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VARs, common factors and the empirical validation of equilibrium business cycle models

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Abstract

Equilibrium business cycle models have typically less shocks than variables. As pointed out by Altug, 1989 and Sargent, 1989, if variables are measured with error, this characteristic implies that the model solution for measured variables has a factor structure. This paper compares estimation performance for the impulse response coefficients based on a VAR approximation to this class of models and an estimation method that explicitly takes into account the restrictions implied by the factor structure. Bias and mean squared error for both factor based and VAR based estimates of impulse response functions are quantified using, as data generating process, a calibrated standard equilibrium business cycle model. We show that, at short horizons, VAR estimates of impulse response functions are less accurate than factor estimates while the two methods perform similarly at medium and long run horizons.

JEL subject classification : E32, C33, C52

Key words and phrases : Dynamic factor models, structural VARs, identification, equilibrium business cycle models.

1 Introduction

The basic econometric tool for empirical validation of macroeconomic models is the Vector Autoregressive Model (VAR). This model is easy to estimate and, once identification restrictions are imposed, it can be used to evaluate the impact of economic shocks on key variables.

In structural VAR macroeconomics, variables are represented as driven by serially uncorrelated shocks, each having a different source or nature, like "demand", "supply", "technology", "monetary policy" and so on. Each variable reacts to a particular shock with a specific sign, intensity and lag structure, summarized by the so called "impulse-response function". Implications of economic theory not used for identification can then be compared with estimation results and tested.

A strong motivation for the use of VARs is that stochastic general equilibrium macroeconomic models have solution that can be represented in VAR form and therefore VAR econometrics provide the tool to bridge theory and data.

The typical theoretical macro model, however, has few shocks driving the key variables in the macroeconomy. In the first generation real business cycle models, for example, one shock – technology – is responsible for volatility of output, consumption and investment both in the short and long-run. In that stylized economy, there is only one source of variation. Other models take into account shocks in preferences or money, but sources of macro variations remain few.

The implication of this feature is that equilibrium business cycle models have reduced stochastic rank (i.e. the spectral density of the observation has reduced rank). A further implication, as observed by Altug, 1989 and Sargent, 1989, is that, when variables are measured with errors, the model for measured variables has a dynamic factor analytic structure. Since it can be easily shown that, with measurement error, the reduced form solution follows a VARMA model, from the estimation point of view, there are two approximations to the measured model that one might consider: a VAR model with a sufficiently large number of lags or a method that takes explicitly into account the restrictions implied by the model (factor model estimation).

Dynamic factor models imply a restriction on the spectral density of the observations whereby the latter can be expressed as the sum of two orthogonal components; the spectral density of the common component, of reduced rank, and the spectral density of the idiosyncratic component, of full rank. The former captures all the covariances of the observations at leads and lags while the latter is diagonal and can therefore represents non cross-correlated measurement error.

The factor literature, which has wide applications in many fields other than economics, has been first introduced in macroeconomics by Sargent and Sims, 1977 and Geweke, 1977 and further developed by Geweke and Singleton, 1981 and Engle and Watson, 1983. Recently, factor models have been rediscovered in macroeconomics as a tool for analyzing large panels of time series (Forni and Reichlin, 1998, Forni, Hallin, Lippi and Reichlin, 2000, Stock and Watson, 2002 and related literature). For empirical evidence of stochastic rank reduction, see Altissimo et al, 2002 on European data and Giannone et al., 2002 on US data.

The objective of this paper is to evaluate the performance of these two alternative approximations under different hypotheses on the size of measurement error. We will perform this comparison by generating data from a simple business cycle model with and without measurement error and comparing impulse response estimates from VAR and factor procedures. This model, although extremely simple, has the basic features of rank reduction, common to more complex business cycle models and it is therefore suited for the controlled experiment performed in this paper.

Measurement errors, we will show, contaminate VAR impulse response functions at all horizons. Under the assumption of poorly autocorrelated measurement error and persistent dynamic in the model economy, the contamination affects contemporaneous and short-term responses in particular, while factor estimates are more precise in the short run and provide a similar degree of precision at all horizons, which, as to be expected, depends on the size of the error. The intuition of this result is that the factor model helps to clean data from measurement error by exploiting the theoretical (and empirical) feature of stochastic rank reduction.

Our results suggest that VAR estimates are more reliable in the medium and long-run than in the short-run.

They also explain the empirical finding that while macroeconomic time series co-move at low and business cycle frequencies, it is more difficult to find evidence of rank reduction at higher frequencies: at low frequencies, economic variables are less contaminated by measurement error and, as a consequence, underlying collinear relations are more evident than at higher frequencies.

The paper is organized as follows. In the first section, we will describe the general linear solution of equilibrium business cycle models and then illustrate a special simple case. In the second section, we discuss VAR and factor estimates with and without measurement errors. In the third, we perform the empirical experiment based on the simple model. In the fourth we explore model selection issues. The last section concludes.

2 A model economy and VAR analysis

2.1 Equilibrium business cycle models

A. General Structure

Let us recall the general structure of an equilibrium business cycle model. In this framework, as it is well known, the problem in the decentralized economy is the same as the social planner's. The latter maximizes the utility of the representative agent:

$$\max\ \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t U(X_t, Y_t) \right]$$

subject to the feasibility constraints::

$$f(X_t, X_{t-1}, \cdots, Y_t, Y_{t-1}, \cdots, S_t, S_{t-1}, \cdots) \leq 0$$

$$S_t = g(\epsilon_t, \epsilon_{t-1}, \cdots)$$

where X_t is the $m \times 1$ vector of endogenous predetermined variables, Y_t is the $n \times 1$ vector of the endogenous non predetermined variables and S_t is the $q \times 1$ vector of exogenous variables (the number of variables considered is therefore $N = m + n + q$). The parameter β defines the discount factor and ϵ_t is a q dimensional i.i.d. normal process with mean 0 and variance Σ_ϵ.

Stated at this level of generality, the model encompasses several examples in the literature, from the simple real business cycle model á la King, Plosser and Rebelo, 1991, to the time-to-build economy á la Kydland and Prescott, 1983 to the model with heterogenous capital (Campbell, 1997). Indicating with small letters the difference between the log of the variables and their non-stochastic steady state, the solution of such models has the following recursive structure:

$$\Psi(L)s_t = \epsilon_t$$

$$C(L)x_t = D(L)s_t$$

$$y_t = \Lambda_1(L)x_t + \Lambda_2(L)s_t$$

where:

$$C(L) = C_0 + C_1 L + \ldots + C_{p_c} L^{p_c}$$

$$D(L) = D_0 + D_1 L + \ldots + D_{p_d} L^{p_d}$$

$$\Lambda_1(L) = \Lambda_{1,1} L + \ldots + \Lambda_{1,p_{\Lambda_1}} L^{p_{\Lambda_1}}$$

$$\Lambda_2(L) = \Lambda_{2,0} + \Lambda_{2,1} L + \ldots + \Lambda_{2,p_{\Lambda_2}} L^{p_{\Lambda_2}}.$$

It should be noticed that this solution form applies even to a larger class of models than those based on the maximization problem described above. As Christiano, 2001, pointed out, more complex models with heterogeneous agents and different information sets, also have the same solution structure. This can be understood by noticing that the length of the filters $\Lambda_1(L)$ and $C(L)$ is determined by the lags of predetermined variables necessary for the determination of the endogenous and the predetermined variables while the filters $\Lambda_2(L)$ and $D(L)$ accomodate for the possibility that endogenous variables are determined on the basis of different information sets.

Defining the vector of all the observables as $w_t = [y_t' \; x_t' \; s_t']'$, the solution, written in its constrained VAR form, is:

$$A(L)w_t = B\epsilon_t \tag{2.1}$$

where:

$$A(L) = \begin{pmatrix} I_n & -\Lambda_1(L) & -\Lambda_2(L) \\ 0 & C(L) & -D(L) \\ 0 & 0 & \Psi(L) \end{pmatrix}$$

and:

$$B = \begin{pmatrix} 0_{(n\times q)} \\ 0_{(m\times q)} \\ I_q \end{pmatrix}$$

The dynamic rank of this system of equations, defined as the rank of the spectral density matrix of w_t is q, with $q < N$. The model, therefore, has reduced dynamic rank.

It is also customary to write the solution in its static state space representation where the vector of state variables includes the lagged predetermined variables, and current and lagged exogenous variables. The latter is defined as $F_t = [x'_{t-1} \dots x'_{t-p_x}\ s'_t \dots s'_{t-p_s}]'$, where $p_x = \max\{p_{\Lambda_1}, p_c\}$ and $p_s = \max\{p_{\Lambda_2}, p_d\}$, while the variables in the vector w_t are expressed as contemporaneous linear combinations of F_t:

$$w_t = \Lambda F_t \tag{2.2}$$

with:

$$H(L)F_t = K\epsilon_t. \tag{2.3}$$

The dimension of the vector of state variables in this static representation is $r = mp_x + q(p_s+1)$ and it therefore depends on the p_x and p_s lags included in the model as well as on q and m. This is also an upper bound for the rank of the contemporaneous variance-covariance matrix of w_t, $\Gamma_w(0) = \mathrm{E}w'w$ and defines the static rank of the system.

Static and dynamic rank reveal different features of the model economies.

Reduced dynamic rank q tells us that only q shocks matter for dynamics and therefore is a consequence of the characteristics of the exogenous forces driving the economy, while the static rank depends in general on the structure of the economy (the zero restrictions on the coefficients of the VAR form) and on the number of lags included[1]. Typically, models with rich dynamics, such as, for example, the time-to-build model á la Kydland and Prescott, 1983, have reduced stochastic rank but may have full static rank while simpler models have both reduced static and dynamic rank.

[1] A different restriction implies rank reduction of the lagged VAR matrices. In this case the solution will have reduced rank representation as in Ahn and Reinsel, 1988, and Velu et al., 1986, or *common features* as defined in Engle and Kozicki, 1993, and, under further restrictions, *common cycles* as in Vahid and Engle, 1993.

Static and dynamic ranks must be thought as restrictions, in principle testable, derived from theory. Moreover, rank reduction has implications for estimation that we will develop below.

To clarify the structure of the model and the role of the filters, as well as the role of rank reduction it will be useful to discuss a specific example of the general model. The same example will be used in the empirical section.

B. *The basic business cycle model*

What we illustrate here is a simplified version of King, Plosser and Rebelo, 1991, which is also the textbook example analyzed by Uhlig, 1998, to which we refer for all details.

The model can be seen as a special case of what discussed in subsection A, where there is only one source of variability – technology –, labor is exogenous, there are no time to build features, agents are homogeneous and have the same information set. We have: $n = 3$, $m = 1$, $q = 1$ and $\Psi(L) = 1 - \psi L$. The only exogenous state variable is productivity, z_t, which, with lagged capital stock k_{t-1}, form the vector of state variables. By using a standard functional form for the utility function, we can write the maximization problem as:

$$\max U = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta} - 1}{1-\eta} \right]$$

subject to:

$$C_t + K_t = Z_t K_{t-1}^{\rho} + (1-\delta) K_{t-1}$$

$$\log(Z_t) = \log \bar{Z} + \psi \log(Z_{t-1}) + \epsilon_t$$

where C_t, K_t define consumption and the capital stock and Z_t is the productivity exogenous process. The parameters β, δ, ρ, η and ψ define, respectively, the discount factor, the depreciation rate, the capital share, the coefficient of relative risk aversion and the autoregressive parameter governing persistence of the technology shock in the productivity equation.

Notice that in this case $p_{\Lambda_1} = p_c = 1$, $p_{\Lambda_2} = p_d = 0$. We have $y_t = [r_t \; c_t \; y_t]'$ where r_t is the real interest rate and y_t is output; moreover, $x_t = k_t$ and $s_t = z_t$ (lower cases define, as before, variables in log and deviation from their non stochastic steady state).

The VAR solution can be written as:

$$A(L) w_t = B \epsilon_t$$

where:

$$A(L) = \begin{pmatrix} I_3 & -\Lambda_1 L & -\Lambda_2 \\ 0 & (1 - CL) & -D \\ 0 & 0 & (1 - \psi L) \end{pmatrix}$$

and:

$$B = \begin{pmatrix} 0_{(4\times 1)} \\ 1 \end{pmatrix}$$

where $A(L)$ can be written as $[I - AL]$. Then w_t has a VAR(1) structure: $[I - AL]w_t = B\epsilon_t$. Obviously, the coefficients of the A and B matrices depend on the deep parameters ρ, β, δ, η, the parameter ψ governing technology and the steady state value of the level of productivity.

The vector of the state variables is $F_t = [k_{t-1}\ z_t]'$ and:

$$F_t = HF_{t-1} + K\epsilon_t.$$

Notice that the number of state variables is less than the dimension of the model and it is equal to two. This also implies that the rank r of $\Gamma_w(0)$, the static rank, is equal to 2. This model therefore has both dynamic and static reduced rank. Notice also that, for this example, the rank of A is equal to 2 so that the static rank is the same as the rank of the autoregressive lagged matrix therefore implying that the model has "common features" (Engle and Kozicki, 1993).

3 Business Cycle Empirics

What is the best estimation procedure to recover the dynamic structure of the model economy? We will here compare two alternative strategies. The first is VAR analysis and consists in estimating a reduced form autoregressive model on w_t, identifying the exogenous shocks using a minimal set of (just-identifying) restrictions and then matching the resulting impulse response functions with the theoretical ones (for a survey of this line of research applied to the study of the effects of monetary policy shocks, see Christiano, Eichenbaum and Evans, 1999). The second exploits explicitly stochastic rank reduction and consists in the estimation of a dynamic factor model. This strategy was first advocated in the macroeconomic literature by Sargent and Sims, 1977 and used for structural analysis by Altug, 1989 and Sargent, 1989. That literature, however, while showing how to test for restrictions on the covariances of the data, did not go as far as showing how to estimate impulse response functions and identifying shocks as in VARs. This is why factor models have not been popular tools for empirical structural and policy analysis. In what follows, we show how to identify (common) shocks and impulse response functions in factor models and compare the estimates with those based on VARs.

3.1 VAR Analysis

For VAR estimation to be feasible, we must have full static rank since the estimation of A requires the inversion of $\Gamma_w(0)$.

As we have seen, the simple model, but this is true for a wide class of models, has reduced static rank and so has the VAR. In the case of the basic model, the matrix

$\Gamma_w(0)$ has rank 2, so that a 5 dimensional VAR cannot be estimated, as $\Gamma_w(0)$ cannot be inverted. Reduced static rank may be implied in simple parameterization of theoretical models, and it has been found empirically in the specific form of common cycles (see Issler and Vahid, 2001).

With static rank reduction and measurement error, we can estimate a VAR for a block of variables of dimension r provided that the VAR representation for that block exists. Alternatively, we can introduce measurement error, and estimate a VAR on the whole system. Let us now analyze the two cases.

A. *No measurement Error*

When variables are cleaned from measurement error, estimation can be performed on a block of w_t, call it w_t^B, so as to obtain a full rank covariance matrix of the variables in the block $\Gamma_w^B(0)$.

Let us analyze this case for the general model and call the dimension of the block N_B. It is easily seen that any block has a VMA representation:

$$w_t^B = \Theta^B(L)\epsilon_t$$

For example, if only the non predetermined variables are included in the block, then:

$$\Theta^B(L) = [\Lambda_1^B(L)C(L)^{-1}D(L) + \Lambda_2^B(L)]\Psi(L)^{-1}$$

For a VAR representation to exist, the following condition must hold.

FUNDAMENTALNESS CONDITION. There exists a $q \times N_B$ matrix of filters $\alpha(L)$ in non-negative powers of L such that:

$$\alpha(L)\Theta^B(L) = I_q.$$

This point has been made by Hansen and Sargent, 1990 and Lippi and Reichlin, 1993. For further insights into this issue, see Forni, Lippi and Reichlin, 2002.

If $p_x = 1$, $p_s = 0$ and the exogenous process is fundamental, this condition is satisfied. Hence, for our simple model the condition holds.

If the fundamentalness condition is satisfied, we can approximate the VMA representation with a finite order VAR:

$$A^B(L)w_t^B = v_t^B$$

where $A^B(L)$ is a finite order $N_B \times N_B$ matrix of filters and
$v_t^B = B^B\epsilon_t$, with B^B being a $N_B \times q$ matrix[2].

Notice that B^B is an orthonormal rotation of the first q principal component of $\Gamma_{v^B}(0)$. Defining as V the $q \times q$ matrix containing its first q eigenvalues and as J

[2]Due to the approximation with a finite order VAR, $\mathrm{E}v_t^B v_t^{B'} = \Gamma_v(0)$ is not exactly of reduced rank.

the $N_B \times q$ matrix of the corresponding eigenvectors, we have: $\epsilon_t = R'J^{-1/2}V'v_t^B$, $\Gamma_{v_B}(0) = VJV' = B^B B^{B'}$, $B^B = VJ^{1/2}R$ where $RR' = I_q$.

The impulse response function are hence given by:

$$w_t^B = A^B(L)^{-1}VJ^{1/2}R\epsilon_t$$

Notice that once we have consistent estimates of $A^B(L)$, the impulse response functions can be consistently estimated since the eigenvalues and the eigenvectors are continuous functions of the matrix entries.

An important remark is that the dimension of the rotation matrix, and hence the degree of indeterminacy due to observational equivalence of alternative structures, depends only on the dimension q of the vector of exogenous shocks and not on the dimension of the subsystem N_B.

B. *Measurement Error*

If the variables have independent measurement error, collinearity disappears and the estimation of the full system is always possible.

Let us assume that measurement error comes in its simplest form, i.e. as a white noise process $\xi_t \sim WN(0, \Gamma_\xi(0))$ orthogonal to the vector of the variables of interest w_t. Let us refer to the simple model. The vector of measured variables is:

$$\tilde{w}_t = w_t + \xi_t. \tag{3.4}$$

A $VAR(p)$ for w_t implies the following $VARMA(p,p)$ model for the measured equation:

$$A(L)\tilde{w}_t = u_t + A(L)\xi_t \tag{3.5}$$

where $u_t = B\epsilon_t$.

Standard results indicate that, since the error term is autocorrelated, $A(L)$ and B are not identified and cannot be estimated consistently by a $VAR(p)$.

Two alternative strategies are available in this situation. The first is to impose the restrictions above and to estimate a factor model, i.e. a model in which each variable, $\tilde{w}_{jt}$, $j = 1, \ldots, n$, is represented as the sum of two stationary, mutually orthogonal, unobservable components: the 'common component', w_{jt}, and the 'idiosyncratic component', ξ_{jt}. The common component is driven by a small number, q, of common 'factors' or common shocks, which are the same for all the cross-sectional units, but are possibly loaded with different coefficients and lag structures. By contrast, the 'idiosyncratic component' is driven by shocks specific to each variable. We will discuss factor model estimation in the next section.

The second alternative is to estimate the model by a VAR of order $\tilde{p} > p$ to approximate the VARMA. Let us analyze this strategy here.

Consider the Wold representation of the measured process:

$$\tilde{w}_t = \tilde{\Theta}(L)\tilde{\epsilon}_t$$

where $\tilde{\epsilon}_t \sim W.N.(0, \Sigma_{\tilde{\epsilon}})$. The impulse response parameters are given by $\tilde{\Theta}(L)\Sigma_{\tilde{\epsilon}}^{1/2}\tilde{R}$ with $\tilde{R}\tilde{R}' = I_{N_B}$ [3].

The parameters $\tilde{\Theta}(L)$ and $\Sigma_{\tilde{\epsilon}}$ are related to those of the uncontaminated process (2.1) and the measurement error by the spectral identity:

$$\tilde{\Theta}(e^{-i\lambda})\Sigma_{\tilde{\epsilon}}\tilde{\Theta}(e^{i\lambda})' = A(e^{-i\lambda})^{-1}B\Sigma_{\epsilon}B'A(e^{i\lambda})'^{-1} + \Gamma_{\xi}(0) \tag{3.6}$$

Notice that, although $\tilde{\Theta}(L)$ can be consistently estimated, one cannot recover $A(L)^{-1}B\Sigma_{\epsilon}^{1/2}$ without recovering $\Gamma_{\xi}(0)$. Hence, there exists no rotation matrix $\tilde{R}$ for which one of the structural shocks has the same impulse response function of the "true" ones. It is interesting to stress that this problem is deeper than the typical identification indeterminacy that pervades the VAR literature (see Christiano, Eichnbaum and Evans, 1999). Even if the researcher knew perfectly the economic model and knew how to choose the appropriate rotation matrix for shocks identification, the presence of measurement error would make inference impossible.

Identity (3.6) shows how measurement errors contaminate the impulse responses parameters of the economic model over the frequency domain. In our example, since the spectral density of the error is constant over λ (white noise error), the spectral density of the measured data $\tilde{w}_t$ is close to the spectral density of w_t at the frequencies where the variance of the non-contaminated process is higher. Hence, if the variance of the process is concentrated in the long-run and at business-cycle frequencies, as in our model economy, then the longer the horizon, the closer the impulse response coefficients of the measured data will be to those of the economic model. This result still holds if measurement error is serially correlated, but less persistent than the signal.

This helps to explain why, empirically, comovement is more clearly detected at long-run frequencies. This feature is also reinforced when the variance of the noise is smaller in the long-run, which is likely to be true empirically.

The empirical performance of the VAR, on the other hand, depends on how well a $VAR(\tilde{p})$ approximates the $VARMA(p,p)$ for a given sample size, which, in turn, depends on the roots of $A(L)$ and on the size of $\Gamma_{\epsilon}(0)$. The more persistent is the VAR and the larger is $\Gamma_{\epsilon}(0)$, the poorer is the approximation.

3.2 Factor model estimation

As Altug, 1989 and Sargent, 1989 have observed, if we add orthogonal measurement error, the model economy has a factor analytic structure. This can be seen by adding measurement error to equation (2.1). We obtain:

[3] Notice that the rotation matrix $\tilde{R}$ is of dimension N_B. The reason is that, because of the presence of measurement error, $\Gamma_{\tilde{\epsilon}}(0)$ is of full rank.

$$\tilde{w}_t = A(L)^{-1}B\epsilon_t + \xi_t = \Theta(L)\epsilon_t + \xi_t \tag{3.7}$$

where ϵ_t is a vector of common shocks of dimension q and ξ_t is an idiosyncratic process of dimension N (see, for example, Sargent and Sims, 1977). Notice that this is a model where the common component, i.e. the component "cleaned" from measurement error, is of reduced dynamic rank q. A different specification can be obtained from a model in which the common component has both reduced dynamic rank q and reduced static rank r. Precisely, this is obtained by adding orthogonal measurement error to equations (2.2) and (2.3):

$$\begin{aligned} \tilde{w}_t &= \Lambda F_t + \xi_t \\ H(L)F_t &= K\epsilon_t \end{aligned} \tag{3.8}$$

where Λ is a $N \times r$ matrix, F_t is $r \times 1$ and ΛF_t represents the "common component" of $\tilde{w}_t$, while, ξ_t is the "idiosyncratic component". The model written in this way, is the static state representation discussed earlier. It can be shown that in the case in which the order of the VAR process for s_t is less or equal than p_s, the filter $H(L)$ in (3.8) is of order 1 so that the states have a VAR(1) representation (on this point, see Giannone, Reichlin and Sala, 2002).

Notice that model (3.8) exhibits "common features", i.e. there exist linear combinations of the observables which are white noise (see Engle and Kozicki, 1993 and Vahid and Engle, 1993). This characteristic of the model can be exploited to improve efficiency in estimation since it implies a more parsimonious representation. We will illustrate this point in the empirical section.

Under the assumption that the "idiosyncratic components", $\xi_{jt}, j = 1, \ldots, n$, are mutually orthogonal, the model is identified and can be estimated by Maximum Likelihood. Let $\eta_t = \tilde{w}_t - E(\tilde{w}_t|\tilde{w}_{t-1}, \ldots, \tilde{w}_1)$ denote the innovations in $\tilde{w}_t$ and $V_t = E(\eta_t\eta_t')$ denote their covariance matrix. Assuming normality of ξ_t and ϵ_t, the log-likelihood function of the model can be written as:

$$\ln L(\tilde{w}_1, \ldots, \tilde{w}_t; \psi) \propto -\sum_{t=1}^{T}(\ln|V_t| + \eta_t' V_t^{-1}\eta_t) \tag{3.9}$$

where ψ is the vector of the unknown parameters.

For each set of parameters, the innovations η_t and their covariance matrix V_t can be computed using the Kalman filter. The log-likelihood can ther be maximized using numerical algorithms[4]. Having obtained an estimate of the parameters, we can easily obtain the impulse response functions. Impulse response functions are defined, up to a rotation of order q, as: $\Theta(L) = A(L)^{-1}B\Sigma_\epsilon^{-1/2}$ for the full static rank model (3.7) and $\Theta(L) = \Lambda H(L)^{-1}K\Sigma_\epsilon^{-1/2}$ for the reduced static rank model (3.8). In the empirical section we will report results for both models.

[4]In this paper, we use the EM algorithm

Table 1: **Calibrated Parameters**

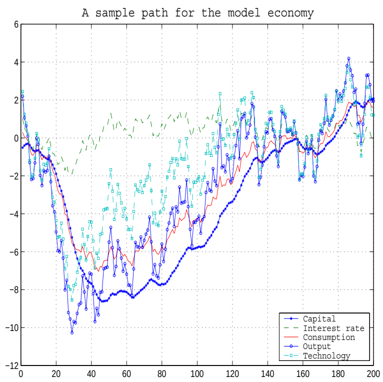
β	.99
ρ	.36
η	1
δ	.025
ψ	.95

4 Empirical comparison

In this section we perform the following experiment. We generate data from the model economy and estimate a VAR and a factor model, with and without measurement error. For each estimation method, we compute impulse response functions and report bias, mean squared errors and confidence bands. The particular model economy is the simple business cycle model where we use the same calibrated parameters as in Uhlig, 1998. They are reported in the Table below [5].

In Figure 1 we show the sample paths of the five variables for one simulation of the model.

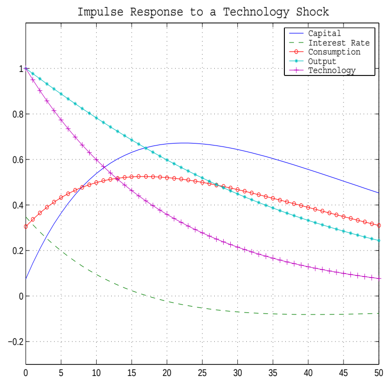
Figure 1. Simulated Path



In Figure 2 we show the theoretical impulse response functions in response to a unitary technology shock generated by the model.

[5]The qualitative results of the paper are robust to perturbation of the calibrated parameters.

Figure 2. Theoretical Impulse Response Functions



The measurement error is generated as:

$$\xi_t \sim i.i.d.N\left(\mathbf{0}_5, \text{diag}[\gamma_r, \gamma_c, \gamma_y, \gamma_k, \gamma_z]\right)$$

with the γ_is calibrated so that the degree of commonality, given by the ratio $\frac{Var(\tilde{w}_t^i)}{Var(w_t^i)} = 1 - \frac{\gamma_i}{Var(w_t^i)}$, is the same for $i = r, \ldots, z$ and is equal to: $VR = [.9, .8]$.

For each size of the measurement error, we generate 500 vector time-series $w_t = (c_t, r_t, y_t, k_t, z_t)'$ for our model economy with a sample size $T = 200$.

A. *Estimation*

As a full-size VAR on w_t cannot be estimated without introducing measurement error, we concentrate on the sub-block $w_t^{yc} = (y_t, c_t)'$.

We estimate the VAR without measurement error on the sub-block w_t^{yc} by assuming to know the lag length (in this case, 1). Given that we have just one shock there are no identification problems in absence of measurement errors.

With measurement error, the model is a VARMA process with two shocks and the latter must be identified structurally. For every simulation, we estimate the VAR with a lag length of $p = 1, ..., 10$ and for every lag length we identify one shock, by choosing one column of the orthonormal rotation matrix $\tilde{R}$:

$$\tilde{R} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}, \qquad \theta \in [-\pi, \pi]. \tag{4.10}$$

so as to minimize the sum of the Euclidean distances between the true and the estimated impulse responses for 10 period after the shock for both y_t and c_t.

For every VR we obtain 500 impulse responses for each lag length $p = 1, ..., 10$.

We choose the optimal lag length p^* as the one that gives the minimum MSE for 10 periods after the shock. It turns out that for every VR the optimal lag length is 7.

The factor model is estimated by Maximum Likelihood. We assume that p and q are known, as we have done for the number of lags in VAR estimates. In Section 5 we discuss in greater detail model selection issues.

B. *Comparison*

A comparison between alternative methods is provided in what follows.

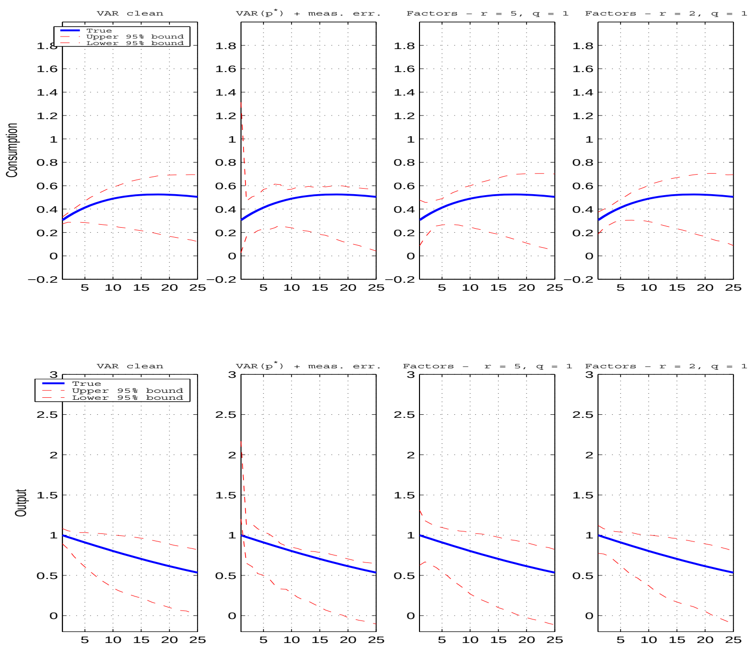
For each value of VR we report one figure and one table,

Consider the results for $VR = 0.9$, reported in Figure 3 and Table 2. From left to right, the columns in Figure 3 display 95% confidence bands for impulse response functions[6] computed respectively, from: (a) the estimation of a VAR(1) on clean data, (b) the estimation of a $VAR(p^*)$ on contaminated data; (c) the estimation of a factor model, taking into account only dynamic rank reduction (i.e. $r = 5$, $q = 1$) and assuming that the correct lag length $p = 1$ is known; (d) the estimation of a factor model, taking into account not only dynamic rank reduction, but also static rank reduction (i.e. $r = 2$, $q = 1$), again assuming to know the correct lag length $p = 1$. The upper part of the figure displays results for consumption, the lower part for output.

The true impulse response functions are reported for comparison (bold lines).

Table 2 displays the means of the empirical distributions for the mean squared error and bias for the four estimation methods, computed at various horizons.

Figure 3. Comparison of Impulse Response Functions - VR = 0.9



top: Consumption - bottom: Output

[6]95% confidence bands are computed from the empirical distribution function by taking the 2.5-th and the 97.5-th percentile.

Table 2: **MSE and Bias - VR** = 0.9

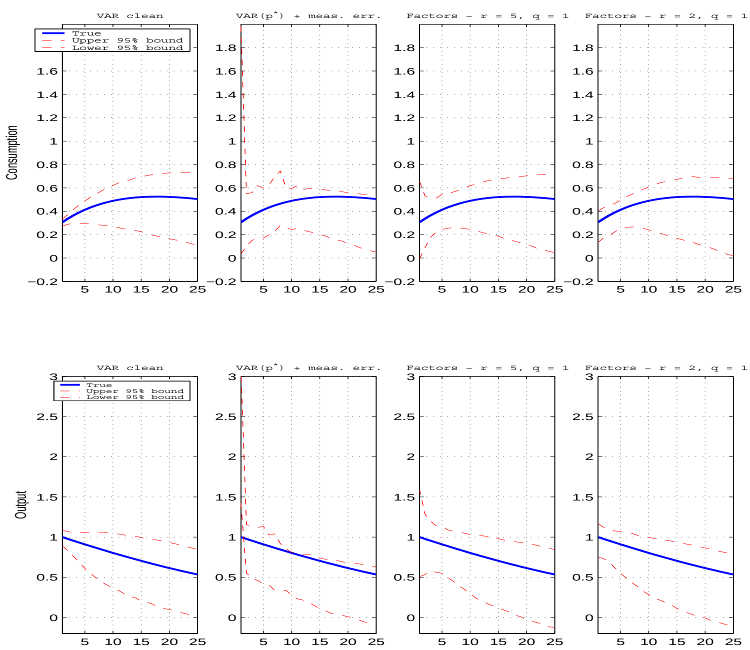
Output

	VAR clean		**VAR meas. err.**		**Factor** $(r=5)$		**Factor** $(r=2)$	
Hor.	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
1	0.0025	-0.0066	0.4655	0.6344	0.0301	-0.0286	0.0093	-0.0431
2	0.0039	-0.0192	0.0213	-0.0713	0.0202	-0.0541	0.0086	-0.0475
3	0.007	-0.0304	0.0261	-0.089	0.0211	-0.0632	0.0098	-0.0525
4	0.011	-0.0403	0.0303	-0.1032	0.0236	-0.0789	0.0123	-0.058
5	0.0154	-0.0492	0.0345	-0.1198	0.0272	-0.0854	0.0157	-0.064
10	0.0358	-0.0829	0.0528	-0.1788	0.0524	-0.1285	0.038	-0.1004
15	0.0485	-0.1049	0.0775	-0.2191	0.0779	-0.1663	0.0616	-0.1415
20	0.0549	-0.1193	0.0929	-0.2398	0.097	-0.1983	0.0823	-0.18
25	0.0571	-0.1275	0.0993	-0.2466	0.1083	-0.2217	0.0973	-0.2106

Consumption

	VAR clean		**VAR meas. err.**		**Factor** $(r=5)$		**Factor** $(r=2)$	
Hor.	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
1	0.0002	-0.0016	0.1273	0.1907	0.0118	-0.0309	0.0025	-0.0173
2	0.0005	-0.0052	0.006	-0.0123	0.0054	-0.02	0.0021	-0.0168
3	0.001	-0.009	0.0067	-0.0217	0.0042	-0.0214	0.002	-0.0172
4	0.0017	-0.0132	0.0065	-0.0261	0.004	-0.0252	0.0022	-0.0184
5	0.0026	-0.0177	0.0079	-0.0299	0.0046	-0.0312	0.0026	-0.0203
10	0.0095	-0.0418	0.0089	-0.0525	0.012	-0.06	0.008	-0.0388
15	0.0178	-0.0658	0.0204	-0.0964	0.024	-0.0917	0.0178	-0.0675
20	0.0257	-0.0869	0.0355	-0.1408	0.0378	-0.1224	0.0302	-0.1004
25	0.0321	-0.1036	0.0502	-0.1759	0.0509	-0.1498	0.0433	-0.1324

Figure 4. Comparison of Impulse Response Functions - VR = 0.8



top: Consumption - bottom: Output

Table 3: **MSE and Bias - VR** = 0.8

Output

	VAR clean		**VAR meas**		**Factor** ($r = 5$)		**Factor** ($r = 2$)	
Hor.	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
1	0.0025	-0.0066	1.4705	1.1318	0.0763	0.0295	0.0093	-0.0344
2	0.0039	-0.0192	0.0363	-0.1044	0.0401	-0.0649	0.0086	-0.0447
3	0.0070	-0.0304	0.0368	-0.1051	0.0288	-0.0657	0.0098	-0.0548
4	0.0110	-0.0403	0.0434	-0.1251	0.0269	-0.0784	0.0123	-0.0647
5	0.0154	-0.0492	0.0494	-0.1345	0.0277	-0.0811	0.0157	-0.0744
10	0.0358	-0.0829	0.0604	-0.2030	0.0507	-0.1251	0.0380	-0.1213
15	0.0485	-0.1049	0.0817	-0.2384	0.0763	-0.1628	0.0616	-0.1641
20	0.0549	-0.1193	0.0927	-0.2510	0.0971	-0.1956	0.0823	-0.1992
25	0.0571	-0.1275	0.0944	-0.2486	0.1104	-0.2199	0.0973	-0.2236

Consumption

	VAR clean		**VAR meas**		**Factor** ($r = 5$)		**Factor** ($r = 2$)	
Hor.	MSE	Bias	MSE	Bias	MSE	Bias	MSE	Bias
	0.0002	-0.0016	0.3343	0.3101	0.0278	-0.0244	0.0025	-0.0256
2	0.0005	-0.0052	0.0110	-0.0085	0.0130	-0.0220	0.0021	-0.0246
3	0.0010	-0.0090	0.0112	-0.0198	0.0079	-0.0185	0.0020	-0.0251
4	0.0017	-0.0132	0.0124	-0.0186	0.0062	-0.0259	0.0022	-0.0267
5	0.0026	-0.0177	0.0131	-0.0278	0.0066	-0.0288	0.0026	-0.0292
10	0.0095	-0.0418	0.0088	-0.0444	0.0131	-0.0584	0.0080	-0.0513
15	0.0178	-0.0658	0.0204	-0.0985	0.0248	-0.0896	0.0178	-0.0827
20	0.0257	-0.0869	0.0377	-0.1552	0.0390	-0.1209	0.0302	-0.1163
25	0.0321	-0.1036	0.0539	-0.1941	0.0526	-0.1486	0.0433	-0.1472

Three important features of the results are to be highlighted.

First, and this demonstrates empirically the results of the previous section, even a small measurement error ($VR = .9$) is sufficient to spoil the inference drawn from the VAR on short-run impulse response coefficients. It is evident that the VAR does not consistently estimate the true contemporaneous response of the system.

Second, in the case of the "contaminated" VAR, as measurement error increases (VR decreases), both the bias and the mean squared error at short time horizons become larger. This result suggests that impulse response coefficients estimated using a VAR are more reliable at medium and long run horizon. As stressed in Section 3.1, this is due to the fact that the variance of the signals is concentrated in the long-run and at business cycle while the variance of the noise is assumed uniformly distributed over frequencies (white noise). This result still holds if measurement error is allowed to be serially correlated but less persistent than the signal.

Third, imposing the additional restriction $r = 2$, which exploit "common features", alows to obtain a more parsimonious representation and therefore improves the precision of the estimation (for a similar result, based on common cycle restrictions, see Vahid and Issler, 2002).

5 Model Selection and Robustness

In this Section we perform some robustness analysis and we show that previous results still hold once we take into account model selection issues. We assume not to know the data generating process and we proceed to the selection of the best specification using information criteria.

We use three standard information criteria, Akaike (AIC), Schwarz (SBIC) and Hannan-Quinn (HQ), for each of the three models under consideration. The first model is the VAR on contaminated variables $\tilde{w}_t^{yc}$; the second is the factor model with dynamic rank reduction and full static rank (3.7) and the third is the factor model with both static and dynamic rank reduction (3.8). The selected model will be the one minimizing one of the following criteria:

$$\text{AIC: } T^{-1}\left[-2\ln L(y\left|\hat{\psi}\right.) + 2(\#\text{ of estimated parameters})\right]$$

$$\text{SBIC: } T^{-1}\left[-2\ln L(y\left|\hat{\psi}\right.) + \ln T(\#\text{ of estimated parameters})\right]$$

$$\text{HQ: } T^{-1}\left[-2\ln L(y\left|\hat{\psi}\right.) + 2\ln\ln T(\#\text{ of estimated parameters})\right]$$

where $L(y\left|\hat{\psi}\right.)$ is the value of likelihood as a function of the estimated parameters.

For the VAR on contaminated variables we focus on the choice of the lag length p. For the factor model with dynamic rank reduction we focus on the simultaneous choice of lag length p and dynamic rank q, while for the factor model with both static and dynamic rank reduction we simultaneously choose p, q and the static rank r.

For each of the three models we compute two summary statistics. First, we report the frequency of selecting each specification. Second, we show the bias and the MSE associated with the impulse response functions computed from the best models as selected by the information criteria.

Let us start with the VAR on the sub-block $\tilde{w}_t^{yc}$. We simulate 500 vector time series from the model as in the previous Section and we compute the optimal lag length using respectively the AIC, the SBIC and the HQ criteria. Table 4 displays the frequency of lag length selection for both $VR = .9$ and $VR = .8$. SBIC and HQ give rather similar results and select, on average, lag length between 2 and 3. The AIC shows higher dispersion and selects on average a higher lag length. It is interesting to notice that for each of the three criteria the probability of selecting $p = 1$ is always very low. This confirms the theoretical results in the previous Sections: in presence of measurement error, $\tilde{w}_t^{yc}$ follows a VARMA(1,1). The VAR approximation of the VARMA requires a larger lag length and this is what is selected by the criteria.

Tables 5 and 6 show results for the simultaneous choice of the lag length p and dynamic rank q in factor model (3.7). For both $VR = .9$ and $VR = .8$ the true lag length $p = 1$ is selected more than 90% of the times by any criteria. According to SBIC and to HQ the most likely model is indeed the true one with $p = 1$ and $q = 1$. Models with $q > 2$ are selected only 10% of the times by HQ and 20% of the times by SBIC. As in the VAR above, AIC tends to privilege richer models, selecting $q > 2$ more than 50% of the times.

Table 4: Choice of lag length in VAR with measurement error

Lag order:	0	1	2	3	4	5	6	7	8	9	10	11	12
VR = .9													
AIC	0	0.2	2.2	9.6	24.4	20.4	16	12.6	6.2	4	1.2	2.2	1
SBIC	0	3.4	56	36.4	3.8	0.2	0.2	0	0	0	0	0	0
HQ	0	0.6	21.2	42.4	23.2	10.4	2	0	0.2	0	0	0	0
VR = .8													
AIC	0	0.4	4.2	18.6	21.2	21.4	13.2	9.6	4.2	2.6	1.8	1.8	1
SBIC	0.2	6.4	61.6	29	2.6	0.2	0	0	0	0	0	0	0
HQ	0	1.6	26.8	43.4	20	6.2	1.6	0.4	0	0	0	0	0

Table 5: Choice of lag length (p) and dynamic rank (q) - $VR = .9$

p	1					2				
q	1	2	3	4	5	1	2	3	4	5
AIC	0.15	0.22	0.21	0.06	0.23	0.002	0.04	0.07	0.003	0.004
SBC	0.36	0.29	0.15	0.04	0.15	0.002	0.004	0	0	0
HQ	0.57	0.27	0.09	0.02	0.04	0.002	0.004	0	0	0

Table 6: Choice of lag length (p) and dynamic rank (q) - $VR = .8$

p	1					2				
q	1	2	3	4	5	1	2	3	4	5
AIC	0.21	0.32	0.17	0.04	0.15	0	0.03	0.05	0.02	0.01
SBIC	0.45	0.34	0.09	0.03	0.09	0	0.002	0.002	0	0
HQ	0.66	0.25	0.04	0.006	0.04	0.002	0	0.002	0	0

Table 7: Simultaneous choice of static (r), dynamic (q) rank and lag length (p)

r	1		2				3						4							
p	1	2	1		2		1			2			1				2			
q	1	1	1	2	1	2	1	2	3	1	2	3	1	2	3	4	1	2	3	4
$VR = .8$																				
AIC	0	0	0.18	0.6	0.08	0.12	0	0.01	0	0	0	0	0	0	0	0	0	0	0.01	0
SBIC	0	0	0.27	0.7	0.01	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0
HQ	0	0	0.33	0.67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$VR = .9$																				
AIC	0	0	0.15	0.62	0.04	0.14	0	0.01	0	0	0.04	0	0	0	0	0	0	0	0	0
SBIC	0	0	0.18	0.75	0.02	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0
HQ	0	0	0.26	0.74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Results for the simultaneous selection of p, q and the static rank r are displayed in Table 7. HQ always selects both the correct lag length $p = 1$ and static rank $r = 2$. SBIC always selects the correct static rank $r = 2$ and selects $p = 1$ more than 95% of the times. AIC, has a tendency to select over parameterized models as in the previous experiment. It is worth noticing that no criteria ever selects underparameterized models (i.e. $r = 1$). A second issue on which we will come back below is that once static rank reduction has been obtained, it becomes more difficult to select the correct dynamic rank $q = 1$.

Let us move to the analysis of the bias and the MSE of the impulse response functions estimated from the best models selected by the information criteria.

In Tables 8 and 9 below we report bias and MSE computed as follows. For each simulation and for each of the three models, we consider the specification selected by HQ, we compute the impulse response functions and the associated bias and MSE. The empirical mean of the distribution of bias and MSE is displayed. For VAR models, identification has been obtained by choosing the rotation matrix $\tilde{R}$ (4.10) minimizing the distance between the true impulse responses and the estimated ones.

For factor models identification has been achieved by extracting the first eigenvector of the variance covariance matrix of the residuals of the state equation in the state-space model. Selecting the model according to information criteria, confirms the results in Section 4, obtained by assuming correct model specification. In the short run, parameters estimated by VARs have larger bias and MSE than those obtained from factor models, while at medium and long run horizons the performance is similar.

Table 8: **MSE and Bias -** $VR = .9$

Output

	VAR meas. err.		**Factor** $(r = 5)$		**Factor** $(r = r^*)$	
Hor.	MSE	Bias	MSE	Bias	MSE	Bias
1	0.5372	0.6967	0.0279	0.0308	0.006	−0.0153
2	0.0299	−0.1055	0.0224	−0.0642	0.0069	−0.0282
3	0.0319	−0.1102	0.0245	−0.0768	0.01	−0.0407
4	0.0341	−0.1114	0.0273	−0.092	0.0145	−0.053
5	0.0357	−0.1256	0.032	−0.1033	0.0199	−0.0651
10	0.0623	−0.198	0.062	−0.1542	0.0502	−0.1228
15	0.0796	−0.2266	0.0913	−0.1956	0.0784	−0.1743
20	0.0858	−0.2357	0.1124	−0.2265	0.1013	−0.2158
25	0.0832	−0.2309	0.1226	−0.2458	0.1167	−0.2444

Consumption

	VAR meas. err.		**Factor** $(r = 5)$		**Factor** $(r = r^*)$	
Hor.	MSE	Bias	MSE	Bias	MSE	Bias
1	0.1551	0.2055	0.0104	−0.021	0.0021	−0.0088
2	0.0092	−0.0151	0.0061	−0.0226	0.002	−0.0122
3	0.012	−0.0134	0.0048	−0.0271	0.0023	−0.0162
4	0.0149	0.0142	0.0049	−0.0324	0.0028	−0.0207
5	0.0126	0.0273	0.0057	−0.0378	0.0036	−0.0257
10	0.0116	−0.0414	0.0151	−0.0729	0.0113	−0.0564
15	0.0287	−0.1132	0.029	−0.1084	0.0236	−0.0922
20	0.0492	−0.17	0.0446	−0.1409	0.0384	−0.1283
25	0.0647	−0.2069	0.0587	−0.1683	0.0532	−0.1604

Table 9: **MSE and Bias -** $VR = .8$

Output

Hor.	**VAR meas. err.** MSE	Bias	**Factor** $(r = 5)$ MSE	Bias	**Factor** $(r = r^*)$ MSE	Bias
1	1.3707	1.1184	0.0963	0.0484	0.0125	−0.0414
2	0.0465	−0.1243	0.0635	−0.1074	0.0134	−0.0539
3	0.0469	−0.1173	0.0564	−0.1082	0.0167	−0.065
4	0.0396	−0.0925	0.0536	−0.1176	0.0213	−0.075
5	0.037	−0.1033	0.0497	−0.1165	0.0266	−0.0842
10	0.0727	−0.2152	0.0647	−0.1558	0.0531	−0.1221
15	0.0913	−0.2457	0.0816	−0.1897	0.0727	−0.1522
20	0.0932	−0.2493	0.0968	−0.2189	0.0839	−0.1759
25	0.0856	−0.2381	0.1086	−0.2413	0.0879	−0.1924

Consumption

Hor.	**VAR meas. err.** MSE	Bias	**Factor** $(r = 5)$ MSE	Bias	**Factor** $(r = r^*)$ MSE	Bias
1	0.6353	0.6224	0.0296	−0.0393	0.0048	−0.0222
2	0.0184	0.0395	0.0148	−0.0231	0.0042	−0.026
3	0.0282	0.0647	0.0092	−0.0295	0.0041	−0.03
4	0.0443	0.1282	0.0105	−0.0374	0.0044	−0.0342
5	0.0384	0.1296	0.0099	−0.0422	0.0052	−0.0387
10	0.0173	−0.0245	0.0177	−0.0739	0.0128	−0.0631
15	0.0392	−0.1305	0.0282	−0.106	0.0237	−0.0889
20	0.0635	−0.1995	0.0407	−0.1376	0.0349	−0.1137
25	0.079	−0.2374	0.0529	−0.1653	0.0441	−0.1354

What we have learned from the model selection experiment?

Empirical results suggest that if one does not take into account static rank reduction and only considers the possibility of dynamic rank reduction (model (3.7)), it is more likely for the correct dynamic rank to emerge. The penalization for not exploiting the rank reduction in the spectral density is so big to become the driving force in the selection of "low q" models. If, on the other hand, one considers static rank reduction explicitely (model (3.8)), then it is less likely to select a model with the correct value for q. In this case, it is the choice of a low static rank that significantly reduces the number of parameters to be estimated, while the gain from the additional reduction in the dynamic rank is limited. In our data generating process, this is due to the very simple relation between the dynamic and the static factor structure. In richer models, with r possibly much larger than q, the dynamic rank reduction will likely become predominant. One should notice, however, that the two factor models perfom very similarly in terms of bias and MSE. The reason is that even when $q > 1$ is selected, there exist one large, dominant shock and one small shock that does not contribute much to the variance of the process. Once this is taken into account, the impulse responses are estimated efficiently.

6 Summary and conclusions

Both theory and empirics suggest that there are fewer macroeconomic shocks than variables. When variables are measured with errors, the measured equations generated by equilibrium business cycle models have a dynamic factor structure and the reduced form follows a VARMA model. We have compared VAR and structural factor estimation as two alternative approximations to these equations using a benchmark business cycle model to generate the data under alternative assumptions on the size of measurement error. We have shown that, at short horizons, VAR estimates of impulse response functions are less accurate than factor estimates while the two methods perform similarly at medium and long run horizons.

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