

# Public Debt and Redistribution with Borrowing Constraints.\*

Florin Bilbiie<sup>†</sup>      Tommaso Monacelli<sup>‡</sup>      Roberto Perotti<sup>§</sup>

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## Abstract

We build a model with financial imperfections where Ricardian equivalence holds when prices are flexible, and the steady-state distribution of consumption is uniform, or labor is inelastic. With different steady-state consumption levels, Ricardian equivalence fails; but the present-value multiplier of tax cuts on consumption is zero, and debt-financed tax cuts are—somewhat paradoxically—contractionary. With sticky prices, Ricardian equivalence always fails. A Robin-Hood, revenue-neutral redistribution to borrowers is expansionary on aggregate activity. A uniform cut in taxes financed with public debt has a positive present-value multiplier on consumption, stemming from intertemporal substitution by the savers, who hold the public debt.

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<sup>†</sup>Paris School of Economics, Université Paris I Panthéon-Sorbonne and CEPR; Address: Centre d'Economie de la Sorbonne, 106-112 Boulevard de l'Hopital 75013 Paris; Email: [florin.bilbiie@parisschoolofeconomics.eu](mailto:florin.bilbiie@parisschoolofeconomics.eu); URL: <https://sites.google.com/site/florinbilbiie/>; Bilbiie thanks Banque de France for financial support through the eponymous Chair at the Paris School of Economics, and New York University–Abu Dhabi for hospitality during part of writing this paper. The views expressed are those of the authors and do not necessarily reflect official views or policies of the Banque de France.

<sup>‡</sup>Università Bocconi, IGIER and CEPR. Email: [tommaso.monacelli@unibocconi.it](mailto:tommaso.monacelli@unibocconi.it). URL: <http://www.igier.uni-bocconi.it/monacelli>. Monacelli gratefully acknowledges the contribution of 2012-15 ERC Grant FINIMPMACRO.

<sup>§</sup>Università Bocconi, IGIER, CEPR and NBER. Email: [roberto.perotti@unibocconi.it](mailto:roberto.perotti@unibocconi.it). URL: <http://www.igier.uni-bocconi.it/perotti>.

*“If fiscal policy is used as a deliberate instrument for the more equal distribution of incomes, its effect in increasing the propensity to consume is, of course, all the greater.”*  
(Keynes, 1936, Book III, Chapter 8, Section II).

## 1 Introduction

The aftermath of the Great Recession has revived a classic debate on the effects of so-called fiscal stimulus programs. This debate has often focused on the role of government debt. Less prominent in the debate is the fact that the rise in public debt in many countries has ensued from stimulus packages that have taken the form of transfers to specific income groups, rather than purchases of goods and services (see, e.g., Oh and Reis, 2011; Giambattista and Pennings, 2011; Mehrotra, 2011). It is therefore important to understand the distributional consequences of stimulus programs, particularly in light of the upward trajectory of public debt implied by such programs.

In this paper we study fiscal stimulus policies in the form of temporary tax cuts. We interpret redistribution as *revenue-neutral* tax cuts to a fraction of the population financed by a tax rise to another; by construction, this policy changes the lifetime income (wealth) of private agents. We interpret public debt as a form of *intertemporal* redistribution that does not affect the lifetime income of agents, and is by construction not revenue-neutral for the government.

We conduct our analysis in a framework featuring heterogenous agents, who differ in their degree of impatience, and imperfect financial markets. This setup, sometimes labeled Borrower-Saver model, has become increasingly popular in the recent literature.<sup>1</sup> The resulting model resembles the classic *Savers-Spenders* (SS henceforth) model of fiscal policy (Mankiw 2000) in which "myopic" households, who merely consume their income, coexist with standard, intertemporally optimizing households.<sup>2</sup> Ours is a variant of the SS

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<sup>1</sup>See for instance Eggertson and Krugman (2012) and Monacelli and Perotti (2012). These models are variants of the RBC-type borrower-saver framework proposed in i.a. Kiyotaki and Moore (1997), and extended to a New Keynesian environment by, e.g., Iacoviello (2005) and Monacelli (2009).

<sup>2</sup>The classic Savers-Spenders model has been extended by, among others, Galí et al. (2004, 2007) and Bilbiie (2008) to include nominal rigidities and other frictions in order to study questions ranging from

model in two respects: first, *both* agents are intertemporal maximizers—so that borrowing and lending take place in equilibrium—but a fraction of agents face a suitably defined borrowing limit; second, the *distribution* of debt/saving across agents is endogenous.

In this model, public debt is intimately related to redistribution. A debt-financed, one-time uniform tax cut today is equivalent to a redistribution from the agents who hold the public debt (savers) to those who do not (borrowers) today, followed by a (potentially persistent) redistribution in the opposite direction from the next period onwards, as debt is repaid. The persistence of the implicit future redistribution depends on the speed at which debt is repaid; this can generate endogenously persistent effects of purely transitory tax cuts.

Since our model features credit market imperfections, it is tempting to think that Ricardian equivalence readily fails, so that (lump-sum) tax cuts produce positive (and possibly large) effects on aggregate demand. We first show that this reasoning can be misleading, because the conclusion hinges on two crucial elements: (i) whether or not the steady-state distribution of consumption across agents is uniform; and (ii) whether or not labor supply is endogenous.

In fact, the baseline version of our model with perfectly flexible prices produces two paradoxical results. First, and despite the presence of borrowing frictions, a tax redistribution that favors the constrained agents (a tax cut to the borrowers financed by a rise in taxes to the savers) is completely neutral on *aggregate* consumption if either labor supply is inelastic, or the steady-state distribution of consumption is uniform (e.g., if the borrowing limit is zero—as implicit in the traditional SS model—and profit income is either zero or redistributed uniformly across agents).<sup>3</sup>

Second, even if the steady-state distribution of consumption is not uniform (so that, e.g., a fraction of agents hold private debt and another fraction a corresponding amount of savings), a tax redistribution generates a *contraction* in aggregate spending.

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the effects of government spending to monetary policy analysis and equilibrium determinacy

<sup>3</sup>Throughout the paper, we abstract from the accumulation of physical capital in order to focus on one source of failure of Ricardian equivalence: sticky prices. We hint to some of the possible implications of capital accumulation in the concluding section.

The intuition for these results is that the steady-state distribution of consumption (and wealth) governs the (intensity of the) income effect on labor supply. When steady-state consumption levels are equalized, the income effects on the agents' individual labor supplies are symmetric. In response to a tax redistribution, borrowers choose to work less and savers to work more in an exactly offsetting way. When the distribution of wealth is such that, realistically, borrowing-constrained agents consume relatively less in steady state, their reduction in labor supply more than compensates the increase in labor supply by the savers, leading to an overall contraction in spending and output.

A uniform tax cut financed by issuing public debt (held by the savers), which is repaid by uniform taxation in the future amounts, *de facto*, to redistributing from savers to borrowers today, and reversing that redistribution in the future (when debt is repaid). Within each period, the same logic of redistribution described above applies, so that either the redistribution is neutral, or it generates paradoxical results: the tax cut today is contractionary and the tax increase tomorrow is expansionary. The key extra element is that these contradicting forces are *exactly* symmetric: the present-value multiplier on consumption is always zero.

A large empirical literature (i.a. Blanchard and Perotti, 2002, Romer and Romer 2010, Perotti 2012, Mertens and Ravn, 2012, Favero and Giavazzi 2012) identifies tax innovations using a variety of approaches and studies their macroeconomic effects. While those studies often disagree as to the magnitude of the multipliers, they all find contractionary effects in response to positive tax shocks, which casts serious doubt on the implications of the flexible-price model summarized above.

Matters are different with nominal price rigidity, and even in the case of a uniform steady-state distribution of consumption. Two elements are typical of the sticky-price environment. First, as firms cannot optimally adjust prices, the increase in borrowers' consumption ensuing from the tax cut generates an increase in labor demand. Second, the rise in the real wage that results from the expansion in labor demand generates, for one, a further income effect on borrowers and hence a further expansion in their consumption; and also a fall in profits, with an additional negative income effect on the savers' labor

supply, that is absent under flexible prices.

In this scenario, we obtain two main results. First, a revenue-neutral tax redistribution is *expansionary* on aggregate spending, as well as inflationary. Second, a debt-financed uniform tax cut generates a current expansion in aggregate spending, followed by a contraction. Crucially, however, the two effects are not symmetric: the *present-value* multiplier of a debt-financed tax cut is positive regardless of how fast debt is repaid, whereas it would be zero under the same conditions if prices were flexible.

The reason why the effect of a uniform tax cut goes beyond the mere sum of its implied redistributive components (from savers to borrowers, today; and from borrowers to savers, in the future) stems from intertemporal substitution: real interest rates fall, since the future *de facto* transfer from borrowers to savers generates a fall in demand and deflation, which boosts savers' consumption today. This effect is stronger, the stronger is the intertemporal substitution channel (the more flexible are prices, or the more aggressive is monetary policy), and disappears when the intertemporal-substitution channel is turned off (when there are no equilibrium fluctuations in interest rates).

In the limit, if the debt-financed uniform tax cut is repaid in the indefinite future through permanently higher (but constant) future taxes, a uniform tax cut has effects that are identical to a one-time redistribution from savers to borrowers, although the two policies are very different in nature. The reason for this equivalence stems from the key role of intertemporal substitution in shaping the effects of public debt. With debt that is repaid by permanently higher (but constant) taxation from tomorrow onwards, intertemporal substitution ceases to matter.

## 2 The model

Below we describe a Borrower-Saver model with two non standard features: first, the borrowers/impatient agents face a non-zero borrowing limit; second, goods markets are monopolistic competitive, and possibly firms set prices in a staggered fashion.

### Households

There is a continuum of households  $[0, 1]$  indexed by  $j$ , all having the same utility function

$$U(C_{j,t}, N_{j,t}) = \ln C_{j,t} - \chi_j \frac{N_{j,t}^{1+\varphi}}{1+\varphi},$$

where  $\varphi > 0$  is the inverse of the labor supply elasticity. The agents differ in their discount factors  $\beta_j \in (0, 1)$  and possibly in their preference for leisure  $\chi_j$ . Specifically, we assume that there are two types of agents  $j = s, b$ , and

$$\beta_s > \beta_b.$$

All households (regardless of their discount factor) consume an aggregate basket of individual goods  $z \in [0, 1]$ , with constant elasticity of substitution  $\varepsilon$ :  $C_t = \left( \int_0^1 C_t(z)^{(\varepsilon-1)/\varepsilon} dz \right)^{\varepsilon/(\varepsilon-1)}$ ,  $\varepsilon > 1$ . Standard demand theory implies that total demand for each good is  $C_t(z) = (P_t(z)/P_t)^{-\varepsilon} C_t$ , where  $C_t(z)$  is total demand of good  $z$ ,  $P_t(z)/P_t$  its relative price and  $C_t$  aggregate consumption.<sup>4</sup> The aggregate price index is  $P_t^{1-\varepsilon} = \int_0^1 P_t(z)^{1-\varepsilon} dz$ .

A  $1 - \lambda$  share is represented by households who are patient: we label them *savers*, discounting the future at  $\beta_s$ . Consistent with the equilibrium outcome (discussed below) that patient agents are savers (and hence will hold the bonds issued by impatient agents), we impose that patient agents also hold all the shares in firms.

Each saver chooses consumption, hours worked and asset holdings (bonds and shares), solving the standard intertemporal problem:

$$\max \mathbb{E}_t \left\{ \sum_{i=0}^{\infty} \beta_s^i U(C_{s,t+i}, N_{s,t+i}) \right\},$$

subject to the sequence of constraints:

$$C_{s,t} + B_{s,t+1} + A_{s,t+1} + \Omega_{s,t+1} V_t \leq \frac{1 + I_{t-1}}{1 + \Pi_t} B_{s,t} + \frac{1 + I_{t-1}}{1 + \Pi_t} A_{s,t} + \Omega_{s,t} (V_t + \mathcal{P}_t) + W_t N_{s,t} - \tau_{s,t}, \quad (1)$$

where  $\mathbb{E}_t \{ \cdot \}$  is the expectations operator,  $C_{s,t}, N_{s,t}$  are consumption and hours worked by the patient agent,  $W_t$  is the real wage,  $A_{s,t}$  is the *real* value at beginning of period

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<sup>4</sup>This equation holds in aggregate because the same static problem is solved by both types of households.

$t$  of total private assets held in period  $t$  ( $1 + \Pi_t \equiv P_t/P_{t-1}$  is the gross inflation rate), a portfolio of one-period bonds issued in  $t - 1$  on which the household receives nominal interest  $I_{t-1}$ .  $V_t$  is the real market value at time  $t$  of shares in intermediate good firms,  $\mathcal{P}_t$  are real dividend payoffs of these shares,  $\Omega_{s,t}$  are share holdings,  $\tau_{s,t}$  are per-capita lump-sum taxes paid by the saver, and  $B_{s,t}$  the savers' holdings of nominal public bonds which deliver the same nominal interest as private bonds.

The Euler equations—for bond and share holdings respectively—and the intratemporal optimality condition are:<sup>5</sup>

$$C_{s,t}^{-1} = \beta_s \mathbb{E}_t \left( \frac{1 + I_t}{1 + \Pi_{t+1}} C_{s,t+1}^{-1} \right) \text{ and } V_t = \beta_s \mathbb{E}_t \left( \frac{C_{s,t}}{C_{s,t+1}} \frac{V_{t+1} + \mathcal{P}_{t+1}}{1 + \Pi_{t+1}} \right), \quad (2)$$

$$\chi_s N_{s,t}^\varphi = \frac{1}{C_{s,t}} W_t. \quad (3)$$

The rest of the households on the  $[0, \lambda]$  interval are impatient (and will borrow in equilibrium, hence we index them by  $b$  for borrowers) face the intertemporal constraint:

$$C_{b,t} + A_{b,t+1} \leq \frac{1 + I_{t-1}}{1 + \Pi_t} A_{b,t} + W_t N_{b,t} - \tau_{b,t}, \quad (4)$$

as well as the additional borrowing constraint (on borrowing in real terms) at all times  $t$ :

$$-A_{b,t+1} \leq \bar{D},$$

$$\chi_b N_{b,t}^\varphi = \frac{1}{C_{b,t}} W_t, \quad (5)$$

$$C_{b,t}^{-1} = \beta_b \mathbb{E}_t \left( \frac{1 + I_t}{1 + \Pi_{t+1}} C_{b,t+1}^{-1} \right) + \psi_t, \quad (6)$$

where  $\psi_t$  takes a positive value whenever the constraint is binding. Indeed, because of our assumption on the relative size of the discount factors, the borrowing constraint will bind in steady state (we discuss this in more detail below).

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<sup>5</sup>These conditions must hold along with the usual transversality conditions.

**Firms** Each individual good is produced by a monopolistic competitive firm, indexed by  $z$ , using a technology given by:  $Y_t(z) = N_t(z)$ . Cost minimization taking the wage as given, implies that real marginal cost is  $W_t$ . The profit function in real terms is given by:  $\mathcal{P}_t(z) = [P_t(z)/P_t] Y_t(z) - W_t N_t(z)$ , which aggregated over firms gives total profits  $\mathcal{P}_t = [1 - W_t \Delta_t] Y_t$ . The term  $\Delta_t$  is relative price dispersion defined following Woodford (2003) as  $\Delta_t \equiv \int_0^1 (P_t(z)/P_t)^{-\varepsilon} dz$ .

**Monetary authority** A monetary authority sets the nominal interest rate in response to fluctuations in expected inflation (we assume for simplicity that target inflation is zero):

$$1 + I_t = \Phi(1 + \mathbb{E}_t \Pi_{t+1}),$$

where  $\Phi(1) = \beta_s^{-1} > 1$ .

**Government** The government issues  $B_{t+1}$  one-period bonds, which are held only by the savers. In order to focus on the effects of taxation and public debt, we abstract from government spending. Hence the government budget constraint reads:

$$B_{t+1} = \left( \frac{1 + I_{t-1}}{1 + \Pi_t} \right) B_t - \tau_t, \quad (7)$$

where  $\tau_t$  are *total* tax revenues, i.e.,  $\tau_t = \lambda \tau_{b,t} + (1 - \lambda) \tau_{s,t}$ .

Notice that the assumption that government spending is fixed implies that exogenous variations in taxes will readily constitute a test of whether Ricardian Equivalence holds in our model.

**Equilibrium** In an equilibrium of this economy, all agents take as given prices (with the exception of monopolists who reset their good's price in a given period), as well as the evolution of exogenous processes. A rational expectations equilibrium is then as usually a sequence of processes for all prices and quantities introduced above such that the optimality conditions hold for all agents and all markets clear at any given time  $t$ .



Specifically, labor market clearing requires that labor demand equal total labor supply,  $N_t = \lambda N_{b,t} + (1 - \lambda) N_{s,t}$ . Private debt is in zero net supply  $\int_0^1 A_{j,t+1} = 0$ , and hence, since agents of a certain type make symmetric decisions:

$$\lambda A_{b,t+1} + (1 - \lambda) A_{s,t+1} = 0.$$

Equity market clearing implies that share holdings of each saver are:

$$\Omega_{s,t+1} = \Omega_{s,t} = \Omega = \frac{1}{1 - \lambda}.$$

Finally, by Walras' Law the goods market also clears. The resource constraint specifies that all produced output will be consumed:

$$C_t = Y_t = \frac{N_t}{\Delta_t}, \quad (8)$$

where  $C_t \equiv \lambda C_{b,t} + (1 - \lambda) C_{s,t}$  is aggregate consumption and  $\Delta_t$  is relative-price dispersion.

All bonds issued by the government will be held by savers. Market clearing for public debt implies:

$$(1 - \lambda) B_{s,t+1} = B_{t+1}. \quad (9)$$

In our model, fiscal policy matters only through the impact of taxes (transfers) on borrowers. Substituting equations (7), (9) and the definition of total taxes in the savers' budget constraint, we obtain:

$$C_{s,t} + A_{s,t+1} \frac{1 + I_{t-1}}{1 + \Pi_t} \leq \frac{\lambda}{1 - \lambda} \tau_{b,t} + A_{s,t} + \frac{1}{1 - \lambda} \mathcal{P}_t + W_t N_{s,t}. \quad (10)$$

Savers internalize the government budget constraint through their public debt holdings, and so recognize that a transfer to borrowers today effectively implies a tax on themselves, today or in the future. In this sense, public debt works as a mechanism to *redistribute wealth* among agents, intra- and inter-temporally. The higher the fraction of borrowers, the more sensitive the consumption of savers to a change in the tax on borrowers (*ceteris paribus*).

Note that the only fiscal variable appearing in the equilibrium conditions is  $\tau_{b,t}$ , the level of taxes on borrowers—without any reference to the aggregate level of taxes or public debt. However, the tax process itself needs to respond to public debt in order to ensure sustainability—but it still matters for the aggregate allocation *only* through its impact on taxes on borrowers. To close the model we need to specify how fast this adjustment takes place, and how the burden of readjustment is shared between savers and borrowers.

## 2.1 Tax Rules and Equilibrium Dynamics

We solve our model locally by log-linearizing it around a zero-inflation steady state, in which the borrowing constraint always binds. In order to check accuracy of this solution method, we perform a series of tests based on Den Haan's (2010) "Dynamic Euler equation test" for different values of some key parameters, including those pertaining to shock processes.<sup>6</sup> To anticipate, that analysis shows that for the baseline calibration described below the constraint keeps binding virtually all the time, and approximation errors are negligible—suggesting that our solution method is valid at least for the baseline calibration we consider.

Henceforth, a small letter denotes log-deviations of a variable from its steady-state value, with two exceptions: taxes/transfers and public debt are in deviations from steady state, as a share of steady-state output  $Y$  ( $t_{j,t} \equiv (\tau_{j,t} - \tau_j) / Y$ ;  $b_t \equiv (B_t - B) / Y$ ) and interest and inflation rates are in absolute deviations from their steady-state values. All log-linearized equilibrium conditions are outlined in Table 1, where  $B_Y \equiv B/Y$  and  $\bar{D}_Y \equiv \bar{D}/Y$ .

In our log linear equilibrium, we assume a general financing scheme whereby taxes on each agent increase to repay the outstanding debt, but only gradually so:

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<sup>6</sup>The results of these accuracy tests are reported in an online Appendix; they consist loosely speaking of measuring how different the solution of our loglinearized method is from that of a method that only uses the loglinearized solution to calculate next period's behavior, while other variables are calculated using the true, nonlinear equations of the model. An important by-product of this analysis is that we obtain, for a given set of parameter values, a measure of how often the constraint stops binding. See Den Haan (2010) for further details.

$$t_{j,t} = \phi_B^j b_t - \epsilon_{j,t}, \quad (11)$$

where  $j = b, s$ , and  $\epsilon_t$  is an exogenous, possibly persistent stochastic process with  $\mathbb{E}_t \epsilon_{t+1} = \rho \epsilon_t$ ,  $\rho \geq 0$ .

This tax rule is general enough to allow taxes on each agent to react to stabilize government debt ( $\phi_B^j \geq 0$  is the debt feedback coefficient), and asymmetric changes in taxation for the two agents ( $\epsilon_{j,t}$  is a random and possibly persistent innovation).

**Table 1. Summary of the Log-Linear Model**

Euler equation, S	$\mathbb{E}_t c_{s,t+1} - c_{s,t} = i_t - \mathbb{E}_t \pi_{t+1}$
Labor supply, S	$\varphi n_{s,t} = w_t - c_{s,t}$
Labor supply, B	$\varphi n_{b,t} = w_t - c_{b,t}$
Budget constraint, B	$\gamma c_{b,t} + \bar{D}_Y (i_{t-1} - \pi_t) = \frac{WN_b}{C} (w_t + n_{b,t}) - t_{b,t}$
Production function	$y_t = n_t$
Phillips curve	$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta_s)}{\theta} w_t$
Government debt	$\beta_s b_{t+1} = B_Y (i_{t-1} - \pi_t) + b_t - t_t$
Lump-sum taxes	$t_t = \lambda t_{b,t} + (1 - \lambda) t_{s,t}$
Tax rule	$t_{j,t} = \phi_B^j b_t - \epsilon_{j,t} \quad (j = b, s)$
Labor market clearing	$n_t = \lambda n_{b,t} + (1 - \lambda) n_{s,t}$
Aggregate consumption	$c_t = \lambda \gamma c_{b,t} + (1 - \lambda \gamma) c_{s,t}$
Resource constraint	$y_t = c_t$
Monetary policy	$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1}$
Note: savers' budget constraint replaced with aggregate resource constraint by Walras' Law.	

## 2.2 Steady State

We focus on a deterministic steady state where inflation is zero. Since the constraint binds in steady state ( $\psi = C_b^{-1} [1 - (\beta_b/\beta_s)] > 0$  whenever  $\beta_s > \beta_b$ ), patient agents are net borrowers and steady-state private debt is  $A_b = -\bar{D}$ ; by debt market clearing, then the patient agents are net lenders and their private bond holdings are  $A_s = \lambda \bar{D} / (1 - \lambda)$ .

To simplify the analysis, we make the further assumption that agents work the same number of hours in steady state:  $N_b = N_s = N$ . This assumption is consistent with the view that there are no wealth effects on long-run hours worked. Specifically, the relative

weight of leisure in the utility function needs to be different across agents,  $\chi_s \neq \chi_b$ , by precisely the amount needed to make (only) steady-state hours identical across groups,  $N_b = N_s = N$ .

The utility weights  $\chi_s$  and  $\chi_b$  consistent with this assumption can be shown to be:

$$\chi_s = \frac{1}{N^{1+\varphi} \left( \frac{1}{1+\mu} + \frac{1}{1-\lambda} \frac{\mu}{1+\mu} + \frac{\lambda}{1-\lambda} R \bar{D}_Y \right)} < \chi_b = \frac{1}{N^{1+\varphi} \left( \frac{1}{1+\mu} - R \bar{D}_Y \right)}, \quad (12)$$

where  $R$  is the net real interest rate obtained from the Euler equation of savers,  $R = I = \beta_s^{-1} - 1$ , and  $\mu \geq 0$  is the steady state *net* markup.

The second equation in (12) determines  $N$  as a function of  $\chi_b$ , and the first determines the  $\chi_s$  that delivers the equalization of hours. Note that  $\chi_s < \chi_b$  (to work the same steady-state hours, savers need to dislike labor less).

The per-group steady state shares of consumption in total consumption are

$$\begin{aligned} \frac{C_b}{C} &\equiv \gamma = \frac{1}{1+\mu} - R \bar{D}_Y \leq 1, \\ \frac{C_s}{C} &= \frac{1-\lambda\gamma}{1-\lambda} \geq 1. \end{aligned}$$

Notice that in the particular case of  $\mu = 0$  and zero private debt limit,  $\bar{D}_Y = 0$ , we have  $\gamma = 1$ , implying that the distribution of steady-state consumption is uniform,  $C_b = C_s = C$ .

## 2.3 Two Special Cases

In the remainder of the paper, we will focus on two fiscal policy arrangements that allow us to obtain analytical solutions: (i) pure redistribution and (ii) a debt-financed tax cut.

### 2.3.1 Pure Redistribution ("Robin Hood").

Consider first a transfer that takes place *within the period*, so that the budget is balanced every period:

$$t_{b,t} = -\epsilon_t, \quad t_{s,t} = \frac{\lambda}{1-\lambda} \epsilon_t. \quad (13)$$

In this scenario  $t_{b,t}$  is exogenous. Taxes on savers adjust to ensure public debt sustainability, but this is irrelevant for the allocation. This experiment is equivalent to having a pure "Robin Hood" policy that taxes savers and redistributes the proceedings to borrowers within the period. Importantly, such a change in taxation is revenue-neutral for the government, but changes the wealth (the lifetime income) of both agents.

### 2.3.2 Uniform Tax Cut Financed with Public Debt

Alternatively, consider a *uniform tax cut* ( $t_{b,t} = t_{s,t} = t_t$ ) of size  $\epsilon_t^B$  to both agents, financed via public debt held by the savers.

Unlike the previous experiment, this policy change is obviously not revenue-neutral, but does not *per se* affect the wealth or lifetime income of agents. To see this, consider, for the sake of simplicity, the government budget constraint log-linearized around a steady state with zero public debt ( $B_Y = 0$ ):

$$\beta_s b_{t+1} = b_t - t_t. \quad (14)$$

The aggregate tax rule  $t_t = \phi_B b_t - \epsilon_t^B$  replaced in the government budget constraint (14) implies the debt accumulation equation:

$$b_{t+1} = (1 - \phi_B) \beta_s^{-1} b_t + \beta_s^{-1} \epsilon_t^B. \quad (15)$$

In order to ensure *debt sustainability*, the response of taxes to debt needs to obey:<sup>7</sup>

$$\phi_B \in [1 - \beta_s, 1]. \quad (16)$$

For the sake of simplicity, we assume that the tax shock has zero persistence:  $\epsilon_{t+i}^B = 0$  for any  $i > 0$ . In that case, the debt accumulation equation implies that taxes at  $t + 1$

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<sup>7</sup>Notice, however, that condition (16) need not hold for taxation of *both* agents, but for the *aggregate* response. Indeed, it would be sufficient if for instance taxes of savers fulfilled (16), i.e.  $\phi_B^s > 1 - \beta_s$  and taxes of borrowers did not respond to debt at all,  $\phi_B^b = 0$ . When the condition is fulfilled, (15) can be solved independently of the rest of the model to determine the path of public debt; this is due to our assumption of zero public debt in steady state, which makes interest payments irrelevant to first order.

are given by:

$$\begin{aligned} t_{t+1} &= \phi_B b_{t+1} = \phi_B (1 - \phi_B) \beta_s^{-1} b_t + \phi_B \beta_s^{-1} \epsilon_t^B \\ &= (1 - \phi_B) \beta_s^{-1} t_t + \beta_s^{-1} \epsilon_t^B. \end{aligned} \quad (17)$$

Equation (17) shows that from the period immediately following the tax cut, the tax process follows an AR(1) process, with persistence  $(1 - \phi_B) \beta_s^{-1}$  and an initial value proportional to the initial tax cut.<sup>8</sup> At any time  $t + i$  for  $i > 1$ , taxes obey:

$$t_{t+i} = (1 - \phi_B)^i \beta_s^{-i} t_t + (1 - \phi_B)^{i-1} \beta_s^{-i} \epsilon_t^B. \quad (18)$$

Alternative values of parameter  $\phi_B$  describe different horizons over which debt stabilization is achieved (and therefore the initial tax cut is reversed), as well as different initial values for the size of the initial tax adjustment. It is useful to consider two extreme cases.

1. *One-period debt stabilization.* In this case,  $\phi_B = 1$ . A cut in taxes today  $\epsilon_t^B$  implies the tax process

$$t_t^{FS} = b_t - \epsilon_t^B; \quad t_{t+1}^{FS} = \beta_s^{-1} \epsilon_t^B; \quad t_{t+i}^{FS} = 0 \text{ for } i > 2. \quad (19)$$

The tax process lives for only one period, as all debt is repaid in the next period. Therefore, the tax adjustment in period  $t + 1$  is *a fortiori* the largest in this case. Recall that, since taxation is uniform but all debt is held by the savers, this experiment is equivalent to a redistribution at time  $t$  of amount  $\epsilon_t^B$  from the savers to the borrowers, followed by a reverse transfer in period  $t + 1$  of  $\beta_s^{-1} \epsilon_t^B$ . Effectively, the government lends to the borrowers.

2. *No debt stabilization.* At the other extreme we have  $\phi_B = 1 - \beta_s$ . This implies that the tax process has a unit root:

$$\begin{aligned} t_t^{NS} &= (1 - \beta_s) b_t - \epsilon_t^B, \\ t_{t+i}^{NS} &= t_t^{NS} + \beta_s^{-i} \epsilon_t^B \text{ for any } i > 1. \end{aligned} \quad (20)$$

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<sup>8</sup>If we did not restrict the tax-cut shock to last for only one period, taxes would follow an ARMA(1, 1) process:  $t_{t+1} = (1 - \phi_B) \beta_s^{-1} t_t + \beta_s^{-1} \epsilon_t^B - \epsilon_{t+1}^B$ .

The increase in taxes from period  $t + 1$  onwards is the longest in this case: taxes increase for the indefinite future by the constant amount  $(\beta_s^{-1} - 1) \epsilon_t^B$ . This experiment amounts to a redistribution from savers to borrowers of size  $\epsilon_t^B$  followed by a *permanent* transfer of  $(\beta_s^{-1} - 1) \epsilon_t^B$  from borrowers to savers from  $t + 1$  onwards.

As the parameter  $\phi_B$  increases, the persistence of the tax process diminishes (the initial tax cut is repaid faster) and (hence) the adjustment in taxes in period  $t + 1$  becomes larger, since the present discounted value of taxes needs to be just enough to ensure repayment of the initial debt.<sup>9</sup>

### 3 Flexible Prices and Ricardian Equivalence

We begin by assuming that prices are fully flexible. We show that, in an environment in which the steady-state levels of consumption of borrowers and savers are different, Ricardian equivalence fails: changes in lump-sum taxes affect the real allocation. However, the predictions concerning the effect of tax cuts are counter-intuitive and contrary to empirical findings—which motivates our further analysis of other deviations from Ricardian equivalence.

Log-linearizing (4) and (5) around the steady state, and combining, we obtain:

$$\begin{aligned} n_{b,t} &= \frac{\gamma(1+\mu) - 1}{\varphi\gamma(1+\mu) + 1} w_t + \frac{1+\mu}{\varphi\gamma(1+\mu) + 1} (\bar{D}_Y r_{t-1} + t_{b,t}), \\ c_{b,t} &= \frac{1+\varphi}{\varphi\gamma(1+\mu) + 1} w_t - \frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu) + 1} (\bar{D}_Y r_{t-1} + t_{b,t}), \end{aligned} \quad (21)$$

where  $r_{t-1} \equiv i_{t-1} - \pi_t$ .

Starting from the steady state, and in response to an increase in taxation, borrowers' hours worked decrease (in equilibrium) with the real wage because of a positive income effect (which disappears when the debt limit is zero and  $\gamma(1+\mu) = 1$ ), and increase with taxes and interest payments.

Denote with a star a variable under flexible prices. Evaluating (21) at flexible prices (i.e., constant real marginal cost  $w_t^* = 0$ ), replacing into the aggregate consumption

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<sup>9</sup>It can be easily shown that  $\sum_{i=0}^{\infty} \beta_s^i t_{t+i} = b_t$  simply by using (18).

definition, solving for savers' consumption at flexible prices, and using (5), we obtain the following expression for *aggregate* consumption (output) under flexible prices:<sup>10</sup>

$$c_t^* = \zeta \left( \bar{D}_Y r_{t-1} + t_{b,t} \right), \quad (22)$$

where

$$\zeta \equiv \frac{\lambda(1-\gamma)}{1-\lambda+\varphi(1-\lambda\gamma)} \frac{\varphi(1+\mu)}{\varphi\gamma(1+\mu)+1} \geq 0.$$

Equation (22) contains a reduced form expression for aggregate consumption as a function of the exogenous tax process for borrowers,  $t_{b,t}$ , and the predetermined real interest rate,  $r_{t-1}$ . Direct inspection of (22) in the case  $\gamma = 1$  (equal steady-state consumption shares) or  $\varphi \rightarrow \infty$  (inelastic labor supply) suggests the following proposition:

**Proposition 1** *When either labor supply is inelastic or steady-state consumption of savers and borrowers are equal, Ricardian equivalence holds—regardless of how high the fraction of borrowers  $\lambda$  and how tight the debt constraint  $\bar{D}_Y$  are.*

The intuition for Ricardian equivalence in the two cases covered by the proposition is simple. When labor supply is inelastic, total consumption trivially equals total endowment, regardless of how that endowment is distributed. When instead labor supply is elastic but steady-state consumption levels are equalized, income effects on agents' individual labor supplies (effects which are governed precisely by the steady-state consumption levels) are fully symmetric; to take one example that we elaborate on below: in response to an increase in their taxes  $t_{b,t}$ , borrowers want to work exactly as many hours more as savers are willing to work less when their taxes fall in order to balance the budget ( $t_{s,t} = -\lambda(1-\lambda)^{-1}t_{b,t}$ ).<sup>11</sup>

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<sup>10</sup>Using also aggregate hours and the equilibrium expression for the hours of the borrower, as well as goods market clearing  $c_t = n_t$ .

<sup>11</sup>Guerrieri and Lorenzoni (2011) discuss the relevance of group-type labor supply effects in order to generate an aggregate recession in a Bewley-type economy in response to a credit supply (or private "deleveraging") shock.



This symmetry breaks up when steady-state consumption levels are different. In the more general case  $\gamma < 1$ , three features of the solution are worth emphasizing.

First, Ricardian equivalence fails: any given change in lump-sum taxes on borrowers produces an effect on aggregate consumption.

Second, with  $\zeta > 0$ , the effect on aggregate spending is paradoxical: a rise (fall) in taxes generates a rise (fall) in consumption.

Third, even when the debt limit is zero ( $\bar{D}_Y = 0$ ), there is still steady-state consumption inequality and Ricardian Equivalence still fails. In order to better understand the effects of redistribution and public debt under flexible prices, consider in turn the two extreme fiscal policy experiments described above, assuming for simplicity that  $\bar{D}_Y = 0$ .

### 3.1 Pure redistribution ("Robin Hood")

Consider the first policy experiment outlined in Section 2.3 above: a within-period, balanced-budget transfer to borrowers financed by taxes on savers (13).

Replacing (13) into (22), and assuming  $\bar{D}_Y = 0$ , the multiplier of the tax cut on consumption reads:

$$c_t^* = -\zeta \epsilon_t < 0. \quad (23)$$

Hence, if  $\gamma < 1$ , consumption, output, and labor hours *fall*: redistributing within the period from the unconstrained to the constrained agents produces a contractionary effect on aggregate activity. The intuition for this, somehow paradoxical, result is simple: the negative income effect on savers resulting from the tax redistribution is larger in absolute value than the positive income effect on borrowers.

### 3.2 Public debt

Consider next a temporary uniform tax cut of size  $\epsilon_t^B$  to both agents, financed via public debt. Under this experiment, equation (22), combined with the aggregate tax rule  $t_t = \phi_B b_t - \epsilon_t^B$ , implies that aggregate consumption obeys (assuming  $\bar{D}_Y = 0$  for simplicity)

for any  $i > 0$ :

$$c_{t+i}^* = \zeta t_{t+i}.$$

Since  $\zeta > 0$ , the prediction under flexible prices is once again that tax cuts cause a contraction in aggregate consumption on impact. Moreover, as taxes increase in the future in order to repay public debt even when the shock is purely transitory, the model also predicts that future consumption will increase along with future taxes.

To summarize, the implications of the model under flexible prices are inconsistent with a large empirical literature documenting that positive tax shocks are contractionary, rather than expansionary.<sup>12</sup>

The reason why tax increases are expansionary in our model is strictly related to each agent's income effect on labor supply: the income effect on savers' labor supply deriving from any given tax change is larger than that on borrowers' labor supply. Therefore, in response to a change of equal size (but of opposite sign) in their taxes, savers wish to increase their labor input more than borrowers want to decrease it.

However, it is worth noticing that the *present-value* multiplier on aggregate consumption is zero. The present value multiplier of a debt-financed tax cut can be written:

$$\begin{aligned} \mathcal{M}_{debt}^* &\equiv \frac{\partial \left( \sum_{i=0}^{\infty} \beta_s^i c_{t+i}^* \right)}{\partial \epsilon_t^B} = \zeta \sum_{i=0}^{\infty} \beta_s^i \frac{\partial t_{t+i}}{\partial \epsilon_t^B} \\ &= -\zeta + \zeta \phi_B \sum_{i=1}^{\infty} (1 - \phi_B)^{i-1} = 0. \end{aligned}$$

The contractionary effects of tax cuts and the expansionary effects of future tax increases sum up to a zero net effect on the present discounted value of consumption and hours worked, regardless of how persistent public debt is. These paradoxical effects of lump-sum tax changes on aggregate consumption under flexible prices motivate our further analysis, which consists of studying a model in which price adjustment is imperfect.

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<sup>12</sup>See, for instance, Romer and Romer (2011), Perotti (2012), Favero and Giavazzi (2012).

## 4 Sticky Prices

We assume a standard Calvo-Yun monopolistic competitive environment in which intermediate good firms adjust their prices infrequently. Savers (who in equilibrium will hold all the shares in firms) maximize the discounted sum of future nominal profits.

In the following we assume that steady-state consumption shares are equalized. This is achieved by assuming that both the debt limit and steady-state profits are zero; the latter in turn is obtained with a sales subsidy  $\sigma = \mu$ , so that profits' share in total output is zero. Note also that under this assumption, the implied weights on leisure in the utility function are equal across agents.<sup>13</sup>

The steady-state symmetry of consumption levels makes aggregation simple, and allows us to isolate the role of sticky prices in generating a failure of Ricardian Equivalence. Under these conditions, the aggregate constant-consumption labor supply curve has the same parameters as the individual ones:  $\varphi n_t = w_t - c_t$ , implying  $w_t = (1 + \varphi) c_t$ . Replacing these equations in the definition of aggregate consumption, solving for consumption of savers, and substituting in the savers' Euler equation we obtain the aggregate demand equation:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1} (i_t - \mathbb{E}_t \pi_{t+1}) - \delta^{-1} \eta (t_{b,t} - \mathbb{E}_t t_{b,t+1}), \quad (24)$$

where  $\delta \equiv 1 - \frac{\lambda \varphi}{1 - \lambda}$  and  $\eta \equiv \frac{\lambda}{1 - \lambda} \frac{\varphi}{1 + \varphi}$ .

Bilbiie (2008) shows that—in a model that is equivalent to ours with  $\bar{D}_Y = 0$ —for values of  $\lambda > 1/(1 + \varphi)$ ,  $\delta$  becomes negative: the aggregate elasticity of intertemporal substitution changes sign, and interest rate cuts become contractionary. In that "inverted aggregate demand logic" region, the monetary policy rule needs to follow an inverted Taylor principle in order to ensure determinacy and rule out sunspot fluctuations. In the

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<sup>13</sup>The alternative to achieve this outcome would be to assume that there are steady-state transfers that redistribute asset income evenly; the assumption we use has the relative merit of being consistent with evidence pointing to the long-run share of pure economic profits being virtually zero (see Rotemberg and Woodford, 1999). We also avoid taking a stand on the amount of *steady-state* redistribution through lump-sum transfers, which is very hard to measure.

remainder of this paper, we focus on parameter values that imply that  $\delta > 0$  so that standard aggregate demand logic holds.<sup>14</sup>

Finally, our Calvo-Yun environment implies a standard forward-looking Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa c_t, \text{ where } \kappa \equiv (1 + \varphi) \frac{(1 - \beta_s \theta)(1 - \theta)}{\theta}. \quad (25)$$

with  $\theta \in [0, 1]$  being the probability that each intermediate producer keeps its price constant in every period.

The model is closed by the following Taylor-type interest rate rule:<sup>15</sup>

$$i_t = \phi_\pi \mathbb{E}_t \pi_{t+1}, \quad (26)$$

where  $\phi_\pi > 1$ .

## 4.1 Pure redistribution ("Robin Hood")

Consider once again the effect of pure redistribution—a transfer  $\epsilon_t$  to borrowers financed by taxes on savers *within the period*. The tax processes are once again given by (13). It is instructive to simplify even further and first consider the case where the shock lasts only one period,  $\mathbb{E}_t \epsilon_{t+1} = 0$ .

Since the model is entirely forward-looking, expected values of consumption and inflation are also zero:  $\mathbb{E}_t c_{t+1} = \mathbb{E}_t \pi_{t+1} = 0$ , and the solution is simply:

$$\begin{aligned} c_t &= -\delta^{-1} \eta t_{b,t} = \mathcal{M}_{red} \epsilon_t, \\ \pi_t &= -\kappa \delta^{-1} \eta t_{b,t} = \kappa \mathcal{M}_{red} \epsilon_t, \\ \text{where } \mathcal{M}_{red} &\equiv \frac{\lambda}{1 - \lambda(1 + \varphi)} \frac{\varphi}{1 + \varphi}. \end{aligned}$$

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<sup>14</sup>In our framework with non-zero debt limit, this result will depend upon the value of the debt limit  $\bar{D}_Y$  (intuitively, even when  $\delta < 0$  an increase in the real rate needs not necessarily be expansionary, because of the contractionary effect on aggregate demand of interest payments on outstanding debt). But the same intuition holds, in that for values of the share of borrowers above that threshold, the effects of the type of fiscal shocks analyzed here are overturned.

<sup>15</sup>In the Appendix, we show that our results are largely robust to considering a Taylor rule that reacts to current, realized inflation  $\pi_t$ .

is the consumption multiplier of redistribution under sticky prices. These expressions suggest the following proposition.

**Proposition 2** *A within-the-period revenue-neutral transitory redistribution from savers to borrowers generates an expansion in aggregate consumption and inflation, as long as the elasticity of aggregate demand to interest rate is negative ( $\delta > 0$ ), i.e.:*

$$\lambda < \frac{1}{1 + \varphi}.$$

To understand the intuition, recall first what happens under flexible prices, if income effects on both agents are equal (they have the same long-run consumption values): labor supply of the borrowers shifts downwards, but labor supply of the savers shifts upwards by the same amount. Labor demand does not change either—so redistribution has no effect.

With sticky prices, and even in the knife-edge case of uniform steady-state consumption, two key ingredients break this neutrality. First, recall that output is demand-determined; the increase in borrowers' consumption generates a demand effect: labor demand increases as some firms are stuck with the old, suboptimally low price. The second key ingredient is the asymmetry between income effects. Faced with an increase in the real wage (marginal cost), the savers recognize that they face an extra negative income effect (that is absent with flexible prices) since their profit income falls. In equilibrium, they will therefore work more than the borrowers are willing to work less, therefore supporting the aggregate expansion in consumption. This income effect is increasing in the fraction of borrowers and decreasing with labor supply elasticity, but only up to the threshold given in the Proposition.<sup>16</sup>

In the more general case when redistribution is persistent (exogenously), with  $\mathbb{E}_t \epsilon_{t+1} = \rho \epsilon_t$ , the responses of inflation and consumption to taxes on borrowers are reported in the following proposition (a proof of which can be found in the Appendix):

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<sup>16</sup>Beyond that threshold (if there are "too many" borrowers or if labor supply is "too inelastic") the effects of all shocks are overturned: the slope of the aggregate IS curve changes sign and interest rate increases become expansionary. See Bilbiie (2008) and Bilbiie and Straub (2011) for a detailed analysis.

**Proposition 3** *In response to an exogenously persistent redistribution from savers to borrowers  $\epsilon_t$ , inflation and consumption (output) follow:*

$$\begin{aligned}\pi_t &= \frac{\beta^{-1}\kappa\delta^{-1}\eta(1-\rho)}{\det}\epsilon_t \\ c_t &= \frac{\delta^{-1}\eta(1-\rho)(\beta^{-1}-\rho)}{\det}\epsilon_t,\end{aligned}$$

where  $\det = (1-\rho)(\beta^{-1}-\rho) + \beta^{-1}\kappa\delta^{-1}(\phi_\pi - 1)\rho$ .

Qualitatively, the responses are the same as above: inflation and consumption increase when there is an exogenous tax cut to borrowers. Quantitatively, it can be easily shown that the multiplier on consumption is *decreasing* with the persistence parameter  $\rho$  (implying that it is always lower than the multiplier derived under zero persistence). This happens because a persistent shock generates two effects. First, it raises expected future inflation and hence—via the monetary policy rule—the real interest rate, which works to reduce savers’ consumption through intertemporal substitution. Second, it enhances the negative wealth effect on labor supply by the savers, inducing them, at the margin, to reduce consumption further.<sup>17</sup>

## 4.2 Public Debt

We next turn to the effects of a uniform tax cut financed via issuing public debt, focusing on purely transitory shocks ( $\mathbb{E}_t\epsilon_{t+1}^B = 0$ ) in order to isolate the role of the endogenous propagation of this tax shock through the debt accumulation process.

Debt is issued in order to finance the tax cut of  $\epsilon_t^B$ , and is repaid gradually through uniform taxes in the future; therefore, although the tax cut itself is purely transitory, its effect can be potentially long-lived because of debt accumulation and its persistence and magnitude are governed by the feedback coefficient  $\phi_B$ .

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<sup>17</sup>Finally, the effects are only slightly different if the monetary authority responds to variations in realized, rather than expected inflation. We show in the Appendix that the effects of redistribution are in that case dampened; intuitively, in response to today’s inflation due to the demand effect, the monetary authority increases the real interest rate which makes savers cut consumption today by intertemporal substitution.

In order to illustrate the mechanism, it is useful to consider the two extreme scenarios described in Section 2.3 above, corresponding, respectively, to  $\phi_B = 1$  and  $\phi_B = 1 - \beta_s$ .

#### 4.2.1 Full Debt Stabilization

In the case  $\phi_B = 1$  a tax cut today is debt-financed and fully repaid in the next period: this amounts to a redistribution from savers to borrowers today, and from borrowers to savers in the next period.

Replacing the tax process (19) in the demand curve (24) we obtain:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1} (i_t - \mathbb{E}_t \pi_{t+1}) - \delta^{-1} \eta b_t + \delta^{-1} \eta (1 + \beta_s^{-1}) \epsilon_t^B. \quad (27)$$

Using (27) one can derive the solutions for consumption and inflation, which are shown in the following Proposition (the proof can be found in the Appendix).

**Proposition 4** *In response to a tax cut financed by public debt which is fully repaid next period by uniform taxation, consumption and inflation are given by:*

$$\begin{aligned} c_t &= \delta^{-1} \eta [1 + \delta^{-1} \kappa (\phi_\pi - 1) \beta_s^{-1}] \epsilon_t^B - \delta^{-1} \eta b_t, \\ \pi_t &= \kappa^2 \delta^{-2} \eta (\phi_\pi - 1) \beta_s^{-1} \epsilon_t^B - \kappa \delta^{-1} \eta b_t, \\ c_{t+1} &= -\delta^{-1} \eta \beta_s^{-1} \epsilon_t^B, \\ \pi_{t+1} &= -\kappa \delta^{-1} \eta \beta_s^{-1} \epsilon_t^B, \\ \text{and } \pi_{t+i} &= c_{t+i} = 0 \quad \forall i \geq 2. \end{aligned}$$

The foregoing results can be explained intuitively as follows. Consider, to start with, the equilibrium values obtained in period  $t+1$ : in present-value terms, they are equal (but of opposite sign) to the responses of consumption and inflation to a pure redistribution described in section 4.1 above, precisely because our experiment is akin to a (reverse) redistribution from borrowers to savers in period  $t+1$ .

In period  $t$ , however, we have two effects. First, the usual effect of redistribution on consumption, summarized by the term  $\delta^{-1} \eta (\epsilon_t^B - b_t)$ ; second, an *additional* effect equal to

$\delta^{-2}\eta\kappa(\phi_\pi - 1)\beta_s^{-1}\epsilon_t^B$ , that is driven by intertemporal substitution by the savers and can be explained as follows.

In period  $t + 1$ , firms are faced with lower demand (due to the reverse redistribution from borrowers to savers) and cut prices, creating deflation. Savers react to this by not changing their consumption at all: the real interest rate does not move since expected inflation at  $t + 2$  is zero.<sup>18</sup> At time  $t$ , the expected deflation implies a cut in the ex-ante real interest rate today, and (since tomorrow's consumption is unchanged) an increase in savers' consumption today—once more, by intertemporal substitution. Finally, in equilibrium firms correctly anticipate lower demand in the future and increase prices today by less than they would if redistribution were not 'reversed' in the future.

Notice that, consistent with this intuition, this reinforcing effect under public debt disappears when either prices are fixed ( $\theta \rightarrow 1$ ), or there are no savers, and hence no intertemporal substitution ( $\lambda = 1$ , which implies  $\delta^{-1} \rightarrow 0$ ), or no endogenous movements in real interest rates ( $\phi_\pi = 1$ ); finally, the effect also disappears when there are no borrowers ( $\lambda = 0 \rightarrow \eta = 0$ ), consistently with Ricardian equivalence.

The above results allow us to compute the *present-value* aggregate consumption multiplier of a debt-financed tax cut in the full-stabilization case,  $\mathcal{M}_{debt}^{FS}$ :

$$\mathcal{M}_{debt}^{FS} \equiv \frac{\partial(c_t + \beta_s c_{t+1})}{\partial \epsilon_t^B} = \delta^{-2}\eta\kappa(\phi_\pi - 1)\beta_s^{-1} > 0, \quad (28)$$

where there is discounting at the steady-state real interest rate (which is determined by the savers' discount factor). Equation (28) immediately implies that the present-value multiplier of public debt is *higher* under sticky relative to flexible prices:

$$\mathcal{M}_{debt}^{FS} > \mathcal{M}_{debt}^* = 0.$$

Moreover, (28) shows that  $\mathcal{M}_{debt}$  is identical to the intertemporal effect described previously, i.e., to the component of the period- $t$  consumption multiplier of a uniform

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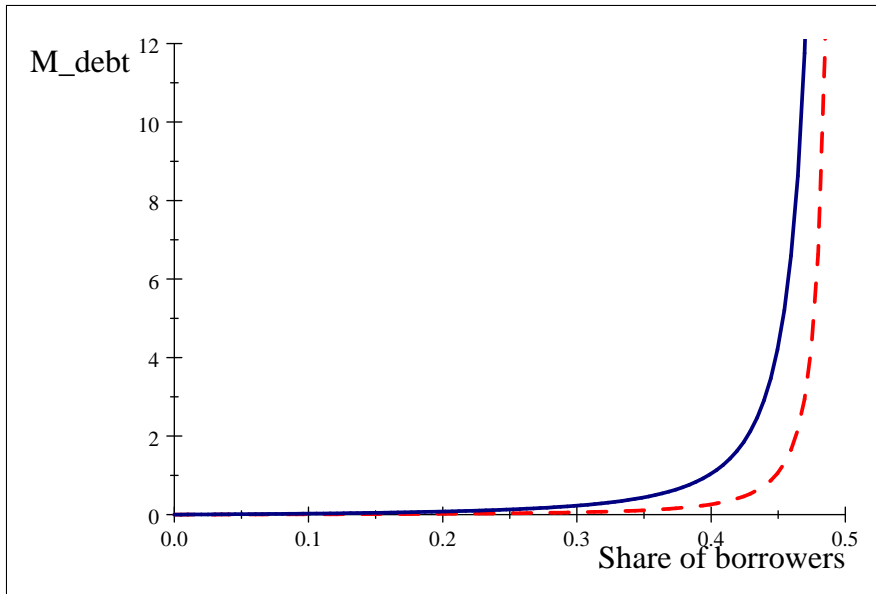
<sup>18</sup>Matters are different when the monetary authority responds to realized, rather than expected inflation, but without affecting the conclusion qualitatively; we discuss this further below



tax cut that is over and above the multiplier due to pure redistribution, and is due to intertemporal substitution.

We can assess the *magnitude* of  $\mathcal{M}_{debt}$  by looking at a parameterization that is standard in the literature, namely: unitary inverse Frisch elasticity  $\varphi = 1$ , average price duration of one year ( $\theta = 0.75$ ), steady-state markup of 0.2 ( $\epsilon = 6$ ), and discount factor of savers  $\beta_s = 0.99$ .

Figure 4.2.1 plots the value of  $\mathcal{M}_{debt}$  for the whole range of the share of borrowers  $\lambda$  for which the elasticity of aggregate demand to the interest rate is positive ( $\delta > 0$ ), namely  $\lambda < 0.5$ . We consider two values of the inflation elasticity of interest rates:  $\phi_\pi = 1.5$  (red dashed line) and  $\phi_\pi = 3$  (blue solid line) respectively; consistent with our analytical results and intuition, the multiplier is uniformly larger for the higher value of  $\phi_\pi$ . At low values of  $\lambda$ , until about 0.4, the multiplier is very small, below 1 percent. But when approaching the threshold beyond which the economy moves to the "inverted" region, the multiplier becomes very large.<sup>19</sup>



<sup>19</sup>For instance, under  $\phi_\pi = 3$ , it is about 4 percent when  $\lambda = 0.45$  and about 12 percent when  $\lambda = 0.47$ . The reason for this abrupt increase is that the elasticity of aggregate demand to real interest rates  $\delta^{-1}$  approaches infinity when  $\lambda$  approaches that threshold value; see Bilbiie (2008) for an elaboration of that point.

Present-value, discounted multiplier as a function of  $\lambda$  for the baseline parameterization, for  $\phi_\pi = 1.5$  (red dashed);  $\phi_\pi = 3$  (blue solid) respectively.

#### 4.2.2 No debt stabilization

When  $\phi_B$  tends to its lower bound given by  $1 - \beta_s$ , the effects of a debt-financed uniform tax cut are almost identical to the effects of pure redistribution. The intuition for this result is simple: when debt repayments are pushed into the far future, savers fully internalize the government budget constraint; taxation in the future is, for them, equivalent to taxation today. But for the borrowers, a tax cut today is disposable income. Therefore, a uniform tax cut becomes equivalent to a pure redistribution within the period when the uniform tax cut is financed with very persistent debt.

Formally, this can be seen by replacing the tax process (20) in the IS curve (24) and first noticing that, since the path of taxes from period  $t + 1$  onwards is constant ((20) implies that  $t_{b,t+i} = E_t t_{b,t+i+1}$  for  $i > 1$ ), all variables go back to steady state in period  $t + 1$ ;<sup>20</sup> Therefore, the tax cut only has an effect at time  $t$ , when the IS curve is:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1} (i_t - \mathbb{E}_t \pi_{t+1}) + \delta^{-1} \eta \beta_s^{-1} \epsilon_t^B,$$

which is the same as the Euler equation (24), obtained for a (purely transitory) redistribution shock of  $t_{b,t} = -\beta_s^{-1} \epsilon_t^B$ . Therefore, the effects of this policy are exactly identical to those obtained in Section 4.1 above when the size of the redistribution is  $\beta_s^{-1} \epsilon_t^B$ , including the present-value multiplier:

$$\mathcal{M}_{debt}^{NS} = \delta^{-1} \eta \beta_s^{-1} = \beta_s^{-1} \mathcal{M}_{red}.$$

It is important to notice that the two policy experiments are very different in nature: one—redistribution—changes the present discounted value of income for both agents, while the other—the other—uniform tax cut—does not. Yet, they have identical effects, because what is important for propagation is the *intertemporal substitution* induced by

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<sup>20</sup>Specifically, for any  $i > 0$  the IS curve becomes:  $c_{t+i} = E_{t+i} c_{t+i+1} - \delta^{-1} (\phi_\pi - 1) E_t \pi_{t+i+1}$ , which together with the Phillips curve implies the unique solution  $c_{t+i} = \pi_{t+i} = 0$ .

changes in taxation over time. With permanently higher taxes from tomorrow onwards, there is no such intertemporal substitution and the only force at work is redistribution today.

### 4.2.3 Endogenously Persistent Debt

When  $\phi_B$  takes on intermediate values—so that debt stabilization is neither immediate nor postponed into the far future—the interplay of income effects, intertemporal substitution by savers and the demand effect due to sticky prices generates different responses that feature endogenous persistence.

In order to solve the model in this more general case, we exploit our previous intuition that a uniform tax cut financed by persistent debt can be reinterpreted as a transfer from savers (the holders of the debt used to finance the tax cut) to borrowers in the period when the tax cut takes place, followed by a—possibly persistent—transfer from borrowers to savers from next period onwards, when debt is being repaid. The model solution from period  $t + 1$  onwards hence closely resembles the solution under a persistent transfer outlined in Proposition 3, while the solution at time  $t$  (when policy is implemented) mirrors that of Proposition 4. The full solution of the model is outlined in the Appendix, and the following Proposition emphasizes the present-value multiplier on consumption.

**Proposition 5** *In response to a one-time uniform tax cut  $\epsilon_t^B$  financed by issuing public debt, the present-value consumption multiplier is*

$$\begin{aligned}\mathcal{M}_{debt} &\equiv \frac{\partial \left( \sum_{i=0}^{\infty} \beta_s^i c_{t+i} \right)}{\partial \epsilon_t^B} \\ &= \frac{\delta^{-2} \eta \beta_s^{-1} \kappa (\phi_{\pi} - 1)}{\phi_B (1 - (1 - \phi_B) \beta_s^{-1}) + (1 - \phi_B) \beta_s^{-1} \delta^{-1} \kappa (\phi_{\pi} - 1)} > 0\end{aligned}$$

Note, to start with, that this solution nests the particular cases of full and no debt stabilization, respectively, when  $\phi_B = 1$  and  $\phi_B = 1 - \beta_s$ .

The key finding is that the present-value multiplier of debt is *positive*, and hence larger than the one under flexible prices, regardless of the value of  $\phi_B$  satisfying (16).<sup>21</sup> The

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<sup>21</sup>This holds as long as we restrict attention to the "standard" region whereby  $\delta > 0$ , and the Taylor principle is satisfied ( $\phi_{\pi} > 1$ ).

intuition for this is similar to the one outlined above in the extreme case of full debt stabilization: the effects of debt go beyond the mere sum of the implied intertemporal redistributions, through intertemporal substitution generated by the movements in the real interest rate. The expectation of a future deflation triggered by the *de facto* reversal of the transfer in the future induces a fall in the long-run real interest rate today, and hence boosts savers' consumption today by intertemporal substitution. Consistent with this intuition, the multiplier collapses to zero when the intertemporal-substitution channel is shut off (i.e. when prices are fixed and  $\kappa = 0$  or  $\phi_\pi = 1$ ). Furthermore, the stronger is this intertemporal substitution channel (i.e., the less sticky are prices—the higher  $\kappa$ —or the more aggressive is monetary policy—the higher is  $\phi_\pi$ ), the larger is the multiplier.<sup>22</sup> Lastly, the intertemporal-substitution channel becomes irrelevant when there is no debt stabilization, for in that case there is in fact no intertemporal substitution: the expansionary effect of the tax cut is due solely to the redistribution from savers to borrowers today.

While the present-value multiplier is positive regardless of the speed of debt repayment  $\phi_B$  (as long as (16) holds), its magnitude depends non-trivially on this parameter. It can be shown that for plausible calibrations (namely, if  $\phi_\pi < 1 + \delta\kappa^{-1}$ , which is 11.754 under our baseline calibration), the present-value multiplier is larger under "no stabilization" than under "full stabilization". The intuition is that in those cases (if monetary policy is not too aggressive, prices are sticky enough) the intertemporal substitution channel present when debt is repaid more abruptly is weaker than the more direct expansionary effect of a redistribution today.

Figure 1 illustrates these findings by plotting the responses of consumption, inflation and public debt to a purely transitory uniform tax cut—for the baseline parameterization described above—under three scenarios. The (blue) solid line corresponds to the "full debt stabilization" scenario, and the red squared line is close to the "no debt stabilization" scenario, both of which are explained at length above. The green line plots the responses obtained for  $\phi_B = 0.5$ . The fall in consumption and inflation in the second

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<sup>22</sup>It is easy to show that  $\mathcal{M}_{debt}$  is increasing in both  $\kappa$  and  $\phi_\pi$ , as long as there is some debt stabilization  $\phi_B > 1 - \beta_s$ .

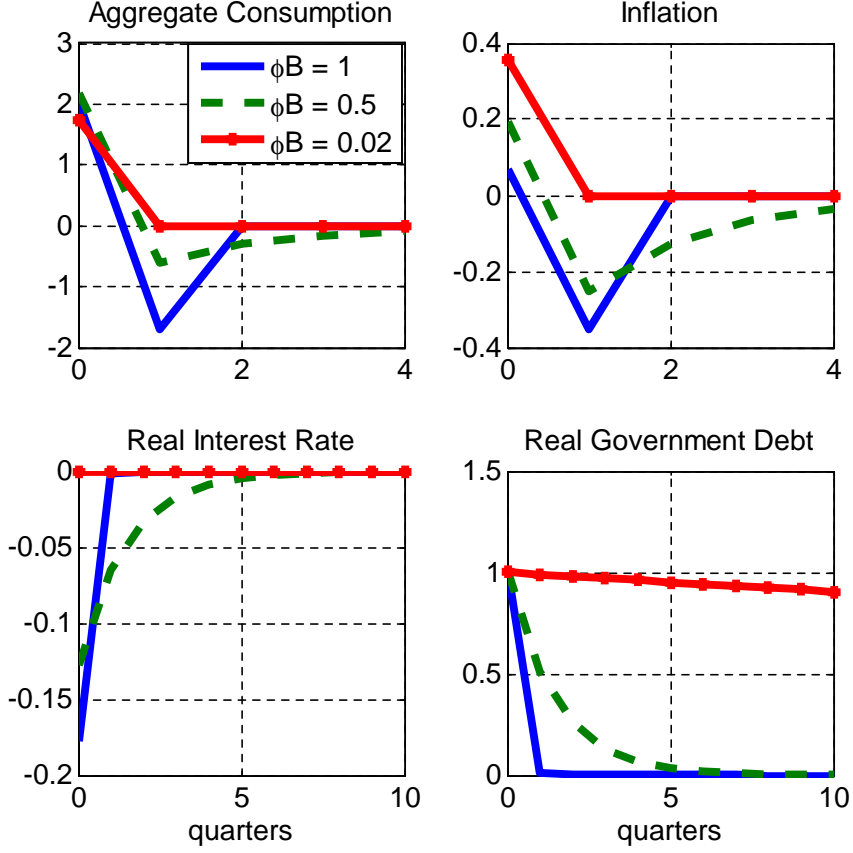


Figure 1: Impulse responses to a debt-financed uniform tax cut, for different debt stabilization coefficients  $\phi_B$ .

period is smaller than under full debt stabilization, but the recession and deflation last longer. The intuition for these intermediary values is similar to that obtained under full stabilization. The only differences are that future transfers from borrowers to savers last longer themselves (recall the effects derived for persistent redistribution), and the initial implicit transfer of period  $t + 1$  is lower (recall the discussion of (18)).

While the results above were derived under the special assumptions that the private debt limit is zero, and steady-state public debt is also zero, we emphasize that they are robust to relaxing those assumptions. The reason is that the main difference, when

relaxing either of those assumptions, is related to interest payments—on either private or public debt, respectively—which turn out to be quantitatively negligible. Results for these—and other—robustness experiments are available in an online Appendix (not for publication).

## 5 Conclusions

We have shown that in an economy with financial imperfections, somewhat surprisingly, Ricardian equivalence holds when prices are flexible and either labor supply is inelastic, or it is elastic but the steady-state (or initial) consumption distribution is uniform. Income effects in that setup are symmetric, so one agent's decision to consume less (and, if labor is elastic, work more) is exactly compensated by another agent's decision to consume more (and work less). When the steady-state distribution of consumption is *not* uniform, Ricardian equivalence does fail, but the effects of changes in lump-sum taxes are paradoxical when judged against the findings of a large empirical literature reviewed above: both a "Robin Hood" redistribution that favors the constrained borrowers and a uniform tax cut financed with public debt are contractionary. Key to this result is the asymmetry in the group-type income effects on labor supply. However, the present-value multiplier of public debt on consumption is always zero.

Under sticky prices, lump-sum tax policies are never neutral, even when the steady-state distribution of consumption is uniform. In this environment, a "Robin Hood" redistribution that favors the borrowers is expansionary on aggregate activity. Unlike under flexible prices, a uniform tax cut financed with public debt has a positive present-value multiplier effect on consumption. In other words, the effects of debt go beyond the mere sum of the implicit intertemporal redistributions (from savers to borrowers today, and vice-versa from tomorrow onwards). Key to this result is intertemporal substitution by the savers: the perspective of a deflationary recession tomorrow triggers a fall in interest rate today, boosting savers' consumption today. For this reason, and although the policy change has no wealth effect *per se* (it does not change the lifetime income of agents), the

present-value multiplier on consumption is positive.

The finding that the present-value multiplier is positive holds regardless of how quickly debt is repaid—although the speed of debt stabilization does influence the magnitude of the multiplier—as long as it is indeed repaid. In the limit, when the tax cut is financed by *permanently* higher taxes from next period onwards (a scenario we label "no debt stabilization"), its effects are identical to those of a "Robin Hood" redistribution today; the reason is that a constant path of taxes from tomorrow onwards generates no intertemporal substitution and hence no effects on aggregate variables beyond the effects of a pure redistribution today.

In order to focus on one source of failure of Ricardian equivalence (sticky prices) we abstracted from another modeling feature that would generate realistic departures from Ricardian equivalence even under flexible prices, namely endogenous investment (for instance, in physical capital). The implications of that assumption have been explored in models with two types of agents elsewhere (see for instance Mankiw, 2000). The interaction of endogenous investment and endogenous borrowing limits is certainly worth exploring, but is beyond the scope of this paper.

## A Appendix

**Proof of Proposition 3.** Rewrite the system as

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t c_{t+1} \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} + \Upsilon_R \epsilon_t, \quad (29)$$

where  $\mathbf{\Gamma} = \begin{bmatrix} \beta_s^{-1} & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & 1 - \beta_s^{-1}\delta^{-1}\kappa(\phi_\pi - 1) \end{bmatrix}$ ,  $\Upsilon_R = \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1 - \rho) \end{bmatrix}$ .

The impulse response functions are calculated as:

$$\begin{aligned} \Omega &= [\rho I - \mathbf{\Gamma}]^{-1} \Upsilon_R \\ &= \frac{1}{\det} \begin{bmatrix} \rho - 1 + \beta_s^{-1}\kappa\delta^{-1}(\phi_\pi - 1) & -\beta_s^{-1}\kappa \\ \beta_s^{-1}\delta^{-1}(\phi_\pi - 1) & \rho - \beta_s^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ -\delta^{-1}\eta(1 - \rho) \end{bmatrix} \\ &= \frac{1}{\det} \begin{bmatrix} \beta_s^{-1}\kappa\delta^{-1}\eta(1 - \rho) \\ \delta^{-1}\eta(1 - \rho)(\beta_s^{-1} - \rho) \end{bmatrix}. \end{aligned}$$

**Proof of Proposition 4.** Note that although the exogenous shock has zero persistence, there is endogenous persistence due to the presence of a state variable, public debt; but that endogenous persistence takes a very special form under our assumption that debt is repaid next period: the effects of the shock will live for two periods only. Therefore, in order to solve the model we must solve for the endogenous variables in periods  $t$  and  $t + 1$ . We do this by solving the model backwards as follows: next period's consumption is determined by the Euler equation at  $t + 1$ :

$$c_{t+1} = \mathbb{E}_{t+1} c_{t+2} - \delta^{-1}(i_{t+1} - \mathbb{E}_{t+1} \pi_{t+2}) - \delta^{-1}\eta b_{t+1} + \delta^{-1}\eta(1 + \beta_s^{-1})\epsilon_{t+1}^B - \delta^{-1}\eta E_t \epsilon_{t+2}^B,$$

which under zero persistence and the assumption that debt is repaid at  $t + 1$  (and so  $i_{t+1} = \mathbb{E}_{t+1} \pi_{t+2} = \mathbb{E}_{t+1} c_{t+2} = 0$ ) delivers:

$$c_{t+1} = -\delta^{-1}\eta\beta_s^{-1}\epsilon_t^B = \mathbb{E}_t c_{t+1},$$

where the second equality holds because the shock  $\epsilon_t^B$  is in the information set at time  $t$ . From the Phillips curve at  $t + 1$ , imposing  $\mathbb{E}_{t+1} \pi_{t+2} = 0$ , we have:

$$\pi_{t+1} = -\kappa\delta^{-1}\eta\beta_s^{-1}\epsilon_t^B = \mathbb{E}_t \pi_{t+1}.$$



The impact multiplier, substituting these expressions in the Euler equation at time  $t$  is:

$$c_t = \delta^{-1}\eta \left[ 1 + \delta^{-1}\kappa(\phi_\pi - 1)\beta_s^{-1} \right] \epsilon_t^B - \delta^{-1}\eta b_t,$$

and inflation is, from the Phillips curve:

$$\pi_t = \kappa^2\delta^{-2}\eta(\phi_\pi - 1)\beta_s^{-1}\epsilon_t^B - \kappa\delta^{-1}\eta b_t.$$

### Effects of Redistribution and Public Debt under a Contemporaneous Taylor Rule

Suppose that the Taylor rule responds to realized inflation:

$$i_t = \phi\pi_t. \tag{30}$$

In the case of *redistribution* with zero persistence, the effects are obtained by merely replacing (30) in the IS curve (24):

$$c_t = \frac{\delta^{-1}\eta}{1 + \delta^{-1}\phi\kappa}\epsilon_t; \quad \pi_t = \frac{\delta^{-1}\eta\kappa}{1 + \delta^{-1}\phi\kappa}\epsilon_t.$$

Redistribution has smaller effects than those obtained under a forward-looking Taylor rule. The reason is that the inflationary effect of redistribution triggers an increase in the real interest rate, which in turn induces savers to consume and work less today (the term  $\delta^{-1}\phi\kappa$  in the denominator). This effect disappears when either prices are fixed ( $\kappa = 0$  and there is no inflation in equilibrium) or there are no savers ( $\lambda \rightarrow 1$  implies that  $\delta^{-1} \rightarrow 0$ ).

Under *public debt* with perfect stabilization ( $\phi_B = 1$ ), the tax process (19) replaced in the IS curve at times  $t$  and  $t + 1$  delivers, respectively:

$$\begin{aligned} c_t &= \mathbb{E}_t c_{t+1} - \delta^{-1}(\phi\pi_t - \mathbb{E}_t \pi_{t+1}) - \delta^{-1}\eta b_t + \delta^{-1}\eta(1 + \beta_s^{-1})\epsilon_t^B, \\ c_{t+1} &= -\delta^{-1}\phi\pi_{t+1} - \delta^{-1}\eta\beta_s^{-1}\epsilon_t^B, \end{aligned}$$

while the Phillips curves are

$$\begin{aligned} \pi_t &= \beta\mathbb{E}_t \pi_{t+1} + \kappa c_t, \\ \pi_{t+1} &= \kappa c_{t+1}, \end{aligned}$$

where we have accounted for the variables returning to steady state from period  $t + 2$  onwards. Solving the above system, we obtain:

$$\begin{aligned} c_{t+1} &= -\frac{\delta^{-1}\eta\beta_s^{-1}}{1 + \delta^{-1}\phi\kappa}\epsilon_t^B, \\ \pi_{t+1} &= -\kappa\frac{\delta^{-1}\eta\beta_s^{-1}}{1 + \delta^{-1}\phi\kappa}\epsilon_t^B, \\ c_t &= \frac{\delta^{-1}\eta[1 + 2\delta^{-1}\phi\kappa + \delta^{-1}\kappa\beta_s^{-1}(\phi - 1)]}{(1 + \delta^{-1}\phi\kappa)^2}\epsilon_t^B - \frac{\delta^{-1}\eta}{(1 + \delta^{-1}\phi\kappa)}b_t, \\ \pi_t &= \frac{\delta^{-2}\kappa^2\eta[\phi + \beta_s^{-1}(\phi - 1)]}{(1 + \delta^{-1}\phi\kappa)^2}\epsilon_t^B - \frac{\delta^{-1}\eta\kappa}{(1 + \delta^{-1}\phi\kappa)}b_t. \end{aligned}$$

The present-value multiplier is:

$$\mathcal{M}_{debt} \equiv \frac{\partial(c_t + \beta_s c_{t+1})}{\partial\epsilon_t^B} = \delta^{-2}\eta\kappa \frac{[\phi + \beta_s^{-1}(\phi - 1)]}{(1 + \delta^{-1}\phi\kappa)^2}.$$

The effects differ from those obtained under a forward-looking rule (in Proposition 4) as follows. There is still deflation in period  $t + 1$  for the same reason as under a forward-looking rule (transfer from saver to borrower). As the real interest rate falls with realized deflation, savers react increasing their consumption at  $t + 1$  relative to  $t + 2$  (when the economy returns to steady state). The expected increase in savers' consumption tomorrow implies that an increase in inflation today—coming from the demand effect of redistribution to borrowers in the first period—will trigger a relatively smaller fall in consumption of savers at time  $t$  relative to the case of pure redistribution—once again, because of intertemporal substitution. In equilibrium, firms correctly anticipate lower demand in the future and increase prices today by less than they would if redistribution were not 'reversed' in the future; so inflation increases by less, reinforcing the effect described previously. The present-value aggregate consumption multiplier of a debt-financed tax cut is positive in this case too, and has the same interpretation as for a forward-looking rule.

#### **Analytical solution with endogenously persistent debt $\phi_B < 1$ .**

Replacing the debt accumulation equation (15) into the Euler equation, we obtain:

$$c_t = \mathbb{E}_t c_{t+1} - \delta^{-1}(i_t - \mathbb{E}_t \pi_{t+1}) + \delta^{-1}\eta\phi_B [\beta_s^{-1}(1 - \phi_B) - 1] b_t + \delta^{-1}\eta(1 + \phi_B\beta_s^{-1})\epsilon_t^B. \quad (31)$$

This is a reduced-form IS curve for a given level of public debt; together with (25) and (26) it can be solved to determine consumption and output as a function of outstanding debt and the fiscal shock. The system to be solved is:

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t c_{t+1} \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} \pi_t \\ c_t \end{bmatrix} + \mathbf{\Psi} b_t + \mathbf{\Upsilon}_B \epsilon_t^B, \quad (32)$$

$$\begin{aligned} \text{where } \mathbf{\Gamma} &= \begin{bmatrix} \beta_s^{-1} & -\beta_s^{-1} \kappa \\ \beta_s^{-1} \delta^{-1} (\phi_\pi - 1) & 1 - \beta_s^{-1} \delta^{-1} \kappa (\phi_\pi - 1) \end{bmatrix}, \\ \mathbf{\Psi} &= \begin{bmatrix} 0 \\ \delta^{-1} \eta \phi_B [1 - \beta_s^{-1} (1 - \phi_B)] \end{bmatrix}; \quad \mathbf{\Upsilon}_B = \begin{bmatrix} 0 \\ -\delta^{-1} \eta (1 + \phi_B^2 \beta_s^{-1}) \end{bmatrix}. \end{aligned} \quad (33)$$

Using the method of undetermined coefficients, we can guess and verify that the solution takes the form:

$$\begin{bmatrix} \pi_t \\ c_t \end{bmatrix} = \mathbf{A}_B b_t + \mathbf{A}_\epsilon \epsilon_t^B,$$

which substituted in the original system (using also the public debt dynamics equation) delivers:

$$\mathbf{A}_B (1 - \phi_B) \beta_s^{-1} b_t + \mathbf{A}_B \beta_s^{-1} \epsilon_t^B = \mathbf{\Gamma} \mathbf{A}_B b_t + \mathbf{\Gamma} \mathbf{A}_\epsilon \epsilon_t^B + \mathbf{\Psi} b_t + \mathbf{\Upsilon}_B \epsilon_t^B.$$

Identifying coefficients:

$$\begin{aligned} \mathbf{A}_B &= [(1 - \phi_B) \beta_s^{-1} \mathbf{I} - \mathbf{\Gamma}]^{-1} \mathbf{\Psi}, \\ \mathbf{A}_\epsilon &= \mathbf{\Gamma}^{-1} (\mathbf{A}_B \beta_s^{-1} - \mathbf{\Upsilon}_B). \end{aligned}$$

$$\begin{aligned} \mathbf{A}_B &= \frac{1}{\det} \begin{bmatrix} (1 - \phi_B) \beta_s^{-1} - 1 + \beta_s^{-1} \kappa \delta^{-1} (\phi_\pi - 1) & -\beta_s^{-1} \kappa \\ \beta_s^{-1} \delta^{-1} (\phi_\pi - 1) & -\phi_B \beta_s^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ \delta^{-1} \eta \phi_B [1 - \beta_s^{-1} (1 - \phi_B)] \end{bmatrix}, \\ \mathbf{A}_B &= -\frac{\delta^{-1} \eta \phi_B [1 - \beta_s^{-1} (1 - \phi_B)]}{\det} \begin{bmatrix} \beta_s^{-1} \kappa \\ \beta_s^{-1} \phi_B \end{bmatrix}, \\ \det &= (1 - (1 - \phi_B) \beta_s^{-1}) \beta_s^{-1} \phi_B + \beta_s^{-2} \kappa \delta^{-1} (\phi_\pi - 1) (1 - \phi_B) > 0 \end{aligned}$$

The multipliers on consumption are:

$$\begin{aligned} \frac{\partial c_t}{\partial \epsilon_t^B} &= \delta^{-1} \eta \left( 1 + \frac{\phi_B \beta_s^{-1} \delta^{-1} \kappa (\phi_\pi - 1)}{\phi_B (1 - (1 - \phi_B) \beta_s^{-1}) + (1 - \phi_B) \beta_s^{-1} \delta^{-1} \kappa (\phi_\pi - 1)} \right), \\ \frac{\partial c_{t+i}}{\partial \epsilon_t^B} &= -\frac{\delta^{-1} \eta \phi_B (1 - (1 - \phi_B) \beta_s^{-1})}{\phi_B (1 - (1 - \phi_B) \beta_s^{-1}) + (1 - \phi_B) \beta_s^{-1} \delta^{-1} \kappa (\phi_\pi - 1)} \phi_B (1 - \phi_B)^{i-1} \beta_s^{-i}, \text{ for } i \geq 1. \end{aligned}$$

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