A number of implications are worth emphasizing. First, strict stabilization of inflation always attains the highest level of welfare. However, the welfare loss of a simple Taylor rule with a mild inflation coefficient is very small. Second, responding to output in a Taylor rule is strongly welfare detrimental. Third, responding to asset prices improves welfare (relative to a Taylor rule) only if this response is negative. Fourth, interest rate smoothing improves the performance of all rules.

5.2 Responding to Inflation vs. Asset Prices

Next we search for a generalization of the results outlined above. Figures 3 and 4 depict the effects on the conditional welfare surface of varying both the inflation coefficient \(\phi_\pi\) and the asset price coefficient \(\phi_q\) in the monetary policy rule (31), respectively without \((\phi_r = 0)\) and with \((\phi_r = 0.9)\) interest rate smoothing. All results shown here correspond to cases in which the coefficient \(\phi_y\) is set equal to zero. As hinted above, rules featuring a positive response to (deviations of ) output (from steady state) are invariably welfare inferior to rules in which the same response is zero. This confirms similar results obtained in Schmitt-Grohé and Uribe (2003). In addition, in the light of the recent policy debate, we are particularly interested in evaluating the relative welfare performance of asset price versus goods price stabilization.

The main result that emerges in both pictures is that, when the inflation coefficient \(\phi_\pi\) is sufficiently low and in the order of magnitude typically assigned in Taylor-type rules (i.e., between 1 and 2), there exists a positive effect on welfare of responding to asset prices. For sufficiently low values of \(\phi_\pi\), in fact, the welfare surface is concave in
the asset price coefficient $\phi_q$, with a maximum reached for values slightly below zero. A more detailed exposition of this result is in Figure 5, where we plot a section of the welfare surface. This corresponds to the effect of varying the asset price coefficient $\phi_q$ for a given value $\phi_\pi = 1.5$ (and still maintaining $\phi_y = 0$), which is the value for the inflation coefficient usually chosen in the specification of a standard Taylor rule. Thus our results suggest that monetary policy should respond negatively (i.e., reduce the nominal interest rate) to increases in the asset price.

The intuition for why, at sufficiently low levels of $\phi_\pi$, monetary policy should respond negatively to asset price rises is as follows. Consider a scenario with productivity shocks, which are the main source of variability in our context. Recall that, in the model, agency costs induce endogenous movements in the price of capital goods relative to consumption goods. Hence, when productivity increases, a higher relative price of capital goods amounts to a procyclical tax. As a result, investment dynamic is distorted below the level it would attain in an environment with frictionless credit markets. In this sense, agency costs restrict investment in a way similar to adjustment costs on capital (see also CF on this point). Thus the monetary authority wishes to lower the interest rate (nominal and, in turn, real) in order to limit the tax distortion that constraints the expansion in investment (with the opposite prescription holding true in the case of negative productivity shocks).

However, it is clear that the benefits of responding to asset price movements tend to vanish when monetary policy turns strongly anti-inflationary. As we see in Figure 3 and 4, at higher levels of $\phi_\pi$, the welfare surface becomes completely flat in $\phi_q$, and the scope for responding to asset prices tends to disappear. In general, the optimal rule prescribes a strong anti-inflationary stance.

5.2.1 Does Stabilizing Inflation Eliminate the Financial Distortions?

The above result bears the question of whether a policy of strict inflation stabilization, which emerges as welfare maximizing in our context, also manages to dampen or even neutralize the credit-market distortions, which manifest themselves in fluctuations of the asset price and/or the finance premium. In Figure 6 and 7 we depict impulse responses
of inflation, output, asset price and finance premium to a productivity shock and to a
government spending shock respectively. We compare the outcome under two rules: (i) a
simple Taylor-type rule, featuring only a mild response to inflation $\phi_\pi = 1.5$ (solid line);
(ii) a strict inflation stabilization rule, which aims at fully stabilizing inflation ($\pi_t = 0$ for
all $t$, dashed line). The crucial point to notice is that, regardless of the underlying shock,
the behavior of both the asset price and the finance premium is only barely affected by
the degree to which monetary policy stabilizes inflation. Thus, under the welfare-optimal
rule which prescribes stable inflation, credit frictions generated by agency costs are clearly
not neutralized.

For one, this result is not surprising. In fact, the monetary authority faces two (cycli-
cal) distortions (i.e., price stickiness and agency costs) and one policy instrument, and
hence cannot simultaneously neutralize both distortions. On the other hand, it suggests
that, in this context, the marginal benefit of neutralizing the price stickiness distortion
largely outweighs the marginal benefit of neutralizing the credit market distortion on a
quantitative ground.

6 Conclusions

We have analyzed optimal interest rate rules in an economy featuring sticky prices and
endogenous agency costs. In our context, and due to the presence of a countercyclical
premium on external finance, credit frictions generate both an effect of persistence (rel-
ative to a model with frictionless credit markets) and an effect of magnification (relative
to a version of our model featuring constant mean investment opportunities for entrepre-
neurs). We have focused our attention on the desirability of including asset prices, along
with inflation, as a separate argument in the policy rule. For sufficiently low values of
the inflation feedback coefficient in the policy rule, and in the range of those typically
assigned in standard Taylor-type rules, responding to a rise in the asset price improves
welfare. In particular, the prescription for the monetary authority is to lower the interest
rate when the asset price rises. This result depends on the fact that, in our context, an

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14See also Dupor (2003) on this point, although in a context in which financial frictions are not explicitly
modelled.
increase in the asset price is isomorphic to an increase in the price of capital relative to consumption goods, and hence acts as an implicit tax on investment. However, when monetary policy focuses on strongly stabilizing inflation, reacting to the asset price plays no significant and independent welfare role in the specification of the interest rate rule. In general, rules with a strong anti-inflationary stance always attain the highest level of welfare.

There are at least two directions that would be worth exploring in future research. First, it would be interesting to significantly expand the stochastic rank of the economy, by including a larger array of shocks as in the studies of Christiano et al. (2005) and Smets and Wouters (2004). Our framework has the advantage of possibly featuring a new set of financial shocks that may be important in explaining business cycle fluctuations. Second, recall that the financial contract between the lender and the borrower is intraperiod. If the contract were intertemporal, movements in inflation would affect the expected real interest rate on the loan, and in turn entrepreneurs’ expected net worth. Inflation variability would in this case increase the value of nominal net worth, thereby reducing the ex-post value of real debt and the cost of the loan. It would be interesting to explore how this motive for inflation variability may conflict with the motive for inflation stabilization rooted in the price stickiness distortion.
The Equilibrium

For any given policy instrument \( \{ R_t \} \) and stochastic processes \( \{ A_t, G_t \} \), a recursive (imperfectly) competitive equilibrium is a sequence of allocations for \( C^h_t, \pi_t, mc_t, N^h_t, q_t, I_t, z_t, C^e_t, \omega_t, nw_t, K_{t+1}, K^e_{t+1} \) which solves the following system of equations:

\[
U^h_t = \beta(1 + R_t) E_t \left\{ U^h_{c,t+1} \frac{P_t}{P_{t+1}} \right\} \tag{35}
\]

\[
U^h_t = \beta E_t \left\{ U^h_{c,t+1} \left( q_{t+1}(1 - \delta) + z_{t+1} \right) \right\} \tag{36}
\]

\[
1 = \beta \gamma E_t \left\{ \left( \frac{z_{t+1} + (1 - \delta)q_{t+1}}{q_t} \right) \frac{q_{t+1} f(\omega_{t+1}, A_{t+1})}{1 - q_{t+1} g(\omega_{t+1}, A_{t+1})} \right\} \tag{37}
\]

\[
nw_t = mc_t A_t F^e_{n,t} + [z_t + q_t(1 - \delta)] K^e_t \tag{38}
\]

\[
K_{t+1} = (1 - \delta)K_t + \eta I_t [1 - \mu \Phi (\omega_t, A_t)] \tag{39}
\]

\[
I_t = \frac{nw_t}{1 - q_t g(\omega_t, A_t)} \tag{40}
\]

\[
K^e_{t+1} = nw_t \left( \frac{f(\omega_t, A_t)}{1 - q_t g(\omega_t, A_t)} \right) - \frac{C^e_t}{q_t} \tag{41}
\]

\[
A_t F(\cdot) = (1 - \eta)C^h_t + \eta C^e_t + \eta I_t + \frac{\theta}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 + G_t \tag{42}
\]

\[
mc_t A_t F^h_{n,t} = -\frac{U^h_{n,t}}{U^h_{c,t}} \tag{43}
\]

\[
mc_t A_t F_{k,t} = z_t \tag{44}
\]

\[
q_t f(\omega_t, A_t) = \frac{f'(\omega_t, A_t)}{g'(\omega_t, A_t)} [q_t g(\omega_t, A_t) - 1] \tag{45}
\]
where we recall that $K_t \equiv (1-\eta)K^h_t + \eta K^e_t$.

B Solution of the Model and Welfare

The set of conditions characterizing the optimal plan can be described as follows:

$$E_t \{ \mathcal{H}(Y_{t+1}, X_{t+1}, X_t) \} = 0$$ \hspace{1cm} (46)

where $E_t$ denotes the mathematical expectations operator, conditional on information available at time $t$, $Y_t$ is the vector of endogenous non-predetermined variables, and $X_t \equiv [x_{1,t}, x_{2,t}]$ is the state vector. Here $x_{1,t}$ denotes the vector of endogenous predetermined variables, while $x_{2,t}$ is the vector of exogenous variables which follows a stochastic process:

$$x_{2,t+1} = F x_{2,t} + \overline{b} \xi_{t+1}, \quad \xi_t \sim i.i.d. N(0, \Sigma)$$ \hspace{1cm} (47)

where the scalars $\overline{\xi}$ and $\overline{b}$ are known parameters, and $\Sigma$ is the variance-covariance matrix of the innovations. The solution of the model is of the form (Schmitt-Grohé and Uribe (2004a)):

$$Y_t = \gamma(X_t, \overline{\xi})$$ \hspace{1cm} (48)

$$X_{t+1} = h(X_t, \overline{\xi}) + \overline{b} \xi_{t+1}$$ \hspace{1cm} (49)

Equation (48) and (49) describe the policy function and the transition function respectively. Our solution method consists in computing a second-order expansion of the functions $\gamma(X_t, \overline{\xi})$ and $h(X_t, \overline{\xi})$ around the deterministic steady state. Schmitt-Grohé and Uribe (2004a) show two results. First, in a first-order approximation of the model, the expected value of any variable belonging to either $Y_t$ or $X_t$ coincides with its value in the non-stochastic steady state. Second, in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady-state value only by a
constant term. Let $\frac{1}{2} \Delta$ be the vector of such constant "correction" terms corresponding to the set of endogenous variables. Hence our measure for the conditional expected value $\mathcal{W}_{0,t}$ can be written:

$$\mathcal{W}_{0,t} = \mathcal{W}_0 + \frac{1}{2} \Delta [\mathcal{W}_0]$$

(50)

where $\mathcal{W}_0 = \frac{U_{C^hN^k}}{1-\beta}$ is the value of $\mathcal{W}_{0,t}$ in the deterministic steady state and $\Delta [\mathcal{W}_0]$ is the (constant) correction element of the vector $\Delta$ corresponding to the endogenous variable $\mathcal{W}_{0,t}$. To analytically compute (and numerically evaluate) the first and second-order derivatives of the relevant policy functions we employ the Matlab codes compiled by Schmitt-Grohé and Uribe, available at the website http://www.econ.duke.edu/~grohe.
References


Table 1. Simple Rules

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>$\pi_t = 0$ all $t$</td>
</tr>
<tr>
<td>TRI</td>
<td>$\phi_x = 1.5 \quad \phi_y = 0 \quad \phi_q = 0$</td>
</tr>
<tr>
<td>TR</td>
<td>$\phi_x = 1.5 \quad \phi_y = 0.5 \quad \phi_q = 0$</td>
</tr>
<tr>
<td>TRI + $\phi_q &lt; 0$</td>
<td>$\phi_x = 1.5 \quad \phi_y = 0 \quad \phi_q = -0.2$</td>
</tr>
<tr>
<td>TRI + $\phi_q &gt; 0$</td>
<td>$\phi_x = 1.5 \quad \phi_y = 0 \quad \phi_q = 0.2$</td>
</tr>
<tr>
<td>TR + $\phi_q &lt; 0$</td>
<td>$\phi_x = 1.5 \quad \phi_y = 0.5 \quad \phi_q = -0.2$</td>
</tr>
</tbody>
</table>

Note: $IT \equiv$ strict consumer price inflation targeting, $TRI \equiv Taylor$ rule with inflation only, $TR \equiv Standard$ Taylor rule (with inflation and output)

Table 2. Welfare Results

<table>
<thead>
<tr>
<th>Rule</th>
<th>No Smoothing ($\phi_r = 0$)</th>
<th>Smoothing ($\phi_r = 0.9$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Welfare</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>IT</td>
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</tr>
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<td>0.003</td>
</tr>
<tr>
<td>TR</td>
<td>-77.43</td>
<td>1.413</td>
</tr>
<tr>
<td>TRI + $\phi_q &lt; 0$</td>
<td>-76.03</td>
<td>0.0013</td>
</tr>
<tr>
<td>TRI + $\phi_q &gt; 0$</td>
<td>-76.033</td>
<td>0.0045</td>
</tr>
<tr>
<td>TRI + output + $\phi_q &lt; 0$</td>
<td>-77.43</td>
<td>1.416</td>
</tr>
</tbody>
</table>

Notes: $\Omega$ is the % fraction of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see equation (34)). Welfare is calculated as conditional to the initial deterministic steady-state.
Figure 1: Impulse Responses to a Total Factor Productivity Shock: No Credit Frictions (solid line, $\mu = 0$) vs. Credit Frictions (dashed line, $\mu = 0.25$, model with $\omega_{m,t} = 1$). Monetary policy rule: $\ln\left(\frac{1+R_t}{1+R}\right) = 1.5 \ln\left(\frac{\pi_t}{\pi}\right)$. 

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$\mu = 0$  
$\mu = 0.25$
Figure 2: Impulse Responses to a Total Factor Productivity Shock: Model with $\omega_{m,t} = 1$ (solid line) vs. Model with endogenous $\omega_{m,t}$ (dashed line). Monetary policy rule: $
abla \ln (\frac{1 + R_t}{1 + \pi}) = 1.5 \ln (\frac{\omega_t}{\pi})$. 

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Figure 3: Effect on Conditional Welfare of Varying the Response to Inflation ($\phi_x$) and to the Asset Price ($\phi_q$). No Interest Rate Smoothing ($\phi_r = 0$).
Figure 4: Effect on Conditional Welfare of Varying the Response to Inflation ($\phi_z$) and to the Asset Price ($\phi_q$). Interest Rate Smoothing ($\phi_r = 0.9$).
Figure 5: Effect on Welfare of Responding to the Asset Price in a Taylor Rule with Fixed Inflation Coefficient ($\phi_z = 1.5$).
Figure 6: Impulse Responses to a Total Factor Productivity Shock: Taylor Rule (solid line) vs. Strict Inflation Stabilization ($\pi_t = 0$ all $t$, dashed line).
Figure 7: Impulse Responses to a Government Spending Shock: Taylor Rule (solid line) vs. Strict Inflation Stabilization ($\pi_t = 0$ all $t$, dashed line).